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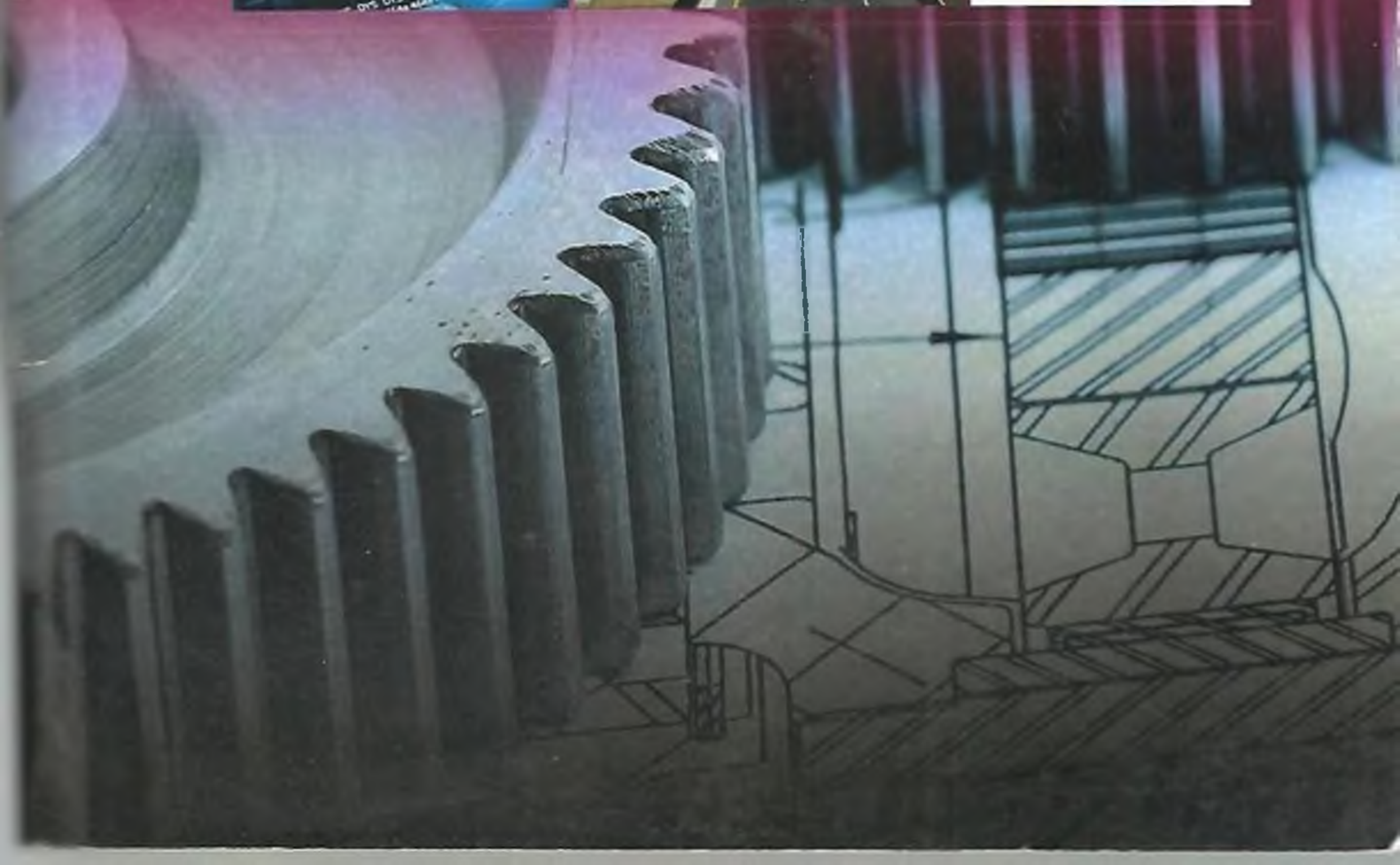

# Response Prediction for Strength and Durability Properties of Laterite-Cement Bricks using the Scheffe Mixture Approach

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**Abstract**

An attempt is made in this paper to present a procedure for selecting constituent proportions for laterite-cement bricks capable of meeting user-defined requirements of strength, cost and durability. The response prediction was developed using the Scheffe's mixture theory. In using this mixture experimental design approach, a three component mixture using water, cement and laterite to produce laterite-cement bricks was carried out with cement content ranging between 8-20 percent by weight of laterite. The physical and geotechnical properties of the laterite sample were determined. The Hydraform Twin-M7 machine was used for the compaction and curing was carried out in a laboratory environment. The compressive strength at the ages of 7 and 28 days were measured using Testometric FS300CT Universal Testing Machine and responses were modeled as a second order quadratic polynomial. The guidelines for development of constraint formulation were carried out for optimization process. An inverse relationship for strength was obtained and compressive strength achievable ranges between 8.40 - 18.32N/mm$^2$. Durability test using ASTM D 559 otherwise called wire brush method was carried out yielding a percentage particle eroded less than 10% which was satisfied within the domain of constituents proportions selected. This procedure is intended to replace the traditional trial method of mixture proportioning which is incapable of developing specifications writing procedure to meet user-defined requirements.

**Keywords:** Bricks, Domain, Durability, Scheffe's theory, Strength, User-defined

## Introduction

The selection of constituent mixture proportions is fundamental for the production of a strong and durable laterite-cement bricks. This is particularly necessary where the laterite-cement brick is to be used as a permanent building walling material, without any protective coatings. This however demands a higher complexity of the mixture design. In order to achieve this, a number of imposed criteria that the mixture must satisfy need to be clearly stated. Useful numerical and optimization tools can similarly be employed to aid the process of satisfying these specified objectives. User defined criteria are usually presented as user-specified requirements. Typical performance criteria could include mechanical properties such as strength and young modulus of elasticity. It could also include durability properties such as abrasion resistance and capillary movement.

