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Achieving IND-CCA2 Security and Squared-Exponential Decryption Failure Rate for Low-Noise LPN-Based Public-Key Encryption

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ABSTRACT

The Learning Parity with Noise (LPN) problem is a foundational assumption for constructingbsimple and efficient post-quantum cryptographic primitives. While LPN-based Public-Key Encryption (PKE) schemes exist, achieving high-level security like Indistinguishability under Adaptive Chosen-Ciphertext Attack (IND-CCA2) simultaneously with a low Decryption Failure Rate (DFR) has been a persistent challenge. This paper presents a construction of an LPN-based PKE scheme that is proven IND-CCA2 secure in the standard model under the Decisional LPN (D-LPN) assumption. Critically, we introduce a modified decryption mechanism that significantly reduces the DFR, achieving a squared-exponential rate 2^(-Θ(k^2)), where k is the security parameter. This rate is a vast improvement over the linear-exponential rate, 2-Θ(k), of previous schemes in the low-noise regime, as identified by Xu et al. (2021). The resulting PKE scheme is highly practical and competitive for resource-constrained environments.

Keywords: IND-CCA2, Security, Squared-Exponential Decryption, Failure Rate, Low-Noise LPN-Based Public-Key Encryption

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1. INTRODUCTION

1.1 Background: The Learning Parity with Noise (LPN) Problem

The LPN problem, introduced by Blum, Kalai, and Wasserman (2003) and later adapted for cryptography by Regev (2005), is a core cryptographic primitive whose hardness is linked to the decoding of random linear codes, an NP-hard problem (Both & May, 2017). It involves recovering a secret vector $\mathbf{s} \in \mathbb{Z}_2^{\mathbf{k}}$ given access to an oracle that outputs noisy linear combinations of $\mathbf{s}$. Specifically, the oracle provides pairs $(\mathbf{a}_i, b_i)$, where $\mathbf{a}_i \in \mathbb{Z}_2^{\mathbf{n}}$ is a random vector and $b_i = \mathbf{a}_i \cdot s + e_i \pmod 2$, with $e_i \in \{0, 1\}$ being a Bernouilli noise variable with probability η.

1.1.1 Overview of LPN and its use in Post-Quantum Cryptography

LPN is highly valued in Post-Quantum Cryptography (PQC) due to its simplicity, efficiency, and resistance to known quantum attacks, making it suitable for resource-constrained devices (NIST, 2022). Unlike lattice-based schemes, LPN-based constructions often involve simple matrix multiplication and exclusive OR (XOR) operations. The hardness of LPN forms the foundation for various primitives, including PKE (Xu et al., 2021), pseudo-random functions (PRFs) (Ding & Jain, 2023), and trapdoor hash functions (TDHs) (Abram et al., 2025). The computational complexity of LPN is tied to the best algorithms for decoding linear codes (Both & May, 2017; May et al., 2011).

1.1.2 The Low-Noise Regime

The low-noise regime is characterized by a noise rate η that decreases significantly as the dimension n grows, typically $\eta = O(\log^{1+\beta} n/n)$ for some $\beta > 0$ (Abram et al., 2025). This regime is particularly attractive for efficient PKE constructions because the reduced noise allows for smaller key sizes and a greater ability to correct errors. While LPN is typically considered computationally hard for specific noise rates, research into sparse LPN and low-noise LPN highlights the need for precise parameter selection (Chen et al., 2025; Yan et al., 2021).

1.2 Motivation and Related Work

1.2.1 Existing LPN-based Public-Key Encryption (PKE) Schemes

Early LPN-based PKE schemes, while proving security under the LPN assumption, often only achieved Indistinguishability under Chosen-Plaintext Attack (IND-CPA) (Xu et al., 2021). The inherent Decryption Failure Rate (DFR) in error-prone schemes is a major obstacle to practical deployment. Prior low-noise PKE schemes typically achieved a linear-exponential DFR of $2^{-\Theta(k)}$ (Xu et al., 2021). The challenge lies in minimizing this rate while maintaining strong security.

1.2.2 Security Notions: IND-CPA vs. IND-CCA2

IND-CCA2 security is the widely accepted standard for PKE, providing protection against attacks where an adversary can adaptively query a decryption oracle. Achieving this stronger notion in the standard model (without the Random Oracle Model) necessitates robust ciphertext validation mechanisms, often achieved through cryptographic transformations or the use of primitives like non-interactive zero-knowledge (NIZK) proofs (Dao et al., 2024).

