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Multi response optimization using desirability functions: Application in dietary food development

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ABSTRACT

Development of dietary food products is one of the most difficult activities in the food industry. It is typically a complex multi-criteria optimization problem. By way of a case study, employing a constrained D-optimal mixture experimental design, this article presents an integrated approach for the development of dietary custard powder from locally available resources. Empirical models were developed, and by the application of numerical multi-response optimization via desirability functions, the optimum combination of ingredients for the custard powder was established. The formulation that produced powdered dietary custard of the highest desirability index of 0.506 was 12.43% biofortified cassava starch, 22.98% yellow corn starch, 29.34% soybean extract, 10% sweet potato extract, 22.25% peanut extract, 1% egg, and 2% milk powder. The quality properties of the optimal powdered custard were: total carbohydrate (67.99%), dietary fiber (0.85%), total fat (14.28%), protein (11.52%), ash content (1.39%), energy value (453.97 Kcal), swelling capacity (0.69% vol), water absorption capacity (1.8%), bulk density 0.763 g/cm^3, calcium (41.25 mg), iron (5.27 mg), potassium (273.1 mg), phosphorus (234.03 mg), magnesium (101.36 mg), zinc (3.19 mg), sodium (733.32 mg), selenium (0.23 mg), vitamin A (0.28 mg), vitamin B1 (0.49 mg), vitamin B2 (0.32 mg), vitamin B3 (7.49 mg), vitamin B5 (0.30 mg), vitamin B6 (0.13 mg), vitamin B12 (0.013 mg), and vitamin C (543.39 mg).

1. Introduction

In many aspects of modern engineering (processes, products, and systems), the need to optimize multiple responses according to several criteria simultaneously is a common problem encountered. Finding the optimum settings of controllable variables (process variables and/or ingredient proportions) that will optimize product quality characteristics (response variables) at the same time is a needed approach in product development. The scientific directions of formulation design, modeling, and optimization are relevant all over the world and are powerful concepts in the successful development of food products that meet consumers' requirements (Bugaev et al., 2022; Satina & Yudina, 2010; Marinina et al., 2020; Ivanova et al., 2018; Shelubkova et al., 2018; Koneva et al., 2016; Korolchenko & Minaylov, 2020; Stankevich et al., 2016).

Food manufacturers are often called upon to develop food products that optimally meet the consumer's organoleptic, nutritional and/or dietary needs. The development of dietary food products is one of the most powerful but difficult activities in the food industry. It is typically a complex multicomponent, multi-criteria optimization problems due to variations in customer dietary needs. Multiple ingredients (the $X_i's$), known as design variables, and several final quality characteristics of the product (the $Y_j's$) have to be dealt with. Both the design variables and the food quality indices are referred to as optimization variables. It is known that when multiple responses are involved, it is usually difficult and complex to optimize them simultaneously, concurrently and concertedly. Integrating multiple criteria into diet design requires careful consideration of balancing multiple factors simultaneously (Muniswamaiah et al., 2020; Pesteh et al., 2019; Alketbi & Mushtaha, 2022; Ding et al., 2020; Laidani et al., 2020; Rathod et al., 2020; Kulikov, 2020; Liu et al., 2020; Meisig et al., 2020; Yuan et al., 2020; Van Dooren, 2018; Gazan et al., 2018; Bashiri et al., 2025a; Bashiri et al., 2025b). Some of the optimization goals and criteria may be conflicting (e.g. some may involve maximization while others are to be minimized), making it difficult to solve them. Since the dietary requirements of consumers vary, optimization goals and criteria for each of the $Y_j's$ will vary for different target consumers. It is practically impossible to establish a generalized optimal blend for all categories of consumers since the dietary needs of consumers vary and there are large number of variables that impact the product quality. Instead, what can be determined are optimal blends for targeted consumers.

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Multi-objective, multi-criteria optimization via desirability functions are structured, organized, and efficient techniques for finding explicit compromises between different multiple responses. Settings of the design variables that yield optimal, or near optimal values for the responses under consideration can be established, even when some of the criteria appear conflicting. Statistical design of experiments is used to collect data. By analyzing the results of designed experiments, one can estimate the response models that relate the factors to the responses and determine the degree of influence of each factor used on the response variable, as well as the interactions between the factors. RSM is a collection of mathematical and statistical techniques notably used for analyzing, modelling and optimizing systems and/or processes in which responses of interest are influenced by several controllable variables. Following analysis of variance (ANOVA) and identification of significant factors, models (i.e., response surface) representing the approximate relationship between responses and design/control factors are obtained. Whenever there are multiple responses of interest to be simultaneously optimized (with different targets), numerical optimization, via desirability functions is employed to obtain the optimal parameter-setting. It is one of the creative ideas that have attracted wide attention in many fields of science, research and development. The aim of the desirability optimization approach is to discover such an input variable set for which all output variables (responses) fall on target values or as close as possible to them. Desirability functions play an increasing role in solving the optimization of process or product quality problems having various quality characteristics to obtain a good compromise between these characteristics. By application of desirability functions, multiple response measures can be combined into one objective function, called overall desirability index; and the optimum combination of optimal ingredients and/or process factors settings, in a defined solution space, can be established. Contour plots are used to characterize the response surface graphically and to determine the optimal parameter-setting. Numerical optimization methods, exploiting desirability function technique, is one of the most frequently used multi-response optimization techniques in practice. When multiple responses are considered, the optimal parameter-setting is obtained by observing overlay contour plots (Alketbi & Mushtaha, 2022; Ding et al., 2020; Laidani et al., 2020; Rathod et al., 2020; Ramalingam et al., 2013; Saha & Alam, 2022; Short & Selvakumar, 2020; Arifin et al., 2020; Rajesh et al., 2020; Wang et al., 2020; Chen et al., 2020; Setiawati & Yusuf, 2020; Yang et al., 2020; Derringer & Suich, 1980; Derringer,1994; Kim & Lin, 2000; Jeong & Kim, 2009; He et al., 2010; Ehrgott, 2005). The approach systematically transforms each response into a scale-free individual desirability index, *d*. Numerical optimization becomes essential when investigating many components with many responses. Multi-response optimization problems require an overall optimization, i.e. simultaneous satisfaction of objective goals with respect to all the quality characteristics. Optimization goals will be assigned to the quality properties (responses) and these goals will be used to construct desirability indices, transforming each to a desirability function/index, d_i (the value of d_i varies between 0 and 1 for any given response). Objective goals may be to maximize, minimize, or target a desired response based on set criteria for all variables. The individual desirability function, is defined differently based on the objective of the response. The individual desirability indexes are then combined by taking geometric mean to obtain an overall desirability index, *D*, having a value in the interval (0; 1), yielding a single objective for multi-response problem (Ehrgott, 2005). Instead of optimizing each response separately, optimal settings for the predictor variables (process variables or ingredients proportions) that satisfy all the responses at once are established. Originally developed by Harrington and later modified by Derringer and Suich, introducing weighted value and assigning relative importance (or priorities) to the individual responses; the desirability function approach is one of the most frequently used multi-response optimization techniques in practice (Derringer and Suich, 1980; Derringer, 1994; Harrington, 1965). Harrington used exponential functions to quantify desirability. Suppose that there are R response functions to simultaneously optimize, denoted by $f_r(x)$ $(r=1.....R)$. For each of the *R* functions, an individual desirability function d is constructed that is high when $f_r(x)$ is at the desirable level (such as a maximum, minimum, or target) and low when $f_r(x)$ is at an undesirable value. Derringer and Suich proposed three forms of these functions, corresponding to the type of optimization goal. For maximization of $f_r(x)$, the function;

$$d_r^{\max} = \begin{cases} 0 & if\ f_r(x) < A \\ \left(\dfrac{f_r(x)-A}{B-A}\right)^w & if\ A \le f_r(x) \le B \\ 1 & if\ f_r(x) > B \end{cases} \tag{1}$$

can be used, where A and B represents the acceptable lower and upper limits for the *ith* response, $f_r(x)$, and *w* represents the weight. A, B, and *w* are to be chosen by the investigator. The bounds of design factors xi are defined when selecting the experimental matrix, so they are known during the optimization process. When w=1, the function is linear. If w>1 then more importance is placed on achieving the target for the response, $f_r(x)$. When w<1, less weight is assigned to achieving the target for the response, $f_r(x)$. When the function is to be minimized, they proposed the function

$$d_r^{\min} = \begin{cases} 0 & if\ f_r(x) > B \\ \left(\dfrac{f_r(x)-B}{A-B}\right)^w & if\ A \le f_r(x) \le B \\ 1 & if\ f_r(x) < A \end{cases} \tag{2}$$

and for target situations,

$$d_r^{target} = \begin{cases} \left(\dfrac{f_r(x)-A}{t_0-A}\right)^{w_1} & if\ A \le f_r(x) \le t_0 \\ \left(\dfrac{f_r(x)-B}{t_0-B}\right)^{w_2} & if\ t_0 \le f_r(x) \le B \\ 0 & otherwise \end{cases} \tag{3}$$

where, t_0 is a targeted value for the *ith* response.

The desirability functions are on the same scale and are discontinuous at the points A, B, and t_0. The values of w, w_1, or w_2 can be chosen so that the desirability criterion is easier or more difficult to satisfy. For example, if w is chosen to be less than 1 in Eq. (2), d_r^{min} is near 1 even if the model $f_r(x)$ is not low. As values of w move closer to 0, the desirability reflected by (2) becomes higher. Likewise, values of *w* greater than 1 will make d_r^{min} harder to satisfy in terms of desirability. These scaling factors are useful when one equation is of greater importance than the others. The graphical representation of desirability functions are presented in Figure 1.

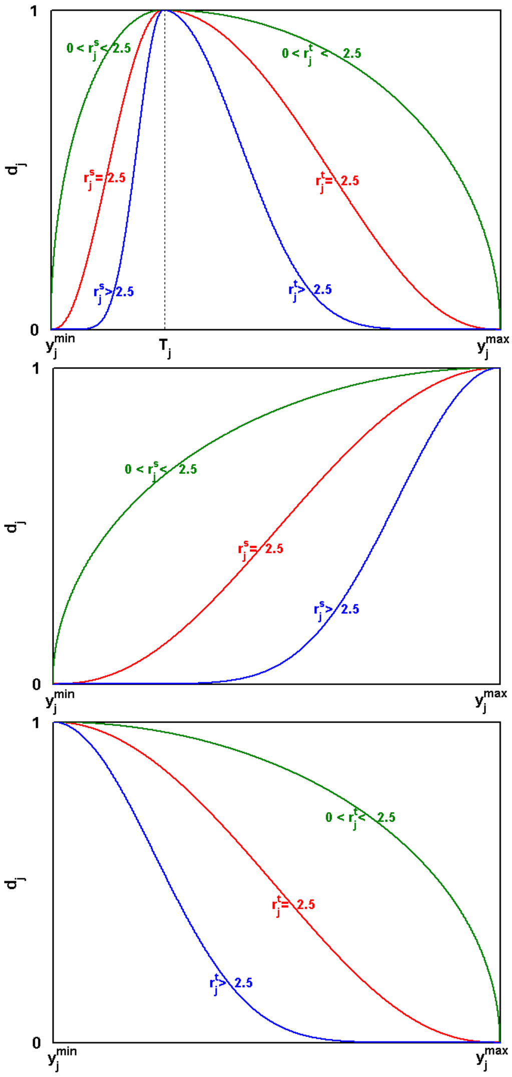

Figure 1. Graphical representation of desirability function

In the case of multi-response optimization, all the responses were then mathematically combined into an overall objective function, *D*, by the use of geometric mean. Once the individual desirability functions are defined for each of the responses, assuming that there are m responses, an overall desirability function is obtained as follows:

$$D = \left(d_1^{r_1} . d_2^{r_2} . d_3^{r_3} .... d_m^{r_m} \right)^{1/(r_1 + r_2 + r_3 + ... + r_m)} \qquad (4)$$

where the r_i*'s* represent the importance of each response. The greater the value of r_i, the more important the response with respect to the other responses. The objective is to now find the settings that return the maximum value of *D*. If any individual desirability di is equal to zero, then the overall desirability may also be equal to zero; hence, a geometric rather than an arithmetic mean is employed. A desirability equal to zero represents a completely unacceptable value of the considered response (quality characteristic). On the contrary, desirability equal to one represents a preferred (ideal) value of the considered response, that is to say, the desirability function value clearly indicates the closeness of a response to its ideal value (Derringer and Suich, 1980). The aim of the desirability optimization approach is to discover such an input variable set for which all output variables (responses) fall on target values or as close as possible to them.