In selecting laterite samples for brick production, certain physical and geotechnical properties of laterite material are required. Gidigasu, (1976) described laterite samples as a light to dark homogeneous, vesicular, unstratified and clinker like soil material consisting mainly of oxides and hydroxides of aluminium, iron, manganese and silica which hardens on extraction and exposure. These laterites samples are similarly described a class of pedogenics where the cementing materials are the sesquioxides content and should normally constitute not less than 50 percent of the mineralogical composition by this definition.

Laterite brick is a locally sourced building material confers technical advantage largely because of the primary requirement of strength. Laterite-cement bricks have a very good thermal property, shock and earthquake resistance (Hydraform, 2014) and particularly impact resistance. Several published research output (Madu, 1984; Osunade and Fajobi, 2000; Awoyera and Akinwumi, 2014; Hydraform, 2014) have tried to confirm the acceptability of its properties for a series of acceptance criteria. These properties include its compressive strength, absorption characteristics and resistance to abrasion. These research reports have also reported its durability properties under exposure to weather and other climatic conditions (Walker, 1995; Guetalla *et al.*, 1995; Heathcote, 2002; Ipinge, 2012;).

Attempts have also been made to improve laterite-cement as a building material for sustainable housing construction through development and manufacturing of compression machines for mechanical stabilization (Adeyemi, 1987 and 2004; NBRRI, 2013; Hydraform, 2014). Stabilization of laterite with cement to produce bricks was investigated (Madu, 1984; Fajobi, 2000; Aguwa, 2009; Osunade and Hydraform, 2014). Stabilization of laterite-cement bricks with pozzolanic material such as Corn Cob Ash (Ogunbode and Apeh, 2012). Stabilization with Locust Waste Bean Ash (Osinubi and Oyelakin, 2013). Stabilization with Coir (Aguwa, 2013). Bentonite Treatment (Amadi *et al.*, 2011). Stabilisation with lime (Singh, 2006; Hydraform, 2014) were studied to improve the performance of the laterite-cement bricks as a building material (Walker, 1995; Ipinge, 2012).

The aim of this study was to investigate an efficient optimization formulation for design of component proportions of cement-laterite composite bricks production to meet user-defined requirements. The objectives of this study were to develop an appropriate domain of mixture combinations and constraint formulation for the optimization process using Genetic Algorithm; determine appropriate mix proportions using Mixture Approach (MA); determine the strength of bricks produced from laterite-cement mixtures and carry out the durability performance test of bricks produced.

*Durability of laterite-cement composite material*

Durability studies have been considered inevitable in the use of laterite-cement bricks because it is believed that there is a reversal of stabilization associated with moisture intrusion within the stabilized materials (Heathcote, 2002). Despite that the method is considered as severe in comparison with actual field performance for building bricks, it is still the most preferred (Walker, 1995; Ipinge, 2012; Heathcote, 2002). Although the addition of cement add to stability, improve resilient properties, reduce excessive cracking and moisture sensitivity, the primary factor influencing the long-term performance of laterite-cement bricks is moisture.

Heathcote, (2002) summarized durability tests for laterite-cement bricks construction based on category source/type as:

i) Wire Brush tests ASTM D559 - indirect test
ii) Spray tests - accelerated and simulation tests
iii) Drip tests -indirect and accelerated tests
iv) Permeability and slake tests-indirect test
v) Strength tests (Wet/Dry Strength Ratio) -indirect test
vi) Surface hardness tests-indirect test

The objective of the ASTM D 559 otherwise called wire brush method which is an indirect test method is to determine the minimum amount of cement required in soil cement to achieve a degree of hardness that is adequate to resist field weathering. Most compressed earth bricks making machine are designed to exert the compactive pressure in uniaxial direction (NBRRI, 2013; Hydraform, 2014; Adeyemi, 2000) or more precisely, it is referred to as a one dimensional compressed soil sample mass (Singh, 2006). The specific volume reduces on compaction because the pore spaces are reduced during mechanical stabilization. The compactive force is also known to determine strength and durability. In general, durability is known to increase exponentially with the degree of compactive effort (Heathcote, 2002).

In an investigation into the erodibility of earth wall units by Heathcote (2002), the report showed that the major climatic factors influencing the erosion of earth walls due to wind-driven rain are:

i) impacting rainfall volume,
ii) drop impact velocity (as determined by wind conditions),
iii) raindrop size and
iv) duration of rainfall.

The laboratory procedure used involved a standard spray test which was modified by introducing a commercially available nozzle, which produces a turbulent spray of individual drops. The erosion rates using this apparatus were found to vary significantly with time, and a correction formula was derived from the experimental results so as to enable a comparison to be made between field and laboratory results.

*Statistical mixture experimental design*

In this design procedure, experimental design points are used and design variables are fitted to these points. Empirical models are fitted for each of the responses to be measured. Refinements of the empirical models are carried out by modifying the response equations after detecting insignificant terms in a model. The final refined equations now form the response prediction equation called fitness functions.

One of the importance of this statistical experimental design procedure is that the responses can be characterized by an uncertainty (variability) which has an important implication for specification writing (FHWA, 1999; Simons *et al.*, 1999). This implies that at least 95 percent of the results are expected to fall within the normal distribution curve or more precisely, with probability $p \leq 0.05$.

