



**DEPARTMENT OF MATHEMATICS
FACULTY OF SCIENCE
OBAFEMI AWOLOWO UNIVERSITY
ILE-IFE, NIGERIA**

CONFERENCE BOOK OF ABSTRACTS

Theme:
**MATHEMATICAL SCIENCES: AN INDISPENSABLE
TRAJECTORY TO SUSTAINABLE DEVELOPMENT**

In Honour of:

**Professor A. G. Adeagbo-Akeyo
Professor Titilola Olakunjo Obilade
Dr. Peter Folorunso Fasogbon
Mr. Adekunle Mojubaade Ogunfidodo**

Tuesday, 20th February 2024

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National Anthem



National Anthem

Stanza 1

Arise, O compatriots
Nigeria's call obey
To serve our fatherland
With love and strength and faith.
The labour of our heroes past Shall never be in vain
To serve with heart and might
One nation bound in freedom
peace and unity.

Stanza 2

O God of creation
Direct our noble cause
Guide our leaders right
Help our youth the truth to know
In love and honesty to grow
And living just and true
Great lofty heights attain
To build a nation where peace and justice reign.

Welcome Messages

On behalf of the Department of Mathematics and the Organizing Committee of this conference, we welcome you to attend this event taking place physically in the Faculty Board Room and Department of Mathematics at Obafemi Awolowo University, Ile-Ife, Nigeria, and over the Internet today, 20th February 2024. We hope that an annual conference, a well-established series of popular and high-quality conferences, will be established in the near future, focusing on the theory and methodology of mathematical sciences and their applications.

This conference aims to:

- Honour our retired colleagues who have served not only the department, faculty, and university but also the entire society at large.
- Provide a high-level international forum for scientists, engineers, and students to gather and present and discuss the latest progress in mathematical sciences research and applications in diverse areas.
- Encourage open discussion and exchange of ideas. We believe that it will extensively promote research in the various fields of mathematical sciences and its applications.

This conference received 53 abstract submissions, which show a sign of a good beginning, despite the economic hardship in the country. After the conference, the submitted papers will be reviewed. After rigorous peer reviews, the accepted papers will be published in a reputable journal. The received abstracts cover many topics in pure and applied mathematics, mathematical statistics and computing, and so on. In addition to the submitted abstracts, the conference includes a welcome address by the Host:

Professor Memudu O. Olatinwo
Head, Department of Mathematics
Obafemi Awolowo University, Ile-Ife, Nigeria.

An address by the Chief Host, Dean, Faculty of Science:

Professor Olufemi Adeyinka Adesina
Dean, Faculty of Science
Obafemi Awolowo University, Ile-Ife, Nigeria.

An opening speech by the Chairman:

Professor Simeon Adebayo Bamire
Vice-Chancellor,
Obafemi Awolowo University
Ile-Ife, Nigeria.

A keynote speech by world-renowned mathematics scholar:

Professor Adewale Roland Tunde Solarin
FMAN, FPNGMS, FNMS
Honorary President, African Mathematical Union

Many volunteers made great contributions toward the success of this conference. We would like to express our sincere gratitude to the alumni of this great department who are in the United Kingdom under the leadership of Dr. Shola Adeyemi for their sponsorship, and many thanks to our amiable Dean of the Faculty of Science, Professor Olufemi Adeyinka Adesina, and Professor Suraju Olusegun Ajadi for linking us up with these wonderful alumni, students of our retired colleagues, and the academic and administrative staff of this department for their cooperation toward the success of this conference. We would also like to sincerely thank all the committee members for their great efforts in organizing the conference. Finally, we would like to thank all the speakers, authors, and participants (both physical and virtual) for their support.

Professor A. T. Ademola



Chairman

PROFESSOR ADEBOYO SIMEON BAMIRE

Vice Chancellor

Obafemi Awolowo University, Ile-Ife, Nigeria



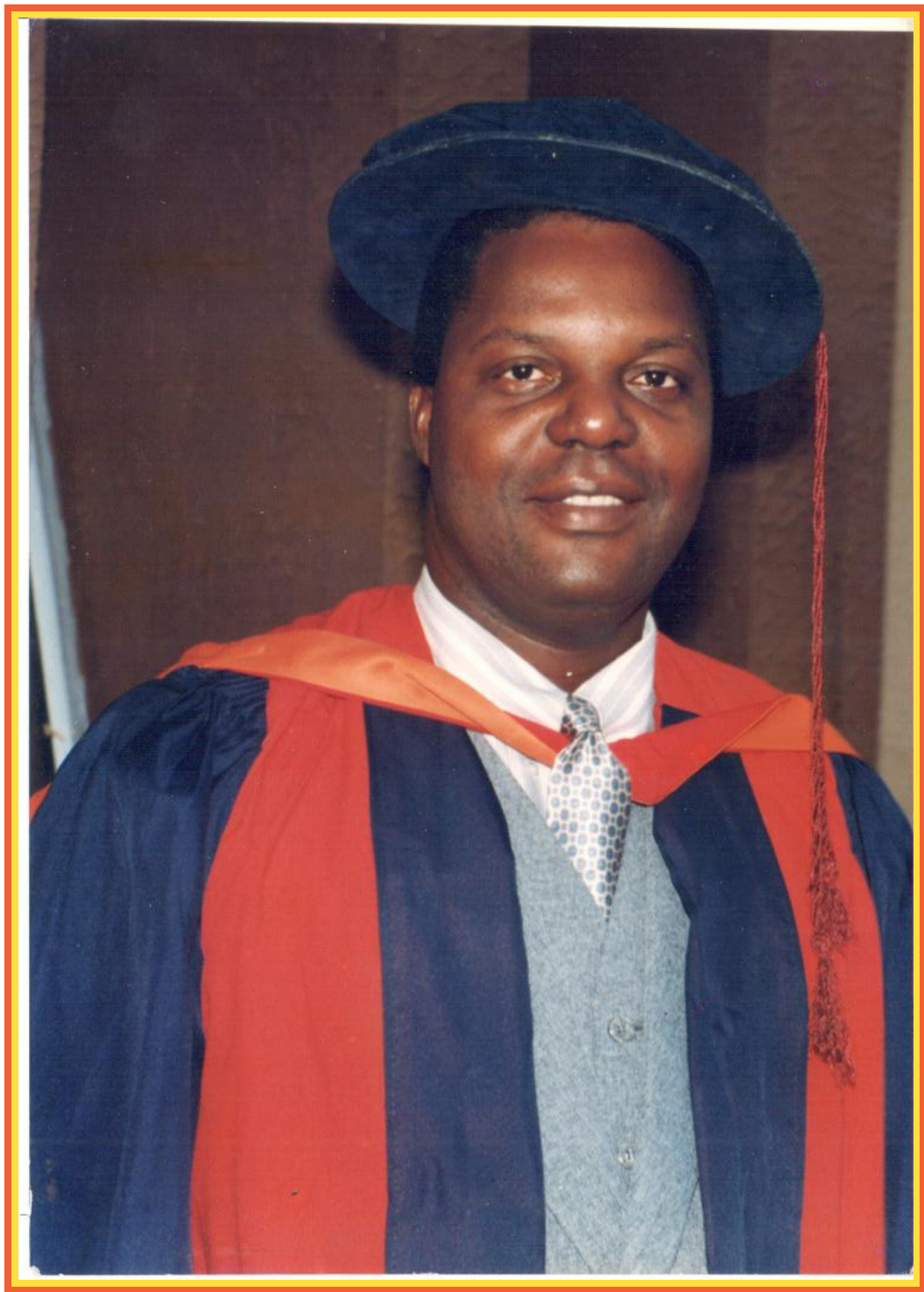
Chief Host

PROFESSOR OLUFEMI ADEYINKA ADESINA
Dean, Faculty of Science
Obafemi Awolowo University, Ile-Ife, Nigeria



Host

PROFESSOR MEMUDU O. OLATINWO
Head, Department of Mathematics
Obafemi Awolowo University, Ile-Ife, Nigeria



Keynote Address

PROFESSOR ADEWALE ROLAND TUNDE SOLARIN
FMAN, FPNGMS, FNMS
Honorary President, African Mathematical Union.