Numerical optimization solutions are generally presented as a table, listed in their order of desirability, detailing the optimal conditions (components proportions and process variables values) that satisfies the set criteria and the overall desirability. The numerical solution can be presented in the form of desirability contour and 3-D Surface plots, optimal bar graph and graphical optimization overlay contour plot; showing the optimized formulation compositions and/or regions that meet specifications (Wendell. 2005; ReliaSoft. 2015; Myers et al., 2016; Dharmaraja & Dipayan, 2018). In the graphical approach to optimization, response models are fitted individually to their respective data. Contour plots are generated and then superimposed to locate one or more regions in the factor space where all the predicted responses attain a certain degree of "acceptability." There can be several candidate points from which the experimenter may choose. However, these plots limit consideration of the control variables to only two. If there are more, then the remaining variables are assigned fixed values. The graphical approach is limited in large systems involving several design variables and responses. Since only two or three design variables can be represented in the same plot, the number of generated plots can be quite large. Furthermore, the graphical approach does not account for the possibility of having correlated responses, which may also be heteroscedastic. Obviously, graphs based on such responses are not very reliable and may adversely affect the finding of optimum conditions (Sung & Vining, 1999).

This article presents an integrated approach for solving complex multicomponent, multi-criteria dietary food production problem using numerical multi-response optimization via desirability functions. By way of a case study, the development of dietary custard powder from some resources was demonstrated. Empirical models were developed, and by the application of numerical multi-response optimization via desirability functions, the optimum combination of ingredients for the custard powder were established. Optimization plots, generated using Design Expert software version 13, were also illustrated.

2. Materials and Methods

2.1. Materials

In this work, development of dietary custard powder was used as an example. There were seven ingredients or input variables: biofortified cassava (X_1), yellow corn (X_2), soybean (X_3), sweet potato (X_4), peanut (X_5), egg (X_6), and milk powder (X_7). The ingredients percentages were identified as the significant controllable factors. Samples were purchased from Kure market in Minna, Niger State, Nigeria.

The preparations of the ingredients were carried out at the department of Agricultural and Bioresources Engineering laboratory. The matured cassava roots were collected from the farm, selected, peeled, washed in clean water and grated with mechanical grater of commercial capacity. Pulp resulted from the grating process which was immediately sieved through a 0.9 mm standard Tyler screen to obtain starch slurry. Repeated decantation was carried out after allowing the starch slurry to sediment in each case. The final starch cake obtained was broken by hand and dried in a convectional cabinet dryer at 45-50 °C for 18 h. The starch was allowed to cool to room temperature, milled and packed in air tight polythene nylon prior to further use. Soya beans seeds were processed into a defatted flour to obtain the extract. Sweet potato flour processing methods involved selection of raw material, cleaning, washing peeling, slicing, drying, milling, packaging and storage. Raw peanut was collected and sorted. It was then washed, grinded and sieved using the sieve cloth, the extract was left in a container for three hours to settle. The extract was separated from the water. The analyses were carried out at the department of Food Science and Technology laboratory at Federal University of Technology, Minna.

2.2. Selection of an appropriate experimental design

An experiment can be defined as a test or a series of tests in which purposeful changes are made to the control variables of a process to help identify the reasons for changes that may be observed in the response variables and to approximate the process function precisely. A five-major-components, two-minor-components, constrained D-optimal mixture experimental design, with 55 randomized experimental runs, was employed for this study. The formulation design constraints were: biofortified cassava starch (10-60%), yellow corn starch (10-50%), soybean extract (5-30%), sweet potato extract (10-40%); peanut extract (5-30%), egg (1-2%) and milk powder (1-2%), respectively. The design matrix for the D-Optimal mixture design is presented in Table 1. The formulation of the blends were based on the design matrix.

2.3. Execution of the experiments and determination of the responses

Twenty-five quality or response variables were evaluated: total carbohydrate, dietary fiber, total fat, protein, ash content, energy value, swelling capacity, water absorption capacity, bulk density, calcium, iron, potassium, phosphorus, magnesium, zinc, sodium, selenium, vitamin A, vitamin B1, vitamin B2, vitamin B3, vitamin B5, vitamin B6, vitamin B12, vitamin C, appearance, texture, taste, flavour, colour, and overall acceptability. The experimental data generated based on the D-Optimal mixture design are presented in Tables 2-5. The quality properties were evaluated using the methods described by the Association of Analytical Chemist (AOAC, 2005). The sensory evaluation of the samples was conducted using a total of forty (40) semi-trained panelists. A 9-point hedonic scale ranging from 9 = like extremely and 1= dislike extremely was used to evaluate the samples.

Table 1. D-Optimal mixture design matrix for the custard formulation experiments

Run	x_1	x_2	x_3	x_4	x_5	x_6	x_7
1	43.5	33.5	5	10	5	2	1
2	37	10	5	40	5	1	2
3	51	10	5	10	21	2	1
4	10	50	13.5	10	13.5	2	1
5	37	10	5	40	5	1.5	1.5
6	10	42	30	10	5	1	2
7	10	10	30	17	30	1.5	1.5
8	37	10	5	40	5	1	2
9	10	27	5	25	30	1	2
10	11	10	30	40	6	2	1
11	29.5	10	17.5	10	30	2	1
12	10	42	30	10	5	2	1
13	10	50	5	27	5	2	1
14	10	42	5	10	30	1.5	1.5
15	31	10	16.17	23.67	16.17	1	2
16	12	10	30	40	5	1	2
17	10	24.5	5	40	17.5	1	2
18	10	42	5	10	30	2	1
19	27	50	5	10	5	1	2
20	42	10	30	10	5	1	2
21	42	10	5	10	30	1.5	1.5
22	10	50	22	10	5	1.5	1.5
23	17.93	15.85	22.41	30.89	9.91	1.5	1.5
24	60	17	5	10	5	1	2
25	10	24.5	17.5	40	5	1	2
26	42	10	30	10	5	1.5	1.5
27	60	10	12	10	5	2	1
28	10	10	30	17	30	2	1
29	10	37	5	40	5	2	1
0	10	10	7	40	30	1	2
31	37	10	5	40	5	2	1
32	12	10	5	40	30	2	1
33	10	29.5	30	10	17.5	1.5	1.5
34	10	50	5	10	22	1	2
35	26	26	5	10	30	1.75	1.25
36	10	11	6	40	30	1.5	1.5
37	27	50	5	10	5	1.75	1.25
38	12	10	30	40	5	1	2
39	10	37	5	40	5	1.5	1.5
40	10	50	5	27	5	1	2
41	16.93	25.1	15.91	23.14	15.91	2	1
42	10	27	30	25	5	1.5	1.5
43	17	10	30	10	30	1	2
44	26	26	30	10	5	1.25	1.75
45	17	10	30	10	30	1	2
46	37	10	5	40	5	2	1
47	43.5	33.5	5	10	5	1.5	1.5
48	42	10	5	10	30	1	2
49	10	10	30	40	7	1.5	1.5
50	42	10	30	10	5	2	1
51	60	10	12	10	5	1.5	1.5
52	10	50	5	18.5	13.5	1.5	1.5
53	26	26	5	10	30	1.25	1.75
54	12	10	5	40	30	2	1
55	26	26	30	10	5	1.75	1.25

x_1 = Biofortified cassava starch (%), x_2 = Yellow corn starch (%), x_3 = Soybean extract (%), x_4 = Sweet potato extract (%), x_5 = Peanut extract (%), x_6 = Egg (%), x_7 = Milk powder (%), D = Desirability

Table 2. Experimental data generated based on the D-Optimal mixture design

Run	y_{cho}	y_{df}	y_{fat}	y_{prot}	y_{ash}	y_{ev}	y_{sc}	y_{wac}	y_{bd}
1	74.16	1.02	9	10.5	1.32	419.64	1.39	2.8	0.63
2	76.62	0.98	8.5	8.4	1.5	416.58	1.2	3	0.66
3	69.05	0.92	13.5	9.45	1.48	435.5	1.06	3.2	0.63
4	72.47	0.88	13.5	8.05	1.5	443.58	0.81	2.6	0.65
5	77.3	1	10	6.3	1	424.4	0.89	3	0.72
6	71.6	1	12	10.5	1.5	436.4	0.96	2.8	0.94
7	58.31	0.94	16.5	18.9	1.55	457.34	0.65	2.5	0.74
8	77.07	0.88	10.1	6.65	1.5	425.78	0.86	2.8	0.65
9	72.37	0.88	14	8.05	1.5	447.68	0.77	3	0.64
10	69.08	0.72	12.5	10.5	3	430.82	0.74	2.6	0.61
11	55.92	0.63	16	11.55	2.5	413.88	0.51	2.8	0.61
12	77.72	0.88	6	11.2	1.2	409.68	0.79	3	0.54
13	84.11	0.74	3.5	6.65	2	394.54	0.53	2.8	0.64
14	63.79	0.77	17.5	11.9	1.04	460.26	0.61	2.2	0.58
15	64.14	0.81	16.5	11.55	1	451.26	0.53	3.2	0.61
16	73.92	0.73	8	11.55	1	413.88	0.78	2.6	0.64
17	81.13	0.87	8	4.9	1.5	416.17	0.53	2.8	0.68
18	73.21	0.88	13.5	7.35	1.46	443.74	0.8	2.4	0.64
19	77.37	0.83	8.5	7	1	416.78	0.24	2.6	0.76
20	76.99	0.72	9	8.05	1.04	421.16	0.66	3.1	0.75
21	72.89	1.06	13	8.05	1	440.76	0.88	2.8	0.66
22	74.75	1	9	8.05	2	412.2	1.09	2.6	0.72
23	72.58	0.97	11	10.85	1	432.22	0.92	3	0.69
24	79.08	0.92	6.5	9.1	1	411.22	1.46	3.2	0.73
25	77.64	0.76	12	5.6	1	440.96	1.16	2.8	0.77
26	80.97	0.88	8.5	5.25	1	421.38	1.24	3.6	0.74
27	88.44	0.91	3	3.85	1	396.16	1.69	2.4	0.6
28	69.08	0.55	16.22	10.15	1	462.9	0.77	1.8	0.8
29	87.98	0.64	3	4.9	1.08	398.52	0.84	2.2	0.62
30	66.29	0.57	14.5	15.4	1.04	457.26	0.79	2.1	0.75
31	86.85	0.82	4	4.73	1	402.32	1.13	3.1	0.71
32	72.46	0.8	15.5	7.7	0.94	460.14	0.09	2.2	0.85
33	75.98	0.52	12.5	7	1	444.42	0.98	2.2	0.74
34	71.25	0.57	12	12.25	0.98	442	0.74	3	0.6
35	73.77	0.88	12	10.15	1	443.68	0.93	2.8	0.9
36	71.92	0.63	11.5	11.55	2	437.38	1.03	3.7	0.86
37	74.38	0.55	8.5	11.55	1.02	420.22	1.48	2	0.6
38	78.43	0.72	9.5	7.35	1	428.62	0.72	2.1	0.65
39	79.56	0.81	5.5	10.15	0.98	408.34	2.23	2.5	0.61
40	82.57	0.71	7	6	0.92	417.28	1.13	2.3	0.64
41	72.09	0.84	14.5	8.57	1	453.14	2.18	2.4	0.72
42	73.64	0.66	9	12.6	1.5	425.96	0.39	2.2	0.65
43	66.15	0.53	18.22	11.2	1.5	473.38	0.89	2.8	0.77
44	67.45	0.74	11	16.8	1.41	436	0.46	2.4	0.67
45	68.18	0.62	15.5	11.2	1	457.02	0.77	2.5	0.77
46	75.14	0.71	2	17.15	1	387.16	2.81	2.5	0.61
47	85.56	0.64	2.5	6.3	1.5	389.94	0.23	2.2	0.56
48	71.57	0.33	12.5	11	2	442.78	0.91	2.6	0.66
49	78.19	0.51	6	10.9	1	410.36	0.52	2.2	0.61
50	73.52	0.43	2.25	10.15	2	354.93	0.48	2.2	0.6
51	81.13	0.72	6	8.75	1	413.52	0.71	2.2	0.58
52	83.01	0.98	5.5	7.35	0.96	410.94	0.68	2.7	0.55
53	68.56	0.84	15	11.1	1.5	453.64	0.45	2.4	0.72
54	77.38	0.72	9	8.38	0.92	424.04	0.67	2	0.67
55	80.76	0.68	6	9.12	1	413.52	0.51	2.6	0.63

