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ORIGINAL ARTICLE

Mathematical model and analysis of the soil-transmitted helminth infections with optimal control

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Abstract

Soil-transmitted helminth diseases are highly prevalent in impoverished regions and pose a significant health burden on the global population. These diseases are primarily transmitted through the contamination of soil with human faces containing parasite eggs. This study presents a novel deterministic mathematical model to comprehensively investigate the dynamics of helminth infection transmission through the soil. The mathematical model exhibits two equilibrium points: the diseases-free equilibrium point (DFE) and the endemic equilibrium point (EEP). The DFE is proven to be locally and globally asymptotically stable when the basic reproduction number is less than one, indicating the potential for disease eradication. Conversely, the EEP is locally asymptotically stable when the basic reproduction number exceeds unity, representing a persistent endemic state. To explore effective intervention strategies for controlling the spread of these infections, optimal control theory is applied. The study incorporates two time-varying control variables derived from sensitivity analysis: the rate of hygiene consciousness in the susceptible class and the rate of hygiene consciousness in the infectious class. Numerical simulations demonstrate that implementing optimal control strategies can successfully curb and mitigate soil-transmitted helminth infections. Overall, this research highlights the importance of proactive and targeted interventions, emphasizing the significance of hygiene education and awareness campaigns. By implementing optimal control measures based on the proposed strategies, the burden of soil-transmitted helminth diseases can be significantly reduced, improving public health in affected regions.

Keywords Helminth infections · Reproduction number · Stability · Sensitivity analysis · Numerical simulations

Mathematics Subject Classification 92B05 · 92-10 · 37N25 · 49J15 · 49K15

Introduction

Soil-transmitted helminth (STH) infections are highly prevalent worldwide, particularly affecting impoverished and marginalized communities. Among the major types of STH infections, Ascaris lumbricoides is responsible for causing ascariasis, trichuriasis, hookworm, and strongyloidiasis. These infections impact over a billion individuals globally, leading to adverse health consequences such as anemia and significant social and economic burdens due to poor educational outcomes (Babatunde et al. 2013).

The global burden of STH infections is substantial, with more than a quarter of the world's population being infected by these parasitic diseases, resulting in significant impacts on human health and disability (De Silva et al. 2003). Ascaris, Trichuris, and hookworm share similar life cycles. The adult stage of these parasites resides in the gastrointestinal system, with Trichuris and Ascaris found in the colon

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and Ascaris and hookworm in the small intestine. These parasites reproduce sexually and produce eggs that are expelled through human feces and deposited in the environment (Pullan et al. 2014).

STH lay their eggs in the faeces of their (human) hosts, where they reside in the intestine. Adult worms lay hundreds of eggs daily in the colon where they reside. These eggs infect the soil in places with poor sanitation. As a result of STH, the presence of hookworm infections can affect the foetus and cause anaemia in the mother during pregnancy, which can result in low birth weight (Van den Driessche and Watmough 2002). Infections with STH are more common in the Americas, China, East Asia, and Sub-Saharan Africa. STH infections are highly common in the tropics and subtropics, where climatic factors play a significant role in the spread of these illnesses (Bethany et al. 2006).

It is also known that soil moisture and relative air humidity have an impact on the growth and survival of eggs and larvae. Soil larval development requires adequate moisture and a warm temperature (Brooker and Michael 2000; De Silva et al. 2003). Poverty, insufficient water supply, sanitation, and poor personal hygiene are also important determinants in the epidemiology of STH (De Silva et al. 2003).

The study of infectious diseases relies heavily on mathematical models. Mathematical modelling is a crucial tool in the epidemiology of infectious diseases. It has been used to investigate and analyze the dynamics of a disease's transmission, predict outbreaks, and assess management measures. This disease has received little attention in the past, which has resulted in a lack of understanding of the mechanisms of transmission. There are few models that have been developed and examined to comprehend the dynamics of helminth infections, nevertheless, to examine the dynamics of soil-transmitted helminth infection, the authors in (Lambura et al. 2020b). Developed and assessed a deterministic compartmental model. They took into account optimal control of the model to sensitize the susceptible population and treatment by mass drug administration and sanitation. The authors of (Truscott et al. 2016). analyzed attempts to model the population-level helminth transmission cycle and the impact of prophylactic chemotherapy on its management and eradication. To study how helminths affect TB and the effects of control measures, the authors of (Lambura et al. 2020a). proposed a compartmental system for the transmission and control of the co-interaction of helminths and tuberculosis. Little attention has been paid to this disease in the past, and this has led to little knowledge of the transmission dynamics. In view of this, we develop a new mathematical model of disease dynamics by extending the work of (Lambura et al. 2020a) in this study by incorporating a hygiene-conscious compartment. These papers provide insights into various aspects of mathematical modelling of soil-transmitted helminth infections, including transmission dynamics, control strategies, and the impact of interventions (Olsen et al. 2021; Pedersen et al. 2021; Truscott et al. 2020; Chaparro-Narváez et al. 2020; Anderson et al. 2019).

To the best of our knowledge, this is the first work on Soil-Transmitted Helminth with the application of optimal control with effects of the rate of hygiene consciousness in the human population. We conducted a qualitative analysis of the model and proposed two time-varying control strategies: the rate of hygiene consciousness in the susceptible class and the rate of hygiene consciousness in the infectious class further understand the impact of hygiene consciousness on the dynamics of the disease. The remainder of the paper is structured as follows: The model formulation and fundamental characteristics are discussed under the method section, the model analysis is presented in the model analysis section, the model's optimal control is discussed, and its analysis are presented under optimal control section, and the concluding remarks are presented in the last section.

Method

To gain a better understanding on the transmission dynamics of the disease, we divide the helminth population into two namely, the human population $N(t)$ and parasite population denoted by $M(t)$. The human population is further subdivided into 5 classes, these are, susceptible $S(t)$, Exposed $E(t)$, Infectious $I(t)$, Hygiene conscious $H(t)$ and Recovered $R(t)$. The total human population is expressed as,

$$N(t) = S(t) + E(t) + I(t) + H(t) + R(t)$$

The recruitment into the susceptible class $S(t)$ is at a rate Λ, the rate at which recovered individuals becomes susceptible after recovery is at a rate γ, the population is then reduced by the *force of infection* λ,

$$\lambda = \frac{\beta M}{K + M},$$

where β is the rate at which contaminated food or skin-penetrating larvae are consumed, causing an infection to spread, and K is the parasite density (carrying capacity). The rate of hygiene awareness and the natural death rate in the susceptible class progressively lower the susceptible population so that,

$$\frac{dS}{dt} = \Lambda - \lambda S - \left(\alpha_1 + \mu_h\right)S + \gamma R.$$

The exposed individuals are generated by the *force of infection* λ. The exposed population is reduced by the progression from exposed compartment to infectious compartment at the rate ρ, and human natural death rate μ_h so that;

$$\frac{dE}{dt} = \lambda S - \left(\rho + \mu_h\right)E.$$

The population of the infected class $I(t)$, is generated by the movement from the exposed class the to infectious class at a rate ρ. The population is further reduced by the rate of hygiene consciousness of infectious individuals α_2, the recovery rate of infectious class due to strong immune system is at a rate q, the recovery rate of infectious individuals due to treatment is at a rate τ, the rate of contamination of the environment is at a rate σ, δ and μ_h represent the disease-induced death rate and natural death rate of human respectfully so that;

$$\frac{dI}{dt} = \rho E - \left(\alpha_2 + q + \tau + \sigma + \delta + \mu_h\right)I.$$

The population of hygiene conscious class $H(t)$, is generated by the hygiene consciousness in the susceptible and infectious compartments at rate α_1 and α_2 respectively. The hygiene conscious population is decreased by recovery rate ε, and human natural death rate μ_h so that;

$$\frac{dH}{dt} = \alpha_1 S + \alpha_2 I - \left(\varepsilon + \mu_h\right)H.$$

The population of the recovered class $R(t)$, is generated by the recovery rate of infectious individuals due to treatment rate τ, recovery rate of infectious individuals due to strong immune system q, and recovery rate of individuals in the hygiene conscious compartment ε. The population is reduced by natural mortality rate μ_h, and the rate at which recovered individuals becomes susceptible again γ so that;

$$\frac{dR}{dt} = (q + \tau)I + \varepsilon H - \left(\gamma + \mu_h\right)R.$$

The parasite population $M(t)$, is generated by the rate of contamination of the environment σ, and it is reduced by clearance rate for parasite μ_m so that;

$$\frac{dM}{dt} = \sigma I - \mu_m M.$$

The illustration above can be represented in the differential Eq. (1), while the pictorial illustration is presented in Fig. 1.