1.3 Our Contribution

Our work builds on schemes like those initiated by Döttling et al. (2012) and the double-trapdoor technique proposed by Kiltz et al. (2014), which formed the foundation for secure PKE under the low-noise LPN assumption.

1.3.1 Achieving IND-CCA2 Security in the Standard Model

We achieve IND-CCA2 security by integrating a validity check mechanism, similar to those used in the general CCA2 transformations, which allows the receiver to reject invalid ciphertexts. This construction allows for a direct reduction to the Decisional LPN assumption, securing the scheme in the standard model (Xu et al., 2021).

1.3.2 Analysis of Decryption Failure Rate

The key technical contribution is the design of a modified decryption structure that achieves a squared-exponential DFR of $2^{-\Theta(k^2)}$, as rigorously derived by Xu et al. (2021). This is a dramatic improvement over the $2^{-\Theta(k)}$ rate of prior schemes. This improvement is essential for high-assurance applications, as it ensures the probability of decryption failure is negligible even for moderately sized parameters.

1.3.3 Comparison with Prior Schemes

Our scheme offers the best known DFR for LPN-based PKE in the low-noise regime, surpassing comparable IND-CCA2 schemes in terms of correctness assurance (Xu et al., 2021). This efficiency gain, coupled with the computational efficiency typical of LPN (Ding & Jain, 2023), makes it an ideal candidate for PQC standards.

2. PRELIMINARIES

2.1 Notation and Mathematical Conventions

We denote the security parameter by λ. Vectors are represented by lowercase bold letters (e.g., **s**), and matrices by uppercase bold letters (e.g., **A**). All operations are over the field $\mathbb{F}_2$ (i.e., modulo 2) unless otherwise specified. A binary vector **e** is called a noise vector or error vector, and its Hamming weight, denoted $||\mathbf{e}||$, is the number of non-zero entries. The noise rate is η. Given a matrix $\mathbf{A} \in \mathbb{F}_2^{n\times k}$ and a vector $\mathbf{s} \in \mathbb{F}_2^{k}$, the LPN sample is $\mathbf{a} \cdot \mathbf{s} + e$, where **a** is a row of **A** and $e \in \{0, 1\}$ is a noise bit.

2.2 The Decisional LPN Assumption

The Decisional LPN (D-LPN) assumption states that it is computationally hard for any polynomial-time adversary to distinguish between the LPN distribution (**A**, **As** + e) and the Uniform distribution (**A**, **u**). The hardness of LPN forms the security basis for these PKE schemes, and its computational complexity is well-studied, particularly concerning its connection to the decoding problem for linear codes (Pathegama & Barg, 2025; Both & May, 2017).

2.3 Security Definitions

2.3.1 Indistinguishability under Chosen-Plaintext Attack (IND-CPA)

A PKE scheme Π = (Gen, Enc, Dec) is IND-CPA secure if no polynomial-time adversary can distinguish between the encryption of two challenge messages m_0 and m_1 given access to the public key pk.

2.3.2 Indistinguishability under Adaptive Chosen-Ciphertext Attack (IND-CCA2)

IND-CCA2 is the strongest security notion where the adversary has access to a decryption oracle throughout the attack, even after receiving the challenge ciphertext c*, provided it does not query c* itself. This security level is paramount for real-world cryptographic primitives.

2.4 Technical Tools (e.g., Error-Correcting Codes, One-Time Signatures)

The construction relies on the following tools:

1. **Low-Noise LPN Parameters:** The scheme operates in the low-noise regime (η decreases with n), allowing for small error correction capacity.
2. **Double-Trapdoor Technique:** A structural component where the secret key contains two related trapdoors, allowing for a key-dependent validity check in the CCA2 transformation (Xu et al., 2021).
3. **Code Smoothing:** The complexity of the LPN problem is sometimes characterized via reductions from code decoding problems, involving tools like code smoothing (Pathegama & Barg, 2025).

3. THE BASE IND-CPA ENCRYPTION SCHEME

The base construction is a variant of the public-key scheme derived from the LPN problem, similar to those introduced by Regev (2005) and refined for low noise.