y_{cho} = Total carbohydrate (%), y_{df} = Dietary fiber (%), y_{fat} = Total fat (%), y_{prot} = Protein (%), y_{ash} = Ash content (%), y_{ev} = Energy value (Kcal), y_{sc} = Swelling capacity (% vol), y_{wac} = Water absorption capacity (%), y_{bd} = Bulk density (g/cm^3)

Table 3. Experimental data generated based on the D-Optimal mixture design continue

Run	y_{ca}	y_{fe}	y_k	y_{phos}	y_{mag}	y_{zn}	y_{sod}	y_{sel}
1	48.3	1.33	17	299.6	107	4.6	991	0.12
2	47.2	1.4	523	299.3	103	3.2	494	0.215
3	45.26	1.06	896	306.1	108	4.1	118	0.17
4	49.41	1.5	28	284.6	102	3.6	1003	0.22
5	49.38	1.06	135	312.8	112	3.6	964	0.415
6	48.31	1.23	178	277	102	4	923	0.12
7	43.26	1.51	198	284	98	4.1	1000	0.315
8	41.28	2.81	206	263	96	3.8	382	0.26
9	40.06	2.61	137	270	102	2.7	989	0.34
10	41.36	1.3	852	270	102	2.6	281	0.255
11	48.94	1.57	183	308	114	2.3	959	0.135
12	49.24	1.91	165	302	112	3	945	0.5
13	48.78	1.89	111	302	110	2.9	920	0.415
14	49.68	1.73	116	312	117	3	986	0.21
15	45.33	1.71	81	312	109	2.8	975	0.275
16	45.3	1.5	283	309	111	2.8	923	0.18
17	48.13	1.73	81	298	110	2.6	1032	0.225
18	47.63	1.67	75	298	109	3	904	0.14
19	46.98	2.2	162	298	109	2.7	521	0.265
20	48.31	2.9	185	278	98	2.8	827	0.235
21	45.32	2.21	249	312	92	2.1	467	0.22
22	46.11	2.84	35	316	98	2.3	1043	0.215
23	43.27	2.22	148	300	100	2.6	945	0.38
24	42.28	1.97	73	306	106	2.8	898	0.25
25	42.14	2.4	100	312	102	2.3	986	0.155
26	40.88	2.46	193	302	100	2.9	986	0.33
27	44.11	3.56	113	296	108	2.7	1024	0.12
28	43.23	2.76	326	310	118	2.8	1632	0.315
29	41.38	5.53	46	293	98	3.1	1060	0.415
30	42.28	2.56	122	270	100	2.7	948	0.385
31	41.63	2.51	120	314	112	2.6	950	0.15
32	38.22	2.73	94	209	84	2.1	1023	0.335
33	38.72	3.39	141	219	90	2.6	937	0.17
34	43.24	4.68	28	308	112	2.8	1024	0.55
35	37.22	3.87	39	284	96	3.2	1043	0.31
36	39.62	3.08	80	274	100	3	1027	0.21
37	39.72	3.72	48	302	100	2.1	1095	0.22
38	36.27	3.14	311	297	98	2.3	292	0.235
39	38.37	3.86	247	299	102	2.6	106	0.22
40	39.33	3.07	135	289	92	1.9	371	0.285
41	36.33	3.15	348	300	92	2.09	155	0.325
42	41.32	3.51	368	284	90	3.4	161	0.305
43	36.72	3.86	463	289	93	3.9	248	0.16
44	36.32	3.83	453	298	102	2.3	346	0.24
45	38.37	3.53	375	300	112	3.4	112	0.185
46	41.68	3.44	384	301	101	2.8	300	0.105
47	33.33	3.72	460	290	97	3	295	0.125
48	43.61	3.72	383	284	102	3	166	0.43
49	47.83	3.77	474	304	114	2.8	306	0.385
50	43.38	3.41	371	283	102	2.3	153	0.275
51	30.16	4.75	343	277	93	1.9	163	0.175
52	47.38	3.94	312	298	113	2	158	0.18
53	38.71	3.94	337	284	108	2.4	150	0.215
54	41.62	3.44	366	293	112	1.9	144	0.14
55	43.72	3.86	364	304	118	2.6	168	0.235

y_{cal} = Calcium (mg), y_{fe} = Iron (mg), y_k = Potassium (mg), y_{phos} = Phosphorus (mg), y_{mag} = Magnesium (mg), y_{zn} = Zinc (mg), y_{sod} = Sodium (mg), y_{sel} = Selenium (mg)

Table 4. Experimental data generated based on the D-Optimal mixture design continue

Run	$y_{vit\ A}$	$y_{vit\ B1}$	$y_{vit\ B2}$	$y_{vit\ B3}$	$y_{vit\ B5}$	$y_{vit\ B6}$	$y_{vit\ B12}$	$y_{vit\ C}$
1	0.39	0.64	0.14	8.34	1.3	0.35	0.03	434.79
2	0.07	0.64	0.1	8.33	0.98	0.28	0.02	456.52
3	0.13	0.52	0.13	9.28	0.72	0.31	0.04	478.26
4	0.12	0.72	0.16	8.11	0.84	0.41	0.02	586.96
5	0.11	0.77	0.15	9.42	0.62	0.6	0.02	434.79
6	0.01	0.81	0.14	10.38	0.71	0.5	0.02	521.74
7	0.08	0.62	0.11	8.72	0.52	0.4	0.03	543.48
8	0.12	0.55	0.11	7.31	0.42	0.3	0.02	652.18
9	0.02	0.48	0.11	6.38	0.42	0.4	0.03	434.79
10	0.11	0.42	0.13	6.37	0.38	0.4	0.03	391.31
11	0.08	0.47	0.12	6.38	0.38	0.5	0.03	413.05
12	0.51	0.47	0.11	7	0.42	0.4	0.04	391.31
13	0.09	0.43	0.13	6.98	0.52	0.5	0.03	652.18
14	0.12	0.38	0.14	6.37	0.42	0.3	0.01	413.05
15	0.18	0.26	0.11	6.34	0.21	0.3	0.01	347.83
16	0.17	0.26	0.13	8.72	0.21	0.3	0.02	369.57
17	0.12	0.31	0.13	7.48	0.3	0.5	0.03	500
18	0.18	0.28	0.14	8.32	0.3	0.4	0.01	521.74
19	0.16	0.38	0.16	8.14	0.7	0.3	0.02	456.52
20	0.04	0.22	0.13	6.27	0.36	0.1	0.01	326.09
21	0.18	0.18	0.1	5.32	0.13	0.1	0.01	304.35
22	0.09	0.16	0.1	5.12	0.23	0.1	0.02	326.09
23	0.49	0.14	0.2	5.12	0.24	0.3	0.01	347.83
24	0.58	0.13	0.21	6.18	0.21	0.2	0.01	391.31
25	0.19	0.12	0.21	5.18	0.22	0.3	0	434.79
26	0.11	0.11	0.18	5.84	0.22	0.2	0	369.57
27	0.22	0.13	0.16	6.08	0.18	0.2	0	391.31
28	0.15	0.12	0.14	6.11	0.16	0.1	0.02	565.22
29	0.06	0.34	0.16	6.31	0.12	0.2	0.02	304.35
30	0.31	0.2	0.26	8.31	0.52	0.4	0.04	347.83
31	0.16	0.2	0.22	6.14	0.2	0.2	0.01	434.79
32	0.37	0.32	0.41	8.33	0.2	0.6	0.01	543.48
33	0.35	0.32	0.4	8.33	0.3	0.5	0.02	391.31
34	0.25	0.21	0.22	6.76	0.2	0.3	0.01	521.74
35	0.14	0.22	0.38	8.59	0.2	0.3	0.02	478.26
36	0.1	0.32	0.32	8.39	0.03	0.04	0.01	434.79
37	0.13	0.28	0.22	6.72	0.01	0.02	0.02	456.52
38	0.15	0.22	0.21	5.63	0.02	0.02	0.03	326.09
39	0.11	0.32	0.22	6.83	0.01	0.02	0.01	347.83
40	0.06	0.24	0.26	5.92	0.02	0.03	0.02	413.05
41	0.03	0.42	0.03	8.41	0.04	0.03	0.03	413.05
42	0.09	0.38	0.03	7.81	0.07	0.01	0.01	434.79
43	0.41	0.22	0.03	7.46	0.01	0.02	0.01	500
44	0.01	0.31	0.04	7.04	0.02	0.01	0.01	434.79
45	0.29	0.28	0.04	8	0.02	0.02	0.01	456.52
46	0.03	0.32	0.02	8.06	0.01	0.01	0.01	456.52
47	0.004	0.32	0.03	7.81	0.01	0.03	0.02	456.52
48	0.09	0.31	0.03	8.16	0.01	0.02	0.01	413.05
49	0.29	0.22	0.02	5.06	0.02	0.02	0.02	434.79
50	0.47	0.26	0.01	6.38	0.02	0.03	0.02	804.35
51	0.21	0.01	0.02	5.36	0.02	0.04	0.01	500
52	0.18	0.01	0.03	8.61	0.02	0.03	0.02	478.26
53	0.38	0.02	0.02	6.01	0.04	0.01	0.01	652.18
54	0.17	0.03	0.02	0.02	0.02	0.01	0.02	434.79
55	0.41	0.03	0.02	0.01	0.02	0.02	0.01	413.05

$Y_{vit\ A}$ = Vitamin A (mg), $Y_{vit\ B1}$ = Vitamin B_1 (mg), $Y_{vit\ B2}$ = Vitamin B_2 (Riboflavin) (mg), $Y_{vit\ B3}$ = Vitamin B_3 (Niacin) (mg), $Y_{vit\ B5}$ = Vitamin B_5 (Pantothenic acid) (mg), $Y_{vit\ B6}$ = Vitamin B_6 (mg), $Y_{vit\ B12}$ = Vitamin B_{12} (mg), $Y_{vit\ C}$ = Vitamin C (mg),

Table 5. Experimental data generated based on the D-Optimal mixture design continue