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \lambda S - \left(\alpha_1 + \mu_h\right)S + \gamma R \\
\frac{dE}{dt} &= \lambda S - \left(\rho + \mu_h\right)E \\
\frac{dI}{dt} &= \rho E - \left(\alpha_2 + q + \tau + \sigma + \delta + \mu_h\right)I \\
\frac{dH}{dt} &= \alpha_1 S + \alpha_2 I - \left(\varepsilon + \mu_h\right)H \\
\frac{dR}{dt} &= (q + \tau)I + \varepsilon H - \left(\gamma + \mu_h\right)R \\
\frac{dM}{dt} &= \sigma I - \mu_m M
\end{aligned} \tag{1}$$

Basic properties of the model

Positivity and boundedness of solution

Here, we show that all the state variable of the model are positive for all time $t > 0$, for the model to be mathematically and biologically meaningful in Ω, given by

$$\Omega = \left\{ (S, E, I, H, R, M) \in R_+^6 : S + E + I + H + R \leq \frac{\Lambda}{\mu_h}, M \leq \frac{\sigma}{\mu_m} \right\} \tag{2}$$

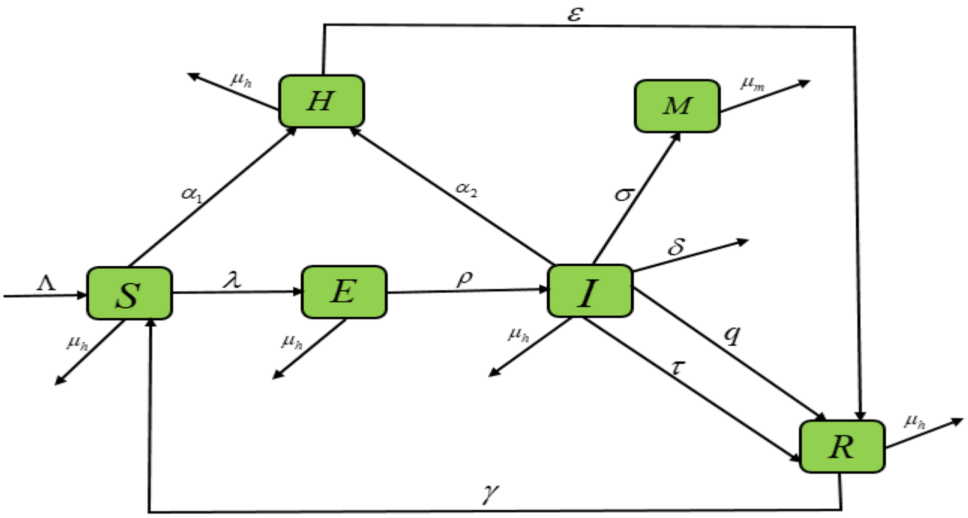


Fig. 1 Flow chart of the of the helminth model

Theorem 1 *Let the initial value for each of the model variable be*

$$\{S(0), E(0), I(0), H(0), R(0), M(0) \geq 0\},$$

then solution of each of the variable are non-negative with $t > 0$.

Proof Let $t_1 = \sup\{t > 0 : S > 0, E > 0, I > 0, H > 0, R > 0, M > 0 \in [0, t]\}$. Thus, $t_1 > 0$.

From the first equation of the model (1),

$$\frac{dS}{dt} = \Lambda - \lambda S - \left(\alpha_1 + \mu_h\right)S + \gamma R.$$

Solving the above equation,

$$\begin{aligned} &\frac{d}{dt}\left\{ S(t)\left[\exp\left(\int_0^t \lambda(\eta)d\eta + \left(\alpha_1 + \mu_h\right)t\right)\right]\right\} \\ &= (\Lambda + \gamma R)\exp\left(\int_0^t \lambda(\eta)d\eta + \left(\alpha_1 + \mu_h\right)t\right). \end{aligned} \tag{3}$$

Integrating the above equation at the range $\left[0, t_1\right]$, we obtainedso that

$$\begin{aligned} &\left\{ S\left(t_1\right)\exp\left[\int_0^{t_1} \lambda(\eta)d\eta + \left(\alpha_1 + \mu_h\right)t_1\right]\right\} - S(0) \\ &= (\Lambda + \gamma R)\int_0^{t_1} \exp\left[\int_0^y \lambda(\eta)d\eta + \left(\alpha_1 + \mu_h\right)y\right]dy, \end{aligned}$$

$$\begin{aligned} S\left(t_1\right) =& S(0)\exp\left[-\left(\int_0^{t_1} \lambda(\eta)d\eta + \left(\alpha_1 + \mu_h\right)t_1\right)\right] \\ &+ \exp\left[-\left(\int_0^{t_1} \lambda(\eta)d\eta + \left(\alpha_1 + \mu_h\right)t_1\right)\right] \\ &\times (\Lambda + \gamma R)\int_0^{t_1} \exp\left[\int_0^y \lambda(\eta)d\eta + \left(\alpha_1 + \mu_h\right)y\right]dy > 0. \end{aligned} \tag{4}$$

In the same vein, we can show that, $E > 0, I > 0, H > 0, R > 0, M > 0$.

Invariant region

Lemma 1 *The region* $\Omega = \left\{(S, E, I, H, R, M) \in R_+^6 : S + E + I + H + R \leq \frac{\Lambda}{\mu_h}, M \leq \frac{\sigma}{\mu_m}\right\}$ *is non-negative invariant and all the solutions are in* R_+^6.

Proof For the total human population, we add the equations to give

$$\frac{dN_h}{dt} = \Lambda - (\delta + \sigma)I - \mu_h N_h \tag{5}$$

Which can be written as

$$\frac{dN_h}{dt} \leq \Lambda - \mu_h N_h$$

The parasite population is given as,

$$\frac{dM}{dt} = \sigma I - \mu_m M \leq \sigma - \mu_m M. \tag{6}$$

So that we have

$$\begin{aligned} &\frac{dN_h}{dt} \leq \Lambda - \mu_h N_h \\ &\frac{dM}{dt} \leq \sigma - \mu_m M. \end{aligned} \tag{7}$$

Following the approach in (Gumel, et al. 2018), thus,

$$\begin{aligned} &0 \leq N_h(t) \leq N_h(0)e^{-\mu_h t} + \frac{\Lambda}{\mu_h}\left(1 - e^{-\mu_h t}\right) \\ &0 \leq M(t) \leq M(0)e^{-\mu_m t} + \frac{\sigma}{\mu_m}\left(1 - e^{-\mu_m t}\right). \end{aligned} \tag{8}$$

Furthermore,

$$\begin{aligned} &0 \leq N_h(t) \leq \frac{\Lambda}{\mu_h} \quad \mathit{if} \quad N_h(0) \leq \frac{\Lambda}{\mu_h} \quad \text{and} \\ &0 \leq M(t) \leq \frac{\sigma}{\mu_m} \quad \mathit{if} \quad M(0) \leq \frac{\sigma}{\mu_m}. \end{aligned} \tag{9}$$

We conclude that, there exist a unique global solution in the domain Ω see Theorem 2.15 in (Stuart and Humphries 1998; Gumel et al. 2018). Thus, the helminth model (1) is a dynamical system on Ω. Conversely, if the solution is outside the region Ω that is,

$$N_h(t) \geq \frac{\Lambda}{\mu_h} \quad \mathit{and} \quad M(t) \geq \frac{\sigma}{\mu_m}. \tag{10}$$

Then, by following the procedure above, $\frac{dN_h}{dt} \leq 0$ *and* $\frac{dM}{dt} \leq 0$. Hence, $N_h(t) \to \frac{\Lambda}{\mu_h}$ and $M(t) \to \frac{\sigma}{\mu_m}$ *as* $t \to \infty$. Thus, the region Ω is attracting.