*The Concept of the Mixture Approach*

The Scheffe mixture polynomial can be used to design and obtain response prediction for mixtures. A triangular Simplex can be used to explore the properties of the mixture where the vertices of the triangle represent numerically, the pure components for a three variable component mixture. When the constraints on the mixture proportions are constructed, then a workable or a feasible region of the experimental region can be defined naturally.

In order to satisfy the requirement of this mixture approach, the constituent proportions are estimated in absolute volume which is fixed and constrained to be summed equal to unity. This is a pre condition for using this method of solution procedure (Simons *et al.,* 1999; Montgomery, 2001). The components in this particular case are water, cement and laterite. The constraint equation therefore is (Montgomery, 2001):

$$\sum_{i=1}^{n} x_i = 1 \qquad (1)$$

$$\text{and} \quad x_i \geq 0$$

The standard form for response prediction of this second order-quadratic polynomial can be expressed (Montgomery, 2001) as:

$$E(y) = \sum_{i=1}^{p} \beta_i x_i + \sum \sum_{i<j}^{p} \beta_{ij} x_i x_j \quad (2)$$

This form of second-order quadratic polynomial is called Scheffé mixture polynomial and the re-parameterized quadratic polynomial equation can be re-written as (Simon *et al.*, 1999; Montgomery, 2001):

$$y_1 = b_1 x_1 + b_2 x_2 + b_3 x_3 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{23} x_2 x_3 \qquad (3)$$

$$where: \qquad x_1 = water, x_2 = cement, x_3 = laterite$$

Here the expressions $x_1x_2$, $x_1x_3$, $x_2x_3$ are the interaction terms while $b_{12}$, $b_{13}$, $b_{23}$ are coefficients of the interaction terms of water, cement and laterite.

In estimating the component proportions for the vertices and all other design points in Simplex lattice design, transformation between pseudo and actual components in the factor space are used to obtain other design points. The is made possible because of the inverse relationship that exists between them, (Onuamah, 2015; Onwuka *et al.*, 2011; Anya and Osadebe, 2015).

*Construction of the Scheffe's [3, 2] augmented Simplex lattice design*

For a second-order quadratic polynomial design, it is possible to make predictions about the full properties within the Simplex by using an augmented [3, 2] lattice design. Mix proportions are fitted at the vertices of the Simplex in a manner as to yield an optimum mixture. Furthermore, additional runs in the interior of the Simplex are included using both axial runs and the entire centroid (Montgomery, 2001; Mama and Osadebe, 2011; Mbadike and Osadebe, 2013). The augmented [3,2] Simplex lattice shown in Fig. 1 consists of ten runs of pure blend (1,0,0), (0,1,0), (0,0,1), (½, ½, 0), binary blends (½, 0, ½), (0, ½, ½), axial blends (⅔, ⅙, ⅙), (⅙, ⅔, ⅙), (⅙, ⅙, ⅔) and the centroid (⅓, ⅓, ⅓).

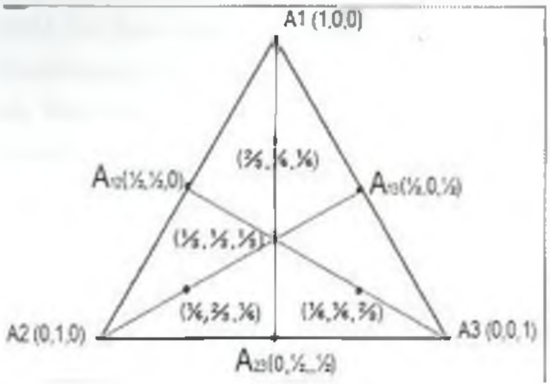


**Fig. 1:** An augmented [3,2] Simplex lattice

Pure and binary blends are however not practicable in its natural form, because the three components have to be mixed together, then a D-optimal design can be used, a procedure also implementable using Design Expert Software (Design Expert, 2000).

*Construction and transformation of variable components*

In an attempt to keep within a practicable compositional boundary, the method of transformation to obtain mixtures for all design points can be used (Scheffe, 1958; Onwuka *et al.*, 2011; Mama and Osadebe, 2011; Mbadike and Osadebe, 2013;). Mixtures, specified in volumetric ratios called mix ratios at a given water cement ratio are fitted at the vertices in a manner as to yield an optimized mix.

A transformation T is used between coded and an actual component in the factor space because the vectors in the factor space of real variables corresponds to the points in the factor space of coded variables. Using this philosophy, the procedure is to assign points to these vectors within the design domain considered. These have inverse relationship (Scheffe, 1958; Mama and Osadebe, 2011) as:

$$Q = TA \quad (4)$$

where $\boldsymbol{T}$ represents a linear transformation at any given point within the factor space between actual and pseudo component vector of variables, and $\boldsymbol{Q}$ represents an identity matrix of the pseudo/coded component variables in the factor space. Multiplying both sides of equation (4) by the inverse, $A^{-1}$ gives:

$$QA^{-1} = TAA^{-1} \quad (5)$$

which yields:

$$A^{-1} = T \quad (6)$$

This methodology can be used to estimate proportions of all other design points within the augmented [3, 2] Simplex lattice points.

**Materials and Methodology**

The laterite sample was sourced within Ilorin environs, Kwara State (KW-31, Elevation 317, and Coordinates 663093, 935109) from the site of an existing burrow pit used for the construction of the 500 housing units with laterite-cement bricks made with Hydraform compressed bricks. The method of disturbed sampling at a depth 0.5m – 1.5m depth for the collection was used. The physical, geotechnical and mineralogical properties of the sample tested are in accordance with BS 1377 (1975) and is shown in Table 1.