Run	Appearance	Texture	Taste	Flavour	Colour	Overall Acceptability
1	7.2	6	5.6	5.8	6.1	6.5
2	7.1	6.2	5.3	5.8	5.9	6.2
3	6.9	5.9	6	5.9	5.8	6.2
4	7.1	6.3	6.3	5.9	5.9	8.6
5	6.7	5.7	5.8	5.8	6.1	6.5
6	6.5	5.8	5.8	5.6	6	6.1
7	6.4	5.7	5.7	5.8	6.1	5.8
8	6.8	6.5	5.7	5.4	5.7	6.2
9	7	6	5.7	5.6	6	6.2
10	6.8	6.1	5.6	6	6.1	6.2
11	6.9	6.3	5.7	5.9	5.9	6.2
12	6.9	6	5.9	5.8	5.8	6
13	6.4	6.1	6.1	5.7	5.9	6.3
14	6.5	6.3	6.1	6	6.3	6.3
15	6.7	6.1	6.2	6.1	6.1	6.4
16	7	6.1	5.8	5.9	6.1	6.6
17	7.1	6.4	6	5.6	6	6.5
18	6.9	6.2	5.9	5.3	6	6.5
19	6.6	6.1	5.5	5.7	5.8	6.4
20	6.3	6	5.9	6	6.1	6.1
21	6.4	5.9	6	5.8	6.3	6.4
22	7.2	5.8	6.5	6	6	6.6
23	6.8	5.7	5.5	5.6	6.2	5.8
24	6.3	5.8	5.7	5.9	5.9	6
25	6.6	6.3	5.9	5.5	5.7	6.2
26	7.1	6.4	5.8	5.9	5.8	6.2
27	7	6.4	6.4	6.1	5.8	7
28	7.3	5.8	5.8	5.5	6.2	6.4
29	6.9	6	5.8	6.2	6	6.4
30	6.7	6.2	6	5.7	5.9	6.4
31	6.7	5.9	6.1	6.1	6	6.6
32	6.4	5.6	6	6.8	6.5	6.6
33	6.2	5.9	6.2	6	6.5	6.4
34	6.6	6	5.7	5.8	6.4	6.4
35	6.5	5.9	5.9	6.1	5.9	6.3
36	7	6.6	5.8	6.4	6.1	6.6
37	6.4	6.3	6.5	5.5	6.1	6.2
38	6.3	5.9	6.1	6.5	6.1	6.4
39	6.5	5.9	6.5	6.5	6.1	6.5
40	6.1	6.1	6	6.3	5.8	6.6
41	6.4	5.9	5.5	6.1	6.1	6.5
42	6.7	5.8	5.4	5.3	5.9	6.4
43	6.8	6.3	5.6	5.9	6	6.4
44	7	6.3	6	5.7	5.9	6.2
45	6.8	6.2	6.2	6.2	5.9	6.7
46	7	6	6.4	5.9	5.7	6.4
47	7	6	6.1	6.3	6.3	6.6
48	7.2	6.6	6.5	6.2	6.1	6.3
49	6.9	6.4	6	6.2	6.1	6.5
50	6.7	6.1	6	6.2	5.7	6.3
51	6.1	6.3	6	6.1	6.1	6.3
52	6.6	5.9	6.1	6.2	6.4	6.4
53	6.3	6.4	6.2	6.5	6.1	6.3
54	6.9	6.2	6.6	6.1	6.1	6.9
55	6.9	5.9	6.3	5.8	5.6	6.6

2.4. Development of predictive mixture/sheffe models for each response

The methodology requires predictive models for each response. The results of the designed experiments were subjected to statistical analyses and through response surface approach, using Design Expert Software (version 13), appropriate Scheffe empirical models that describe each response were fitted by least squares method. The fitted quality models in terms of l_pseudo components for each response in this case study, most of which proved to be statistically significant (P<0.05), were presented as Equations 5 – 35 (Table 6).

Table 6. The fitted quality models in terms of l_pseudo components for each response

Compounds	**Models**
Total Carbohydrate	$\left. y_{cho} = 79.76x_1 + 80.40x_2 + 67.97x_3 + 81.72x_4 + 57.01x_5 \right\} \quad (5)$
Dietary Fiber	$\left. \begin{aligned} y_{df} =\ & 1.07x_1x_6 + 0.9718x_1x_7 + 0.7519x_2x_6 + 0.6367x_2x_7 + 0.1101x_3x_6 + 0.5224x_3x_7 + 0.5277x_4x_6 \\ & - 0.5815x_4x_7 + 0.6433x_5x_6 - 0.5018x_5x_7 - 0.7944x_1x_3x_6 - 0.3730x_1x_3x_7 + 3.18x_1x_4x_7 - 2.07x_1x_6x_7 \\ & + 1.77x_2x_3x_6 + 1.75x_2x_3x_7 + 2.51x_2x_4x_7 + 0.6382x_2x_5x_6 + 1.86x_2x_5x_7 + 0.5277x_2x_6x_7 + 1.60x_3x_4x_6 \\ & + 3.02x_3x_4x_7 + 2.37x_3x_5x_7 - 3.54x_3x_6x_7 + 0.7338x_4x_5x_6 + 4.66x_4x_5x_7 + 1.93x_4x_6x_7 + 6.87x_5x_6x_7 \\ & + 15.74x_1x_3x_6x_7 - 12.36x_2x_5x_6x_7 - 17.40x_4x_5x_6x_7 \end{aligned} \right\} \quad (6)$
Total Fat	$\left. y_{fat} = 5.74x_1 + 6.95x_2 + 12.31x_3 + 6.05x_4 + 23.79x_5 \right\} \quad (7)$
Protein	$\left. y_{prot} = 7.69x_1 + 7.64x_2 + 12.84x_3 + 7.76x_4 + 13.10x_5 \right\} \quad (8)$
Ash Content	$\left. \begin{aligned} y_{ash} =\ & 0.1753x_1x_6 + 1.18x_1x_7 + 2.83x_2x_6 + 0.7724x_2x_7 - 1.87x_3x_6 + 5.16x_3x_7 + 1.54x_4x_6 + 1.65x_4x_7 + 7.25x_5x_6 \\ & + 0.8953x_5x_7 + 10.71x_1x_3x_6 - 7.51x_1x_3x_7 - 3.53x_1x_5x_6 - 4.05x_1x_5x_7 + 1.12x_1x_6x_7 + 1.51x_2x_3x_6 - 4.59x_2x_3x_7 \\ & - 3.45x_2x_4x_6 - 13.65x_2x_5x_6 + 1.97x_2x_5x_7 - 1.96x_2x_6x_7 + 12.41x_3x_4x_6 - 9.00x_3x_4x_7 - 7.36x_3x_5x_6 - 7.03x_3x_5x_7 \\ & - 7.38x_3x_6x_7 - 13.25x_4x_5x_6 + 10.35x_5x_6x_7 - 40.93x_1x_5x_6x_7 + 25.23x_2x_3x_6x_7 - 18.00x_2x_5x_6x_7 \end{aligned} \right\} \quad (9)$
Energy Value	$\left. \begin{aligned} y_{energy} =\ & 389.09x_1x_6 + 417.51x_1x_7 + 415.43x_2x_6 + 416.82x_2x_7 - 411.11x_3x_6 \\ & + 451.24x_3x_7 + 405.57x_4x_6 + 415.53x_4x_7 + 497.34x_5x_6 + 491.56x_5x_7 \end{aligned} \right\} \quad (10)$
Swelling Capacity	$\left. \begin{aligned} y_{sc} =\ & 1.57x_1x_6 + 0.7952x_1x_7 + 1.07x_2x_6 + 0.7790x_2x_7 - 0.3312x_3x_6 \\ & + 0.6514x_3x_7 + 1.38x_4x_6 + 1.06x_4x_7 + 0.1782x_5x_6 + 0.6686x_5x_7 \end{aligned} \right\} \quad (11)$
Water Absorption Capacity	$\left. \begin{aligned} y_{wac} =\ & -28.55x_1 + 141.96x_2 + 1275.01x_3 + 445.85x_4 + 147.59x_5 - 241.47x_1x_2 - 2387.30x_1x_3 - 882.18x_1x_4 \\ & - 225.41x_1x_5 - 2715.37x_2x_3 - 1189.73x_2x_4 - 535.77x_2x_5 - 3372.01x_3x_4 - 2809.94x_3x_5 - 1187.77x_4x_5 \\ & + 3190.16x_1x_2x_3 + 1854.25x_1x_2x_4 + 530.08x_1x_2x_5 + 4272.49x_1x_3x_4 + 4046.72x_1x_3x_5 + 1085.53x_1x_4x_5 \\ & + 4507.73x_2x_3x_4 + 3251.06x_2x_3x_5 + 1800.81x_2x_3x_5 + 4801.43x_3x_4x_5 + 453.13x_1x_2\left(x_1 - x_2\right) \\ & + 1574.31x_1x_3\left(x_1 - x_3\right) + 127.98x_1x_4\left(x_1 - x_4\right) + 314.16x_1x_5\left(x_1 - x_5\right) + 1077.83x_2x_3\left(x_2 - x_3\right) \\ & + 224.49x_2x_4\left(x_2 - x_4\right) - 337.54x_2x_5\left(x_2 - x_5\right) - 736.54x_3x_4\left(x_3 - x_4\right) - 394.00x_3x_5\left(x_3 - x_5\right) \\ & - 510.99x_4x_5\left(x_4 - x_5\right) \end{aligned} \right\} \quad (12)$
Bulk Density	$\left. \begin{aligned} y_{bd} =\ & 0.5931x_1x_6 + 0.6775x_1x_7 + 0.5587x_2x_6 + 0.7053x_2x_7 + 0.5822x_3x_6 \\ & + 0.8584x_3x_7 + 0.7077x_4x_6 + 0.6168x_4x_7 + 0.8823x_5x_6 + 0.6892x_5x_7 \end{aligned} \right\} \quad (13)$
Calcium	$\left. \begin{aligned} y_{ca} =\ & 45.29x_1x_6 + 45.98x_1x_7 + 50.33x_2x_6 + 45.38x_2x_7 + 45.17x_3x_6 + 41.01x_3x_7 + 36.19x_4x_6 \\ & + 43.13x_4x_7 + 43.68x_5x_6 + 37.78x_5x_7 - 45.43x_1x_6x_7 - 24.73x_2x_6x_7 - 4.33x_3x_6x_7 \\ & + 28.34x_4x_6x_7 + 17.40x_5x_6x_7 \end{aligned} \right\} \quad (14)$
Phosphorus	$\left. \begin{aligned} y_{phos} =\ & 292.22x_1 + 298.76x_2 + 117.42x_3 + 301.02x_4 + 201.35x_5 + 287.82x_1x_3 + 165.72x_1x_5 \\ & + 307.69x_2x_3 + 173.97x_2x_5 + 303.95x_3x_4 + 515.27x_3x_5 - 2494.50x_2x_3x_5 \end{aligned} \right\} \quad (17)$
Potassium	$\left. y_{pot} = 303.25x_1 + 22.84x_2 + 447.22x_3 + 251.57x_4 + 211.42x_5 \right\} \quad (16)$
Iron	$\left. \begin{aligned} y_{fe} =\ & 117.65x_1 - 498.49x_2 - 7265.52x_3 - 2406.79x_4 + 826.91x_5 + 859.50x_1x_2 - 13720.20x_1x_3 \\ & + 4981.56x_1x_4 - 1793.54x_1x_5 + 14979.91x_2x_3 + 5939.19x_2x_4 - 699.10x_2x_5 + 19082.69x_3x_4 \\ & + 13104.29x_3x_5 + 3430.12x_4x_5 - 17629.97x_1x_2x_3 - 4217.67x_1x_2x_4 + 1620.33x_1x_2x_5 \\ & - 24524.48x_1x_3x_4 - 20506.27x_1x_3x_5 - 2128.18x_1x_4x_5 - 25409.54x_2x_3x_4 - 14538.43x_2x_3x_5 \\ & - 5827.10x_2x_4x_5 - 26015.32x_3x_4x_5 - 1631.69x_1x_2\left(x_1 - x_2\right) - 8426.61x_1x_3\left(x_1 - x_3\right) \\ & + 2186.66x_1x_4\left(x_1 - x_4\right) + 524.56x_1x_5\left(x_1 - x_5\right) - 7289.94x_2x_3\left(x_2 - x_3\right) - 2047.68x_2x_4\left(x_2 - x_4\right) \\ & + 3009.93x_2x_5\left(x_2 - x_5\right) + 4468.72x_3x_4\left(x_3 - x_4\right) + 3256.31x_3x_5\left(x_3 - x_5\right) + 4303.30x_4x_5\left(x_4 - x_5\right) \end{aligned} \right\} \quad (15)$