Model analysis

Existence of the disease-free equilibrium point (DFE)

The disease-free equilibrium point (DFE) is the state where there is absence of helminth infections in the population. The DFE is obtained by setting the right hand side of all the equations in model (1), and the variable $E, I,$ *and* M to zero. Therefore, the DFE denoted by $\left(\xi_0\right)$ is given

$$\xi_0 = (S^*, E^*, I^*, H^*, R^*, M^*) = \left(\frac{\Lambda}{\alpha_1 + \mu_h}, 0, 0, 0, 0, 0\right) \quad (11)$$

To investigate the stability of the DFE point, it is important to obtain the threshold parameter called the *Basic reproduction number* R_0. This is obtained by using the next generation matrix technique presented in (Van den Driessche and Watmough 2002). It follows that the matrices F and V represents the new infections terms and the remaining transfer terms respectively, given by

$$F = \begin{bmatrix} 0 & 0 & 0 & \frac{\beta S^*}{K} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and}$$

$$V = \begin{bmatrix} \left(\rho + \mu_h\right) & 0 & 0 & 0 \\ -\rho & \left(\alpha_2 + q + \tau + \sigma + \delta + \mu_h\right) & 0 & 0 \\ 0 & -\alpha_2 & \left(\varepsilon + \mu_h\right) & 0 \\ 0 & -\sigma & 0 & \mu_m \end{bmatrix}$$

Thus, the basic reproduction number, denoted by $R_0 = \rho\left(FV^{-1}\right)$, where ρ is the spectral radius or largest eigenvalues, is given by

$$R_0 = \frac{\beta\Lambda\sigma\rho}{\mu_m K\left(\alpha_1 + \mu_h\right)\left(\rho + \mu_h\right)\left(\alpha_2 + q + \tau + \sigma + \delta + \mu_h\right)}. \quad (12)$$

The estimated number of secondary infection cases that would develop as a result of primary infections in a population that is totally susceptible is known as the basic reproduction number R_0 (Van den Driessche and Watmough 2002). It is a threshold measure helpful in figuring out how quickly a disease is spreading. If R_0 is less than one, the disease will not spread if a few infected people are put into an entirely vulnerable population. On the other hand, if R_0 is more than 1, the disease will eventually spread since there will be an increase in the population of infected people with each generation (Omonu et al. 2019). Using Theorem 2 of (Van den Driessche and Watmough 2002). We establish the following result below.

Local stability of the disease-free equilibrium point

Theorem 2 *The disease-free equilibrium* ξ_0 *of the helminth model is locally asymptotically stable if* $R_0 < 1$, *and unstable if* $R_0 > 1$.

Proof To prove theorem 2 above, we obtained the Jacobian Matrix of system (1) evaluated at the DFE ξ_0, given by

$$J\left(\xi_0\right) = \begin{bmatrix} -B_1 & 0 & 0 & 0 & \gamma & -\frac{\beta\Lambda}{KB_1} \\ 0 & -B_2 & 0 & 0 & 0 & \frac{\beta\Lambda}{KB_1} \\ 0 & \rho & -B_3 & 0 & 0 & 0 \\ \alpha_1 & 0 & \alpha_2 & -B_4 & 0 & 0 \\ 0 & 0 & B_5 & \varepsilon & -B_6 & 0 \\ 0 & 0 & \sigma & 0 & 0 & \mu_m \end{bmatrix}$$

where

$$B_1 = \alpha_1 + \mu_h, B_2 = \rho + \mu_h, B_3 = \alpha_2 + q + \tau + \sigma + \delta + \mu_h,$$
$$B_4 = \varepsilon + \mu_h, B_5 = q + \tau, B_6 = \gamma + \mu_h.$$

The eigenvalues of $J\left(\xi_0\right)$ are the solutions of the characteristic polynomial, expressed as,

$$\lambda^6 + a_0\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_{5,} \quad (13)$$

where

$$a_0 = B_1 + B_2 + B_3 + B_4 + B_6 + \mu_m$$

$$a_1 = B_1\left(B_2 + B_3 + B_4 + B_6 + \mu_m\right) + B_2\left(B_3 + B_4 + B_6 + \mu_m\right) + B_3\left(B_4 + B_6 + \mu_m\right) + B_4\left(B_6 + \mu_m\right) + B_6\mu_m$$

$$a_2 = B_1B_2\left(B_3 + B_4 + B_6 + \mu_m\right) + B_3\left(B_1 + B_2\right)\left(B_4 + B_6 + \mu_m\right) + B_4\left(B_1 + B_2\right)\left(B_6 + \mu_m\right) + B_1B_6\mu_m$$
$$+ B_6\left(B_2\mu_m + B_3B_4\right) + \mu_m\left(B_3B_4 + B_3B_6 + B_4B_6\right) - \left(\frac{\beta\Lambda\rho\sigma + \alpha_1\varepsilon\gamma KB_1}{KB_1}\right)$$

$$a_3 = B_1B_2\left(B_3B_4 + B_3B_6 + B_4B_6 + \mu_m\left(B_3 + B_4 + B_6\right)\right) + B_3\left(B_1 + B_2\right)\left(B_4B_6 + \mu_m\left(B_4 + B_6\right)\right) + B_1B_4B_6\mu_m$$
$$+ B_4B_6\mu_m\left(B_2 + B_3\right) - \left(\frac{\beta\Lambda\rho\sigma\left(B_1 + B_4 + B_6\right) + \alpha_1\varepsilon\gamma KB_1\left(B_2 + B_3 + \mu_m\right)}{KB_1}\right)$$

$$a_4 = B_1B_2B_3B_4\left(B_6 + \mu_m\right) + B_1B_2B_6\mu_m\left(B_3 + B_4\right) + B_3B_4B_6\mu_m\left(B_1 + B_2\right) - \frac{\beta\Lambda\rho\sigma\left(B_1B_4 + B_1B_6 + B_4B_6\right)}{KB_1}$$
$$- \alpha_1\varepsilon\gamma\left(B_2B_3 + \mu_m\left(B_2 + B_3\right)\right)$$

$$a_5 = \frac{\left(B_1B_4B_6 - \alpha_1\varepsilon\gamma\right)\left(1 - R_0\right)}{KB_1}.$$

By following the Routh-Hurwitz procedure (Hassan et al. 2022), which states that all roots of the polynomial (13) have negative real parts provided that the coefficient $a_i > 0$, for $i = 1, 2, 3, 4, 5$. Hence, for $a_5 > 0$; then $R_0 < 1$. Therefore, the disease-free equilibrium of model (1) is locally asymptotical stable.

Global stability of the disease-free equilibrium point

The helminth model (1) is written as

$$\begin{aligned} \frac{dX}{dt} &= F(X, Z) \\ \frac{dZ}{dt} &= G(X, Z), \qquad F(X, 0) = 0, \end{aligned} \tag{14}$$

where $X = (S, R)$ and $Z = (E, I, H, M)$ with $X \in R^2_+$ denoting the number of uninfected compartments and $Z \in R^4_+$, denoting the total number of infected compartments. Let $\xi_0 = (X^*, 0)$ represents the disease-free equilibrium point of the system (1).