**Table 1:** Properties of the laterite Sample

| | Physical and Geotechnical Properties | Value |
|---|---|---|
| i) | Liquid limit (LL %) | 49.00 |
| ii) | Plastic limt (PL %) | 30.60 |
| iii) | Plasticity Index (PI %) | 18.40 |
| iv) | Specific gravity | 2.64 |
| v) | Linear Shrinkage (mm) | 10.10 |
| vi) | Maximum Dry Density (kg/m$^3$) | 1821.00 |
| vii) | Optimum Moisture Content (OMC %) | 14.06 |
| viii) | Colour | Reddish Brown |
| ix) | Condition of Sample | Air Dry |
| x) | Group Index | 4.00 |
| xi) | Soil Classification | A-2-7 |
| **Mineralogical Propoerties** | | |
| i) | Iron Oxide Content ($Fe2O3$) (%) | 18.01 |
| ii) | Sesquioxide Content (%) | 42.21 |

A starting set of mixture proportions was carried out using the absolute volume method within a domain of 8 – 20 percent cement content selected using a starting water/cement ratio of 0.5 which was later revised to produce a mix that would produce one cubic meter of the mixture at maximum dry density. The specimen samples were mixed with pan mixer and compacted using a hydraulically compressed M7-Twin Hydraform brick moulding machine. The factored brick samples were cured and tested at 7 and 28 days to obtain compressive strengths and other mechanical properties using a Testometric Universal Testing Machine Model FS300CT. A total of 238 specimens were cast representing seventeen (17) specimens each for the ten (10) design points and an additional four (4) points for testing statistical significance with minimum of 30 specimens tested each day for compressive strength. The testing plan described by ASTM C 170-90 was used.

*Example of estimation of constituent proportions using the Mixture method*

The mix proportion was calculated for the selected workable design domain and fitted to the design points as described in Fig. 1. In order to satisfy the equality condition in equation (1), the constituent proportions were estimated in absolute volume and constrained to be equal to unity. The practical interpretation of the equality constraint in equation (1) can now be expressed in the estimation of the absolute volumes of each of the mixture factors (Neville, 1999; Aguwa, 2009) as:

$$\frac{cement}{\rho_{cement} \times 1000} + \frac{water}{\rho_{water} \times 1000} + \frac{laterite}{\rho_{laterite} \times 1000} = 1 \quad (7)$$

*where*: $\rho = specific\ gravity$

Using an example of cement content of 20% of the dry weight of laterite, the mix ratio can be expressed as 1:5. Here, a starting water/cement ratio can be adopted as 0.5, which represents the assumed starting water required for the hydration of cement to produce a maximum dry density of the laterite cement mix. This of course, will later be replaced with mixing water at optimum moisture content. These steps are:

i) The ratio 1:5 represents one (1) part of cement and five (5) parts of laterite and water represent 0.5 by weight of cement. This ratio can be expressed as water:cement:laterte ratio 0.5:1:5. The laterite content can be expressed as Laterite, L=5*C. Subsequently, the water required based on the adopted initial water/cement ratio can similarly be expressed as Water, W=0.5*C

ii) The equation which satisfies the equality constraint condition of equation (7) can be re-written as:

$$\frac{0.5C}{1000}+\frac{C}{3.15\times 1000}+\frac{5C}{2.64\times 1000}=1$$

collecting the like term and solving for the unknown Cement C, the solution can be obtained as: Cement, C = 368.81kg/m$^3$ ; Water, W = 0.5*C = 184.41kg/m$^3$ and Laterite, L = 5*C = 1844.07kg/m$^3$

The remaining constituent proportions for other ratios of cement to laterite for 8% and 14% cement content corresponding to the points in the factor space can be calculated in like manner. An excel relative referencing address can be used to implement all the other quantities as designated within the augmented [3,2] Simplex lattice points shown in Fig. 2.

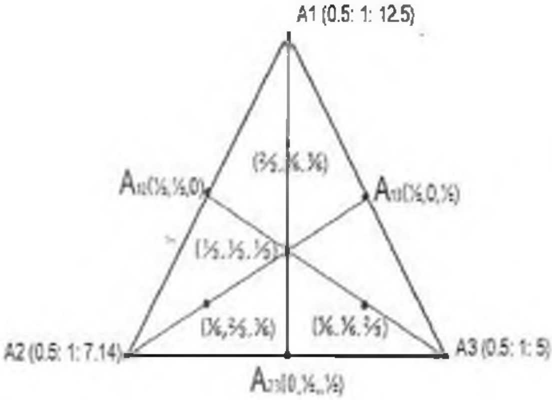


**Fig. 2** An augmented [3,2] Simplex lattice point for the pure, binary, control and center points

*Example of transformation from pseudo component variable to actual factor variable*

In an array form, the pure components are $A_i$ = [0.5, 1, 12.5; 0.5, 1, 7.14; 0.5, 1, 5]$^T$ and the pseudo components for the binary points are $B_{Binary}$=[.5,.5,0; .5,0,.5; 0,.5,.5]$^T$ which are fitted to the vertices and mid points respectively in Fig. 2 can be written in matrix form as:

$$A_i=\begin{bmatrix}0.5000 & 0.5000 & 0.5000\\ 1.0000 & 1.0000 & 1.0000\\ 12.5000 & 7.1400 & 5.0000\end{bmatrix} \quad \text{and } B_{Binary}=\begin{bmatrix}0.5000 & 0.5000 & 0\\ 0.5000 & 0 & 0.5000\\ 0 & 0.5000 & 0.5000\end{bmatrix}$$