Magnesium

$$\left.\begin{aligned} y_{mag} &= 105.03x_1x_6 + 99.75x_1x_7 + 104.80x_2x_6 + 104.63x_2x_7 + 109.92x_3x_6 \\ &+ 96.50x_3x_7 + 99.54x_4x_6 + 104.64x_4x_7 + 104.47x_5x_6 + 105.75x_5x_7 \end{aligned}\right\} \tag{18}$$

Zinc

$$\left.\begin{aligned} y_{zinc} &= 3.56x_1x_6 + 3.14x_1x_7 + 3.85x_2x_6 + 2.54x_2x_7 + 2.00x_3x_6 + 3.52x_3x_7 \\ &+ 1.94x_4x_6 + 2.34x_4x_7 + 2.47x_5x_6 + 3.24x_5x_7 - 4.44x_1x_6x_7 - 5.03x_2x_6x_7 \\ &+ 1.95x_3x_6x_7 + 4.70x_4x_6x_7 + 2.03x_5x_6x_7 \end{aligned}\right\} \tag{19}$$

Sodium

$$\left.\begin{aligned} y_{sod} &= 780.25x_1 + 1389.25x_2 + 498.27x_3 + 4313.54x_4 + 5933.95x_5 - 1619.76x_1x_2 - 166.79x_1x_3 - 8112.25x_1x_4 \\ &- 10810.91x_1x_5 - 372.87x_2x_3 - 8711.81x_2x_4 - 9625.31x_2x_5 - 10487.83x_3x_4 - 12924.31x_3x_5 - 16971.43x_4x_5 \\ &- 10893.05x_1x_2x_3 - 1.16E + 05x_1x_2x_4 - 766.97x_1x_2x_5 + 68530.82x_1x_3x_4 + 19540.88x_1x_3x_5 - 22842.95x_1x_4x_5 \\ &+ 15054.45x_2x_3x_4 + 28488.61x_2x_3x_5 + 20502.80x_2x_4x_5 + 77883.09x_3x_4x_5 \end{aligned}\right\} \tag{20}$$

Selenium

$$\left.\begin{aligned} y_{selen} &= 0.0670x_1x_6 + 0.2812x_1x_7 + 0.3634x_2x_6 + 0.3196x_2x_7 + 0.4496x_3x_6 + 0.0098x_3x_7 + 0.2884x_4x_6 \\ &+ 0.2375x_4x_7 + 0.1309x_5x_6 + 0.4728x_5x_7 + 0.2213x_1x_6x_7 - 0.8578x_2x_6x_7 + 0.5179x_3x_6x_7 \\ &+ 0.5469x_4x_6x_7 - 0.3529x_5x_6x_7 \end{aligned}\right\} \tag{21}$$

Vitamin B2 (Riboflavin)

$$\left.\begin{aligned} y_{vit\ \mathrm{B2}} &= 0.2660x_1 + 0.4423x_2 + 1.40x_3 + 1.10x_4 + 1.10x_5 - 0.9919x_1x_2 - 2.70x_1x_3 \\ &- 2.34x_1x_4 - 2.16x_1x_5 - 3.04x_2x_3 - 2.28x_2x_4 - 2.33x_2x_5 - 4.87x_3x_4 - 4.94x_3x_5 \\ &- 3.11x_4x_5 + 0.0341x_1x_2x_3 - 15.55x_1x_2x_4 + 4.37x_1x_2x_5 + 20.77x_1x_3x_4 + 6.07x_1x_3x_5 \\ &- 8.83x_1x_4x_5 + 5.81x_2x_3x_4 + 14.14x_2x_3x_5 + 0.3313x_2x_4x_5 - 10.91x_3x_4x_5 \end{aligned}\right\} \tag{24}$$

Vitamin B5 (Pantothenic acid)

$$\left. y_{vit\ \mathrm{B5}} = 0.3367x_6 + 0.3030x_7 + 0.6547x_6x_7 - 0.1477x_6x_7\left(x_6 - x_7\right) \right\} \tag{26}$$

Vitamin B6

$$\left.\begin{aligned} y_{vit\ B6} &= 0.1554x_1x_6 + 0.1056x_1x_7 + 0.2940x_2x_6 + 0.2103x_2x_7 + 0.1862x_3x_6 \\ &+ 0.0583x_3x_7 + 0.1427x_4x_6 + 0.4148x_4x_7 + 0.4564x_5x_6 + 0.1655x_5x_7 \end{aligned}\right\} \tag{27}$$

Vitamin A

$$\left.\begin{aligned} y_{vit\ A} &= 0.2420x_1x_6 + 0.7569x_1x_7 + 0.2895x_2x_6 + 0.3197x_2x_7 + 2.06x_3x_6 - 1.67x_3x_7 + 0.3212x_4x_6 \\ &+ 0.7819x_4x_7 + 0.5883x_5x_6 + 0.0946x_5x_7 + 0.5413x_1x_2x_6 - 1.26x_1x_2x_7 - 2.23x_1x_3x_6 - 1.41x_1x_3x_7 \\ &- 0.7545x_1x_4x_6 - 2.70x_1x_4x_7 - 1.14x_1x_5x_6 - 1.36x_1x_5x_7 - 2.48x_1x_6x_7 - 2.24x_2x_3x_6 + 2.25x_2x_3x_7 \\ &- 0.9864x_2x_4x_6 - 2.08x_2x_4x_7 - 1.15x_2x_5x_6 + 0.0442x_2x_5x_7 - 2.56x_2x_6x_7 - 3.97x_3x_4x_6 + 2.09x_3x_4x_7 \\ &- 4.14x_3x_5x_6 + 4.94x_3x_5x_7 - 11.39x_3x_6x_7 - 0.5820x_4x_5x_6 - 0.9728x_4x_5x_7 + 0.0824x_4x_6x_7 - 12.01x_5x_6x_7 \\ &+ 5.87x_1x_2x_6x_7 - 24.10x_1x_3x_6x_7 + 4.70x_1x_4x_6x_7 + 28.34x_1x_5x_6x_7 + 21.70x_2x_3x_6x_7 + 5.67x_2x_4x_6x_7 \\ &+ 26.63x_2x_5x_6x_7 + 21.40x_3x_4x_6x_7 + 37.87x_3x_5x_6x_7 + 17.94x_4x_5x_6x_7 \end{aligned}\right\} \tag{22}$$

Vitamin B3 (Niacin)

$$\left.\begin{aligned} y_{vit\ B3} &= 7.43x_1x_6 + 6.29x_1x_7 + 8.07x_2x_6 + 6.91x_2x_7 + 3.77x_3x_6 \\ &+ 7.56x_3x_7 + 5.70x_4x_6 + 7.40x_4x_7 + 7.25x_5x_6 + 8.06x_5x_7 \end{aligned}\right\} \tag{25}$$

Vitamin B1

$$\left.\begin{aligned} y_{vit\ \mathrm{B1}} &= 0.0173x_1x_6 - 0.0688x_1x_7 + 0.9587x_2x_6 - 0.0111x_2x_7 - 0.5188x_3x_6 + 3.47x_3x_7 + 1.06x_4x_6 \\ &- 0.6323x_4x_7 - 1.58x_5x_6 + 0.1107x_5x_7 + 0.8167x_1x_2x_6 + 1.87x_1x_2x_7 + 1.80x_1x_3x_6 - 5.10x_1x_3x_7 \\ &- 1.24x_1x_4x_6 + 3.84x_1x_4x_7 + 4.77x_1x_5x_6 + 1.06x_1x_5x_7 + 1.83x_1x_6x_7 + 0.5577x_2x_3x_6 - 2.81x_2x_3x_7 \\ &- 2.69x_2x_4x_6 + 2.24x_2x_4x_7 + 1.83x_2x_5x_6 + 0.8233x_2x_5x_7 - 1.45x_2x_6x_7 + 0.2884x_3x_4x_6 - 4.05x_3x_4x_7 \\ &+ 4.36x_3x_5x_6 - 5.69x_3x_5x_7 + 4.59x_3x_6x_7 + 1.12x_4x_5x_6 + 2.37x_4x_5x_7 - 0.1339x_4x_6x_7 + 14.58x_5x_6x_7 \\ &- 7.25x_1x_2x_6x_7 - 15.66x_1x_3x_6x_7 + 2.49x_1x_4x_6x_7 - 34.66x_1x_5x_6x_7 - 9.51x_2x_3x_6x_7 + 3.50x_2x_4x_6x_7 \\ &- 22.37x_2x_5x_6x_7 - 8.28x_3x_4x_6x_7 - 25.77x_3x_5x_6x_7 - 24.02x_4x_5x_6x_7 \end{aligned}\right\} \tag{23}$$

Vitamin B12

$$\left.\begin{aligned} y_{vit\ \mathrm{B12}} &= 0.0016x_1x_7 + 0.0201x_2x_6 + 0.0118x_2x_7 + 0.1045x_3x_6 + 0.2576x_3x_7 - 0.0426x_4x_6 - 0.0205x_4x_7 \\ &- 0.0087x_5x_6 - 0.0111x_5x_7 + 0.0802x_1x_2x_6 + 0.0623x_1x_2x_7 - 0.1100x_1x_3x_6 - 0.4182x_1x_3x_7 + 0.1300x_1x_4x_6 \\ &+ 0.1207x_1x_4x_7 + 0.2125x_1x_5x_6 + 0.0585x_1x_5x_7 + 0.0845x_1x_6x_7 - 0.0722x_2x_3x_6 - 0.4014x_2x_3x_7 + 0.1335x_2x_4x_6 \\ &+ 0.0797x_2x_4x_7 + 0.0121x_2x_5x_6 + 0.0216x_2x_5x_7 + 0.0684x_2x_6x_7 + 0.0251x_3x_4x_6 - 0.3165x_3x_4x_7 - 0.1361x_3x_5x_6 \\ &- 0.4099x_3x_5x_7 - 0.5664x_3x_6x_7 + 0.1538x_4x_5x_6 + 0.2508x_4x_5x_7 + 0.2135x_4x_6x_7 - 0.0157x_5x_6x_7 - 0.4355x_1x_2x_6x_7 \\ &+ 0.5619x_1x_3x_6x_7 - 0.5366x_1x_4x_6x_7 - 0.4556x_1x_5x_6x_7 + 0.6165x_2x_3x_6x_7 - 0.6957x_2x_4x_6x_7 - 0.0733x_2x_5x_6x_7 \\ &+ 0.3892x_3x_4x_6x_7 + 1.44x_3x_5x_6x_7 - 0.7389x_4x_5x_6x_7 \end{aligned}\right\} \tag{28}$$