Chairman, Organising Committee

PROFESSOR ADELEKE TIMOTHY ADEMOLA

Department of Mathematics

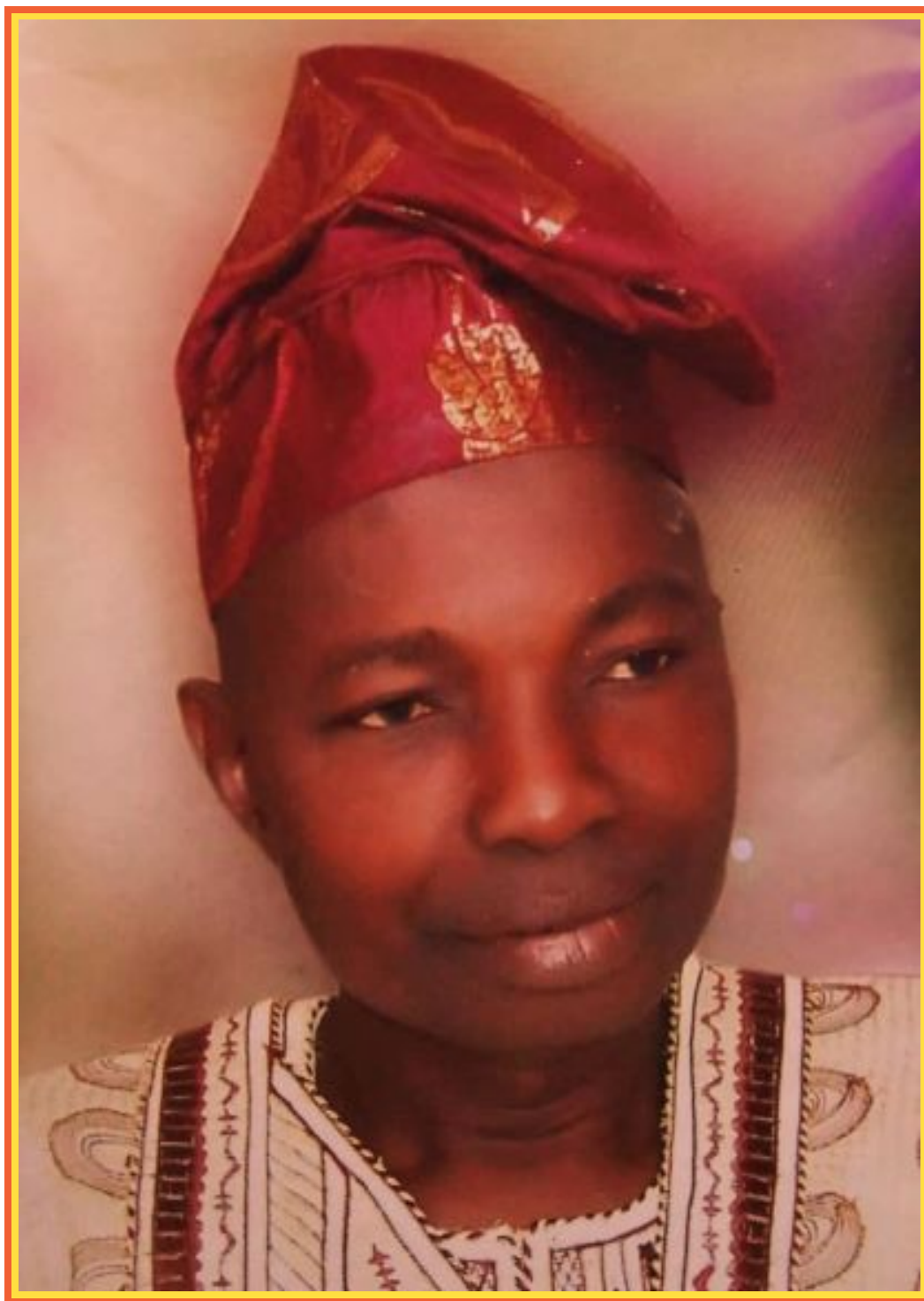
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MR. ADEKUNLE MOJUBAADE OGUNFIDODO
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Obafemi Awolowo University, Ile-Ife
Retired - 14th August 2018

Instructions for Virtual and Physical Presentations

- **Oral Presentation Time:** 15 minutes, including 10 minutes for presentation, and 5 minutes for questions and answers.
- **Presentation Form:** Physical and virtual presentations will be conducted. The virtual presentation will be held online using Zoom Meeting as the platform. Please download and install the Zoom software before the session using the following links:
https://www.bing.com/search?q=zoom+download&cvid=358e4d0979324ab2a6159408a49ec893&gs_lcrp=EgZjaHJvbWUqBggBEAAYQDIGCAAQRrg5MgYIARAAGEAyBggCEAAYQDIGCAMQABhAMgYIBBAAGEAyBggFEAAYQDIGCAYQABhAMgYIBxAAGEAyBggIEAAYQNIBCDQ3MThqMGo0qAIAAIA&FORM=ANAB01&PC=EDGEEDB
- Every session has a Zoom Meeting ID. Please find your session and the corresponding ID. The password is: xxxxxxx
- The presenter is required to enter the meeting 10 minutes before the session starts, and inform the session chairs that he/she is present.
- When you enter the meeting, please mute your speaker in the Zoom Meeting. Before your presentation, please mute it.
- The general presentation software such as Microsoft PowerPoint and Adobe Reader can be used. Please use the screen sharing function of the Zoom Meeting to share your slides.
- Physical presenters must submit the soft copy of their slides to the appropriate session an hour before the session starts.

Technical Sessions

Technical Session I	Venue:	Seminar Room MBB 213
Applied Mathematics(AM) One:	–	Abstract No: 1.1 - 1.11
Time:	–	1.00 - 3.45 pm
Chairman:	–	Professor S. O. Ajadi
Rapporteur:	–	Dr. B. A. Olokuntoye
Panelists : Prof. A. P. Akinola	Dr. A. S. Borokinni	Mr. Y. T. Lawal

S/N	Abstract No.	Abstract Title	Time
1.	1.1	A Within Host Spatial Mathematical Model Investigating the Dynamics of Malaria Parasite Influence on Airway of Asthmatic Humans. - <i>Ogunmiloru and Idowu</i>	1.00 - 1.15 pm
2.	1.2	SmartHIV Manager: The Evolution of Patient Flow Modelling from Research through Development and Innovation. - <i>Adeyemi & Demir</i>	1.15 - 1.30pm
3.	1.3	Integration of Modified Classical Conjugate Gradient Methods for Unconstrained Optimization. - <i>Onuoha et al.</i>	1.30 - 1.45 pm
4.	1.4	Iterative Method for the Numerical Solution of Optimal Control Model for Mosquito and Insecticide. - <i>Adamu et al.</i>	1.45 - 2.00 pm
5.	1.5	Optimized Hybrid One-Step Method for Efficient Direct Numerical Integration of Second-Order Initial Value Problems. - <i>Oludare & Ojo</i>	2.00 - 2.15 pm
6.	1.6	Stationarity in Prophet Model Forecast: Performance Evaluation Approach. - <i>Omotoye & Rotimi</i>	2.15 - 2.30 pm
7.	1.7	Mises Flow Rules for Distortion Gradient Plastic Materials in Finite Deformation. - <i>Borokinni et al.</i>	2.30 - 2.45 pm
8.	1.8	Numerical and Statistical Analysis of the Influence of Temperature-dependent Thermophysical Properties of Tetra-Hybrid Nanofluid Along a Vertical Porous Surface With Suction. - <i>Aselebe et al.</i>	2.45 - 3.00 pm
9.	1.9	MHD Casson Flow Over a Non-linear Convective Inclined Plate With Chemical Reaction and Arrhenius Activation Energy. - <i>Akindele et al.</i>	3.00 - 3.15 pm
10.	1.10	An Improved Ceaser Cipher. - <i>A. Hassan</i>	3.15 - 3.30 pm
11.	1.11	On a University Time Tabling Problem. - <i>Dikko et al.</i>	3.30 - 3.45 pm

Technical Session II
Applied Mathematics(AM) Two:
Time:
Chairman:
Rapporteur:
Panelists : Prof. S. S. Okoya

Venue: Seminar Room MBA 213
 Abstract No: 1.12 - 1.22
 3.45 - 6.15 pm
 Professor O. P. Layeni
 Dr. A. A. Aderogba
 Dr. O. O. Fadodun Mr. R. A. Adetona

S/N	Abstract No.	Abstract Title	Time
1.	1.12	Runge-Kutta Like Method for the Solution of Optimal Control Model of Real Investment and Fish Management. - <i>Adamu et al.</i>	3.45 - 4.00 pm
2.	1.13	On the Differential Equations of Artificial Intelligence Systems: Ifa Divination Corpus. - <i>Bamidele Oluwade</i>	4.00 - 4.15 pm
3.	1.14	Bifurcation Analysis of the Impact of Prep on HIV/AIDS Dynamics With Inflow of Infectives Immigrants. - <i>Oladejo & Odebiyi</i>	4.15 - 4.30 pm
4.	1.15	Role of Magnetic Field and Heat Source/Sink on Fluid Flow in a Channel. - <i>Saifullahi Yusuf</i>	4.30 - 4.45 pm
5.	1.16	Legendre Collocation Method for Six-order Boundary Value Problems. - <i>Oyedepo et al.</i>	4.45 - 5.00 pm
6.	1.17	Modelling and Simulation of Drilling Mud Rheological Properties. - <i>Opadiran & Okoya</i>	5.00 - 5.15 pm
7.	1.18	Path Planning of Multi-Rotor UAVs via Enhanced A* Particle Swarm Optimization Algorithm. - <i>Adisa & Oyekan</i>	5.15 - 5.30 pm
8.	1.19	Axial Force Influence on Dynamic Response to Moving Load of Shear Beam Resting on Elastic Foundation. - <i>Ajijola et al.</i>	5.30 - 5.45 pm
9.	1.20	Sensitivity Analysis of Pertussis Transmission Dynamics - <i>Edogbanya & Johnson</i>	5.45 - 6.00 pm
10.	1.21	A Novel Approach to Environmental Monitoring With Fused Satellite Data Using Sparse Convolutional Neural Network. - <i>Oyebamiji Oluwole</i>	5.45 - 6.00 pm
11.	1.22	Lyapunov Stability and Sensitivity Analysis of a Recurrent Malaria Dynamics. - <i>Abimbade S. F., Olaniyi S., & Ajala O. A.</i>	6.00 - 6.15 pm

Technical Session III**Math. Stat. and Compu.(MSC)****Time:****Chairman:****Rapporteur:****Panelists :** Dr. O. K. Agunloye

