International Journal of Applied Mathematics

Volume 38 No. 12s, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

ADAPTIVE RANGE-BASED VOLATILITY MODELLING OF OIL AND GAS STOCK RETURNS IN NIGERIA

[1]Mohammed Tanimu, [2]Audu Isah, [3]Yisa Yakubu, and [4]Mathew Adetutu

[1,2,3,4]Department of Statistics, Federal University of Technology, Minna, Nigeria

Correspondence Email: motentic@gmail.com, tanimumphd214@st.futminna.edu.ng

Abstract

In this article, a range based volatility model that integrated robust hybrid range-based estimators with exogenous variables was proposed for oil and gas sector. The study employed historical daily data on opening, closing, high, and low prices of stocks from Chevron, Conoil, Oando Plc, and Total Energies, spanning 1st January, 2012 to 12th September, 2025, alongside daily Brent crude oil prices as an exogenous variable. The model and exogenous influences on volatility was examined. The study showed that Range-Based Generalised Autoregressive Conditional Heteroskedasticity with Exogenous variable (RB-GARCH-X) models, especially the non-zero drift version, captured volatility in Nigeria's oil and gas sector effectively, while the adaptive RB-GARCH-X model that adapt to both drift-free and drift-present conditions performed best for highly volatile stocks. The results also revealed that range-based estimation techniques improved model robustness against microstructure noise and estimation errors in both drift-free and drift-present conditions. Parameter estimates indicated strong volatility persistence (β: 0.71–0.99), moderate short-term effects (α: 0.03–0.21), and significant influence of crude oil prices (γ: ≈2.0E-07 to -1.25E-07). The adaptive model provided a more effective balance between short- and long-term effects, demonstrating its robustness for stocks characterised by heightened instability like the Nigeria Oil and Gas stock.

Keywords: Adaptive RB-GARCH, RB-GARCH-X, Rogers–Satchell estimator, Oil and Gas, Nigeria.

1. Introduction

The oil and gas sector, which accounts for a significant portion of Nigeria's gross domestic product and government revenue is subject to frequent and severe price volatility. This volatility driven by both domestic and international factors poses risks to fiscal stability and macroeconomic planning [1, 2, and 3]. Models such as Autoregressive Integrated Moving Average, Generalized Autoregressive Conditional Heterosckedasticity (ARIMA-GARCH), Exponential Generalized Autoregressive Conditional Heterosckedasticity (EGARCH), Threshold Generalized Autoregressive Conditional Heterosckedasticity (TGARCH) and Multivariate Generalized Autoregressive Conditional Heterosckedasticity (MGARCH) (Constant Conditional Correlation (CCC) and Dynamic Conditional Correlation (DCC)) were employed to capture volatility patterns in oil prices and stock returns with varying degrees of success [4,5, and 6]. The recent advancements in volatility modelling included the incorporation of exogenous variables to enhance predictive accuracy and account for spill over effects [7, 8, 9, and 10]. Furthermore, the application of range-based estimation techniques has improved model robustness against microstructure noise and estimation errors [11, 12, and 13].

Volatility in Nigeria's oil and gas sector has significant implications for economic planning, investment stability, and fiscal sustainability. Although, models such as ARIMA-GARCH and EGARCH have been used to forecast oil price and stock return volatility, they often rely on return-based measures and do not sufficiently incorporate external economic

Received: August 09, 2025

Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)
drivers [4, and 5]. Moreover, existing studies tend to overlook the potential benefits of range-based volatility estimators which offer improved precision and predictive power [3, 14, and 15].

The volatility of oil prices has been particularly pronounced following global crises. For example, after the COVID-19 pandemic caused Brent crude prices to fall to USD 41.76 per barrel in 2020, prices rebounded to USD 70.68 per barrel in 2021 [16]. However, Russia's invasion of Ukraine in 2022, coupled with sanctions by Western countries, severely restricted global oil supply chains pushing crude prices to USD 122.71 per barrel by June 2022 [17]. In response to these pressures, Organization of the Petroleum Exporting Countries (OPEC) and its allied non-OPEC oil-producing countries (OPEC+), a coalition formed to influence global oil production and prices by coordinating output levels, implemented substantial supply cuts in 2022 and 2023 to stabilise global oil markets. Despite these efforts, oil prices declined towards the end of 2022 driven by recession fears and tightening global monetary policies [18]. Crude oil production in Nigeria rose to 1,507 Barrels per Day per Thousand (BBL/D/1K) in July 2025 from 1,505 BBL/D/1K in June 2025, with an average production of 1,819.81 BBL/D/1K recorded from 1973 to 2025, peaking at 2,475.00 BBL/D/1K in November 2005 and dropping to a record low of 675.00 BBL/D/1K in February 1983 [19]. Nigeria stands as one of Africa's leading oil producers supported by 15 operational pipelines and an average daily output of approximately 1.5 million barrels in 2023 securing its position as the fifteenth-largest oil producer globally. The petroleum industry remains a cornerstone of Nigeria's economy, contributing around 5.5% to the nation's Gross Domestic Product (GDP) and accounting for nearly 92% of its total export value [20]. Nigeria holds approximately 206.5 trillion cubic feet of proven natural gas reserves, making it the largest in Africa [16]. However, significant volume of associated gas are still flared or re-injected due to infrastructural deficiencies and limited market access [21].