ξ_0 is globally asymptotically stable equilibrium for the model (1) if it satisfies condition W_1 and W_2 below.

$$\begin{aligned} &W_1 : \frac{dX}{dt} = F(X, 0),\ X^* \text{ is globally asymptotically stable} \\ &W_2 : \frac{dZ}{dt} = D_z G(X^*, 0)Z - \widehat{G}(X, Z), \widehat{G}(X, Z) \geq 0 \text{ for all } (X, Z) \in \Omega \end{aligned}$$

where $D_z G(X^*, 0)$ is the Jacobian of $G(X, Z)$ taking in (E, I, H, M) and evaluated at $(X^*, 0) = \left(\left(\frac{\Lambda}{\alpha_1+\mu_h}, 0\right), 0\right)$. If the system (14) satisfies the conditions, then the following theorem holds.

Theorem 3 *The equilibrium points* $\xi_0 = (X^*, 0)$ *of the system* (14) *is globally asymptotically stable if* $R_0 \leq 1$, *and condition* W_1 *and* W_2 *are satisfied.*

Proof From the system (1) we obtained $F(X, Z)$ and $G(X, Z)$.

$$F(X, Z) = \begin{pmatrix} \Lambda - \frac{\beta MS}{K+M} - (\alpha_1 + \mu_h) + \gamma R \\ (q + \tau)I + \varepsilon H - (\gamma + \mu_h)R \end{pmatrix}$$

$$G(X, Z) = \begin{pmatrix} \frac{\beta MS}{K+M} - (\rho + \mu_h)E \\ \rho E - (\alpha_2 + q + \tau + \sigma + \delta + \mu_h)I \\ \alpha_1 S + \alpha_2 I - (\varepsilon + \mu_h)H \\ \sigma I - \mu_m M \end{pmatrix}$$

Now, we consider $\frac{dX}{dt} = F(X, 0)$ the reduced system. From condition W_1,

$$\begin{aligned} \frac{dS}{dt} &= \Lambda - (\alpha_1 + \mu_h)S \\ \frac{dR}{dt} &= 0 \end{aligned} \tag{15}$$

$X^* = \left(\frac{\Lambda}{\alpha_1+\mu_h}, 0\right)$ is a globally asymptotically stable equilibrium point for the reduced system. $\frac{dX}{dt} = F(X, 0)$ in (15). We note that this asymptotic dynamic is independent of the initial condition in Ω. Therefore, the convergence of the solution of the reduced system (15) is global in Ω. Hence, we compute

$$D_z G(X^*, 0) = \begin{pmatrix} -(\rho + \mu_h) & 0 & 0 & \frac{\beta\Lambda}{K(\alpha_1+\mu_h)} \\ \rho & -(\alpha_2 + q + \tau + \sigma + \delta + \mu_h) & 0 & 0 \\ 0 & \alpha_2 & -(\varepsilon + \mu_h) & 0 \\ 0 & \sigma & 0 & -\mu_m \end{pmatrix}$$

$$\widehat{G}(X, Z) = \begin{pmatrix} \beta\left(\frac{\Lambda}{K(\alpha_1+\mu_h)} - \frac{MS}{K+M}\right) \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Here, since $\frac{MS}{K+M} \leq \frac{\Lambda}{K(\alpha_1+\mu_h)}$, it is clear that $\widehat{G}(X, Z) \geq 0$ for all $(X, Z) \in \Omega$. Therefore, by Lasalle invariance principle (La Salle 1976), this proves that the disease-free equilibrium is globally asymptotically stable.

Existence of endemic equilibrium point

In this section, we will explore the existence of the endemic equilibrium point. The endemic equilibrium point is a positive steady state solution where the disease persists in the population. In this case, the infected variables are non-zero. To derive the endemic equilibrium point, we solve the system (1) in terms of the force of infection (λ), with

$$\lambda = \frac{\beta M}{K + M} \tag{16}$$

Therefore, we have;

$$
\begin{aligned}
S^{**} &= \frac{\Lambda B_2B_3B_4B_6}{B_2B_3B_4B_6\left(\lambda^{**}+B_1\right)-\lambda^{**}B_4B_5\gamma\rho-\lambda^{**}\alpha_2\varepsilon\gamma\rho-B_2B_3\alpha_1\gamma\varepsilon}\\
E^{**} &= \frac{\lambda^{**}\Lambda B_3B_4B_6}{B_2B_3B_4B_6\left(\lambda^{**}+B_1\right)-\lambda^{**}B_4B_5\gamma\rho-\lambda^{**}\alpha_2\varepsilon\gamma\rho-B_2B_3\alpha_1\gamma\varepsilon}\\
I^{**} &= \frac{\lambda^{**}\Lambda B_4B_6\rho}{B_2B_3B_4B_6\left(\lambda^{**}+B_1\right)-\lambda^{**}B_4B_5\gamma\rho-\lambda^{**}\alpha_2\varepsilon\gamma\rho-B_2B_3\alpha_1\gamma\varepsilon}\\
H^{**} &= \frac{\Lambda B_6\left(B_2B_3\alpha_1+\lambda^{**}\alpha_2\rho\right)}{B_2B_3B_4B_6\left(\lambda^{**}+B_1\right)-\lambda^{**}B_4B_5\gamma\rho-\lambda^{**}\alpha_2\varepsilon\gamma\rho-B_2B_3\alpha_1\gamma\varepsilon}\\
R^{**} &= \frac{\Lambda\left(\lambda^{**}B_4B_5\rho+B_2B_3\alpha_1\varepsilon+\lambda^{**}\alpha_2\rho\varepsilon\right)}{B_2B_3B_4B_6\left(\lambda^{**}+B_1\right)-\lambda^{**}B_4B_5\gamma\rho-\lambda^{**}\alpha_2\varepsilon\gamma\rho-B_2B_3\alpha_1\gamma\varepsilon}\\
M^{**} &= \frac{\lambda^{**}\Lambda B_4B_6\rho\sigma}{\mu_m\left(B_2B_3B_4B_6\left(\lambda^{**}+B_1\right)-\lambda^{**}B_4B_5\gamma\rho-\lambda^{**}\alpha_2\varepsilon\gamma\rho-B_2B_3\alpha_1\gamma\varepsilon\right)}
\end{aligned}
\qquad (17)
$$

Substituting M^{**} into the force of infection

$$\lambda^{**}=\frac{\beta M^{**}}{K+M^{**}}$$

We obtained

$$f(\lambda^{**})=A_1\lambda^{**2}+A_2\lambda^{**}=0 \qquad (18)$$

where

$$
\begin{aligned}
A_1 &= B_4B_6\left(B_2B_3\mu_mK+\Lambda\rho\right)-\mu_mK\gamma\rho\left(B_4B_5+\alpha_2\varepsilon\right)\\
A_2 &= B_2B_3\mu_mK\left(B_4B_6\left(1-R_0\right)-\alpha_1\gamma\varepsilon\right)
\end{aligned}
$$

Solving the quadratic $f(\lambda^{**})$ give the roots $\lambda^{**}=0$ and $\lambda^{**}=-B_2B_3\mu_mK\left(B_4B_6\left(1-R_0\right)-\alpha_1\gamma\varepsilon\right)<0$ whenever $R_0<1$. Furthermore, for $R_0>1$, the root $\lambda^{**}=-B_2B_3\mu_mK\left(B_4B_6\left(1-R_0\right)-\alpha_1\gamma\varepsilon\right)>0$, this confirm the existence of a unique endemic equilibrium when $R_0>1$. This result is summarized below.