Dr. A. J. Saka

Dr. S. O. Ezeah

Venue: Computing Lab.: Room MBA 216

– Abstract No: 2.1 - 2.11

– 1.00 - 3.45 pm

– Professor A. A. Olosunde

– Dr. I. O. Ayodeji

S/N	Abstract No.	Abstract Title	Time
1.	2.1	Volatility Modeling and Forecasting using Range-based GARCH Models. - <i>Ogunwole & Agunloye</i>	1.00 - 1.15 pm
2.	2.2	A New Improved Estimator For Population Distribution Function Using Auxiliary Variable Under Simple Random Sampling. - <i>Rabiu & Abubaka</i>	1.15 - 1.30 pm
3.	2.3	Mathematical Approaches to Enhance Energy Efficiency in Construction for Sustainable Development in Nigeria. - <i>Yusuf Ahmad</i>	1.30 - 1.45 pm
4.	2.4	Model for the Prediction of Cholera Incidence. - <i>James et al.</i>	1.45 - 2.00 pm
5.	2.5	Backtesting Methods for Evaluating the Viability of Stock Prices in Nigeria. - <i>Taiwo et al.</i>	2.00 - 2.15 pm
6.	2.6	Digital Distraction and Personality Trait as Correlate of Knowledge Retention of Undergraduate Mathematics in Tertiary Institutions in Ondo State. - <i>Adekolu and Olupona</i>	2.15 - 2.30 pm
7.	2.7	Omiyale: Sounding the Mathematics of the Rain and Flood Disaster in Ibadan City, Nigeria. - <i>Olusegun Stephen Titus</i>	2.30 - 2.45 pm
8.	2.8	Semiparametric Lognormal Shared Frailty Modeling of Diabetes Patients with Acute Coronary Syndrome. - <i>Adeleke et al.</i>	2.45 - 3.00 pm
9.	2.9	A Comparative Analysis of Option Pricing Models: Assessing Their Applicability in Valuing European Options. - <i>Akin-Alamu & Oyekan</i>	3.00 - 3.15 pm
10.	2.10	Prediction of Covid-19 Pandemic Using Machine Learning Markov Chain Models. - <i>Bisiriyu & Agunloye</i>	3.15 - 3.30 pm
11.	2.11	Efficiency Study of Presence of Non-Response on the Family of Binary-Type Estimator for Estimating Population Mean under Simple Random Sampling. - <i>S. Dahiru, B. W. Afolabi, & G. I. Onwuka</i>	3.30 - 3.45 pm

Group 1

Applied Mathematics(AM)

1.1 A Within Host Spatial Mathematical Model Investigating the Dynamics of Malaria Parasite Influence on Airway of Asthmatic Humans

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Abstract

In this work we present a spatial within-host mathematical model that explores the dynamics between the malaria parasite and airway-related processes in asthmatic individuals. Asthma, a chronic respiratory condition characterized by airway constriction, inflammation, and excessive mucus production, can be further complicated by the presence of a malaria parasite infection. To investigate this coexistence, we develop a mathematical model that captures the spatial distribution of smooth cells in airway constriction, inflamed immune cells, and mucus formation in goblet cells within the respiratory system of asthmatic individuals infected with the malaria parasite. Qualitatively, we establish the model boundedness of the positive global solution of the spatial model using appropriate theorem. Finally, we employ the finite difference scheme coded in python computational software to simulate the model, and elucidate the complex interplay between the malaria parasite and the host's immune response, examining how the parasite affects airway constriction, inflammation, and mucus formation. Our findings contribute to a deeper understanding of the pathophysiology of asthma-malaria infection and highlight the importance of considering both diseases in tandem for comprehensive management strategies.

Keywords: Model, existence and boundedness, spatial, finite difference

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-

1.2 SmartHIV Manager: The Evolution of Patient Flow Modelling from Research through Development and Innovation

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Abstract

This abstract delineates the evolution of patient flow modelling within the context of the SmartHIV Manager a comprehensive healthcare innovation tailored for HIV patient care and service management, [Adeyemi, et al].

The research phase commences with an exploration of healthcare systems specific to HIV care and management, employing established methodologies. Queueing theory is applied to analyse hospital operations, resource allocation, and patient pathways modelling [Adeyemi, et. al] within the realm of HIV patient management.

In the development phase, the integration of electronic health records (EHRs), systematic reviews, expert opinions and the implementation of simulation models tailored to HIV patient flow become pivotal. These models, rooted in discrete-event simulation, offer a dynamic representation of the HIV patient journey, facilitating scenario analyses and optimizing resource allocation with a computer environment.

The innovation phase introduces the SmartHIV Manager's unique features, notably the integration of predictive analytics. Machine learning algorithms leverage comprehensive datasets to predict HIV patient flow patterns, enabling proactive resource allocation and mitigating potential bottlenecks. Real-time monitoring systems, empowered by advanced sensors and analytics, dynamically adjust workflows to enhance efficiency in HIV care and management.

A distinctive innovation within the SmartHIV Manager is the implementation in a web application using human-centric design principles to deliver an easy-to-use, real-time planning platform. This approach goes beyond logistical considerations, ensuring that the patient experience is central to the optimization of HIV care workflows.

In conclusion, the SmartHIV Manager exemplifies the evolution of patient flow modelling, showcasing a dynamic interplay between research, development, and innovation. Continuous advancements, interdisciplinary collaboration, and a commitment to patient-centric solutions underscore the ongoing journey of optimizing HIV patient care through refined patient flow.

Keywords: Modelling, patient flow, queueing theory, discrete-event simulation, innovation

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1.3 Integration of Modified Classical Conjugate Gradient Methods for Unconstrained Optimization

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Abstract

The integration of modified classical conjugate gradient methods (CGMs) for unconstrained optimization represents a crucial and evolving area of research within the field of optimization algorithms. Over time, numerous studies have put forth diverse modifications and novel approaches to enhance the effectiveness of classical CGMs. These modifications aim to address specific challenges and improve the overall performance of optimization algorithms in unconstrained scenarios. This ongoing research endeavors to unify various classical CGMs aimed at addressing unconstrained optimization challenges while fostering a deeper understanding of their synergies. Classical CGMs have demonstrated efficacy in optimization tasks, and the strategic modification of these methods has yielded a number of diverse approaches. This paper focuses on consolidating these modified variants, specifically emphasizing those exhibiting similarities in their numerators. The integration process involves systematically merging the advantageous aspects of these modified methods to develop not only innovative but also more resilient approaches to unconstrained optimization problems. The ultimate goal of this unification effort is to leverage the strengths inherent in different approaches and create a cohesive framework that significantly enhances overall optimization performance. To thoroughly assess the efficacy of the integrated methods, a series of comprehensive performance tests are conducted. These tests include a meticulous comparison of outcomes with those of classical CGMs, providing valuable insights into the relative strengths and weaknesses of the modified approaches across diverse optimization scenarios. The evaluation criteria encompass convergence rates, and computational efficiency. The conclusive outcome demonstrates that the unified approaches consistently outperform individual methods across all evaluation criteria.

Keywords: Classical conjugate gradient methods, optimization problems, performance tests, convergence rates, computational efficiency

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1.4 Iterative Method for the Numerical Solution of Optimal Control Model for Mosquito and Insecticide

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Abstract

A linear multistep method is transformed into an iterative method based on Patade and Bhalekar technique for the numerical solution of optimal control problem modelled for mosquito and insecticide management using forward-backward sweep methods via Pontryagin's principle. Stability and convergence analysis of the iterative method is carried out and it is found to be stable, convergent and of order four. A MATLAB code using MATLAB 2018a is written for the implementation of the new iterative technique. The result obtained by the method clearly shows that, the population of mosquitoes can be minimized to a large extent using the new iterative method while reducing the harmful effect of the insecticide which subsequently reduce the spread of malaria.

Keywords: Malaria, mosquito, insecticide, optimal control problem, mathematical model.

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1.5 Optimized Hybrid One-Step Method for Efficient Direct Numerical Integration of Second-Order Initial Value Problems

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Abstract

This work develops and analyzes an efficient optimized hybrid block one step method for integrating general second-order ODE IVPs. Leveraging the strengths of hybrid and block methodologies, the method delivers an accurate implicit

numerical integrator. Further, an enhanced adaptive step-size formulation is presented, improving performance on benchmark problems.

Keywords: One step, hybrid, block method, error constant, optimization strategy, order.

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1.6 Stationarity in Prophet Model Forecast: Performance Evaluation Approach

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Abstract

Stationarity is a common assumption in time series analysis. However, the influence of stationarity on the properties and behavior of the forecasting model could be significant and sometimes cause misleading results. In this paper, the effects of stationarity on the forecast obtained from a non-stationary time series using the Prophet Model algorithm in R studio, a scalable forecasting model by Facebook based on a generalized additive model is examined and assessed. We compare the forecast obtained from the non-stationary data to the stationary version resulting from the first differencing method. The forecasts obtained

from both non-stationary data and stationary data are not significantly different from their respective actual values and also indicate strong correlation coefficients, although performance metrics results from RMSE, MAE, and coverage indicate that the Prophet model performs better with stationary data.

Keywords: Forecast, Prophet, stationarity, performance metrics

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1.7 Mises Flow Rules for Distortion Gradient Plastic Materials in Finite Deformation

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Abstract

This work presents a framework for which gradient theories of plastic materials under large deformation can be studied using the principle of maximum plastic dissipation. It is assumed that the deformation gradient admits the Kroner-Lee decomposition into a product of elastic and plastic distortions. The principle of virtual power, frame-indifference, and the free-energy imbalance are used to obtain the force balances, constitutive relations, and plastic flow laws. Rate-independent gradient theory is studied, and it is shown that the theory mimics the classical theory through the Mises flow relations for gradient materials.

Keywords: Mises flow rule, maximum plastic dissipation, large deformation, plastic flow rule, distortion gradient plasticity.