Nigerian government has prioritised the development of gas infrastructure, with notable projects including the Nigeria Liquefied Natural Gas (NLNG) expansion on Bonny Island and the Ajaokuta-Kaduna-Kano (AKK) pipeline. These initiatives aim to harness Nigeria's gas potential for domestic consumption and export, particularly to Europe and Asia through future links with the Trans-Saharan Gas Pipeline project [21]. The Nigerian oil and gas sector remains a critical pillar of the national economy, albeit one fraught with persistent challenges and transformative opportunities. Domestic disruptions, infrastructural deficits, and global energy transition dynamics continue to shape sectoral outcomes. However, the implementation of the Petroleum Industry Act (PIA), expansion of refining and gas infrastructure, and strategic engagement with global energy trends offer pathways to sustainable sectoral growth. Thus, understanding and modelling volatility within this evolving landscape is crucial for Nigeria's broader macroeconomic stability and energy security in the face of a rapidly changing global energy environment.

The volatility of oil prices impact macroeconomic variables such as Gross Domestic Product (GDP), interest rate, and exchange rates, further compounding economic uncertainty [1, 22, 23, and 24]. Also, Brent crude which is a major global benchmark for crude oil is widely used as a pricing standard for two-thirds of the world's internationally traded crude oil supplies [16]. Yet, many models fail to consider these exogenous influences explicitly, resulting in less reliable forecasts. To improve volatility estimation, range-based estimators such as those developed by Parkinson [25], Garman-Klass [26], Rogers-Satchell [27], and Bollerslev *et al.*,[28] have been introduced. While these estimators demonstrated enhanced efficiency by incorporating high-low and open-close price ranges, none was universally robust across both drift-free and drift-present conditions. For example, Parkinson's estimator is efficient only

Volume 38 No. 12s, 2025
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

under zero drift, while Rogers-Satchell accommodated drift. This study addressed this gap by proposing a range-based volatility model that integrates robust hybrid range-based estimators with exogenous variables for oil and gas sector. The proposed model effectively capture the dynamics of Chevron, Conoil, Oando Plc, and Total Energies time series data in the Nigeria oil and gas sector. The effectiveness of the proposed estimator in capturing volatility patterns was evaluated, and the role of exogenous variables in volatility estimation assessed.

2. Material and Method

This study utilised daily historical secondary data comprising opening, closing, high, and low stock prices from leading and consistently traded oil and gas companies listed on the Nigerian Stock Exchange. The selected companies are Chevron, Conoil, Oando Plc, and Total Energies. The dataset spans from 1st January, 2012 to 12th September, 2025. The exogenous variable employed was the daily global crude oil price (Brent Oil Price) covering the same period. These datasets were obtained from the Nigerian Exchange Group [29] and the Central Bank of Nigeria [30] daily rates portal. The datasets were harmonised and cleaned to ensure its suitability for analysis.

2.1 Range-Based Generalized Autoregressive Conditional Heteroscedastic Model (RB-GARCH-X)

The (RB-GARCH-X) model with exogenous variable proposed in this study modified the GARCH model introduced by Bollerslev [31]. The Bollerslev [31] GARCH model is given in equation (1).

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (1)$$

Where σ_t^2 denote the conditional volatility of the series at time t, ω denote a constant term that ensures the variance remains positive and provides a baseline level of volatility, $\alpha\epsilon_{t-1}^2$ capture the impact of past shocks (the lagged squared error term). A large α indicate that volatility reacts strongly to new information or market shocks, and $\beta\sigma_{t-1}^2$ capture the persistence of volatility through time. A high β means volatility is highly persistent with shocks taking a long time to decay.

Equation (1) was further extended into the GARCH-X model by incorporating exogenous variables, as evidenced in [32]. This study replaced the lagged squared error term (ϵ_{t-1}^2) used in equation (1) with a more robust estimator that account for both zero and non-zero drift, in contrast to Molnár's model [33], which was limited to zero drift. The proposed RB-GARCH-X model integrated an exogenous variable to capture external influences on Nigerian oil and gas stock returns from Molnár's Equation [33] in equation (2).

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, \sigma_{p,t-i}^2 + \sum_{j=1}^{p} \beta_j\, h_{t-j} \quad (2)$$

where: h_t denote the conditional volatility at time t, α_0 denote a constant term ensuring the variance is always positive, $\sum_{i=1}^{q} \alpha_i\, \sigma_{p,t-i}^2$ denote the ARCH part, where past squared shocks (innovations) directly impact today's volatility. The coefficients α_i measure how strongly new information or past shocks affect volatility, $\sum_{j=1}^{p} \beta_j\, h_{t-j}$ denote the GARCH part capturing the persistence of volatility by linking today's variance to past variances. The coefficients β_j measure how long shocks remain influential over time, and $\sigma_{p,t-i}^2$ is the Parkinson range-based variance estimator for day $t - i$.

$$\hat{\sigma}_{p,t-i}^2 = \frac{\left[\log\left(\frac{H_{t-i}}{L_{t-i}}\right)\right]^2}{4\, Log(2)} \quad (3)$$

Received: August 09, 2025

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

where: H_{t-i} denote the highest stock price of the asset on day $t - i$, L_{t-i} denote the lowest stock price of the asset on day $t - i$, and $4\,Log(2)$ is a scaling factor derived under the assumption of a Brownian motion with zero drift, making the estimator unbiased for variance.