Vitamin C

$$\left.\begin{aligned} y_{vit\ \mathrm{C}} &= 446.59x_1x_6 + 345.12x_1x_7 + 935.28x_2x_6 + 215.50x_2x_7 + 1729.95x_3x_6 + 691.02x_3x_7 - 483.17x_4x_6 \\ &+ 763.71x_4x_7 - 110.17x_5x_6 + 603.76x_5x_7 - 1055.74x_1x_2x_6 + 940.11x_1x_2x_7 - 1215.67x_1x_3x_6 \\ &- 890.28x_1x_3x_7 + 1891.00x_1x_4x_6 - 100.43x_1x_4x_7 + 832.13x_1x_5x_6 - 157.95x_1x_5x_7 - 3736.55x_2x_3x_6 \\ &+ 161.05x_2x_3x_7 + 449.18x_2x_4x_6 - 43.08x_2x_4x_7 + 130.36x_2x_5x_6 + 690.35x_2x_5x_7 - 559.53x_3x_4x_6 \\ &- 1483.19x_3x_4x_7 - 1137.23x_3x_5x_6 - 392.77x_3x_5x_7 + 3160.71x_4x_5x_6 - 1302.33x_4x_5x_7 \end{aligned}\right\} \tag{29}$$

Appearance

$$\left.\begin{aligned} y_{appear} &= 6.87x_1x_6 + 6.54x_1x_7 + 6.80x_2x_6 + 6.36x_2x_7 + 7.16x_3x_6 \\ &+ 6.48x_3x_7 + 6.66x_4x_6 + 6.95x_4x_7 + 6.59x_5x_6 + 6.97x_5x_7 \end{aligned}\right\} \quad (30)$$

Texture

$$\left.\begin{aligned} y_{tex} &= 6.18x_1x_6 + 5.46x_1x_7 + 6.94x_2x_6 + 5.60x_2x_7 + 2.20x_3x_6 + 5.67x_3x_7 + 6.89x_4x_6 + 8.18x_4x_7 + 8.42x_5x_6 \\ &+ 6.46x_5x_7 + 1.95x_1x_2x_6 + 2.78x_1x_2x_7 + 6.66x_1x_3x_6 + 1.94x_1x_3x_7 - 2.41x_1x_4x_6 - 2.18x_1x_4x_7 - 4.53x_1x_5x_6 \\ &+ 2.96x_1x_5x_7 + 2.17x_1x_6x_7 - 4.44x_2x_3x_6 - 0.8033x_2x_3x_7 - 3.75x_2x_4x_6 - 1.54x_2x_4x_7 - 5.76x_2x_5x_6 \\ &+ 0.6295x_2x_5x_7 - 0.0924x_2x_6x_7 + 5.52x_3x_4x_6 - 4.24x_3x_4x_7 + 1.87x_3x_5x_6 - 0.0142x_3x_5x_7 + 17.82x_3x_6x_7 \\ &- 6.58x_4x_5x_6 - 4.78x_4x_5x_7 + 3.14x_4x_6x_7 - 4.71x_5x_6x_7 - 5.09x_1x_2x_6x_7 - 30.65x_1x_3x_6x_7 - 18.01x_1x_4x_6x_7 \\ &- 3.21x_1x_5x_6x_7 - 30.37x_2x_3x_6x_7 - 12.49x_2x_4x_6x_7 + 11.48x_2x_5x_6x_7 - 32.45x_3x_4x_6x_7 - 28.43x_3x_5x_6x_7 \\ &+ 11.95x_4x_5x_6x_7 \end{aligned}\right\} \quad (31)$$

Colour

$$\left.\begin{aligned} y_{colour} &= 5.20x_1 + 6.86x_2 - 21.10x_3 + 20.88x_4 + 7.99x_5 + 0.2985x_1x_2 + 52.00x_1x_3 - 29.07x_1x_4 - 0.9171x_1x_5 \\ &+ 50.35x_2x_3 - 31.91x_2x_4 - 5.37x_2x_5 + 22.74x_3x_4 + 48.39x_3x_5 - 32.19x_4x_5 - 60.77x_1x_2x_3 + 98.58x_1x_2x_4 \\ &- 1.88x_1x_2x_5 - 8.42x_1x_3x_4 - 56.74x_1x_3x_5 + 35.29x_1x_4x_5 - 28.08x_2x_3x_4 - 55.82x_2x_3x_5 + 41.98x_2x_4x_5 \\ &- 34.78x_3x_4x_5 + 5.67x_1x_2\left(x_1 - x_2\right) - 20.00x_1x_3\left(x_1 - x_3\right) + 24.89x_1x_4\left(x_1 - x_4\right) - 0.0103x_1x_5\left(x_1 - x_5\right) \\ &- 36.23x_2x_3\left(x_2 - x_3\right) + 17.63x_2x_4\left(x_2 - x_4\right) + 3.88x_2x_5\left(x_2 - x_5\right) + 57.29x_3x_4\left(x_3 - x_4\right) + 53.33x_3x_5\left(x_3 - x_5\right) \\ &- 22.48x_4x_5\left(x_4 - x_5\right) \end{aligned}\right\} \quad (34)$$

Taste

$$\left.\begin{aligned} y_{taste} &= 5.76x_1 + 6.24x_2 + 0.9719x_3 + 6.76x_4 + 6.86x_5 - 0.4953x_1x_2 + 9.15x_1x_3 - 1.71x_1x_4 \\ &- 0.0854x_1x_5 + 8.06x_2x_3 - 1.50x_2x_4 - 2.47x_2x_5 + 7.40x_3x_4 + 10.68x_3x_5 - 3.39x_4x_5 \\ &+ 7.94x_1x_2x_3 - 57.54x_1x_2x_4 - 0.6347x_1x_2x_5 - 9.23x_1x_3x_4 - 34.70x_1x_3x_5 + 43.60x_1x_4x_5 \\ &- 15.73x_2x_3x_4 + 0.9947x_2x_3x_5 + 0.1117x_2x_4x_5 - 31.80x_3x_4x_5 \end{aligned}\right\} \quad (32)$$

Flavour

$$\left.\begin{aligned} y_{flavor} &= 5.62x_1 + 4.79x_2 + 1.17x_3 + 3.36x_4 + 2.59x_5 + 3.01x_1x_2 + 9.59x_1x_3 + 5.48x_1x_4 \\ &+ 6.60x_1x_5 + 10.38x_2x_3 + 8.71x_2x_4 + 7.70x_2x_5 + 16.29x_3x_4 + 18.17x_3x_5 + 12.77x_4x_5 \\ &- 11.02x_1x_2x_3 + 51.22x_1x_2x_4 + 7.09x_1x_2x_5 - 42.47x_1x_3x_4 - 27.19x_1x_3x_5 + 26.02x_1x_4x_5 \\ &- 44.08x_2x_3x_4 - 18.51x_2x_3x_5 - 27.99x_2x_4x_5 - 63.17x_3x_4x_5 \end{aligned}\right\} \quad (33)$$

Overall Acceptability

$$\left.\begin{aligned} y_{overall} &= 24.48x_1 - 76.83x_2 - 1296.45x_3 - 483.82x_4 + 297.36x_5 + 145.18x_1x_2 + 2461.11x_1x_3 \\ &+ 1021.81x_1x_4 - 594.62x_1x_5 + 2672.00x_2x_3 + 1168.40x_2x_4 - 412.91x_2x_5 + 3546.90x_3x_4 \\ &+ 2103.17x_3x_5 + 458.07x_4x_5 - 3135.64x_1x_2x_3 - 737.39x_1x_2x_4 + 638.55x_1x_2x_5 \\ &- 4628.46x_1x_3x_4 - 3547.55x_1x_3x_5 - 73.01x_1x_4x_5 - 4737.64x_2x_3x_4 - 2222.98x_2x_3x_5 \\ &- 869.60x_2x_4x_5 - 4720.99x_3x_4x_5 - 266.78x_1x_2\left(x_1 - x_2\right) - 1485.70x_1x_3\left(x_1 - x_3\right) \\ &+ 405.41x_1x_4\left(x_1 - x_4\right) + 284.33x_1x_5\left(x_1 - x_5\right) - 1326.88x_2x_3\left(x_2 - x_3\right) - 472.17x_2x_4\left(x_2 - x_4\right) \\ &+ 687.41x_2x_5\left(x_2 - x_5\right) + 724.50x_3x_4\left(x_3 - x_4\right) + 543.33x_3x_5\left(x_3 - x_5\right) - 1021.69x_4x_5\left(x_4 - x_5\right) \end{aligned}\right\} \quad (35)$$

2.5. Multiple response optimization

Multiple response optimization

Since many ingredients and quality properties were involved in the dietary custard example, optimization goals and constraints, which defines the individual objective functions, were set about each of the responses (i.e. quality characteristics); within the experimental region. It is at this point that multi-disciplinary collaboration of food scientists, nutritionists, dietician, and food engineer, is needed. Setting of optimization goals typically requires the knowledge of the dietary needs of consumers, in terms of the quality properties (responses). Consumers can be categorized as infants, children, adults, pregnant women, lactating mothers, sport men/women, convalescent, etc. The dietary requirements of these categories of consumers are not the same. For instance, to optimize for infants and children, the goal may be to maximize protein, whereas maximization of protein may not be the right setting for pregnant women. In addition, the proportion of each of the ingredients that maximizes a response may not be the same for maximizing another response. Furthermore, the objective goal of one response may be diametrically in opposition to the objective goal of the others. In the dietary custard powder, for instance, the objective goal for moisture content, naturally, should be minimization; while for other responses like protein, vitamin B6, or vitamin C; depending on the targeted consumer, may be either maximization or targeting a particular value. In the dietary custard example, a generalized optimization was carried out using the optimization constraints presented in Table 6. Some of the ingredients and quality properties were allowed to range within their pre-established experimental constraints in the design matrix.

Numerical optimization via desirability function technique

Numerical optimization methods, exploiting desirability function technique was employed to mathematically combine the individual response desirability measures into one overall index and to find the formulation that produces maximum overall desirability.

3. Results and Discussion

Based on the optimization constraints (Table 7), nine desirability formulation solutions (component proportions) were found as summarized in Table 8. The quality properties of the formulated optimal dietary custard are presented in Tables 9 - 12. Solution 1 (with the comment "selected") is the one that best meet the specified criteria. The maximum value of combined overall desirability index was 0.506. The indicated ingredient proportions values are the factor settings that result in the highest desirability index and indicate the design space/response surface with the best acceptable outcome. The goal of numerical optimization is to establish the optimal condition. Based on the generalized optimization carried out, we have nine local optima, with solution 1 being the best.

So, from the optimization solutions obtained in this work (Tables 8-11), to achieve dietary custard of highest optimal quality; the following conditions must be strictly adhered to: Biofortified cassava starch (12.43%), Yellow corn starch (22.98%), Soybean extract (29.34%), Sweet potato extract (10%), Peanut extract (22.25%), Egg (1%), and Milk powder (2%). If all these conditions are adhered to, the following are expected to be the quality properties of the dietary custard: Total Carbohydrate (67.99%), Dietary fibre (0.85%), Total fat (14.28%), Protein (11.52%), Ash content (1.39%), Energy value (453.97Kcal), Swelling capacity (0.69% vol), Water absorption capacity (1.8%), Bulk density 0.763g/cm3, Calcium (41.25mg), Iron (5.27mg), Potassium (273.1mg), Phosphorus (234.03mg), Magnesium (101.36mg), Zinc (3.19mg), Sodium (733.32mg), Selenium (0.23mg), Vitamin A (0.28mg), Vitamin B1 (0.49mg), Vitamin B2 (0.32mg), Vitamin B3 (7.49mg), Vitamin B5 (0.30mg), Vitamin B6 (0.13mg), Vitamin B12 (0.013mg), and Vitamin C (543.39mg). The interpretation of the overall desirability index $D = 0.506$ means that all the goals that were set during the optimization process were 50.6% achieved.

The value of the desirability index gotten is really not a big issue to worry about. If the desirability index gotten is low (maybe < 0.4 or < 40%), it only suggests that the goals are too strict or mutually exclusive. The more the goals are relaxed, the closer the desirability index approaches 1. In multi-response, multi-criteria optimization study; lots of function mapping and enveloping are involved. Desirability index is simply a mathematical method of finding the optimum set of conditions, in terms of proportions (or percentages) of the ingredients and values of the process parameters; that will satisfy all the set targets and achieve an optimized product with the best quality characteristics.