Theorem 4 *The helminth model (1) has a unique endemic equilibrium whenever* $R_0>1$, *and no endemic equilibrium otherwise.*

Sensitivity analysis

In this section, we carried out the sensitivity analysis of the basic reproduction number in order to determine the impact of each of the model parameters that contribute to the transmission of helminth infection. We used the approach presented in (Peter et al. 2020b). To determine the sensitivity index of the basic parameters, we use the normalized forward sensitivity index of a variable, 'p', that depends differentially on the parameter, 'g', is defined as,

$$\Im_g^p=\frac{\partial p}{\partial g}\times\frac{g}{p}$$

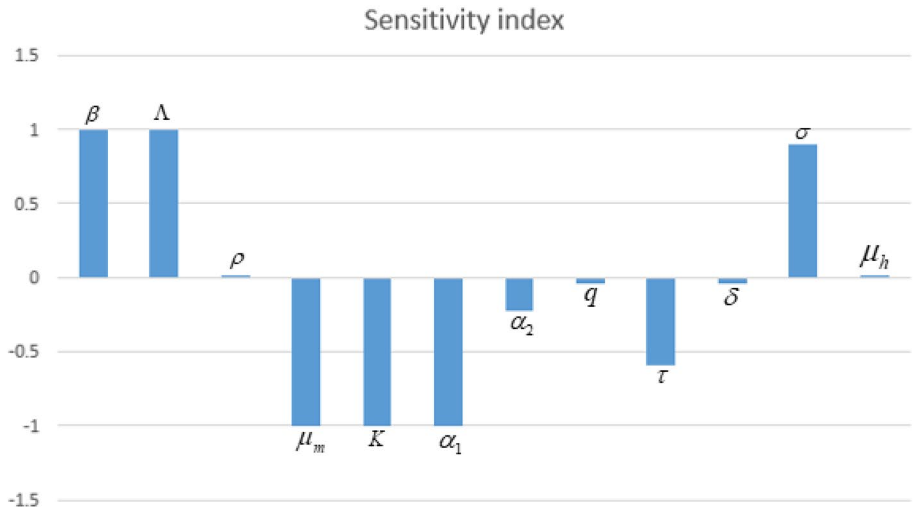


Fig. 2 Sensitivity index of the basic Reproduction number R_0

Therefore, the sensitivity index of the basic reproduction number $\left(R_0\right)$ with respect to the parameter'g' is given by

$$
\begin{aligned}
\mathfrak{I}_g^{R_0} &= \frac{\partial R_0}{\partial g} \times \frac{g}{R_0}\\
\mathfrak{I}_\beta^{R_0} &= \frac{\Lambda\rho\sigma}{\mu_m K\left(\alpha_1+\mu\right)\left(\rho+\mu_h\right)\left(\alpha_1+q+\sigma+\tau+\delta+\mu_h\right)} \times \frac{\beta\mu_m K\left(\alpha_1+\mu\right)\left(\rho+\mu_h\right)\left(\alpha_1+q+\sigma+\tau+\delta+\mu_h\right)}{\beta\Lambda\rho\sigma}\\
\mathfrak{I}_\beta^{R_0} &= 1.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\mathfrak{I}_\Lambda^{R_0} &= 1\\
\mathfrak{I}_\rho^{R_0} &= \frac{\mu_h}{\rho+\mu_h} = 0.0004564\\
\mathfrak{I}_\sigma^{R_0} &= \frac{\alpha_2+q+\tau+\delta+\mu_h}{\alpha_2+q+\sigma+\tau+\delta+\mu_h} = 0.8993\\
\mathfrak{I}_{\mu_m}^{R_0} &= -1\\
\mathfrak{I}_K^{R_0} &= -1\\
\mathfrak{I}_{\alpha_1}^{R_0} &= -\frac{\alpha_1}{\alpha_1+\mu_h} = -0.9995\\
\mathfrak{I}_{\mu_h}^{R_0} &= -\frac{\mu_h\left(3\mu_h^2+2\mu_h\left(\alpha_1+\alpha_2+q+\sigma+\tau+\delta+\rho\right)+\left(\alpha_2+q+\sigma+\tau+\delta\right)\left(\alpha_1+\rho\right)+\alpha_1\rho\right)}{\left(\alpha_1+\mu_h\right)\left(\rho+\mu_h\right)\left(\alpha_2+q+\sigma+\tau+\delta+\mu_h\right)} = 0.0009638\\
\mathfrak{I}_{\alpha_2}^{R_0} &= -\frac{\alpha_2}{\left(\alpha_2+q+\sigma+\tau+\delta+\mu_h\right)} = -0.2237\\
\mathfrak{I}_q^{R_0} &= -\frac{q}{\left(\alpha_2+q+\sigma+\tau+\delta+\mu_h\right)} = -0.0399\\
\mathfrak{I}_\tau^{R_0} &= -\frac{\tau}{\left(\alpha_2+q+\sigma+\tau+\delta+\mu_h\right)} = -0.5965\\
\mathfrak{I}_\delta^{R_0} &= -\frac{\delta}{\left(\alpha_2+q+\sigma+\tau+\delta+\mu_h\right)} = -0.0391
\end{aligned}
$$

Interpretation of the Sensitivity indices Fig. 3 shows the reproduction number's sensitivity indices in relation to the fundamental factors. If the values of the parameters with positive indices increase, it means that they have a considerable effect on accelerating the spread of the disease. Therefore, if the value of the parameters with positive indices increases, the basic reproduction number also does. As their values increase while others' values remain constant, the parameters with negative indices, on the other hand, are in charge of reducing the burden of the disease. In other words, if the value of the parameters with negative indices rises, the reproduction number decreases.

Optimal control

Helminths infections impose a great burden on poor populations in the developing world, yet robust, low-cost and effective public health interventions are available to relieve that burden and provide a better quality of life for people in poor settings. The concept of optimal control theory is an essential tool in epidemiological modeling to determine the best control strategies in combating the spread of an infection, such as discussed in (Peter et al. 2022). Based on the results gotten from the sensitivity analysis, we will carry out an analysis to determine an optimal intervention strategy on two time-varying controls, namely; the rate of hygiene consciousness in the susceptible class $\left(\alpha_1(t)\right)$ and the rate of hygiene consciousness of infectious individuals $\left(\alpha_2(t)\right)$. Using these controls, the helminth model (1) becomes

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \frac{\beta MS}{K+M} - \left(\alpha_1(t)+\mu_h\right)S + \gamma R\\
\frac{dE}{dt} &= \frac{\beta MS}{K+M} - \left(\rho+\mu_h\right)E\\
\frac{dI}{dt} &= \rho E - \left(\alpha_2(t)+q+\tau+\sigma+\delta+\mu_h\right)I\\
\frac{dH}{dt} &= \alpha_1(t)S + \alpha_2(t)I - \left(\varepsilon+\mu_h\right)H\\
\frac{dR}{dt} &= (q+\tau)I + \varepsilon H - \left(\gamma+\mu_h\right)R\\
\frac{dM}{dt} &= \sigma I - \mu_m M
\end{aligned} \tag{19}$$

For this, we consider the objective functional

$$J\left[\alpha_1,\alpha_2\right] = \int_0^{t_f} \left[D_1 I + \frac{1}{2}\left(D_2\alpha_1^2(t) + D_3\alpha_2^2(t)\right)\right]dt \tag{20}$$

where t_f is the final time, and the coefficients $D_1, D_2,\ and\, D_3$ are the weight factors to help balance each term in the integrand (20), so that none of the terms dominates. The term $D_1 I$ represents the cost associated with monitoring infected individuals in all stage, while the term $D_2\alpha_1^2(t) +\ D_3\alpha_2^2(t)$ represents the cost associated with the public health education campaign to educate the public on personal hygiene. Our aim is to minimize infection as well as keeping the cost of the control strategies low. The goal is to find an optimal control pair $\alpha_1(t)$and $\alpha_2(t)$ such that

$$J\left[\alpha_1^*,\alpha_2^*\right] = \min_{\alpha_1,\alpha_2\in\Delta} J\left[\alpha_1,\alpha_2\right] \tag{21}$$

where the control set (Δ) is defined as

$$\Delta = \left\{ \left(\alpha_1(t),\alpha_2(t)\right) \in L^1\left(0,t_f\right) \middle| a_1 \le \alpha_1(t) \ge b_1, a_2 \le \alpha_2(t) \ge b_2 \right\}$$

Analysis of the optimal control model

Pontryagin's Maximum Principle (Pontryagin et al. 1962) reveals the prerequisite that an ideal control pair must meet with respect to the control pair $\alpha_1(t)$ and $\alpha_2(t)$, this principle transforms problems (19) and (20) into a problem of minimizing a Hamiltonian $\left(\mathrm{H}^{\circ}\right)$ pointwise.