It is obtained as [34] in equation (4). Given that the price S_t of a financial asset follow a geometric Brownian motion (GBM):

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{4}$$

where: S_t denote the stock price (here, an oil and gas company's stock) at time t, $\mu S_t dt$ denote the drift term, representing the expected return or average growth rate of the stock over a small time interval dt, $\sigma S_t dW_t$ denote the diffusion term capturing random shocks or volatility in stock prices. Here, σ is the volatility parameter, and dW_t is the increment of a Wiener process (standard Brownian motion)

Equation (4) is the stochastic differential equation (SDE) describing the Geometric Brownian Motion (GBM) widely used to model stock prices. This model assumes that oil and gas stock prices evolve continuously over time driven by a predictable growth component (μ) and an unpredictable random component ($\boldsymbol{\sigma} dW_t$) reflecting market uncertainty, news, or shocks in global crude oil prices

Taking logs of equation (4) we get:

$$dlogS_t = \left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t. \tag{5}$$

For estimating volatility with the Parkinson method [25], we assume zero drift ($\mu = 0$) for simplicity, as it has minimal effect on high-low range over short intervals. Those:

$$d(\log S_t) \approx \sigma dW_t \tag{6}$$

So, $log\ S_t \sim N(0, \sigma^2 t)$

Now, define the High-Low Range

Let:

H_t=High price over interval $[t, t+1]$

L_t=Low price over interval $[t, t+1]$

Then the log range is:

$$R_t = \log\left(\frac{H_t}{L_t}\right) \tag{7}$$

From the properties of Brownian motion the expected squared log-range is:

$$E[R_t^2] = 4log(2) \cdot \sigma^2 \tag{8}$$

Rearrange to solve for σ^2

$$\sigma^2 = \frac{E[R_t^2]}{4log(2)} \tag{9}$$

In practice, replace the expected value with the observed squared log-range:

$$\hat{\sigma}_{p,t}^2 = \frac{\left[\log\left(\frac{H_t}{L_t}\right)\right]^2}{4\,Log(2)} \tag{10}$$

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ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

This estimator in equation (10) uses only the high and low prices and is up to 5 times more efficient than using squared returns but when the drift is zero. To address this shortfall, Rogers-Satchel [27] proposed a new range based estimator given as follows:

$$\hat{\sigma}^2_{RS,t-1} = \ln\left(\frac{H_t}{C_t}\right) \ln\left(\frac{H_t}{O_t}\right) + \ln\left(\frac{L_t}{C_t}\right) \ln\left(\frac{L_t}{O_t}\right) \tag{11}$$

where; O_t denote Opening price, H_t denote High price, L_t denote Low price, C_t denote Closing price

2.1.1 Rogers-Satchell estimator

To show that:

$$\hat{\sigma}^2_{RS,t-1} = \ln\left(\frac{H_t}{C_t}\right) \ln\left(\frac{H_t}{O_t}\right) + \ln\left(\frac{L_t}{C_t}\right) \ln\left(\frac{L_t}{O_t}\right) \tag{12}$$

Let the log-price of an asset [34]:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{13}$$

This is a geometric Brownian motion with drift, so:

$$S_t = S_0 \cdot e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t} \tag{14}$$

where; S_0 denote initial stock price at time t = 0, μ denote the drift (average rate of return), σ denote the volatility (size of fluctuations), and W_t denote the standard Brownian motion at time t.

Equation (14) is the closed-form solution to the stochastic differential equation.

Unlike Parkinson's estimator (which assumes zero drift), the Rogers-Satchell estimator is robust to non-zero drift. It is derived by taking expectations over the path of log-prices, resulting in an estimator unbiased under drift.

To show that $\hat{\sigma}^2_{RS,t-1} = \ln\left(\frac{H_t}{C_t}\right) \ln\left(\frac{H_t}{O_t}\right) + \ln\left(\frac{L_t}{C_t}\right) \ln\left(\frac{L_t}{O_t}\right)$