The transformation procedure of equations (4) – (6) can be used to obtain the mix ratios for the binary points $B_{ij}$'s as: $A_{ij} = A_i * B_{Binary}$

$$\begin{bmatrix}0.5000 & 0.5000 & 0.5000\\ 1.0000 & 1.0000 & 1.0000\\ 12.500 & 7.1400 & 5.0000\end{bmatrix}\cdot\begin{bmatrix}0.5000 & 0.5000 & 0\\ 0.5000 & 0 & 0.5000\\ 0 & 0.5000 & 0.5000\end{bmatrix} = \begin{bmatrix}0.5000 & 0.5000 & 0.5000\\ 1.0000 & 1.0000 & 1.0000\\ 9.8200 & 8.7500 & 6.0700\end{bmatrix}$$

In like manner, the transformation of the factor variables for the control and centre points into real component variables can similarly be carried out to obtain the mix ratios corresponding to these control and centre points.

The domain of 8-20 percent cement was used (Osunade, 1995; Aguwa, 2009; Hydraform, 2014) because:

i) it represents a cement content percent of laterite where curvature can be detected and also produce a durable mix;
ii) maximum of 20 percent cement content would enable extrusion from the hydraform machine mould with minimum friction on the wearing plate; and
iii) the limits would maintain plastic bonds of the laterite.

*Determination of Optimum Moisture Content and methodology for revised mixing water determination*

The procedure as described in BS 1377 (1975) was employed for the determination of mixing water required to produce maximum dry density mixture using the 4.5kg rammer heavy compaction because: the machine compactive effort is $10MN/m^2$ (Hydraform, 2014). The stages involved are:

i) determine the quantity of mixing water that produces the maximum dry density of compacted soil-cement mixture.
ii) substitute this quantity of mixing water (at optimum moisture content) with the assumed starting mixing water.
iii) while adopting a statistical significance with probability $p \leq 0.05$, carry out a response prediction for water required against the ratio of cement:laterite (the variable) to obtain a linear relationship for correcting variability. The resulting expression was used to recalculate the actual mixing water required and is used to replace the starting mixing water.
iv) the proportions was revised to reflect the summation of all the absolute volumes equal to unity in equation (7) by using the same procedure described in section 3.1 using the revised water/cement ratio. The resulting revised design is as shown in Tables 2(a) and (b)

**Table 2(a):** Design matrix at Optimum Moisture Content using an augmented [3, 2] Simplex lattice by weight

| S/no. | Coordinate Points | | Pseudo component ratios $x_1$=water, $x_2$=cement, $x_3$=laterite X1 | X2 | X3 | Actual components ratios $x_1$ water | $x_2$ Cement | $x_3$ Laterite | Actual component mixes, $kg/m^3$ (0% sand replacement) water | cement | laterite |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| 1 | | A1 | 1 | 0 | 0 | 1.83 | 1.00 | 12.50 | 265.75 | 145.33 | 1816.63 |
| 2 | PURE | A2 | 0 | 1 | 0 | 1.09 | 1.00 | 7.14 | 264.69 | 243.32 | 1737.29 |
| 3 | | A3 | 0 | 0 | 1 | 0.78 | 1.00 | 5.00 | 261.26 | 334.06 | 1670.30 |
| 4 | | A12 | ½ | ½ | 0 | 1.46 | 1.00 | 9.82 | 265.66 | 181.90 | 1786.22 |
| 5 | BINARY | A13 | ½ | 0 | ½ | 1.31 | 1.00 | 8.75 | 265.45 | 202.25 | 1769.70 |
| 6 | | A23 | 0 | ½ | ½ | 0.94 | 1.00 | 6.07 | 263.55 | 281.44 | 1708.35 |
| 7 | | C1 | ⅙ | ⅔ | ⅙ | 1.16 | 1.00 | 7.68 | 265.03 | 227.79 | 1749.40 |
| 8 | CONTROL | C2 | ⅔ | ⅙ | ⅙ | 1.53 | 1.00 | 10.36 | 265.71 | 173.11 | 1793.44 |
| 9 | | C3 | ⅙ | ⅙ | ⅔ | 1.01 | 1.00 | 6.61 | 264.22 | 260.80 | 1723.88 |
| 10 | CENTRE | O | ⅓ | ⅓ | ⅓ | 1.24 | 1.00 | 8.21 | 265.28 | 214.37 | 1760.00 |

*The highlighted are the upper and the lower limits on the domains of constituent proportions by weight

*The quantities in columns 9, 10, 11 are the respective unit weights per $m^3$ of the mixture proportions for water, cement and laterites respectively

*A1, A2, A3 represent pure blends, A12, A13, A23 represent binary blends, C1, C2, C3 represent control points and O represents centre point fitted in the factor space.