The Hamiltonian is given by;

$$\begin{aligned}
\mathrm{H}^{\circ} = {} & D_1 I + \frac{1}{2}\left(D_2\alpha_1^2(t) + D_3\alpha_2^2(t)\right) + \lambda_1\left[\Lambda - \frac{\beta MS}{K+M} - \left(\alpha_1(t)+\mu_h\right)S + \gamma R\right]\\
& +\lambda_2\left[\frac{\beta MS}{K+M} - \left(\rho+\mu_h\right)E\right] + \lambda_3\left[\rho E - \left(\alpha_2(t)+q+\tau+\sigma+\delta+\mu_h\right)I\right]\\
& +\lambda_4\left[\alpha_1(t)S + \alpha_2(t)I - \left(\varepsilon+\mu_h\right)H\right]\\
& +\lambda_5\left[(q+\tau)I + \varepsilon H - \left(\gamma+\mu_h\right)R\right] + \lambda_6\left[\sigma I - \mu_m M\right],
\end{aligned} \tag{22}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5,\ and \lambda_6$ are the adjoint functions with respect to the state variables in (19), by applying the Pontryagin's Maximum Principle (Pontryagin et al. 1962) and the existence result for the optimal control pair $\alpha_1(t)$ and$\alpha_2(t)$, the theorem below is established.

Theorem 5 *There exists an optimal control pair* $\alpha_1^*(t)$ *and* $\alpha_2^*(t)$, *and corresponding solution* $(S^*, E^*, I^*, H^*, R^*,\ and\ M^*)$ *that minimize* $J\left(\alpha_1(t),\alpha_2(t)\right)$ *over* Δ. *Furthermore, there exist adjoint functions,* λ_i ,$(i = 1,2,3,\ldots,6)$ *such that*;

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= \left(\frac{\beta M^*}{K+M^*} + \left(\alpha_1(t)+\mu_h\right)\right)\lambda_1 - \frac{\beta M^*}{K+M^*}\lambda_2 - \alpha_1(t)\lambda_4\\
\frac{d\lambda_2}{dt} &= \left(\rho+\mu_h\right)\lambda_2 - \rho\lambda_3\\
\frac{d\lambda_3}{dt} &= -D_1 + \left(\alpha_2(t)+q+\tau+\sigma+\delta+\mu_h\right)\lambda_3 - \alpha_2(t)\lambda_4 - (q+\tau)\lambda_5 - \sigma\lambda_6\\
\frac{d\lambda_4}{dt} &= \left(\varepsilon+\mu_h\right)\lambda_4 - \varepsilon\lambda_5\\
\frac{d\lambda_5}{dt} &= -\gamma\lambda_1 + \left(\gamma+\mu_h\right)\lambda_5\\
\frac{d\lambda_6}{dt} &= \frac{\beta KS^*}{(K+M^*)^2}\left(\lambda_1-\lambda_2\right) + \mu_m\lambda_6,
\end{aligned} \tag{23}$$

with transversality conditions

$$\lambda_i\left(t_f\right) = 0, \quad i = 1,2,3,\ldots,6. \tag{24}$$

The following characterization holds;

$$\begin{aligned}
\alpha_1^*(t) &= \min\left[\max\left(a_1, \frac{1}{D_2}\left(\lambda_1-\lambda_4\right)S^*, b_1\right)\right]\\
\alpha_2^*(t) &= \min\left[\max\left(a_2, \frac{1}{D_3}\left(\lambda_3-\lambda_4\right)I^*, b_2\right)\right].
\end{aligned} \tag{25}$$

Proposition 1 *Corollary 4.2 of* (Fleming and Reshel 1975) *gives the existence of an optimal control pair* $\left(\alpha_1(t)\ and\ \alpha_2(t)\right)$ *due to the convexity of the integrand of* J *with respect of*$\left(\alpha_1(t)\ and\ \alpha_2(t)\right)$, *a prior boundedness of the state solutions, and the local Lipschitz property of the model* (19) *with respect to the variables.*

Proof of the theorem 5 Using Pontryagin's Maximum Principles, we have,

Table 1 Description of the model variables and parameters

Variable	Description
$S(t)$	Susceptible class
$E(t)$	Exposed class
$I(t)$	Infectious class
$H(t)$	Hygiene conscious class
$R(t)$	Recovered class
$M(t)$	Parasite population

Parameter	Description	Value	Source
Λ	Recruitment rate in the susceptible class	$\frac{1}{18250}\ day^{-1}$	Lambura et al. (2020a)
α_1	Rate of hygiene consciousness in the susceptible class	0.4	Assumed
α_2	Rate of hygiene consciousness in the infectious class	0.6	Assumed
β	Intake rate of egg in contaminated food or larvae that have penetrated the skin	0.4	Assumed
ρ	Progression rate from exposed class to infectious class	$\frac{1}{10}\ day^{-1}$	Lambura et al. (2020a)
q	Recovery rate of infectious individuals perhaps due to strong immune system	$\frac{1}{28}\ day^{-1}$	Lambura et al. (2020a)
τ	Recovery rate of infectious individuals due to treatment	$\frac{16}{30}\ day^{-1}$	Assumed
γ	Rate at which recovered individuals becomes susceptible again	$\frac{1}{7}\ day^{-1}$	Lambura et al. (2020a)
σ	Rate of contamination of the environment	$0.09\ day^{-1}$	Lambura et al. (2020a)
ε	Recovery rate of hygiene conscious infectious individuals	0.03	Assumed
μ_h	Natural death rate for humans	$\frac{1}{(60*365)}\ day^{-1}$	Lambura et al. (2020a)
μ_m	Clearance rate for parasites	$\frac{13}{37500}\ day^{-1}$	Lambura et al. (2020a)
K	Density of the parasites (carrying capacity)	$10^5 parasites$	Lambura et al. (2020a)
δ	Diseases-induced death rate	$\frac{35}{1000} day^{-1}$	Lambura et al. (2020a)

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= -\frac{\partial H^{\circ}}{\partial S}, \quad \lambda_1\left(t_f\right) = 0,\\
\frac{d\lambda_2}{dt} &= -\frac{\partial H^{\circ}}{\partial E}, \quad \lambda_2\left(t_f\right) = 0,\\
\frac{d\lambda_3}{dt} &= -\frac{\partial H^{\circ}}{\partial I}, \quad \lambda_3\left(t_f\right) = 0,\\
\frac{d\lambda_4}{dt} &= -\frac{\partial H^{\circ}}{\partial H}, \quad \lambda_4\left(t_f\right) = 0,\\
\frac{d\lambda_5}{dt} &= -\frac{\partial H^{\circ}}{\partial R}, \quad \lambda_5\left(t_f\right) = 0,\\
\frac{d\lambda_6}{dt} &= -\frac{\partial H^{\circ}}{\partial M}, \quad \lambda_6\left(t_f\right) = 0,
\end{aligned} \tag{26}$$

and considering the optimality condition;

$$\frac{\partial H^{\circ}}{\partial \alpha_1} = 0 \ and \ \frac{\partial H^{\circ}}{\partial \alpha_2} = 0. \tag{27}$$

The optimal control pair $\left(\alpha_1^* \ and \ \alpha_2^*\right)$ can be solved for, subject to the state variables. Taking into account the bounds on the control, the characterization in (15) can be obtained. This yield;

For the control,$\alpha_1^*(t)$, we obtained

$$\frac{\partial \mathrm{H}^{\circ}}{\partial \alpha_1} = D_2\alpha_1(t) - \lambda_1 S^* + \lambda_4 S^* = 0$$

This implies that

$$\alpha_1^*(t) = \frac{1}{D_2}\left(\lambda_1 - \lambda_4\right)S^* \tag{29}$$

Similarly, for the control,$\alpha_2^*(t)$, we obtained

$$\frac{\partial \mathrm{H}^{\circ}}{\partial \alpha_2} = D_3\alpha_2(t) - \lambda_3 I^* + \lambda_4 I^*$$