Then, we define the Rogers-Satchell estimator as:

$$\hat{\sigma}^2_{RS,t} = (x - r)(x) + (y - r)(y) \tag{15}$$

Expanding equation (15) gives:

$$\hat{\sigma}^2_{p,t} = x(x - r) + y(y - r) = x^2 - xr + y^2 - yr \tag{16}$$

Now since:

$$ln(H_t/C_t) = ln(H_t/O_t) - ln(C_t/O_t) = x - r \tag{17}$$

$$ln\,(L_t/C_t) = ln\,(L_t/O_t) - ln\,(C_t/O_t) = y - r \tag{18}$$

Substituting gives:

$$\hat{\sigma}^2_{RS,t-1} = \ln\left(\frac{H_t}{C_t}\right) \ln\left(\frac{H_t}{O_t}\right) + \ln\left(\frac{L_t}{C_t}\right) \ln\left(\frac{L_t}{O_t}\right) \tag{19}$$

Since real financial assets may exhibit varying levels of drift and non-drift, the study proposed the Adaptive Range-Based Estimator (ARBE) that linearly combine both Rogers-Satchell [27] and Parkinson [25] estimators based on drift-sensitive weight $\lambda_t \in [0,1]$ to balance bias and efficiency dynamically.

2.2 Adaptive Range-Based Estimator (ARBE)

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ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

The Parkinson [25] estimator is highly efficient under zero drift but biased when drift or jumps are present. On the other hand, Rogers–Satchell [27] estimator is more Robust to drift and directional trends, but less efficient when drift is negligible. Therefore, this study developed a hybrid estimator (ARBE) that: retains efficiency in low-drift conditions (like Parkinson) [25]; but is robust to drift, trends, and jumps (like Rogers–Satchell), [27]; incorporates all available price points: high, low, open, and close; and can adapt dynamically to the presence or absence of drift.

The proposed ARBE estimator is a weighted average of the Parkinson and Rogers Satchell's estimator, where the weight is data-driven and adaptive to the market's drift and volatility structure. It is given in equation (20)

$$\hat{\sigma}^2_{ARBE,t} = \lambda_t \,.\, \hat{\sigma}^2_{RS,t} + (1-\lambda_t)\hat{\sigma}^2_{P,t} \quad (20)$$

where: $\hat{\sigma}^2_{RS,t}$ denote the Rogers–Satchell estimator, $\hat{\sigma}^2_{P,t}$ Parkinson estimator, $\lambda_t \in [0,1]$ adaptive weight based on the strength of drift. This is a weighted average (convex combination), ensuring: $0 \le \hat{\sigma}^2_{ARBE,t} \le max(\hat{\sigma}^2_{RS,t}\,, \hat{\sigma}^2_{p,t})$ such that: When drift is low, $\lambda_t \to 0$, implying that $ARBE$ favours Parkinson estimator and when drift is high, $\lambda_t \to 1$, implying that $ARBE$ favours Rogers–Satchell estimator. Thus, the estimator adapts to the underlying market conditions.

The drift-to-volatility ratio (also called the signal-to-noise ratio) is defined as:

$$\delta_t = \frac{r_t}{\hat{\sigma}_t} \quad (21)$$

where: $r_t = lo\,g\left(\frac{C_t}{O_t}\right)$: log return over the period t, $\hat{\sigma}_t$: Volatility estimate (from Rogers–Satchell), when drift is very small$(\delta_t \approx 0)$, $set\ \lambda_t \approx 0$: trust Parkinson, when drift is large, set $\lambda_t \approx 1$: trust Rogers-Satchell estimator.

A smooth, bounded function commonly used is the logistic (sigmoid):

$$\lambda_t = \frac{\delta_t^2}{1+\delta_t^2} \quad (22)$$

This satisfies: $\delta_t = 0 \Rightarrow \lambda_t = 0$ with $|\ \delta_t\ | \to \infty \Rightarrow \lambda_t \to 1$, δ_t is always in [0, 1],

from equation (22),

$$\lambda_t = \frac{\delta_t^2}{1+\delta_t^2} = \frac{\mathrm{r}_t^2}{\mathrm{r}_t^2+\hat{\sigma}_t^2} \quad (23)$$

We obtain:

$$\hat{\sigma}^2_{ARBE,t} = \left(\frac{\mathrm{r}_t^2}{\mathrm{r}_t^2+\hat{\sigma}_t^2}\right)\hat{\sigma}^2_{RS,t} + \left(\frac{\hat{\sigma}_t^2}{\mathrm{r}_t^2+\hat{\sigma}_t^2}\right)\hat{\sigma}^2_{P,t} \quad (24)$$

Now, the Range-Based GARCH-X is obtained by substituting equation (24) into equation (2) and introducing exogenous variable(s). Thus, we have (25):

$$\sigma_t^2 = \omega + \alpha\hat{\sigma}^2_{ARBE,t-1} + \beta\sigma^2_{t-1} + \delta \mathrm{x}_{\mathrm{t-1}} \quad (25)$$

where: σ_t^2 denote conditional variance of returns at time t, ω is the constant term, α denote coefficient on the adaptive range based estimator, $\hat{\sigma}^2_{ARBE,t-1}$ denote adaptive range-based volatility proxy, β denote GARCH term (lagged conditional variance), δ denote coefficient on the exogenous variable, $\mathrm{x}_{\mathrm{t-1}}$ denote exogenous variable observed at time $t-1$ (lagged global crude oil prices).