**Table 2(b)**: Design matrix at Optimum Moisture Content using an augmented [3, 2] Simplex lattice by volume

| S/no. | Coordinate Points | | Pseudo component ratios $x_1$=water, $x_2$=cement, $x_3$=laterite | | | Actual components ratios | | | Actual component mixes, $m^3$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | X1 | X2 | X3 | $x_1$ water | $x_2$ Cement | $x_3$ Laterite | $x_1$ water | $x_2$ cement | $x_3$ laterite |
| (1) | (2) | | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) |
| 1 | | A1 | 1 | 0 | 0 | 1.83 | 1.00 | 12.50 | 0.266 | 0.046 | 0.688 |
| 2 | PURE | A2 | 0 | 1 | 0 | 1.09 | 1.00 | 7.14 | 0.265 | 0.077 | 0.658 |
| 3 | | A3 | 0 | 0 | 1 | 0.78 | 1.00 | 5.00 | 0.261 | 0.106 | 0.633 |
| 4 | | A12 | ½ | ½ | 0 | 1.46 | 1.00 | 9.82 | 0.266 | 0.058 | 0.677 |
| 5 | BINARY | A13 | ½ | 0 | ½ | 1.31 | 1.00 | 8.75 | 0.266 | 0.064 | 0.670 |
| 6 | | A23 | 0 | ½ | ½ | 0.94 | 1.00 | 6.07 | 0.264 | 0.089 | 0.647 |
| 7 | | C1 | ⅙ | ⅔ | ⅙ | 1.16 | 1.00 | 7.68 | 0.265 | 0.072 | 0.663 |
| 8 | CONTROL | C2 | ⅔ | ⅙ | ⅙ | 1.53 | 1.00 | 10.36 | 0.266 | 0.055 | 0.679 |
| 9 | | C3 | ⅙ | ⅙ | ⅔ | 1.01 | 1.00 | 6.61 | 0.264 | 0.083 | 0.653 |
| 10 | CENTRE | O | ⅓ | ⅓ | ⅓ | 1.24 | 1.00 | 8.21 | 0.265 | 0.068 | 0.667 |

*The highlighted are the upper and the lower limits on the domains of constituent proportions by volume

*The quantities in columns 9,10,11 are divided by the respective unit weights of 1000, 3150 and 2640kg/m³ for water, cement and laterites respectively

Similarly, using a statistical significance with probability $p \leq 0.05$, re-calculate a new revised response prediction for water requirement (response), in column (9) of Table 2(a) against the ratio of cement/laterite (as the variable) which is the ratio of column (10)/column (11) of Table 2(a) to obtain a revised linear relationship which reflects the equality constraint of equation (1). This revised response prediction for water is shown in equation (8).

$$Water = 269.5 - 36.93 * \left(\frac{Cement}{Laterite}\right) \quad (8)$$

v) Similarly, using a statistical significance $p \leq 0.05$, a perfect linear relationship for response prediction for laterite quantity (the response) in column (11) against cement (the variable) in column (10) of Table 2(a) can be carried out. The resulting predictive response for laterite quantity is presented in equation (9). This response satisfies the condition equality condition of equation (1).

$$Laterite\ quantity = 1927 - 0.7767 * Cement \quad (9)$$

*Development of constraint for domains on the constituent proportions*

The domain of the highlighted constituent proportions in Table 2(a) can be used for building constraints on the bounds for the propotions to yield 1m³ of compacted volume. For example, The vertices P1 and P3 represent the lower and upper limits respectively on water, cement and laterite. This is shown in row (1) and row (3), columns (9), (10) and (11). From Table 2(a), dividing the proportions by the respective unit weight for lower and upper limits of water (1000 kg/m³) gives 0.261 and 0.266. Similarly for cement, by dividing the proportions by the unit weight of cement (3150 kg/m³) gives 0.046 and 0.106 and for laterite, dividing the proportions by the unit weight of laterite (2640 kg/m³) gives 0.633 and 0.688. The domains of lower and upper limits on the proportions is represented in equation (10)

$$\left.\begin{array}{l} 0.261 \leq x_1 \leq 0.266 \\ 0.046 \leq x_2 \leq 0.106 \\ 0.633 \leq x_3 \leq 0.688 \end{array}\right\} \quad (10)$$

**Results and discussion**

The modeling of response predictions for laterite-cement mixes for strength at 7, 28 days and cost were carried out using the second-order quadratic polynomial in equation (3). The results have shown that strength still remains the primary response prediction for describing all other measured properties and is therefore adopted here. For example the bricks with higher strength yield higher Young's modulus of elasticity and lower strain. Similarly, the brick with higher strength corresponds with higher cost.

*Description of the response prediction using the augmented {3, 2} Mixture model*

The models for strength at 7 and 28 days including cost that adequately explain the fitted data are shown in Tables 4(a) - (c). These responses from the input data in Table 2(b), columns (9), (10) and (11) were analyzed using Design Expert software where the runs are randomized so as to avoid extraneous variables in the experiment (Simon *et al.*, 1999; Montgomery, 2001). Replicate mixes are also required and added to provide an estimate of repeatability or statistical significance of the fitted coefficients. The run order for the data is shown in Appendix A.

A low value of $p \leq 0.05$ statistical significance shows that a model and the coefficients are significant and should be included in the model. Contour plots produced can then be used to identify the conditions that give the extremum visually. The response prediction equations obtained reflected the form of the statistical method. By default, the Mixture method does not include the intercept because in the Scheffe quadratic polynomial expression, the polynomial equation has been re-parameterized and therefore the constant term eliminated. The interaction terms that are not included in the model also shows that they are not significant because probability $p \geq 0.05$. The response predictions are as shown in Equations (11) and (12) for inverse relationship for strength at 28-day, 7-day respectively. No transformation for cost and this is shown in Equation (13). The cost of material, machine moulding and labour forms the basis for all the all-inclusive cost build-up for production of the brick. This however, is a dynamic process as it is continually influenced by market inflation rate

$$\frac{1}{fc_{28}} = -3.54724 * \text{Water} + 0.10341 * \text{Cement} + 1.53865 * \text{Laterite} \quad (11)$$

$$\frac{1}{fc_{7}} = -4.13545 * \text{Water} + 0.21151 * \text{Cement} + 1.79349 * \text{Laterite} \quad (12)$$

$$\text{Cost} = -9.48243 * \text{Water} + 236.04554 * \text{Cement} + 24.41443 * \text{Laterite} \quad (13)$$