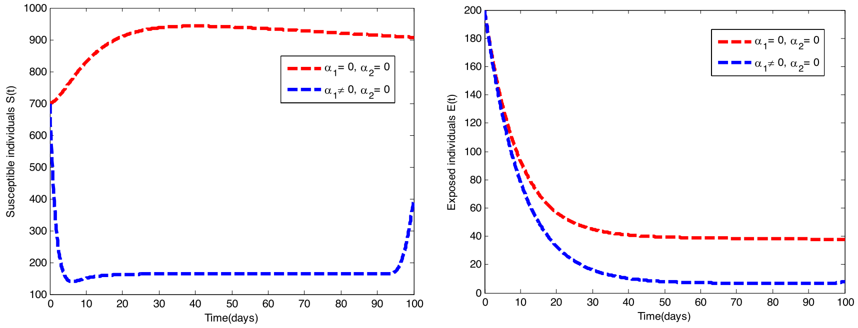
This implies that

$$\alpha_2^*(t) = \frac{1}{D_3}\left(\lambda_3 - \lambda_4\right)I^* \tag{30}$$

It is observed that the optimality conditions obtained by taking the derivatives of the Hamiltonian with respect to the controls only hold in the interior of the control set.

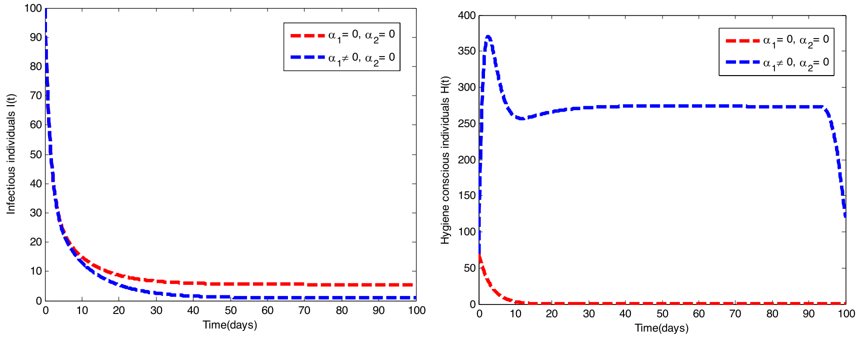
This end the proof.

Fig. 3 Simulation of the model showing the effect of the rate of hygiene consciousness in the susceptible class (α_1) on the transmission dynamics of helminth disease



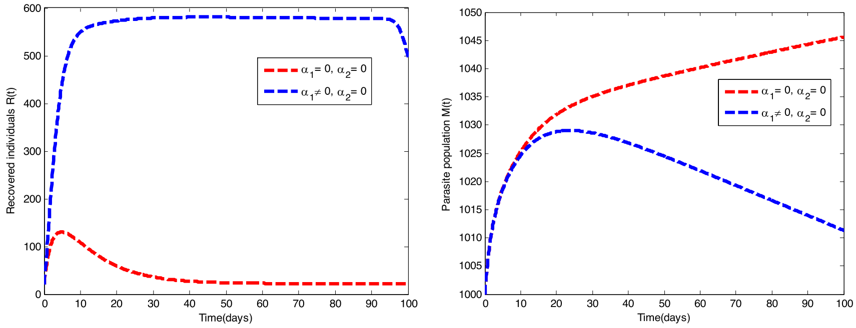
(a) Optimal control graph of the susceptible individuals against time.

(b) Optimal control graph of exposed individuals against time.



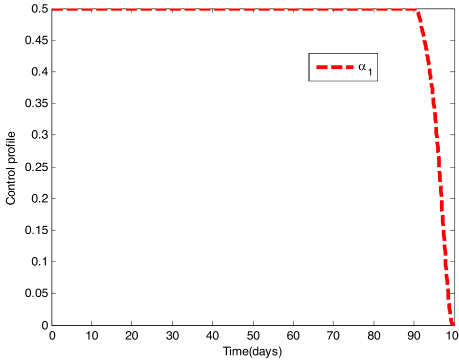
(c) Optimal control graph of the infectious individuals against time.

(d) Optimal control graph of the hygiene conscious individuals against time.



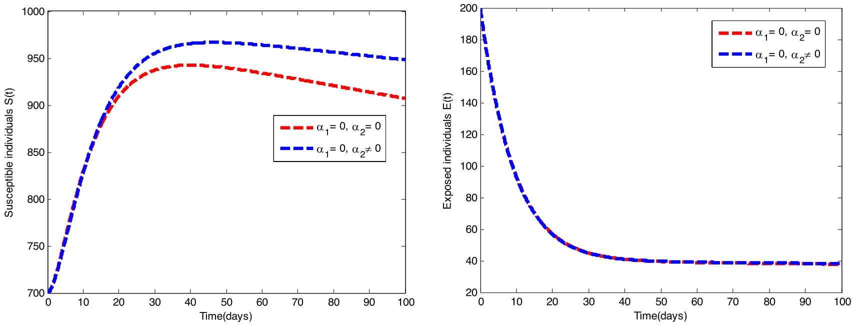
(e) Optimal control graph of the recovered individuals against time.

(f) Optimal control graph of the parasite population against time.



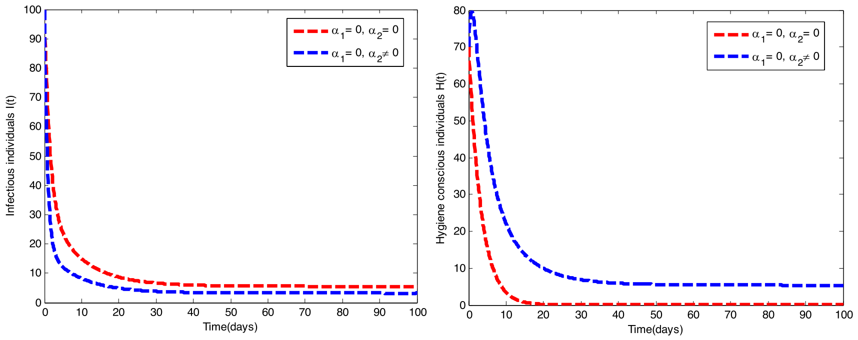
(g) Control profile graph of α_1

Fig. 4 Simulation of the model showing the effect of the rate of hygiene consciousness in the infectious class $\left(\alpha_2\right)$ on the transmission dynamics of helminth disease



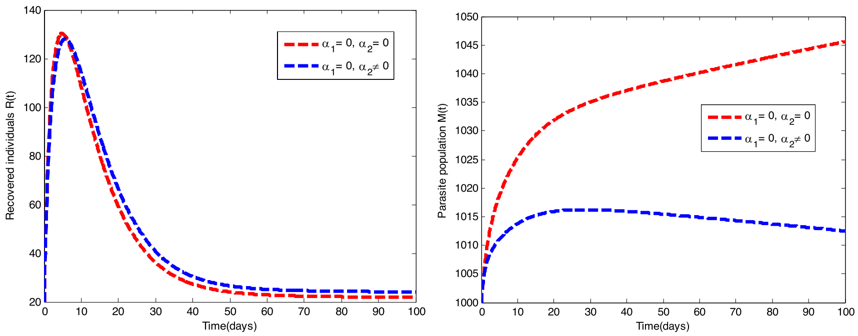
(a) Optimal control graph of the susceptible individuals against time.

(b) Optimal control graph of exposed individuals against time.



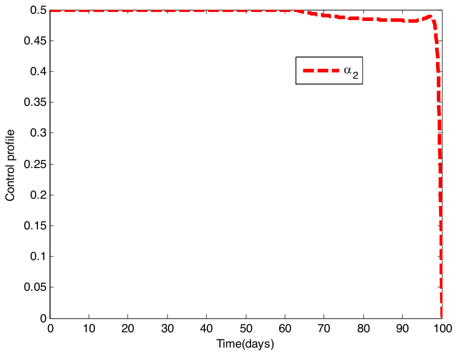
(c) Optimal control graph of the infectious individuals against time.