2.3 Model Assumptions

Received: August 09, 2025

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ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

i. Zero Conditional Mean: $E[r_t \mid F_{t-1}] = 0$

ii. Conditional Normality: $r_t \mid F_{t-1} \sim N(0, H_t)$

iii. Log-prices follow a continuous-time process:

$$dlnP_t = \mu_t dt + \sigma_t dW_t \quad (26)$$

where Parkinson estimator assumes $\mu_t = 0$, but RS and ARBE relax this.

iv. Daily high, low, open, and close prices are available for constructing ARBE.

v. Exogenous variables X_{t-1} are measurable at time $t-1$ and may affect volatility.

vi. Parameter Constraints: $\omega_i > 0, \alpha_i \geq 0, \beta_i \geq 0$ and $\alpha_i + \beta_i < 1$ for covariance stationarity.

vii. Positive Definiteness: $H_t \succ 0$ is guaranteed by diagonal structure and positive components.

This approach capture intra-day price movements more accurately than return-based measures. This model allow the conditional variance to be influenced not only by the adaptive range-based estimator and past variance, but also by an external or exogenous factor (for example: interest rate, global crude oil prices, exchange rate, Inflation rate, and policy dummy). The objective of the model is to improve the forecast performance and the goodness of fit of Range GARCH models by replacing the squared return term with a less noisy volatility proxy: the high-low price range, the Parkinson estimator and the Rogers-Satchell estimator.

3. Results

This section present the results of the analysis carried out. The analysis evaluated the model's effectiveness in capturing the dynamics of Nigeria oil and gas stocks by integrating range-based estimators with exogenous variables. Table 1 present the descriptive statistics for the companies' stock returns.

Table 1: Descriptive Statistics of Chevron, Conoil, Oando and Total Stock Returns

Oil and Gas Stock	Returns	Mean	Median	Std. Dev.	Skewness	Kurtosis	Observations
Chevron	RCLOSE	0.000405	0.000000	0.020748	0.218799	23.36097	3038
	RHIGH	0.000405	0.000000	0.026582	0.063101	15.25752	3038
	RLOW	0.000405	0.000000	0.025839	1.017213	21.06467	3038
	ROPEN	0.000405	0.000000	0.022867	0.253368	16.20021	3038
	LOGRANGE	0.007850	0.000000	0.025645	3.783924	18.70041	3038
Conoil	RCLOSE	0.000754	0.000000	0.031327	0.102048	6.957713	2927
	RHIGH	0.000743	0.000000	5.739794	-0.00236	9.050323	2927
	RLOW	0.000723	0.000000	5.734089	-0.00245	9.049778	2927
	ROPEN	0.000743	0.000000	0.451343	0.021427	1449.038	2927
	LOGRANGE	0.028770	0.000000	0.047782	9.799004	216.8422	2927
Oando	RCLOSE	0.000282	0.000000	0.039161	0.106969	4.106149	3343
	RHIGH	0.000287	0.000000	1.936981	-0.00497	61.90079	3343
	RLOW	0.000279	0.000000	1.935548	-0.00534	61.89849	3343
	ROPEN	0.000286	0.000000	0.045692	0.241224	6.859817	3343

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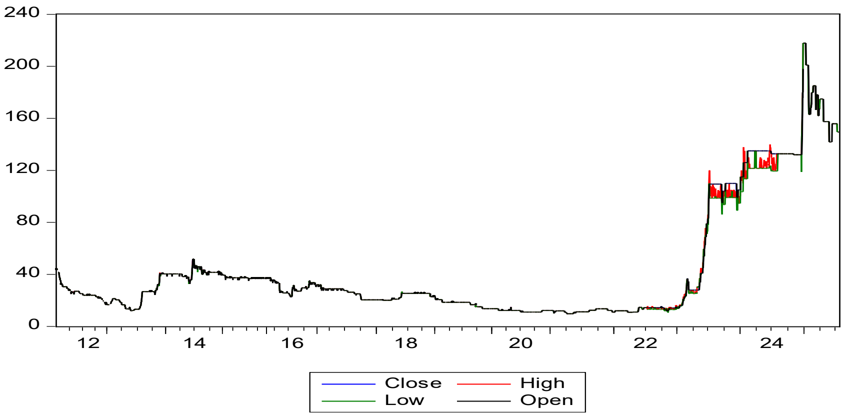
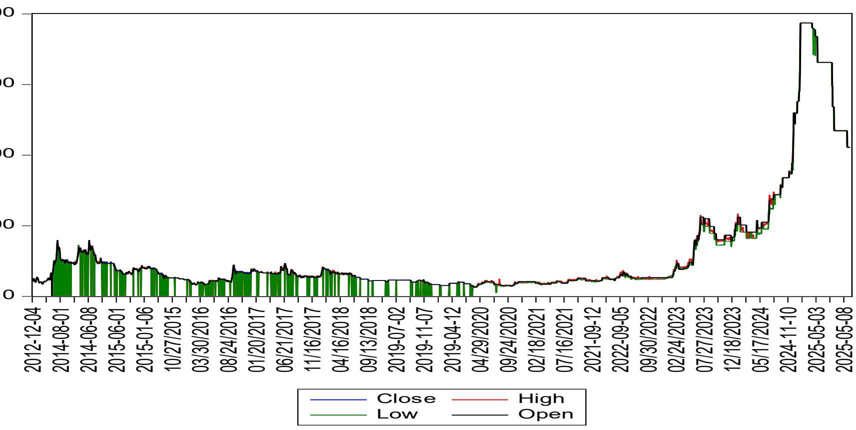
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