*Optimization formulation*

The optimization formulation for selecting mixture proportions to meet user-defined requirements can be implemented using a Matlab solver. Sample mix design for using this procedure is presented in Appendix B.1. Similarly, an analytical method is also proposed in Appendix B.2. Optimization formulation is presented as:

*Minimize f(x) = -3.54724*Water + 0.10341*Cement + 1.53865*Laterite strength at 28 days in Equation (11)*

*Subject to inequalities:*

$x_{il} \leq x_i \leq x_{iu}$   *upper and lower levels on the variables*

$\sum_{i=1}^{3} a_i x_i \leq c_i$   *cost*

*Equalities:*

$x_1 + x_2 + x_3 = 1$   *absolute volumes must be equal to 1*

$-a_i x_{2i} + x_{3i} = 0$   *ratio of cement: laterite*

$x_{1i} = y_{1i}$   *water requirement in Equation (8)*

*Durability of the laterite-cement bricks*

The result of ASTM D 559-03 otherwise called wire brush test is presented in Table 3. Although the method is considered severe in comparison with actual field performance for building bricks (Walker, 1995; Ipinge, 2012; Heathcote, 2002), it still remains the most preferred. The major disadvantage is the length of time required for the test (Heathcote, 2002; ASTM D 559-03). It measures the amount of particles eroded as a percentage of the original oven-dry mass of laterite-cement brick specimen. The basic requirement of acceptability is that the percentage mass eroded should not exceed 10 percent and this is satisfied based on the limits or the feasible domain selected.

**Table 3:** Result of durability test by wire brush

| BRICK CODE COLUMN | 12 Cycles | | Weight Loss % |
|---|---|---|---|
| | Initial weight (grammes) | Final weight (grammes) | |
| A1 | 1962 | 1838 | 6.32 |
| A2 | 1975 | 1885 | 4.56 |
| A3 | 1993 | 1942 | 2.56 |
| A12 | 1958 | 1868 | 4.60 |
| A13 | 1964 | 1871 | 4.74 |
| A23 | 2003 | 1901 | 5.09 |
| C1 | 1995 | 1903 | 4.61 |
| C2 | 1998 | 1912 | 4.30 |
| C3 | 2020 | 1916 | 5.15 |
| 0 | 1980 | 1896 | 4.24 |

**The highlighted are the upper and the lower limits on the domains of constituent proportions by volume*

**A1, A2, A3 represent pure blends. A12, A13, A23 represent binary blends. C1, C2, C3 represent control points and O represents centre point fitted in the factor space.*

*Comparative compressive strength results using the Scheffe Mixture approach*

A comparative result of compressive strength computed using the Genetic Algorithm example in Appendix B.1 is shown in Table 4. It shows that the measured properties of bricks produced are largely dependent on the quantity of cement and compactive effort (Osunade and Fajobi, 2000; Aguwa, 2009; Awoyera and Akinwumi, 2014; Hydraform, 2014). Similarly, production of bricks within 8 - 20 percent cement content design domain has shown reasonable results that would guide on quality brick production that would be durable and this can be adopted as a useful guide in specification writing for mass housing development. The compressive strength values are well above the minimum requirement of 2.8N/mm$^2$ in accordance with NIS (2004).

**Table 4:** Comparative compressive strength results using Mixture Approach Design

| | | | Hydraform (2014) | Aguwa (2009)[a] | Guettala et al (2005)[b] | Awoyera & Akinwumi (2014)[c] | Experimental Result |
|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| S/No. | Cement Content(%) | Compactive effort MN/m2 | 10 | 4 | 15 | 2 | 10 |
| 1 | 4 | COMPRESSIVE STRENGTH N/mm$^2$ | - | 1.9 | - | - | - |
| 2 | 5 | | 3 | - | 15.4 | - | - |
| 3 | 6 | | - | 3.5 | - | - | - |
| 4 | 7 | | 5 | - | - | - | - |
| 5 | 8 | | - | 5.1 | - | 2.3 | 8.4 |
| 6 | 10 | | 8 | 6.1 | 18.4 | 3.49 | 9.6 |
| 7 | 12 | | - | 6.5 | - | 3.86 | 11.17 |
| 8 | 14 | | - | 7.1 | - | - | 12.79 |
| 9 | 15 | | 10 | - | - | - | - |
| 10 | 16 | | - | 8.3 | - | - | 14.55 |
| 11 | 18 | | - | 9.2 | - | - | 18.32 |
| 12 | 20 | | 12 | 9.6 | - | - | - |
| 13 | 25 | | 14 | - | - | - | - |

** The highlighted header row represents the compactive effort in MN/m$^2$*

** The column number (8) was estimated using the example in Appendix B.1*

[a] using 2.5kg rammer ; [b] a hydraulically compressed machine in Algeria and [c] using Cinva Ram

**Conclusions and Recommendations**

In using the method of mixture proportioning, it has been shown that statistically designed composite bricks satisfying user specified requirements is practicable. Similarly, responses capable of achieving user-defined requirements can be developed and thus specification writing for site production is possible. The GA stochastic method and the analytical procedures presented in the Appendix are implementable computationally. Compressive strength and compactive effort still represent major factors in predicting the properties of the bricks moulded.