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1.8 Numerical and Statistical Analysis of the Influence of Temperature-dependent Thermophysical Properties of Tetra-Hybrid Nanofluid Along a Vertical Porous Surface With Suction

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Abstract

The impact of thermophysical properties of Tetra hybrid nanofluid cannot to overemphasized in the improvement of the rate of heat transmission in the heat exchangers, automobile, and solar storage. These features can be tampered with when nanoparticles are been mixed with the base fluid to produce an improved heat carrier fluid for the system. This study explored both numerical and statistics methods to investigate the influence of the temperature dependent thermophysical properties on dissipative movement $TiO_2 - SiO_2 - ZnO - Fe_2O_3/PAO$ Tetra hybrid nanofluid in the presence of suction along a vertical porous outward. The system of governing Partial Differential Equations (PDEs) was formulated and later transformed into the system of coupled nonlinear third order Ordinary Differential Equations (ODEs) by similarity techniques. The nonlinear third order boundary value problems of ODEs were reduced into the system of first order initial value problems, and the unknown values of initial conditions were determined by shooting approach. The fourth order Runge-Kutta method explored to solve the ODEs with the aid of Maple 18.0 software package. The impact of the thermophysical parameters of $TiO_2 - SiO_2 - ZnO - Fe_2O_3/PAO$ Tetra hybrid nanofluid on the velocity and temperature profile were displayed graphically, similarly, their effects on the skin friction coefficient and Nusselt number were shown through tables and the relationship between the independent variables (thermophysical parameters)

and the dependent variables (skin friction coefficient and Nusselt number) was analyzed by linear regression using Python 3.0 language. It was found that as variable thermal conductivity parameter upsurges both the skin friction coefficient and Nusselt number intensify at the rate of **0.011697519** and **8.043581616**. As both the viscosity and density parameters escalate the velocity profile upsurges due to the presence of buoyancy force, conversely, the temperature profile declines as variable specific heat capacity parameter increases.

Keywords: Tetra hybrid nanofluid, shooting technique, numerical method, statistics approach, thermophysical properties.

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1.9 MHD Casson Flow Over a Non-linear Convective Inclined Plate With Chemical Reaction and Arrhenius Activation Energy

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Abstract

This study examines the Magnetohydrodynamics (MHD) Casson flow passing over a nonlinear convective inclined plate with incorporating a chemical reaction and Arrhenius activation energy. However, the magneto-hydrodynamic flow of two-dimensional radiative Casson fluid (CF) across a non-linear convective inclined plate in the existence of heat generation is addressed theoretically and numerically. The Arrhenius activation energy and chemical reaction are two additional impacts that have been added to the innovative nature of the model. By applying the appropriate transformations, PDEs (partial differential equations) were converted into coupled ODEs (ordinary differential equations) in terms of similarity variables combined with the boundary conditions. The finite difference method (FDM) and MAPLE 18.0 software were used to solve the resultant equations numerically. According to the findings, the thermal Grashof number and the mass Grashof number of the nano-fluid flow model were able to improve the strength of the drag coefficient, the rate of heat transfer at the surface of the plate, and the Sherwood number. Additionally, there was a reduction in the velocity gradient as the magnetic field strength increased. The concentration decreases when a chemical reaction is present, but it improves as the activation energy rises. These findings will help engineers create devices with strong heat and mass transfer rates. The results were compared to previously published research to assess their validity and discovered a large degree of consistency.

Keywords: MHD, thermal Grashof number, mass Grashof number, chemical reaction, Arrhenius activation energy.

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1.10 An Improved Ceaser Cipher

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Abstract

The conventional Caesar cipher, traditionally reliant on alphabet letters, has undergone a modification in this research. The innovation involves the combination of both alphabets and special characters, resulting in a strong cipher text. This alteration serves a dual purpose: facilitating uncomplicated decryption for the intended recipient while presenting a complex challenge for potential intruders, by expanding the character set to encompass special characters, the cipher attains a maximum complexity. This not only enhances the security of the encoded message but also fortifies the resistance against unauthorized deciphering attempts. The strategic utilization of special characters introduces an additional layer of security, rendering the cipher less susceptible to conventional decryption methods. The primary objective remains to create a cryptographic system that strikes a delicate balance between accessibility and security. The intentional inclusion of special characters not only diversifies the encoding possibilities but also contributes to the creation of a dynamic cipher system. This adaptive approach ensures that the communication remains safeguarded against adversaries seeking to compromise the confidentiality of the information. In essence, this evolved Caesar cipher becomes a potent tool in the hands of the sender and a formidable enigma for those attempting unauthorized access.

Keywords: Ciphertext, encryption, decryption, cryptography, crypt-analysis.

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1.11 On a University Time Tabling Problem

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Abstract

The University timetabling problem is a combinatorial Optimization problem that requires the assignment of time and resources to academic events while satisfying various constraints and optimizing specific objectives. In this paper we model a university course allocation timetabling problem using Faculty of Science, University of Ibadan as a case study. We further use a python code to generate and automate the process.

Keywords: Schedule, resource allocation, heuristic timetabling, optimization algorithm, software code

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1.12 Runge-Kutta Like Method for the Solution of Optimal Control Model of Real Investment and Fish Management

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Abstract

A one step linear multistep method with highbrid point is transformed into Runge-Kutta Like Method (RKLM) using Daftardar-Gejji and Gafari series, based on Patade and Bhalekar approach. Stability properties of the RKLM is tested and found to be stable and convergent. Forward-backward sweep algorithms for the RKLM are written, and MATLAB code for the implementation of the RKLM also written. The RKLM is used to solve physical optimal control problems on investment and fish management using the forward-backward sweep algorithms via Pontryagin's principle. The results obtained in the first problem shows that, as the investment decreases, the capital first increase in order to increase production before the capital depreciate. The result obtained in the second problem shows that, when the weight parameter of the fish is higher, the harvesting rate get to zero faster, and the total fish mass reach maximum level quicker. The results obtained show that, forward-backward sweep methods together with RKLM can effectively solve optimal control problems.

Keywords: First Boubaker polynomials, fish, investment, model, optimal control problem.

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1.13 On the Differential Equations of Artificial Intelligence Systems: Ifa Divination Corpus

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(being Abstract of paper intended for presentation at the conference organized by the Department of Mathematics, Obafemi Awolowo University, Ile-Ife, Nigeria in honour of four retiring senior academic staff, 20th February, 2024. Theme: Mathematical Sciences: An Indispensable Trajectory to Sustainable Development).

Abstract

(Dedicated to all my past teachers in the Department of Mathematics, Obafemi Awolowo University, Ile-Ife, Osun State, Nigeria.)

The (qualitative) theory of differential equations (TDE) is one of the most applicable areas of mathematics. Of particular importance is the concept of autonomy of differential equations (DE). This may be described, for instance, by the first order autonomous ordinary DE with a polynomial nonlinear part given by (Oluwade, 1999; Oluwade and Afuwape, 2004)

$$x' = f(x) = \sum a_i x^i \quad i = 0, 1, 2, \dots, n-1 \quad \text{and} \quad a_{n-1} \neq 0. \quad (1)$$

In artificial intelligence (AI), autonomy plays a central role with respect to intelligent agents (Russell and Norvig, 1995). Among others, the concept of autonomy is used to describe the operational performance of batteries in engineering systems such as solar energy applications. Ifa Divination Corpus (IDcorp) presents an interesting synergy between TDE and AI. From the perspective of AI, IDcorp is a natural language processing (NLP) system (Thanaki, 2017; Oluwade, 2003). And from the lens of the qualitative TDE, the books (technically called 'odus' or 'odu ifa') of IDcorp can be represented by a combination of the set $S = \{\text{Attractor } (A), \text{Repellor } (R), \text{Positive Shunt } (P), \text{Negative Shunt } (N)\}$, which is the set of phase portraits of equation (1) when $n = 2$. That is, each of the 256 odus can be modelled as a phase portrait when $n = 8$. IDcorp practice was conceived and codified by Orunmila, a Yoruba pioneer philosopher and spiritual leader. IDcorp also serves as the origin of the Yoruba counting system - the Yoruba vigesimal number system (base 20), as attested to by the first book, called Eji Ogbe (Longe, 2009A; Longe, 2009B; Abimbola, 1975; Abimbola, 1977A). There is a 1 – 1 correspondence between the divinities and the 256 books. There are 16 major odus and 240 (i.e. $16P_2$) minor odus. Every book has thousands of chapters where each chapter is a poem. The typical Ifa divination process via the divining chain technique has four sequential stages, namely: casting of divining chain, statement of the signature, reading of relevant poem and interpretation (Longe, 1983; Oluwade and Longe, 2003; Abimbola, 1977B). In an earlier paper (Oluwade and Longe, 2003), the authors presented a representation of IDcorp in terms of the discrete structure of computer science. That is, the algebraic characteristics of the IDcorp was shown to have some similarities with the 8-bit Extended Binary Coded Decimal Interchange Code (EBCDIC) of systems programming, of order 256 (Oluwade and Longe, 2003; Oluwade, 2004). In a recent study (Olagunju *et al.*, 2023), the authors presented additional properties of IDcorp. It was shown, among others, that the set of 16 major odus forms an abelian group under matrix addition mod 2. In the present paper, aspects of the AI representation of IDcorp are presented from the perspectives of the qualitative TDE. For example, following the work of Oluwade (2005), it is shown that one of the odus is the phase portrait x_1, x_2, \dots, x_n defined by ANNRPAR, where $x_i \in \{A, R\}$ if x_i is a convex side of a half nut of the divining chain and $x_i \in \{P, N\}$ if otherwise, where $i = 1, 2, \dots, 8$.

Keywords: Differential equations, artificial intelligence, ifa divination corpus, EBCDIC code, phase portrait.