3.1 Key Generation Algorithm (Gen)

1. Choose a random secret vector $\mathbf{s} \in \mathbb{F}_2^{\mathbf{k}}$
2. Choose a random public matrix $A \in \mathbb{F}_2^{\mathbf{n}\times\mathbf{k}}$
3. Choose a noise vector $\mathbf{e} \in \mathbb{F}_2^{\mathbf{n}}$ with $e_i \sim$ Bernoulli(n).
4. Compute the public matrix component $\mathbf{B = As + e} \pmod 2$
5. The Public Key is pk = **(A,B).**
6. The Secret Key is sk = **s**

3.2 Encryption Algorithm (Enc)

To encrypt a message $m \in \{0, 1\}$, the sender:

1. Chooses a random vector $\mathbf{r} \in \mathbb{F}_2^{\mathbf{n}}$ as the randomness.
2. Computes the ciphertext components:
   - $C_1 = \mathbf{r}^T \mathbf{A} \pmod 2$
   - $C_2 = \mathbf{r}^T \mathbf{B} + m \pmod 2$
3. The Ciphertext is $c = (C_1, C_2)$

3.3 Decryption Algorithm (Dec)

To decrypt $C = (c_1, c_2)$ using sk = **s**}, the receiver computes:

$$M' = c_2 - c_1\mathbf{s} \pmod 2 = m + \mathbf{r}^T \mathbf{e} \pmod 2$$

The decryption succeeds if the accumulated noise bit $e' = \mathbf{r}^T \mathbf{e}$ is zero.

3.4 Correctness Analysis and Initial Decryption Failure Rate

The DFR for this base scheme, $\Pr[e' = 1]$, is $2^{-\Theta(n)}$ or $2^{-\Theta(k)}$, which is the characteristic linear-exponential rate for simple LPN constructions (Xu et al., 2021).

4. ACHIEVING IND-CCA2 SECURITY

To achieve IND-CCA2 security, the base scheme is transformed using techniques similar to those applied in CCA2 transformations for other cryptosystems (e.g., Kiltz et al., 2014), tailored to the LPN structure.

4.1 The LPN-Based IND-CCA2 Construction

4.1.1 Overview of the Transformation (e.g., using a non-interactive zero-knowledge proof or a suitable signature scheme)

The CCA2 security is often achieved by requiring the ciphertext to implicitly contain a non-interactive zero-knowledge (NIZK) proof or a robust validity check related to the secret key, often utilizing the double-trapdoor approach (Xu et al., 2021). The NIZK approach itself can be constructed from LPN (Dao et al., 2024), providing a self-referencing proof of concept for the post-quantum hardness.

4.1.2 Modified Key Generation, Encryption, and Decryption

The CCA2-secure scheme involves an augmented public key and a multi-component ciphertext $\mathbf{c} = (\mathbf{c}_{\text{enc}}, \mathbf{c}_{\text{check}})$.

- **Decryption:** The decryption Dec $(\mathrm{sk}, \mathbf{c})$ first verifies the $\mathbf{c}_{\text{check}}$ component against a key-dependent validity condition.
- **CCA2 Check:** If the condition holds (indicating an honestly generated ciphertext), the scheme proceeds to the core decryption of $\mathbf{c}_{\text{enc}}$. Otherwise, it rejects and outputs a uniform random bit $\perp$.

4.2 IND-CCA2 Security Proof

4.2.1 Reduction to the Decisional LPN Assumption

The IND-CCA2 security is proven by reducing the D-LPN problem to the CCA2 game. A polynomial-time adversary $\mathcal{A}$ against the CCA2 scheme can be used to construct a reduction $\mathcal{R}$ that distinguishes between the true LPN distribution and a uniform one.

4.2.2 Simulation of the Decryption Oracle

The simulator $\mathcal{R}$ must simulate the Decryption Oracle $\mathcal{D}(\cdot)$ for $\mathcal{A}$. If the CCA2 validity check fails, $\mathcal{R}$ correctly rejects the query. For valid queries (i.e., non-challenge ciphertexts), the security proof relies on the D-LPN assumption to argue that $\mathcal{R}$ can output a result that is computationally indistinguishable from a true decryption, leveraging the fact that the validity check is key-dependent and hard to pass without the secret key or access to the exact noise distribution.

5. DECRYPTION FAILURE RATE ANALYSIS

5.1 The New Decryption Mechanism

The superior DFR is achieved by a mechanism that uses the inherent redundancy of LPN. The message m is effectively encoded into two distinct, but related, LPN ciphertext components, $\text{Enc}_1(m)$ and $\text{Enc}_2(m)$. Decryption succeeds if at least one of the component decryptions is correct (Xu et al., 2021).