	LOGRANGE	0.029769	0.022728	0.029152	1.576999	6.997716	3343
Total Energies	RCLOSE	0.000371	0.000000	0.023434	0.007191	9.504677	2987
	RHIGH	0.000371	0.000000	5.129111	-0.00197	13.84042	2987
	RLOW	0.000371	0.000000	5.126546	-0.00201	13.83413	2987
	ROPEN	0.000371	0.000000	0.490252	-0.55869	1485.611	2987
	LOGRANGE	0.027237	0.013334	0.080205	30.34799	1078.876	2987

Table 1 showed generally low mean returns across all firms with values close to zero (Chevron: 0.000405; Conoil: 0.000754; Oando: 0.000282; Total Energies: 0.000371), and medians consistently at 0.000000, suggesting limited average daily gains. The standard deviations indicated varying degrees of volatility, highest for Oando (0.039161) and Conoil (0.031327) compared to Chevron (0.020748) and Total Energies (0.023434). Skewness values are generally close to zero, though Chevron's logrange (3.78) and Total Energies' logrange (30.35) suggested strong asymmetry in price movements. Kurtosis values were exceptionally high across all stocks particularly for Conoil's open returns (1449.04) and Total Energies' open returns (1485.61), implying heavy-tailed distributions and extreme return events. The logrange, a proxy for volatility clustering show Oando (0.029769) and Conoil (0.028770) with the highest averages, while Chevron's logrange was the lowest (0.007850). These findings suggested that while all four stocks exhibited non-normal return distributions characterised by volatility clustering, Oando and Conoil display greater return fluctuations, whereas Chevron remained relatively more stable. Figures 1(a, b, c & d) and 2 ((a, b, c & d) are time plots of the selected Nigeria oil and gas stock prices and their respective stock returns.



(a) CHEVRON



Received: August 09, 2025

International Journal of Applied Mathematics
Volume 38 No. 12s, 2025
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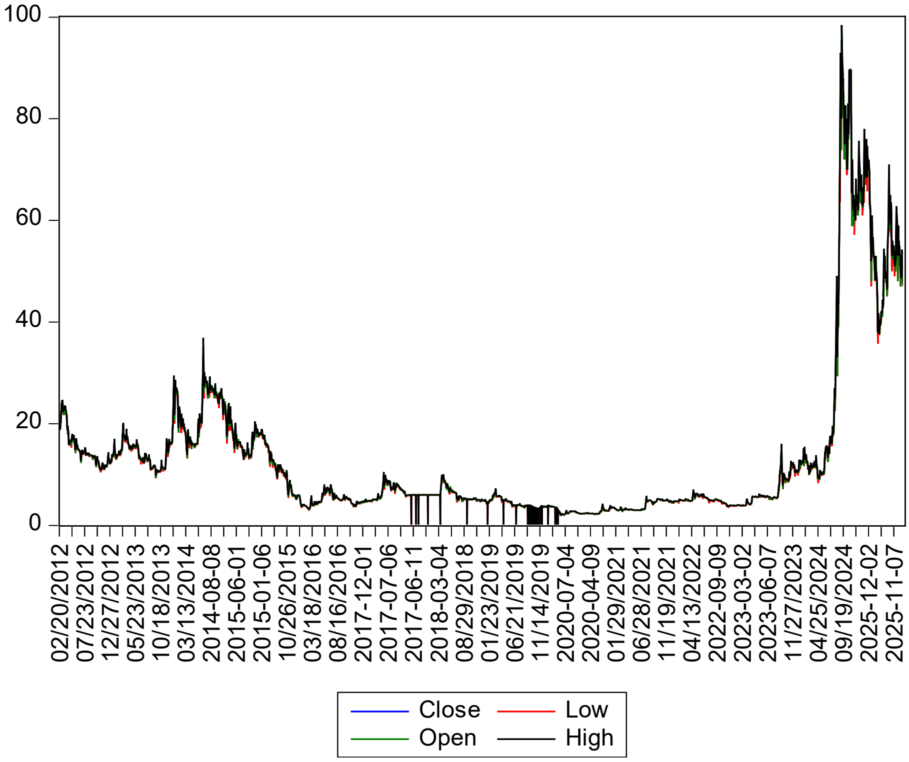
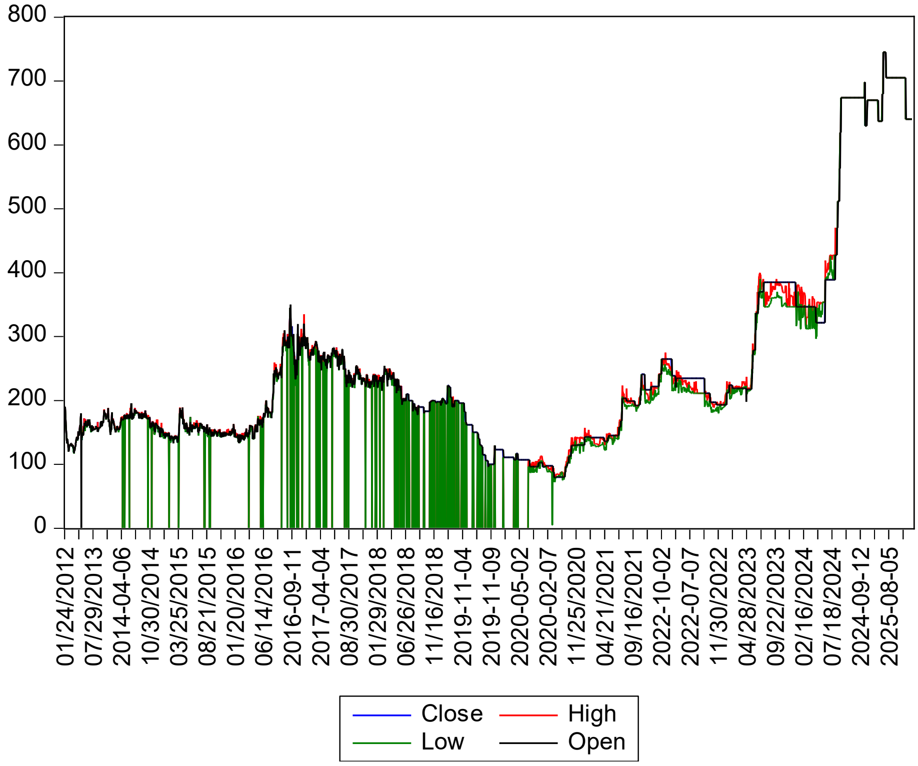
(b) CONOIL