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1.14 Bifurcation Analysis of the Impact of Prep on HIV/AIDS Dynamics With Inflow of Infectives Immigrants

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Abstract

To investigate the impact of Pre-exposure prophylaxis (PrEP) on the dynamical spread of HIV/AIDS with direct influx of HIV-infected immigrants in the population, a deterministic mathematical model comprising of six compartments, incorporating PrEP and the presence of infective immigrants was proposed. However, as long as infective immigrants continue to enter the population, achieving a disease-free equilibrium becomes unattainable. Consequently, the model possesses only the endemic equilibrium which also exhibit a

forward bifurcation. The global asymptotic stability of the endemic equilibrium was confirmed through the application of a quadratic Lyapunov function and LaSalle's invariance principle. The findings indicate that the disease persists when there is a direct influx of infective immigrants. Further numerical simulations demonstrate that despite the influx of infective immigrants, PrEP can mitigate the spread of HIV transmission in the population with individuals? migrants who are infected.

Keywords: Pre-exposure prophylaxis, effective immigrants, forward bifurcation, Lyapunov function.

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1.15 Role of Magnetic Field and Heat Source/Sink on Fluid Flow in a Channel

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Abstract

This paper presents analytical study of the combined effect of magnetic field and heat source/sink on fully developed natural convection flow in a channel. The governing equations are obtained and solved for temperature, velocity, Nusselt number and skin friction by perturbation method. Result shows that temperature increases for positive values of heat source parameter as a result of increased kinetic energy of the molecules. velocity profile is observed to decreased with increasing value of magnetic field parameter.

Keywords: Heat source, perturbation, natural convection, magnetic field.

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1.16 Legendre Collocation Method for Six-order Boundary Value Problems

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Abstract

The study aims to employ Legendre polynomials as basis functions for numerically solving six-order. The goal of the study is to use Legendre polynomials as basis functions to numerically solve six-order Boundary Value Problems(BVPs). Using collocation points, the method transforms the differential problem into a set of algebraic equations, which are subsequently resolved by matrix inversion. Three test problems are discussed for validation. The proposed method yields results that are accurate and quite close to the exact solution. Additionally, the accuracy and effectiveness of the procedure are described using tables and figures. In comparison to other methods in the literature, the results obtained using the proposed method demonstrate that it is more accurate and effective.

Keywords: Legendre polynomials, boundary value problems, collocation approach, approximate solution, absolute error.

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1.17 Modelling and Simulation of Drilling Mud Rheological Properties

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Abstract

Drilling mud and oil-based fluids play a crucial role in drilling operations. Second-grade fluid model can help simulate and analyze the flow behaviour of drilling mud under different conditions aiding design of drilling operations and selection of appropriate mud formulations. This study, therefore, aims at analyzing the flow and heat transfer phenomena as well as the interplay between radiation effects and viscous dissipation on the drilling mud as it flow around the drilling pipe in the wellbore wall. We described a non-linear radiation effect and viscous dissipation, which can result in intricate temperature and viscosity profiles that further influence the flow characteristics of the mud. The second-grade fluid flow was modeled with a system of non-linear coupled partial differential equations, which was non-dimensionalized and reduced to a set of nonlinear coupled ordinary differential equations using stream functions and series approximation. The numerical solution to the governing equations was obtained by the standard trapezoidal technique on the Maple Platform. An increasing influence of radiation in the wellbore was found to increase the flow(velocity) and temperature of the mud. While, a quantitative increase in the dissipation effect marginally affects the flow and temperature profiles. Comparisons of the obtained results with existing reports in the literature were also examined.

Keywords: Drilling mud, non-linear radiation, second-grade fluid, numerical solution, natural convection flow, viscous dissipation.

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1.18 Path Planning of Multi-Rotor UAVs via Enhanced A^* Particle Swarm Optimization Algorithm

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Abstract

In this work, we studied how practical high-quality flight paths are crucial for the effective functioning of an Unmanned Area Vehicle (UAV). We proposed an enhanced path planning algorithm for multi-rotor UAVs using a combination of the A^* algorithm and Particle Swarm Optimization (PSO). Through the A^* algorithm we obtain an initial path for UAVs based on heuristic search by leveraging a combination of information about the current state of the UAV, known as the “start” state, and the desired goal state. Then PSO optimizes the initial path by considering various factors, such as the desired objectives (e.g., obstacle avoidance, energy efficiency, smoothness), constraints, and performance metrics. By iteratively refining the path based on these factors, the PSO algorithm aims to find an optimized path that exhibits improved quality and performance compared to the initial path generated by the A^* algorithm. The simulation and comparison of the enhanced A^* PSO is conducted using MATLAB R2007b and the path planning results show that the algorithm outperforms other path planning algorithms. The optimized path of multi-rotor UAVs obtained through this approach contributes to advancing the capabilities and effectiveness of UAV systems in various practical scenarios.

Keywords: A^* algorithm, multi-rotor UAVs, particle swarm optimization, unmanned area vehicle.

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1.19 Axial Force Influence on Dynamic Response to Moving Load of Shear Beam Resting on Elastic Foundation

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Abstract

Aim: The aim of this study is to determine the influence of axial force on the deflection of shear beam resting on elastic foundation when traversed by moving load.

Method: The governing equations are simultaneous partial differential equations and the main objective of this study is to obtain a closed form solution to this class of dynamical problem. In order to obtain this solution, the method of integral transformation was used. In particular, finite Fourier series was used to reduce the equations to coupled second order ordinary differential equations. Thereafter, the simplified simultaneous equations were then solved by Laplace transformation in conjunction with convolution theory to obtain the solution.

Result: The closed form solution obtained was analyzed to obtain the effects of axial force and foundation stiffness on the response of shear beam when under the action of the moving load and the results were displayed in plotted curves. Analysis in this study revealed that the response amplitude of the shear beam decreases with increasing values of axial force. The same result and analysis were obtained when values of foundation stiffness were increased.

Conclusion: The axial force influence on dynamic response to moving load of shear beam resting on elastic foundation has been examined. This study has provided vital information on the effects of axial force and foundation stiffness on the response of shear beam resting on elastic foundation and under moving load. From the graphs, interesting results were obtained. Increase in the values of structural parameters reduces the transverse displacement of the Shear beam. Practically speaking, increase in the values of structural parameters reduces the resonance risk of the vibrating system.

Keywords: Axial force, shear beam, moving load, elastic foundation.

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1.20 Sensitivity Analysis of Pertussis Transmission Dynamics

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Abstract

Pertussis generally known as whooping cough is an airborne infectious disease which spreads through respiratory droplets and vaccine preventable. Whooping cough primarily affects infants and young children. Due to the disruption in basic routine immunization programs caused by the recent COVID-19 pandemic, the disease reemergence stands as a double burden and a significant global health concern for both developed and developing countries. In this paper, a comprehensive analysis of pertussis transmission dynamics is presented via a five (5) non-linear mathematical model. The developed model considers the impact of Vaccination, Treatment and maternally derived immunity on disease spread. The stability of the systems equilibrium point was analyzed and the basic reproduction number R_0 obtained using the next generation matrix method. The Normalized forward sensitivity index analysis for R_0 was conducted to find the most crucial parameters to be targeted upon for disease intervention strategy. MATLAB 2021a was used to carry out numerical simulations to validate theoretical findings, corresponding results interprets that periodic vaccination from birth, disease treatment in adults, vaccination of pregnant women will specifically lower the basic reproduction number and in turn cushion or eradicate disease spread.

Keywords: Pertussis, steady state, basic reproduction number, sensitivity analysis.

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1.21 A Novel Approach to Environmental Monitoring With Fused Satellite Data Using Sparse Convolutional Neural Network

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Abstract

Monitoring tailing dams and water reservoirs for structural damages is crucial for developing risk assessment models and emergency response plans to better prepare for potential disasters or incidents. Remote sensing data has proven highly effective in monitoring water leakage and dam movement. The change in vegetation levels using spectra indices is a reliable indicator of limited or excessive water. This paper presents a novel technique for spatio-temporal data modelling that integrates Sparse Convolutional Neural Networks (SCNN) and Long Short-Term Memory (LSTM) networks. Bayesian optimization is used to optimize the model. The model is designed to efficiently capture spatial dependencies in data while preserving computational resources through sparsity. The memory-retaining capabilities of LSTM networks are used to address temporal correlations. The hybrid method was applied to Normalized Difference Vegetation Index (NDVI) data derived from multisensor data fusion techniques for selected reservoirs across the UK. Our empirical results show that the proposed methods are efficient and accurate for predicting and identifying short and longer-term changes in vegetation. This information is useful for monitoring dam structures to ensure their stability and prevent catastrophic failures.

Keywords: Bayesian optimization, cloud masks, water reservoirs, NDVI, SCNN-LSTM model.

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1.22 Lyapunov Stability and Sensitivity Analysis of a Recurrent Malaria Dynamics

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Abstract

In this study, a mathematical model of malaria featuring recrudescence, relapse, and reinfection is presented and analysed. The well-posedness of the governing model is established through the theory of positivity and boundedness of solution with Lipschitz condition. The model exhibits backward bifurcation property in the presence of reinfection. However, global asymptotic dynamics of the model around equilibrium points in the absence of reinfection are examined through suitably developed Lyapunov functions. In addition, the influence of model parameters on the basic reproduction number is investigated using the normalized forward sensitivity index.

Keywords: Mathematical model, recurrent malaria, reinfection, backward bifurcation, Lyapunov function, sensitivity analysis.

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Group 2

Mathematical Statistics and Computing(MSC)

2.1 Volatility Modeling and Forecasting using Range-based GARCH Models

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Abstract

This paper compares forecast performance of symmetric and asymmetric GARCH models with symmetric and asymmetric range-based GARCH models. Of particular interest is investigation of whether inclusion of range in the conditional variance equation has significant impact on forecast performance of the range-based GARCH models. The models examined in this paper are GARCH(1,1), TAR(1,1), RGARCH(1,1), and RTAR(1,1) models. The forecast performance of these models is evaluated by several loss functions. Using range-based GARCH models, the results of data analysis showed that allowing for a leptokurtic error distribution leads to significant improvements in volatility forecasts compared to using the normal distribution. This result holds for daily, weekly as well as monthly forecast horizon.