Figures 2-6 shows the numerical optimization solution desirability bar graph, contour plot, mix-mix plot, 3D surface plot and 3D mix-mix plot for the optimal dietary custard.

Table 7. Optimization constraints for the dietary custard

Name	Goal	Lower Limit	Upper Limit	Lower Weight	Upper Weight	Importance
Bio-Cassava Starch	in range	10	60	1	1	3
Yellow Corn Starch	in range	10	50	1	1	2
Soybean Extract	maximize	20	30	1	1	5
Sweet Potato Extract	in range	10	40	1	1	3
Peanut Extract	maximize	10	30	1	1	2
Egg	in range	1	2	1	1	3
Milk Powder	maximize	1	2	1	1	5
Total Carbohydrate	in range	55.92	88.44	1	1	3
Dietary Fiber	maximize	0.33	1.06	1	1	5
Total Fat	minimize	2	18.22	1	1	3
Protein	target = 18	3.85	18.9	1	10	5
Ash Content	in range	0.92	3	1	1	3
Energy Value	target = 470	354.93	473.38	1	10	5
Swelling Capacity	maximize	0.09	2.81	1	1	3
WAC	in range	1.8	3.7	1	1	3
Bulk Density	in range	0.54	0.94	1	1	3
Calcium	maximize	30.16	49.68	1	1	3
Iron	maximize	1.06	5.53	1	3	3
Potassium	maximize	17	896	1	1	4
Phosphorus	in range	209	316	1	1	3
Magnesium	maximize	84	118	1	1	3
Zinc	maximize	1.9	4.6	1	5	3
Sodium	in range	106	1632	1	1	3
Selenium	in range	0.105	0.55	1	1	3
Vitamin A	maximize	0.004	0.58	1	5	5
Vitamin B1	maximize	0.01	0.81	1	5	5
Vitamin B2	maximize	0.01	0.41	1	1	5
Vitamin B3	maximize	0.01	10.38	1	1	5
Vitamin B5	maximize	0.01	1.3	1	1	5
Vitamin B6	maximize	0.01	0.6	1	5	5
Vitamin B12	maximize	0	0.04	1	5	5
Vitamin C	maximize	304.35	804.35	1	5	5
Appearance	in range	6.1	7.3	1	1	3
Texture	in range	5.6	6.6	1	1	3
Taste	In range	5.3	6.6	1	1	3
Flavour	in range	5.3	6.8	1	1	3
Colour	in range	5.6	6.5	1	1	3
Overall Acceptability	in range	5.8	8.6	1	1	3

Table 8. The optimal formulation conditions (component proportions) for the dietary custard

No	x_1	x_2	x_3	x_4	x_5	x_6	x_7	D	
1	**12.430**	**22.980**	**29.339**	**10.000**	**22.251**	**1.012**	**1.988**	**0.506**	**Selected**
2	11.094	13.969	29.646	13.563	28.729	1.199	1.801	0.498	
3	10.000	17.434	28.925	13.881	26.761	1.460	1.540	0.486	
4	10.203	24.747	30.000	10.018	22.032	1.566	1.434	0.474	
5	11.658	25.392	29.529	10.000	20.421	1.375	1.625	0.472	
6	21.881	26.135	26.259	11.482	11.243	1.776	1.224	0.383	
7	15.263	19.103	24.649	27.264	10.721	1.571	1.429	0.374	
8	16.173	18.887	22.721	27.415	11.804	1.808	1.192	0.371	
9	26.140	12.152	20.193	27.844	10.670	1.484	1.516	0.316	

x_1 = Biofortified cassava starch (%), x_2 = Yellow corn starch (%), x_3 = Soybean extract (%), x_4 = Sweet potato extract (%), x_5 = Peanut extract (%), x_6 = Egg (%), x_7 = Milk powder (%), D = Desirability

Table 9. The optimal quality properties of the powdered dietary custard

No	y_{cho}	y_{df}	y_{fat}	y_{prot}	y_{ash}	y_{ev}	y_{sc}	y_{wac}	y_{bd}	D	
1	**67.987**	**0.854**	**14.282**	**11.517**	**1.390**	**453.967**	**0.690**	**1.800**	**0.763**	**0.506**	**Selected**
2	65.360	0.866	16.196	12.171	1.252	459.520	0.638	3.610	0.750	0.498	
3	66.345	0.822	15.565	11.917	1.213	452.585	0.582	1.870	0.732	0.486	
4	67.959	0.706	14.326	11.555	0.954	444.998	0.573	1.905	0.710	0.474	
5	68.705	0.757	13.776	11.359	1.129	445.530	0.632	1.800	0.725	0.472	
6	73.104	0.761	10.518	10.194	1.091	421.719	0.793	1.813	0.645	0.383	
7	74.109	0.795	10.102	10.022	1.133	424.942	0.842	1.800	0.682	0.374	
8	74.079	0.778	10.219	9.951	1.345	421.480	0.869	1.800	0.667	0.371	
9	74.993	0.940	9.429	9.622	0.987	422.902	0.921	3.700	0.682	0.316	

Table 10. The optimal quality properties of the dietary custard continue

No	y_{ca}	y_{fe}	y_k	y_{phos}	y_{mag}	y_{zn}	y_{sod}	y_{sel}	D	
1	**41.249**	**5.268**	**273.080**	**234.032**	**101.356**	**3.191**	**733.324**	**0.232**	**0.506**	**Selected**
2	41.753	10.327	304.516	267.532	102.497	3.322	962.711	0.262	0.498	
3	43.288	13.668	288.533	250.239	103.862	3.154	1131.209	0.274	0.486	
4	43.151	11.907	266.394	222.078	104.533	2.901	833.073	0.262	0.474	
5	42.018	3.635	264.643	229.118	103.404	2.958	793.158	0.242	0.472	
6	42.846	7.943	266.170	278.341	105.269	2.721	373.116	0.280	0.383	
7	43.027	5.935	283.230	292.756	103.472	2.937	767.683	0.315	0.374	
8	43.052	5.530	277.540	294.283	104.182	2.737	760.357	0.317	0.371	
9	41.605	5.733	305.726	306.260	102.816	2.952	1296.486	0.311	0.316	

Table 11. The optimal quality properties of the dietary custard continue

No	$y_{vit\ A}$	$y_{vit\ B1}$	$y_{vit\ B2}$	$y_{vit\ B3}$	$y_{vit\ B5}$	$y_{vit\ B6}$	$y_{vit\ B12}$	$y_{vit\ C}$	D	
1	0.280	0.488	0.324	7.492	0.297	0.129	0.013	543.385	0.506	**Selected**
2	0.275	0.504	0.194	7.295	0.219	0.170	0.023	493.317	0.498	
3	0.276	0.482	0.264	6.799	0.159	0.216	0.025	485.293	0.486	
4	0.349	0.361	0.362	6.631	0.156	0.223	0.024	471.899	0.474	
5	0.383	0.324	0.345	6.920	0.171	0.189	0.020	494.388	0.472	
6	0.446	0.156	0.139	6.402	0.201	0.215	0.020	430.206	0.383	
7	0.456	0.109	0.169	6.420	0.157	0.211	0.014	424.502	0.374	
8	0.331	0.187	0.154	6.129	0.215	0.217	0.022	436.712	0.371	
9	0.534	0.060	0.270	6.568	0.157	0.197	0.010	439.940	0.316	

Table 12. The optimal quality properties of the dietary custard continue

No	Appearance	Texture	Taste	Flavour	Colour	Overall Acceptability	*D*	
1	6.606	6.096	6.187	6.074	6.366	6.377	0.506	**Selected**
2	6.737	5.835	6.039	5.822	6.264	8.600	0.498	
3	6.765	5.721	6.058	5.740	6.394	8.600	0.486	
4	6.765	5.804	6.272	6.085	6.427	8.562	0.474	
5	6.701	5.895	6.180	6.046	6.425	6.049	0.472	
6	6.832	5.804	5.928	5.929	6.500	5.800	0.383	
7	6.777	5.677	5.325	5.490	6.329	5.800	0.374	
8	6.816	5.722	5.373	5.589	6.281	5.879	0.371	
9	6.766	5.605	5.730	5.754	6.323	6.800	0.316	

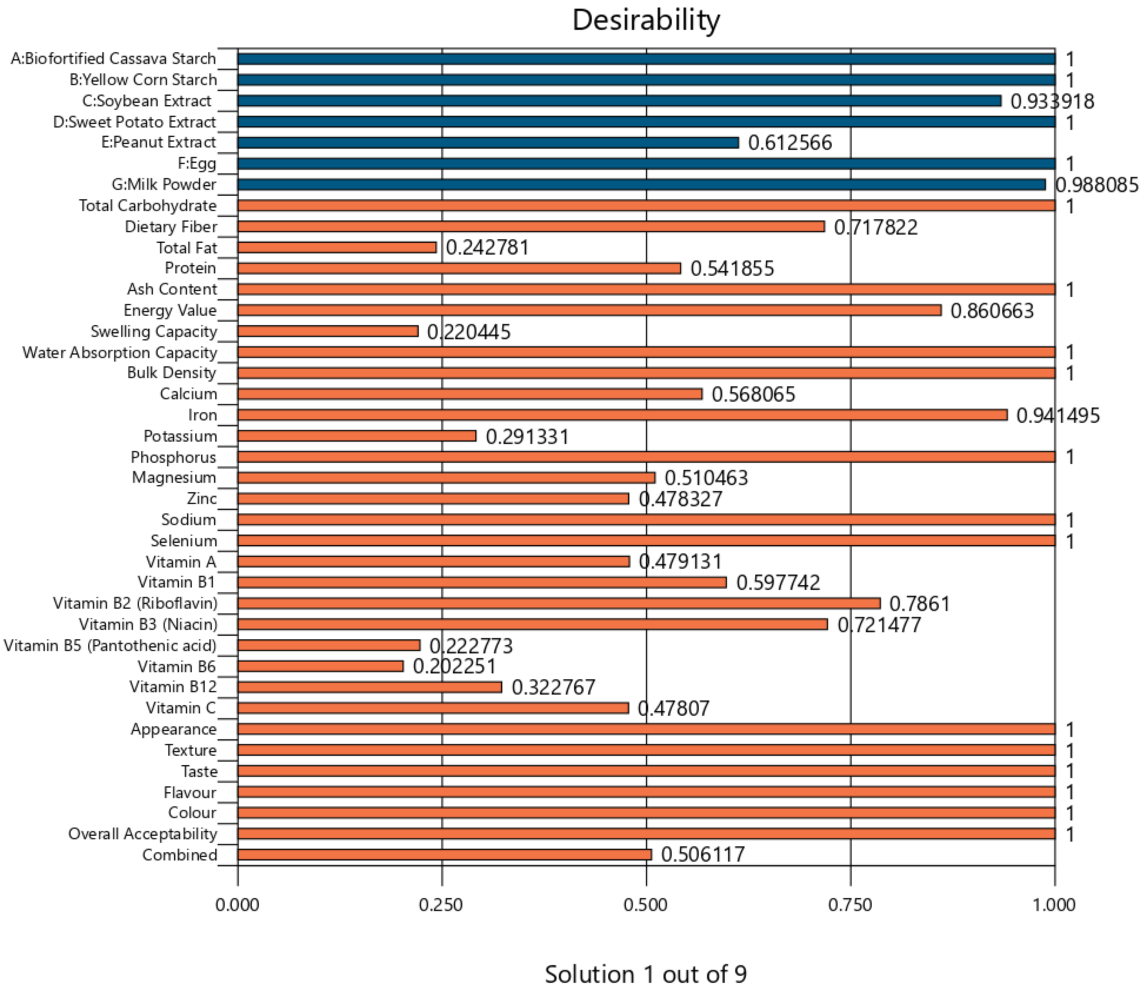

Figure 2. Numerical optimization solution desirability bar graph for the optimal dietary custard

The numerical optimization bar graph presents a graphical view of each optimal solution. Optimal factor settings are shown with red bars while optimal response prediction values are displayed in blue. Responses with models, but no goals are shown with gray. Presenting the numerical optimization bar graph is a good way to explore how the factor settings affect the response. The numerical optimization solution desirability mix-mix, 3D surface and 3D mix-mix plots show all responses (Figures 3-6). Presenting the numerical optimization graphs is an effective way to explore the effects of factor settings (i.e. ingredients proportions) on the responses. Note that Design Expert software flags the optimal point with identification of composition and other pertinent statistics.