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APPENDIX A: **Mixture proportions (per $m^3$) and responses- RUN ORDER**

| Std Order | Run Order | Block | Component 1 A:Water $m^3$ | Component 2 B:Cement $m^3$ | Component 3 C:Laterite $m^3$ | Response 1 fc 7 $N/mm^2$ | Response 2 fc 28 $N/mm^2$ | Response 3 Cost Naira |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 | Block 1 | 0.266 | 0.058 | 0.677 | 6.09 | 8.406 | 27.6214 |
| 6 | 2 | Block 1 | 0.264 | 0.089 | 0.647 | 11.079 | 14.445 | 34.3934 |
| 1 | 3 | Block 1 | 0.266 | 0.046 | 0.688 | 7.627 | 8.4 | 25.1481 |
| 7 | 4 | Block 1 | 0.265 | 0.072 | 0.663 | 10.558 | 11.976 | 30.736 |
| 11 | 5 | Block 1 | 0.265 | 0.068 | 0.667 | 8.22299 | 10.551 | 29.8244 |
| 10 | 6 | Block 1 | 0.265 | 0.068 | 0.667 | 10.349 | 10.359 | 29.8244 |
| 3 | 7 | Block 1 | 0.261 | 0.106 | 0.633 | 12.508 | 16.653 | 37.9975 |
| 5 | 8 | Block 1 | 0.265 | 0.064 | 0.67 | 8.792 | 10.432 | 29.0014 |
| 9 | 9 | Block 1 | 0.264 | 0.083 | 0.653 | 10.959 | 12.991 | 32.9842 |
| 14 | 10 | Block 1 | 0.265 | 0.068 | 0.667 | 8.47996 | 10.8807 | 29.8244 |
| 12 | 11 | Block 1 | 0.265 | 0.068 | 0.667 | 8.22299 | 10.551 | 29.8244 |
| 2 | 12 | Block 1 | 0.265 | 0.077 | 0.658 | 9.884 | 13.249 | 31.7929 |
| 13 | 13 | Block 1 | 0.265 | 0.068 | 0.667 | 8.73693 | 11.2104 | 29.8244 |
| 8 | 14 | Block 1 | 0.266 | 0.055 | 0.679 | 8.337 | 10.452 | 27.0266 |

**APPENDIX B.1: Example of optimization of component mixes to meet user defined requirement using the Genetic Algorithm method**

Problem statement: To obtain mix proportions to achieve prescribed 28days strength for laterite cement brick. The data input for this requirement are as stated:

i) Use cement content of 8% representing ratio 1:12.5 of cement to laterite
ii) The equality constraint of the sum of all the absolute volumes must be equal to 1
iii) The total cost should not exceed ₦30:00 per brick

The objective function for strength at 28 days from Table 4(a) is:

$$Strength, \quad \frac{1}{fc_{28}} = -3.54724 \cdot water + 0.10341 \cdot cement + 1.53865 \cdot laterite$$

The response prediction for cost of producing one brick for MX-0 from Table 4(c) is:

$$MX - 0\%; \; Cost = -9.48243 \cdot water + 236.04554 \cdot cement + 24.41443 \cdot laterite$$

The constraint on the ratio of cement to laterite which is to be 1:12.5 can be constructed as:

$$\frac{x_3}{x_2} = 12.5; \qquad where: x_2 = cement \; and \; x_3 = laterite.$$

This can be re-written as:

$x_3 = 12.5x_2$ and re-arranging gives the linear relationship $-12.5\,x_2 + x_3 = 0$ and multiplying by their respective unit weights per cubic metre, $3150 kg/m^3$ and

iv) Substitute the absolute volumes of the respective quantities in the equation relating strength at 28days in Table 4a to obtain the compressive strength at 28 days

v) Calculate the inverse or reciprocal of the value obtained in (iv)

vi) Calculate the cement laterite ratio and cement percentage per $m^3$ of mix

vii) Now calculate the cost per brick or per $m^2$

Substitute the values in the problem statement:

i) Using a value of cement within the suggested limit (absolute volume = 0.057) represents 179.55kg of cement, that is (0.057 x 3150 = 179.55kg), where unit weight of cement is 3150kg/$m^3$

ii) The corresponding absolute volume of laterite from equation 11(a) relating the calculated cement quantity is: $laterite = (1927 - 0.7767 * cement)$ which gives (1927-(0.7767*179.55))/2640 = 0.6771 and the weight of laterite is 0.6771*2640 = 1787.54352kg/$m^3$

iii) The corresponding quantity of water from equation 10(a) relating the calculated cement/laterite ratio is: $water = 269.5 - 36.93 * \frac{cement}{laterite}$. this substitution gives = (269.5-(36.93*(179.55/1787.54352))) = 265.791kg/$m^3$. The absolute volume of water is 266.55/1000 = 0.265791 ≈ 0.266

iv) Substituting the absolute volumes of all the constituent materials in equation 5(a)

$$1/fc_{28} = -3.54724 * water + 0.10341 * cement + 1.53865 * laterite =$$

$$1/fc_{28} = -3.54724 * 0.266 + 0.10341 * 0.057 + 1.53865 * 0.6771 = 0.104891$$

v) The inverse is 9.5337N/$mm^2$

vi) The cement laterite ratio is $179.55/1787.54352$ which represents ratio 1:10

vii) The cost function in equation 5(c) for MX - 0 is:

$Cost = -9.48243 * water + 236.04554 * cement + 24.41443 * laterite$ which can be substituted to yield:

cost = (-9.48243*0.266 + (236.04554*0.057 + (24.41443*0.0.6771))) = N27.46 per brick < N30.00