Keywords: Volatility, conditional variance, range-based GARCH, leptokurtic distribution, loss function and forecast horizon

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2.2 A New Improved Estimator For Population Distribution Function Using Auxiliary Variable Under Simple Random Sampling

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Abstract

To gain insight into various phenomena of interest, cumulative distribution function (CDF) can be used to analyze survey data. The purpose of this study was to present an efficient estimator for estimating a population cumulative function using auxiliary variable under simple random sampling. The properties of the proposed estimator bias and mean square error (MSE) were derived up to first order degree of approximation using Taylor's series expansion. The proposed estimator was compared theoretically and empirically with the adjusted estimators in the literature. The proposed estimator was found to be better than adjusted estimators based on the percentage relative efficient (PRE) gain and MSE under some condition.

Keywords: Auxiliary variable, simple random sampling, cumulative distribution function.

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2.3 Mathematical Approaches to Enhance Energy Efficiency in Construction for Sustainable Development in Nigeria

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Abstract

This research addresses the imperative of integrating mathematical methodologies to optimize energy efficiency within the Nigerian construction sector, aligning with the broader goals of sustainable development. The introduction delineates the urgency of mitigating energy consumption in construction and underscores the potential impact of mathematical models on enhancing sustainability. The research objectives are framed to develop robust mathematical models, identify key variables influencing energy efficiency, integrate mathematical approaches into decision-making, evaluate environmental and economic impacts, and provide actionable recommendations for sustainable development in Nigerian construction. A comprehensive literature review examines existing research, emphasizing the significance of sustainable construction practices, the impact of materials and design on energy efficiency, and the need for interdisciplinary approaches. The integration of mathematical modeling in construction is explored, highlighting its potential to optimize various facets of the construction lifecycle. The methodology employs primary data

collection through surveys and interviews with diverse stakeholders, while the instrumentations include structured questionnaires and interview guides. Descriptive statistics are employed to analyze the collected data, incorporating measures of central tendency and dispersion to provide a quantitative overview of energy efficiency practices. Findings indicate a collective recognition of critical factors influencing energy efficiency, challenges in current construction practices, and a consensus on the importance of specific design features. Stakeholders demonstrate a high awareness of sustainable practices, and challenges in implementation are acknowledged. The conclusion synthesizes the research findings, emphasizing the importance of addressing challenges, promoting interdisciplinary collaboration, and advocating for sustainable practices. The recommendations propose actionable steps, including promoting collaboration, enhancing education, integrating mathematical models, incentivizing sustainable practices, and advocating for supportive policies. In essence, this research contributes valuable insights to the discourse on sustainable development in the Nigerian construction sector. By leveraging mathematical approaches, stakeholders have the potential to usher in a transformative era of energy-efficient practices, aligning with global aspirations for a more sustainable future.

Keywords: Mathematical approaches, enhance energy efficiency, sustainable development in Nigeria

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2.4 Model for the Prediction of Cholera Incidence

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Abstract

Cholera remains a significant health challenge in Nigeria, particularly in North - western Nigeria, where poor sanitation, contaminated water sources, and limited healthcare access contribute to its spread. This paper investigates the modeling and forecasting of the cholera cases in one of the hospital in Kebbi State, North-western Nigeria over five years. Quarterly data, from 2002 to 2019 was analyzed. The Box and Jenkins method was used. Stationarity was confirmed through KPSS and ADF tests. ARIMA model (2, 1, 2) was chosen based on the lowest AIC, indicating its suitability for predicting cholera cases. The study reveals a closer relationship between mean, median, maximum, and minimum values from the first quarter of 2011 to the fourth quarter of 2020.

The ARIMA (2, 1, 2) model predicts a gradual increase in cholera cases in the next two years. This research emphasizes the importance of sanitation improvements and hygiene education to mitigate cholera outbreaks in Kebbi State.

Keywords: Cholera, ARIMA, prediction.

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2.5 Backtesting Methods for Evaluating the Viability of Stock Prices in Nigeria

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Abstract

Backtesting is a method that is utilized to evaluate the effectiveness of a trading strategy or forecasting strategy by applying it to data that has been collected in the past. Backtesting is a technique that is used to analyze how well a model or strategy would have performed on historical data. This evaluation is done with the purpose of assisting analysts and researchers in comprehending the possible usefulness of the model or strategy in everyday situations. The purpose of this article is to evaluate the viability of the stock prices of a few Nigerian banks by combining the stylized GARCH model with the Long-short term memory (LSTM) model. On the basis of the Box-Jenkins method for analyzing time series, the models were identified, the parameters were estimated by employing the method of maximum likelihood, the models were evaluated by utilizing their mean square errors, the models were utilized to forecast, and the forecasts were evaluated by employing the backtesting methods.

Keywords: GARCH models, long-short term memory (LSTM), backtesting, value at risk (VaR), expected shortfall (ES).

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2.6 Digital Distraction and Personality Trait as Correlate of Knowledge Retention of Undergraduate Mathematics in Tertiary Institutions in Ondo State

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Abstract

This study examines the relationship between digital distractions, personality traits as correlates of knowledge retention of Mathematics undergraduates in Nigerian Universities. The study adopted a descriptive research design of the survey type, the population of the study comprised of 175 undergraduates who were selected from Tertiary Institutions in Ondo State using simple random sampling technique. Data collected for this study was analyzed using descriptive and inferential statistics such as frequency counts, percentages and mean and Regression analysis. The result of the study revealed that there is high level of digital distraction with a mean score of ($= 3.45$) and high level of knowledge retention with a mean score of ($= 2.96$) among mathematics undergraduates in Tertiary Institutions in Ondo State. The level of openness records a mean of ($= 3.27$), conscientiousness ($= 3.35$), agreeableness ($= 2.85$), extraversion ($= 2.75$) and emotional instability ($= 2.48$) was high among undergraduates in Tertiary Institutions in Ondo State. Furthermore, the study revealed that there is a relationship between digital distraction and knowledge retention of mathematics undergraduates in Tertiary Institutions in Ondo State [$F(174) = 20.146; p_i 0.05$]. There is no significant relationship between personality traits and knowledge retention of mathematics undergraduates in Tertiary Institutions in Ondo State [$F(174) = 0.026; p_i 0.05$]. Based on the findings of the study, It was recommended among others that undergraduates are encouraged to minimize the number of times they spend on their

digital devices as this would help improve their knowledge retention and boost their confidence to embrace any mathematical challenges.

Keywords: Digital distraction, personality traits, knowledge retention, undergraduates.

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2.7 Omiyale: Sounding the Mathematics of the Rain and Flood Disaster in Ibadan City, Nigeria

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Abstract

Research on Yoruba popular music has tended to draw attention to its interest in romance and the accumulation of wealth. Inadequate attention has been paid to its engagement with the environment. Ibadan, the capital of Oyo State in Nigeria has witnessed perennial flood disasters, and this has been a concern of Yoruba musicians. This article is an ethnographic investigation of the place of music in the rain and flood disasters of Ibadan as addressed in the songs of Yoruba musicians, Kollinton Ayinla, Foyeke Ajangila, Ebenezer Obey and Agbada Owo. And it is based on ecomusicology theory. The article describes the connections between the volume and measures of rain fall, the cultural past, when indigenous knowledge was respected and flooding was avoided, and the present, when it was forgotten, and flooding became a constant. Based on the very specific, local context of Ibadan, I argue that calls for the return of indigenous knowledge as propagated in popular music could be a way forward

in environmental crises currently experienced around the globe.

Keywords: Omiyale, environmental degradation, flood and rain, popular music, performance studies, measurements.

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2.8 Semiparametric Lognormal Shared Frailty Modeling of Diabetes Patients with Acute Coronary Syndrome

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Abstract

Modelling and estimation of common risks in lifetimes of individuals with similar features say organs, medical centers, and in short, clusters are best done using shared frailty models. A number of publications have been devoted to estimation methods in shared frailty but few to the use of pseudo-full-likelihood(PFL) method of estimation and application into the patients and acute coronary syndrome. This study aims to model the survival time as well as obtain cluster-specific frailty starting with a simulation study based on semi-parametric lognormal shared frailty distribution using the PFL method and obtaining a prediction interval for a random effect. To explore this further, we used data from UK Biobank to investigate the existence of shared frailty in diabetes patients characterized by hypoglycemia and history of ACS (STEMI and NSTEMI) and also to see how the observed variabilities affect the hazards. The results suggest the presence of frailty influence within the clusters and indicate cluster time dependence in the study population.

Keywords: Acute coronary syndrome, diabetes mellitus, lognormal frailty model, pseudo-full-likelihood, shared frailty model.

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2.9 A Comparative Analysis of Option Pricing Models: Assessing Their Applicability in Valuing European Options

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Abstract

In this work, we did a comparative analysis of the Black-Scholes and Binomial Option Pricing models to evaluate their effectiveness in valuing European options. The comparative analysis involves an examination of the performance of both models under various market conditions, interest rates, and time to maturity. Historical market data is employed to simulate pricing outcomes and rigorously assess the accuracy of each model's predictions. Furthermore, the research explores the practical implications of the findings for investors and option traders. It investigates which model performs more reliably in different market scenarios, offering insights into when and how these models should be employed for optimal decision-making in option trading and risk management. By the conclusion the results shows that the error incurred by the Binomial model is small, therefore makes the Binomial model good for pricing European options. This will help with equipping readers to make more informed choices when valuing European options. This work contributes to the ongoing discourse in the financial industry concerning the refinement and adaptation of option pricing techniques to suit dynamic market conditions.