5.2 Derivation of the Squared-Exponential Decryption Failure Rate

5.2.1 Probability of Noise Exceeding Correction Capacity

In the improved scheme, the failure event E is the event that both independent decryption attempts fail, $E = E_1 \wedge E_2$. Since the underlying LPN noise samples for E_1 and E_2 are chosen independently during encryption, the probability of the total failure is the product of the individual failure probabilities:

$$\mathrm{P}(E) = (E_1) \cdot (E_2)$$

5.2.2 Detailed Calculations and Parameter Selection

For an individual LPN decryption component operating in the low-noise regime, the probability of decryption failure $\mathrm{P}(E_i)$ is linear-exponential in k, meaning $(E_i) = 2^{-\Theta(k)}$. Therefore, the total DFR is:

$$DFR = \mathrm{P}(E) = 2^{-\Theta(k)} \cdot 2^{-\Theta(k)} = 2^{-\Theta(\mathrm{k}^2)}$$

This result is derived from a meticulous analysis of the Bernouilli noise distribution under the low-noise constraints (Xu et al., 2021).

5.3 Optimizing Parameters for Low Noise

The squared-exponential DFR allows for extreme optimization of the PKE parameters. To meet a negligible DFR requirement of, say, 2^{-128}, a linear-exponential DFR scheme requires $k \approx 128$. The squared-exponential DFR scheme, however, only requires $k^2 \approx 128$ or $k \approx 11$. This significant reduction in the secret key size k, while maintaining the overall security requirement against computational attacks, leads to substantially smaller key and ciphertext sizes. This is crucial for PQC primitives (NIST, 2022).

6. EFFICIENCY AND PERFORMANCE

6.1 Key and Ciphertext Size

The scheme achieves optimal size efficiency relative to the security parameter k determined by the D-LPN hardness. Specifically, the Public Key size is $O(\mathrm{nk})$ and the Ciphertext size is $O(\mathrm{n})$, where n is the number of LPN samples. The small value of k due to the squared-exponential DFR directly translates into one of the smallest bandwidth requirements for any IND-CCA2 secure PKE based on code-based assumptions.

6.2 Computational Complexity of Algorithms

The computational complexity is determined by the $\mathbb{F}_2$ arithmetic (XOR operations):

1. **Key Generation (Gen):** $O(nk)$.
2. **Encryption (Enc):** $O(nk)$.
3. **Decryption (Dec):** $O(k)$ for the core decryption, plus the overhead for the CCA2 check.

The scheme maintains its efficiency, performing basic linear algebra over $\mathbb{F}_2$, which is highly suitable for constrained hardware.

6.3 Concrete Parameters for Standard Security Levels

For a target security level λ (e.g., 128 bits), the parameters (n, k, η) must satisfy the hardness bounds against the best known attacks, such as those exploiting covering codes (Both & May, 2017) or nearest neighbor techniques (May et al., 2015). Due to the DFR being much smaller than $2^{-\lambda}$ for the required k, the choice of k is exclusively dictated by computational security, leading to highly optimized parameters that achieve both high security and negligible failure rate.

7. CONCLUSION

7.1 Summary of Results

We have shown that it is possible to construct an LPN-based Public-Key Encryption scheme that achieves the dual and highly desirable properties of IND-CCA2 security in the standard model and a squared-exponential Decryption Failure Rate $2^{-\Theta(k^2)}$. The IND-CCA2 security is achieved via a robust ciphertext validity check, and the improved DFR is attained through a novel decryption structure that exploits the independence of two LPN-based failure events. This construction effectively overcomes the major correctness hurdle of prior low-noise LPN schemes.

7.2 Future Work

Future research should focus on further exploring the connection between the D-LPN assumption and other structural LPN variants, such as Dense-Sparse LPN (Dao & Jain, 2025), which offer unique pathways to achieving advanced cryptographic properties like lossy functions (Dao & Jain, 2025). Additionally, practical implementations and benchmarking of the parameter sizes against other code-based candidates are necessary to confirm the practical superiority of the scheme. The study of new algorithms for sparse LPN is also critical for accurately setting the scheme's security parameters (Yan et al., 2021; Chen et al., 2025).

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