(d) Optimal control graph of the hygiene conscious individuals against time.



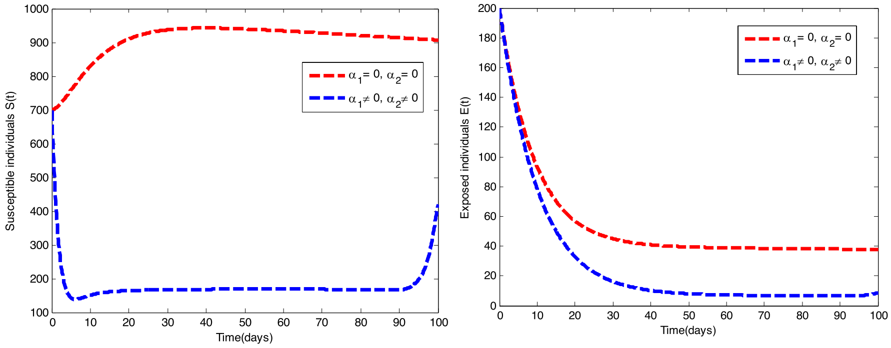
(e) Optimal control graph of the recovered individuals against time.

(f) Optimal control graph of the parasite population against time.



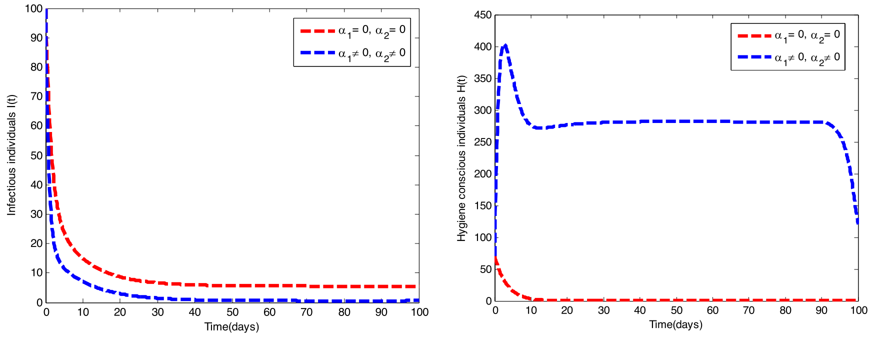
(g) Control profile graph of α_2

Fig. 5 Simulation of the model showing the effect of the rate of hygiene consciousness in the susceptible class $\left(\alpha_1\right)$ and rate of hygiene consciousness in the infected class $\left(\alpha_2\right)$ on the transmission dynamics of helminth disease



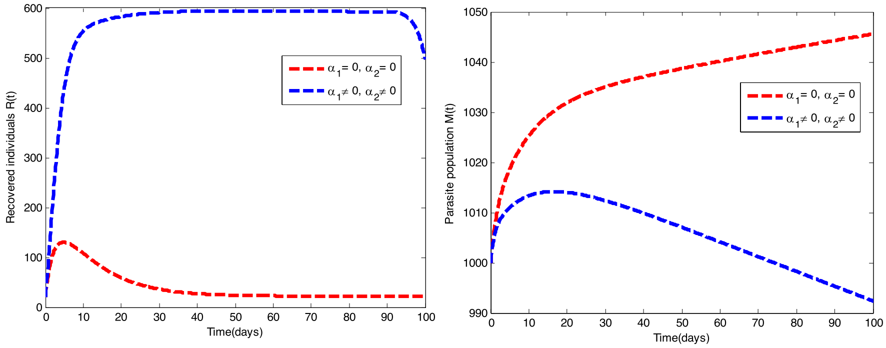
(a) Optimal control graph of the susceptible individuals against time.

(b) Optimal control graph of exposed individuals against time.



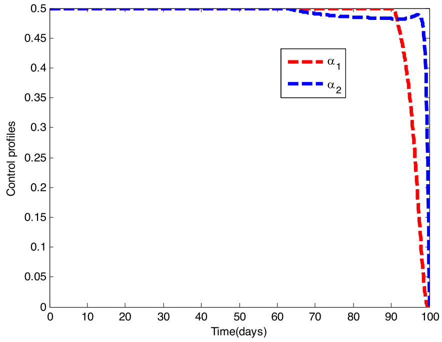
(c) Optimal control graph of the infectious individuals against time.

(d) Optimal control graph of the hygiene conscious individuals against time.



(e) Optimal control graph of the recovered individuals against time.

(f) Optimal control graph of the parasite population against time.



(g) Control profiles graph of α_1 *and* α_2

Numerical simulation for the helminth model with optimal control

The numerical simulations for the optimal control model are performed in this part utilizing the forward–backward sweep method described in (Abioye et al. 2020; Abioye et al. 2021; Ayoola et al. 2021; Peter et al. 2020a, b, 2021, 2022). and the parameter values listed in Table 1. The procedure starts with an initial guess on the control variables. The adjoint system (23) is then solved backward in time while the state system (19) is simultaneously solved forward in time using the Runge–Kutta fourth order approach. By updating the controls and adding the new state and adjoint system values to the characterization, the procedure is repeated until convergence is reached. The initial conditions used are $S(0) = 700$, $E(0) = 200$,$I(0) = 100$,$H(0) = 70$,$R(0) = 20$,$M(0) = 1000$.

Result and discussion

Figure 3 depicts the simulations of the effect of the rate of hygiene consciousness in the susceptible class $\left(\alpha_1\right)$ on each of the compartments. It is observed that when the control $\left(\alpha_1\right)$ is applied, the susceptible individuals, exposed individuals, infectious individuals, and the parasite population decreases while the recovered individuals and hygiene conscious individuals increases significantly.

Figure 4 depicts the simulations of the effect of the rate of hygiene consciousness in the infectious class $\left(\alpha_2\right)$ on each of the compartments. It is observed that when the control $\left(\alpha_2\right)$ is applied, the infectious individuals and the parasite population decreases over time. The exposed individuals experiences no change when the control $\left(\alpha_2\right)$ was applied, while the susceptible individuals, hygiene conscious individuals and the recovered individuals increases slightly.

Figure 5 depicts the simulations of both the effects of the rate of hygiene consciousness in the susceptible class $\left(\alpha_1\right)$ and the rate of hygiene consciousness in the infectious class $\left(\alpha_2\right)$ on each of the compartments. When both controls $\left(\alpha_1\right)$ *and* $\left(\alpha_2\right)$ was applied, it is observed that there is a significant drop in the number of susceptible individuals, exposed individuals, infectious individuals and the parasite population to without the two control strategies. It is also observed that the recovered individuals recorded is a significant increase compared to without the control strategies.

Conclusion

In this study, we present a deterministic mathematical model for the dynamics of helminth infections transmitted through soil. After qualitatively analyzing the model, we incorporated two time-varying controls to prevent the diseases from spreading further. The findings from the study shows that, the helminths model has a locally asymptotically stable disease-free equilibrium whenever the basic reproduction number is less than unity. The model also has a unique endemic equilibrium whenever the basic reproduction number is greater than unity. The sensitivity analysis performed on the basic reproduction number of the model indicates that some parameters are very significant in reducing the burden of helminth infections if their values are on increasing, such parameters are the clearance rate for parasites, rate of hygiene consciousness for the susceptible individuals, rate of hygiene consciousness for the infectious individuals, recovery rate of infectious individuals due to treatment etc. This will help policy makers in making effective decision on combating the spread of helminth infections. The numerical simulations reveal that if both control strategies are properly and effectively implemented, the burden of helminth infections will be mitigated. The study can be extended in numerous ways, such as introducing age structured model to gain more insight into the dynamics of helminth infections in various age groups. Also, the co-dynamics of soil-transmitted helminths infections and other infectious diseases can be investigated.

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Data availability Not applicable.

Declarations

Conflict of interest The authors declare no potential conflict of interest.

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