Keywords: Black-Scholes option model, Binomial option pricing model, European option.

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2.10 Prediction of Covid-19 Pandemic Using Machine Learning Markov Chain Models

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Abstract

The Covid-19 is a global pandemic, which swept across the world from 2019 to 2020. It presented a severe threat to mankind, creating a perilous situation leading to economic downturn throughout the period it lasted. Numerous researchers explored different mathematical models to study the dynamics of the pandemic. The present study employs a Markov chain model, based on a 5-element state space for the highest wave of Covid-19 and a 4-element state space for the lowest wave. The objective of this paper is to examine predictive ability of Markov chain model for Covid-19 spread using both the highest and lowest waves of the pandemic. We demonstrated that a machine learning Markov chain model provides a robust and accurate prediction for Covid-19 spread.

Keywords: Covid-19, Markov Chain, state space, machine learning technique, prediction.

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2.11 Efficiency Study of Presence of Non-Response on the Family of Binary-Type Estimator for Estimating Population Mean under Simple Random Sampling

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Abstract

In statistical estimation processes, accurate estimation of parameters is crucial. Incorporating valid auxiliary information can improve estimators by considering correlated variables. This study focused on binary type estimators for predicting population mean under SRSWOR in scenarios with non-response. The Mean Square Error (MSE) and Bias characteristics of the proposed estimators were derived, along with identifying the optimal MSE. The proposed family of estimators exhibited higher efficiency compared to existing estimators, as supported by both theoretical and empirical investigations.

Keywords: Auxiliary variable, bias, mean square error, binary-type estimator, non-response.

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Group 3

Pure Mathematics(PM)

3.1 Hyers-Ulam-Rassias Stability of the Nonlinear Second-Order Differential Equations With Forcing Term

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Abstract

In this paper, we consider the Hyers-Ulam-Rassias stability of nonlinear second-order differential equations with a forcing term. By using the Bihari inequality and the Gronwall-Bellman-Bihari integral inequality, we obtain new criteria for the Hyers-Ulam-Rassias stability of the considered nonlinear second-order differential equations.

Keywords: Hyers-Ulam-Rassias stability, second-order differential equations, the Bihari inequality, the Gronwall-Bellman-Bihari integral inequality.

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3.2 Strongly Continuous Semigroup on Decoherence-Free Subalgebra of Quantum Markov Semigroup

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Abstract

In this work, we shall be concerned with strongly continuous semigroup on decoherence-free subalgebra of quantum Markov semigroup. A derivation for the infinitesimal generator of decoherence-free subalgebra was first established. The derivation was then used to establish a theorem for the characterization of a strongly continuous semigroup on decoherence-free subalgebra of quantum Markov semigroup.

Keywords: Strongly continuous semigroup, decoherence-free subalgebra, quantum Markov semigroup, infinitesimal generator.

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3.3 Analysis of Weak Associativity in Some Hyper-Algebraic Structures That Represent Dismutation Reactions

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Abstract

In this paper, some chemical systems of Tin (Sn), Indium (In) and Vanadium (V) which are represented by hyper-algebraic structures (S_{Sn}, \oplus) , (S_{In}, \oplus) and (S_V, \oplus) were studied. The analyses of their algebraic properties and the probabilities of elements in dismutation reactions were carried out. It was shown that in the dismutation reactions, the left nuclear (N_λ) -probability, middle nuclear (N_μ) -probability and right nuclear (N_ρ) -probability for each of the hyper-algebraic structures (S_{Sn}, \oplus) , (S_{In}, \oplus) and (S_V, \oplus) is less than 1.000. This implies that, (S_{Sn}, \oplus) , (S_{In}, \oplus) and (S_V, \oplus) are non-associative hyper-algebraic structures. Also, from the results obtained for FLEX-probability, it was shown that, (S_{Sn}, \oplus) , (S_{In}, \oplus) and (S_V, \oplus) have flexible elements because the values of their FLEX-probabilities are 1.000 each. Hence, (S_{Sn}, \oplus) , (S_{In}, \oplus) and (S_V, \oplus) are flexible.

Keywords: Hypergroup, polygroup, polyloop.

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3.4 On the Extension of Γ_1 -Nonderanged Permutation Group

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Abstract

A Γ_1 -non deranged permutation group $\mathcal{G}_p^{\Gamma_1}$ ($p \geq 5$ and $p = \text{prime}$) is a permutation group such that $\mathcal{G}_p^{\Gamma_1} = \{\omega_i | 1 \leq i \leq p-1\}$ where $\omega_i = ((1)(1+i)_{mP} (1+2i)_{mP} \dots (1+(p-1)i)_{mP})$. In this paper, an extension of $\mathcal{G}_p^{\Gamma_1}$ denoted by $\mathbb{G}_p^{\Gamma_1}$ is given in such a way that $\omega_p \in \mathcal{G}_p^{\Gamma_1}$ and an operation of addition '+' and multiplication '.' is define on the extended version $\mathbb{G}_p^{\Gamma_1}$ to show that $\mathbb{G}_p^{\Gamma_1}$ together with either addition '+' or multiplication '.' is an Abelian group. Also, the extended version $(\mathbb{G}_p^{\Gamma_1}, '+', '.')$ is both a commutative ring with identity and a vector space.

Keywords: Group, permutation group, nonderanged permutation group, commutative ring, vector space.

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3.5 On the Uniform Stability and Ultimate Boundedness of Solutions to a Class of Second-order Neutral Functional Differential Equations with Delay

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Abstract

This paper presents criteria for the asymptotic stability of the zero forcing term and, for a nonzero forcing term, boundedness of solutions to a class of neutral functional differential equations of order two using the direct method of Lyapunov. The equation is first reduced to a system of first-order differential equations, and then an appropriate Lyapunov's functional is obtained. This functional is used to establish conditions that ensure stability, uniform asymptotic stability of the zero solutions, and, for the nonzero forcing term, uniform boundedness and uniform ultimate boundedness of solutions for the discussed nonlinear neutral differential equation with delay. Special cases are provided to demonstrate the credibility and dependability of these results.

Keywords: Uniform stability, boundedness, second-order, neutral differential equation, Lyapunov functional.

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3.6 On Distributional Henstock-Kurzweil-Stieltjes- \diamond -Integral for Inner Product space-valued functions on Time scales

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Abstract

In this paper, the concept of time scales calculus is introduced in the study of distributional Henstock-Kurzweil-Stieltjes integral in inner product space. Some of the basic properties of the distributional Henstock-Kurzweil-Stieltjes- \diamond -integral for inner product space-valued functions on time scales are stated and proved. Applications of fixed point theorems and Volterra integral equation are studied on time scales.

Keywords: Distributional, Henstock integral, Kurzweil integral, Stieltjes integral, inner product space, time scales.

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3.7 On the Characterization of Some Variants of Inverse Properties in Conjugate Loop

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Abstract

A conjugate loop is a two-sided loop that obeys the identity $x(yx^{-1}) = (xy)x^{-1}$. In this work, necessary and sufficient(s) conditions for conjugate loop to possess variants of inverse property (left, right, weak, cross, automorphic, anti-automorphic) were established.

Keywords: Left, right, weak, cross, automorphic, anti-automorphic inverse properties, conjugate loop

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3.8 On Semi-symmetric (α, β, γ) -inverse Quasigroup

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Abstract

A quasigroup (Q, \cdot) will be called an (α, β, γ) -inverse quasigroup, if there exist fixed permutations α, β and γ of Q , such that $(x \cdot y)\alpha \cdot x\beta = y\gamma \forall (x, y) \in Q \times Q$. Examples were given to illustrate that a quasigroup can have more than one (α, β, γ) -inverse property. Consequently, for a set Δ_Q of such triples, it was shown that if the semi-symmetry law holds in (Q, \cdot) , it induces a binary operation on Δ_Q for which Δ_Q is a group. Interestingly, this leads to an isomorphism between Δ_Q and the autotopism group of (Q, \cdot) .

Keywords: Weak, cross, m -, (r, s, t) -, (α, β, γ) -, inverse properties, semi-symmetric.

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3.9 Partial Differential Formulation of Covid-19

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Abstract

Inspired by the second Fick's law in physics and following Fisher's equation, we formulate a reaction-Diffusion Partial Differential Equations(PDE) Model to predict the Spatio-Temporal spread of the Covid-19 epidemic. The formulated seird model has the condition that the susceptible class is subject to logistic growth in the absence of the disease and on introduction of a time-delay from the susceptible class s to the infective class i , seird is transformed to an sird model which is then analyzed. First we prove the existence and boundedness of solutions and investigated their long-time behaviour, and show that the trivial equilibrium E_0 is always unstable, also that for $R_0 < 1$ and $R_0 \leq 1$ that the disease free equilibrium E_f is locally and globally stable asymptotically stable respectively, finally for $1 < R_0 \leq 3$ and $R_0 > 1$ that the endemic equilibrium is locally and globally asymptotically stable respectively.

Keywords: Susceptible class, infective class, long-time behaviour, equilibrium.