The graphical optimization solution overlay contour and mix-mix plots for the optimal dietary custard are presented in Figures 7 and 8. The graphical optimization solution overlay contour and mix-mix plots give the summary or details of the optimal conditions and quality properties of the optimal dietary custard. The contours are plotted at the limits specified by the "set criteria". The bright yellow defines the acceptable factor settings while the grey defines the unacceptable factor settings. If intervals are included in the criteria, then a blend of the acceptable and unacceptable colors is used to show where the interval limits are unacceptable. The overlay plots provide a window into regions where experimenters can meet specifications for multiple responses. Thus, it facilitates the development of a "quality by design space,"

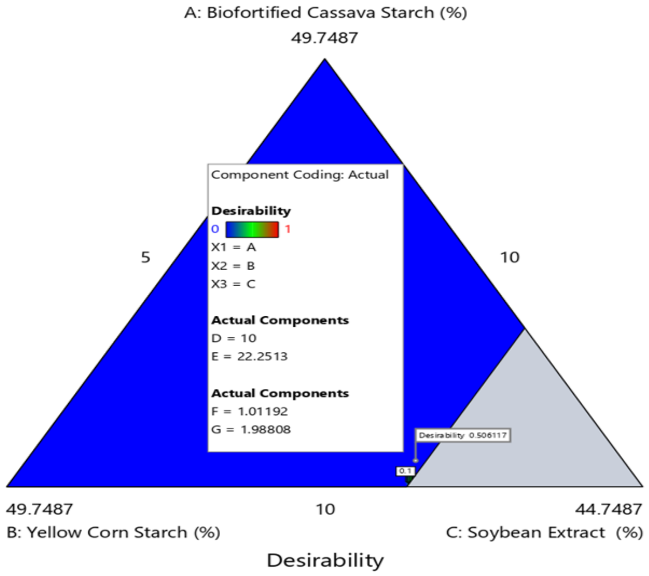

Figure 3. Numerical optimization solution desirability contour plot for the optimal dietary custard

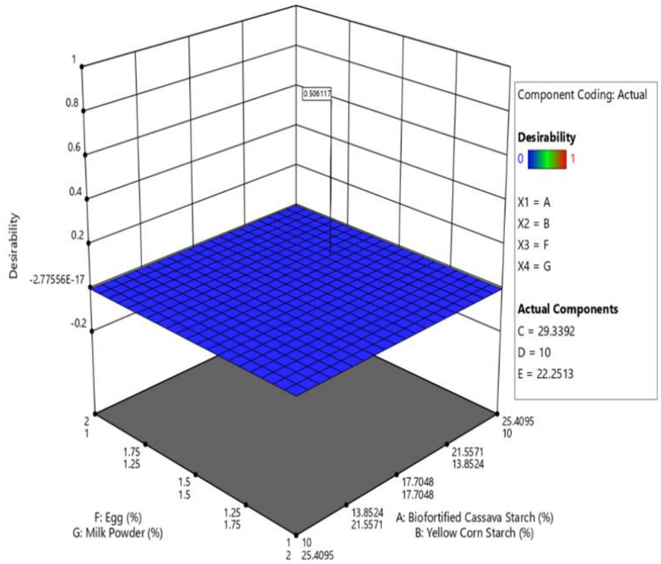

Figure 6. Numerical optimization solution desirability 3D mix-mix plot for the optimal dietary custard

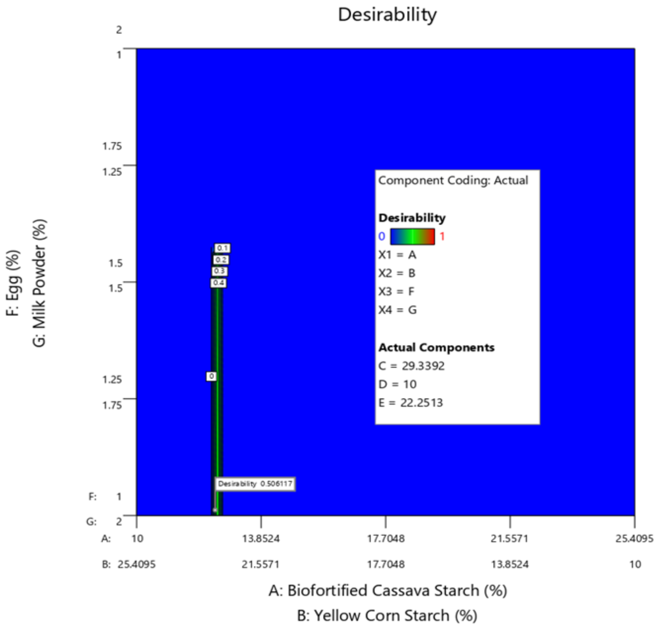

Figure 4. Numerical optimization solution desirability mix-mix plot for the optimal dietary custard

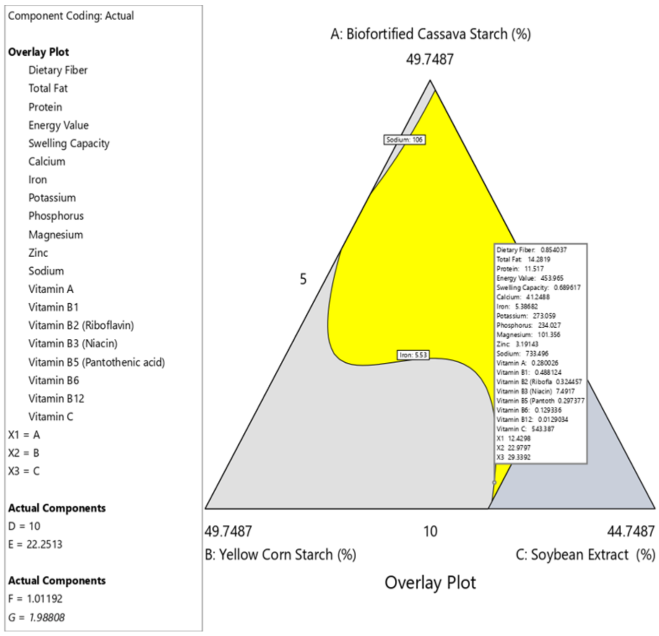

Figure 7. The graphical optimization solution overlay contour plot for the optimal dietary custard

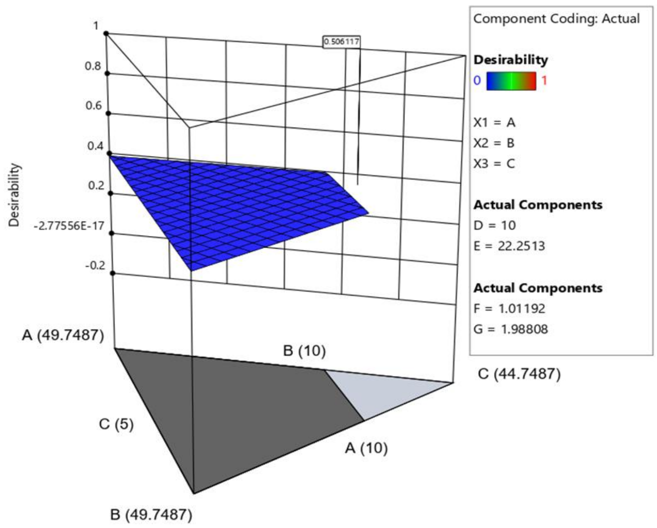

Figure 5. Numerical optimization solution desirability 3D surface plot for the optimal dietary custard

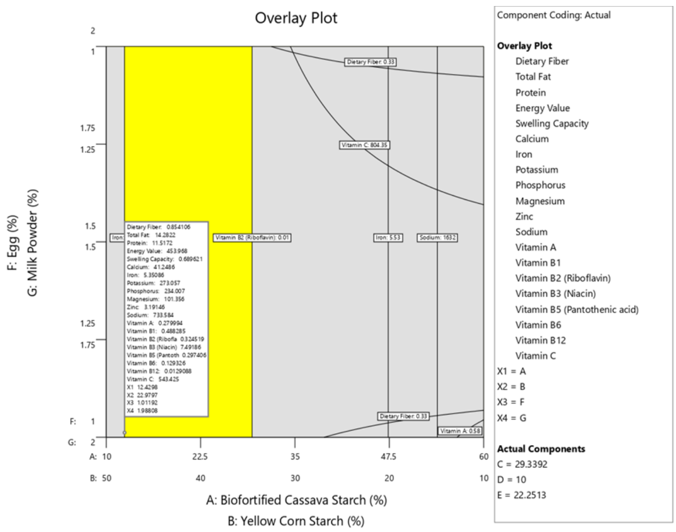

Figure 8. Graphical optimization solution overlay mix-mix plot for the optimal dietary custard

4. Conclusion

This study presents a novel procedure for optimizing products with multiple responses. By way of a case study, the article presents the development of dietary custard powder from blends of biofortified cassava starch, yellow corn starch, soybean extract, sweet potato extract, peanut extract, egg, and milk powder. A seven-component constrained; D-optimal mixture experimental design was employed. The formulation that produced powdered dietary custard of the highest desirability index of 0.506 was 12.43 % biofortified cassava starch, 22.98 % yellow corn starch, 29.34 % soybean extract, 10 % sweet potato extract, 22.25 % peanut extract. 1 % Egg, and 2 % milk powder. These are the formulation conditions that will satisfy the formulation of custard based on the optimization settings employed during the optimization process. The quality properties of this optimal powdered dietary custard were: Total Carbohydrate (67.99 %), Dietary Fiber (0.85 %), Total Fat (14.28 %), Protein (11.52 %), Ash Content (1.39 %), Energy Value (453.97 Kcal), Swelling Capacity (0.69 % vol), Water Absorption Capacity (1.8 %), Bulk Density 0.763g/cm3, Calcium (41.25 mg), Iron (5.27 mg), Potassium (273.1 mg), Phosphorus (234.03 mg), Magnesium (101.36 mg), Zinc (3.19 mg), Sodium (733.32 mg), Selenium (0.23 mg), Vitamin A (0.28 mg), Vitamin B1 (0.49 mg), Vitamin B2 (Riboflavin) (0.32 mg), Vitamin B3 (Niacin) (7.49 mg), Vitamin B5 (Pantothenic acid) (0.30 mg), Vitamin B6 (0.13 mg), Vitamin B12 (0.013 mg), and Vitamin C (543.39 mg). Blends of biofortified cassava starch, yellow corn starch, soybean extract, sweet potato extract, peanut extract. egg, and milk powder have been employed for the custard formulation; as against the conventional custard which are typically made from mono-cereals (e.g. corn, millet, sorghum, wheat, rice, or oats) and consist essentially of carbohydrate. There are lots of resources in Africa from which value-added custard can be formulated.

Ethical statement

The ethical committee of the Federal University of Technology, Minna gave the approval for the implementation of the research (Assign Number: 000061).

Conflict of interests

The author declares no competing interests

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Data availability

All experimental data generated during this study and the results of statistical data analysis are included in this manuscript.

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