3.10 On the Operator Fourier Transform of the Harish-Chandra Schwartz Algebra

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Abstract

We establish a K -type decomposition of the Harish-Chandra Schwartz algebra $\mathcal{C}^p(G)$, for any real-rank 1 reductive group G with a maximal compact subgroup K and $0 < p \leq 2$. This decomposition is then used to give an infinite-matrix-realization of the operator Fourier image $\mathfrak{F} : \mathcal{C}^p(G) \rightarrow \mathcal{C}^p(\hat{G})$ of $\mathcal{C}^p(G)$ as a Frechet multiplication algebra in which every member of $\mathcal{C}^p(\hat{G})$ consists of a countable block-matrices of the form

$$((\mathfrak{F}_B(\vec{\alpha})_{(\gamma,m)}(\Lambda) \otimes \mathfrak{F}_H(\vec{\alpha})_{(\gamma,l)}(Q : \chi : \nu))_{\gamma \in F, (l,m) \in \mathbb{Z}^2})_{FC\hat{K}, |F| < \infty}$$

for every $\alpha \in \mathcal{C}^p(G)$. This proves Trombi's conjecture for G of real rank 1 and the technique leads to a proof of the fundamental theorem of harmonic analysis for any arbitrary real-rank reductive group G .

Keywords: Reductive groups, Harish-Chandra Schwartz algebra, operator Fourier transform, K -type decomposition.

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3.11 Existence of Fixed Points via C^* –Algebra-Valued Simulation Functions

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Abstract

In this paper, the ideas of C^* –algebra-valued simulation functions and some corresponding fixed point results are discussed. It is noted that the notions studied in this work unify, complement and generalize a handful of related concepts in the literature.

Keywords: C^* –algebra, C^* –algebra-valued metric space, C^* –algebra-valued simulation function, fixed point.

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3.12 On Fixed Point Results of Hybrid F –contractive Operators

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Abstract

The survey of available literature shows that most fixed point results in generalized metric spaces correspond with their equivalent ones in metric or quasi-metric spaces. To provide an interesting research in these spaces, this paper employs a recent method for analysing fixed point findings in G –metric space to introduce a novel concept called Jaggi-type hybrid $(\phi - F)$ –contraction and establish some fixed point results for this class of contractions in the framework of generalized metric space. Some special cases of the results obtained herein with respect to the corresponding literature are highlighted and discussed.

Keywords: F –contraction, G –metric, fixed point, hybrid contraction.

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3.13 On Fixed Point of Quasi Contraction With Application to Integral Equation

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Abstract

It is observed from the surveyed literature that there is no sufficient study of quasi-weakly contractive operators in the context of b -metric-like spaces. From this background information, this paper introduces a new unified notion of quasi-weakly contractive operator in b -metric-like space and examines the existence and uniqueness of invariant points of such operators. The idea put forward herewith subsumes a few known results in the literature. A few corollaries which reduce our findings to other famous ideas are presented and discussed.

Keywords: Fixed point, b -metric-like, integral equation.

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3.14 Fixed Point Results of Hardy-Rogers-Type Contraction in Generalized Metric Space via Interpolation

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Abstract

In this manuscript, a new concept of interpolative contraction, namely interpolative Hardy-Rogers-type (G - μ)-contraction is introduced and some fixed point results in generalized metric space that are not deducible from their akin in metric space are obtained. The preeminence of this class of contractions is that it complements and subsumes a few corresponding notions in the literature. Consequently, substantial examples are constructed to validate the assumptions of our obtained theorems and to show their distinction from corresponding results.

Keywords: G -metric, fixed point, Hardy-Rogers contraction, interpolation.

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3.15 On Stability, Controllability and Feedback Control of Nonlinear Autonomous Systems

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Abstract

We review the Lyapunov - Razumikhin method employed in the theory of Stability and Controllability of Nonlinear autonomous systems. The results of the stability analysis were obtained using the Lyapunov indirect approach to examine the local stability of the equilibrium point using Jacobian linearization.

Keywords: Controllability, feedback control, Lyapunov indirect method.

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3.16 On the Full Scalar-Valued Bochner Theorem on Real Reductive Groups

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Abstract

The major results of Barker leading to the spherical Bochner theorem and its (spherical) extension, were possible through the spherical transform theory of Trombi-Varadarajan and were greatly restricted by the non-availability of the full (non-spherical) Harish-Chandra Fourier transform theory on a general connected semisimple Lie group, G . Sequel to the recently announced results of Oyadare where the full image of the Schwartz-type algebras, $\mathcal{C}^p(G)$, under the full scalar-valued Fourier transform is computed as $\mathcal{C}^p(\widehat{G}) := \{(\widehat{\xi}_1)^{-1} \cdot h \cdot (\widehat{\xi}_1)^{-1} : h \in \bar{\mathcal{Z}}(\mathfrak{F}^\epsilon)\}$ with $\bar{\mathcal{Z}}(\mathfrak{F}^\epsilon)$ given as the Trombi-Varadarajan image of $\mathcal{C}^p(G//K)$ ($0 < p \leq 2$), the present paper gives the full scalar-valued Bochner theorem for G by lifting the earlier results of Barker to full non-spherical status. An extension of the Bochner theorem to all of $\mathcal{C}^p(G)$, $1 \leq p \leq 2$, is then established. It is also conjectured that every positive-definite distribution T on G which corresponds to a Bochner measure μ on \mathfrak{F}^ϵ extends uniquely to an element of $\mathcal{C}^p(G)'$ if and only if T can be expressed as a finite sum of derivatives of a class of functions exclusively parameterized by members of \mathfrak{F}^ϵ and $\text{supp}(\mu) \subset \mathfrak{F}^\epsilon$ (with $\epsilon = (\frac{2}{p}) - 1$ for $1 \leq p \leq 2$). This gives the non-spherical abstract version of the extension theorem for any positive-definite distribution on G . Our results confirm the one-to-one correspondence between tempered invariant positive-definite distributions and the Bochner measures of the case $SU(1, 1)/\{\pm 1\}$ (as

earlier computed by Barker) for all G .

Keywords: Spherical Bochner theorem, Tempered invariant distributions, Harish-Chandra's Schwartz algebras.

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3.17 Fourier Transform of Schwartz Algebras on Groups in the Harish-Chandra Class

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Abstract

It is well-known that the Harish-Chandra transform, $f \rightarrow Hf$, is a topological isomorphism of the spherical (Schwartz) convolution algebra $\mathcal{C}^p(G//K)$ (where K is a maximal compact subgroup of any arbitrarily chosen group G in the Harish-Chandra class and $0 < p \leq 2$) onto the (Schwartz) multiplication algebra $\bar{\mathcal{Z}}(\mathcal{F}_\epsilon)$ (of \mathfrak{w} -invariant members of $\mathcal{Z}(\mathcal{F}_\epsilon)$, with $\epsilon = (2/p) - 1$). This is the Trombi and Varadarajan theorem. Also the full Harish-Chandra Plancherel formula on G is known for all of $\mathcal{C}^2(G) =: C(G)$. In order to then understand the structure of Harish-Chandra transform more clearly and to compute the image of $\mathcal{C}^p(G)$ under it (without any restriction) we derive an absolutely convergent series expansion (in terms of known functions) for the Harish-Chandra transform by an application of the full Plancherel formula on G . This leads to a computation of the image of $\mathcal{C}(G)$ under the Harish-Chandra transform which is seen as a concrete realization of Arthur's result and is easily extended to all of $\mathcal{C}^p(G)$ in much the same way as it is known in the work of Trombi and Varadarajan.

Keywords: Spherical Bochner theorem, tempered invariant distributions, Harish-Chandra's Schwartz algebras.

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3.18 A Fixed Point Theorem of Quasi-Contraction in C^* -Algebra-Valued Metric

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Abstract

The concept of C^* -algebra is of great importance in quantum theory and statistical mechanics. A lot of work has been done in efforts to refine the well known Banach contraction principle in C^* -algebra valued metric space. However, none of these investigations has attempted to establish Quasi contraction in aforementioned notions. In the present paper, we propose this idea in C^* -algebra valued metric space and prove a fixed point theorem in such space. The result obtained in this work extends a few corresponding ones in the corresponding literature.

Keywords: Metric spaces, C^* -algebra valued metric spaces, fixed point, quasi contraction.

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3.19 Common Fixed Points of Enriched Contractions in Banach Spaces

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Abstract

This paper presents comprehensive study on the existence and uniqueness of fixed point and common fixed points for general class of enriched contractions in Banach spaces. We prove some fixed point theorems as well as common fixed point theorems for these general class of enriched contraction definitions, using Jungck-Schaefer and other iterative techniques. Having in mind, the wide range of applicability of Banach contraction principle and some of its generalizations and extensions, this paper gives a more robust contraction conditions, in respect of V. Berinde *et al.* [6]. Our results generalize and extend results of Banach [2], Jungck [8], Akram *et al.* [1], Olatinwo *et al.* [13], Berinde *et al.* [6], and many other related results in the literature.

Keywords: Enriched Akram-Jungck contraction, Jungck-Schaefer iteration, positively homogeneous function, common fixed point and Banach spaces.

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3.20 On Continuous Dependence of the Fixed Point for Modified Kirk-Mann Iterative Process in Banach Spaces

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Abstract

Let E be a Banach space and T be a self map defined on E satisfying certain contractive conditions. In 2008 Olatinwo introduced a Kirk-Mann type iterative process and employed it to obtain some results on continuous dependence of the fixed point of T . For $x_0 \in E$, define the sequence $\{x_n\}_{n=0}^{\infty}$ by

$$x_{n+1} = \sum_{i=0}^k \alpha_{n,i} T^i x_n, \quad n = 0, 1, 2, \dots, \quad \sum_{i=0}^k \alpha_{n,i} = 1$$

$\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$, $\alpha_{n,i} \in [0, 1]$, where k is a fixed integer.

In this paper, we introduce and employed a more general Kirk-Mann type iterative process to obtain some results on continuous dependence of fixed point, and also remove the constraint that $\sum_{i=0}^k \alpha_{n,i} = 1$.

Keywords: Kirk-Mann type iterative process, continuous dependence, fixed point.

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