(c) OANDO

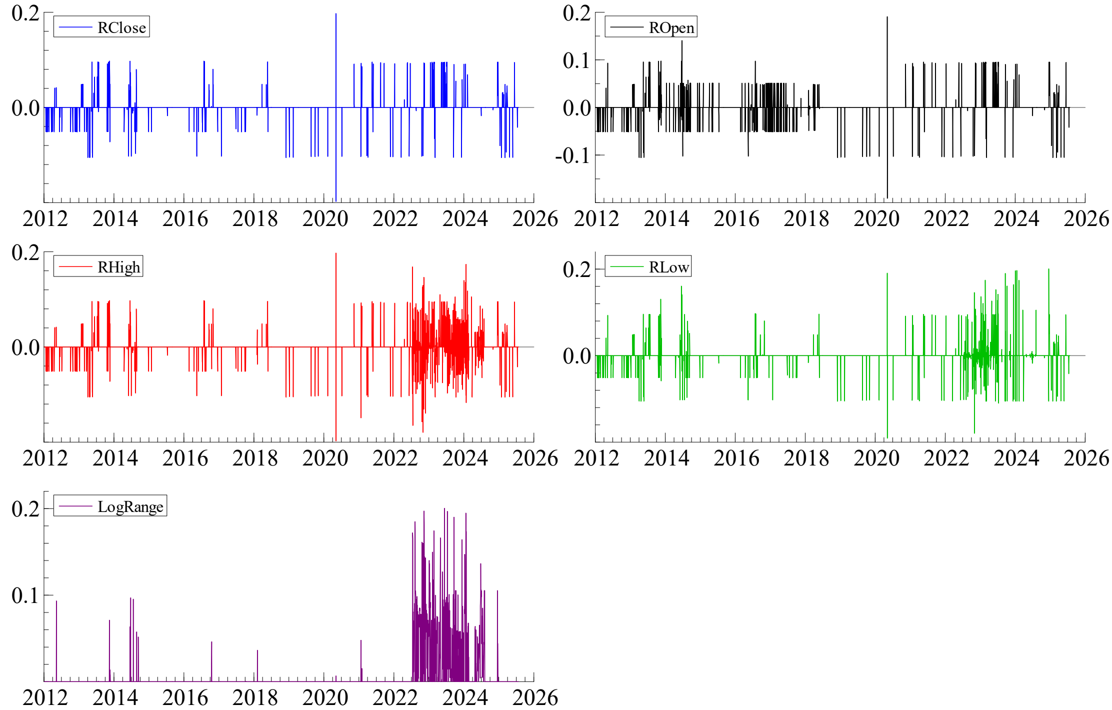


(d) TOTAL

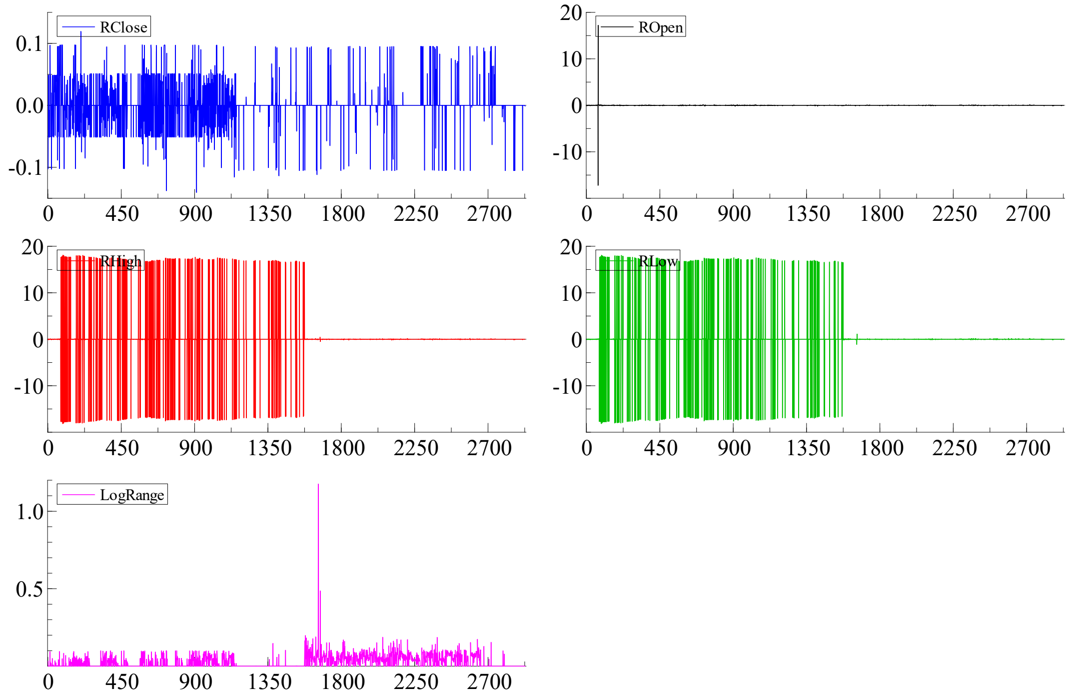
Figures 1(a, b, c & d): Time Plots of Chevron, Conoil, Oando and Total Stock Prices

Figures 1(a, b, c & d) show the time plot for Chevron, Conoil, Oando and Total Energies Stock Prices respectively, while on the horizontal axis is the study period while the vertical

ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)
axis show the prices of the study stocks. The figures show that all the study stock experienced both upward and downward trend, the upward trend is largely stable towards the end of each study period.



(a) CHEVRON



(b) CONOIL

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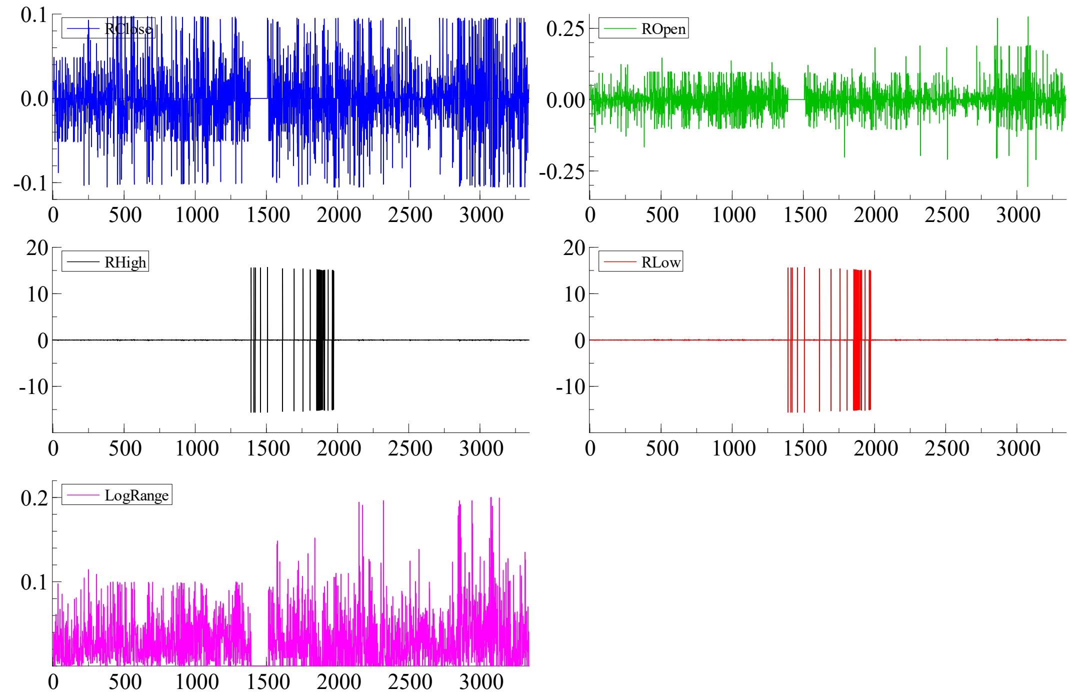
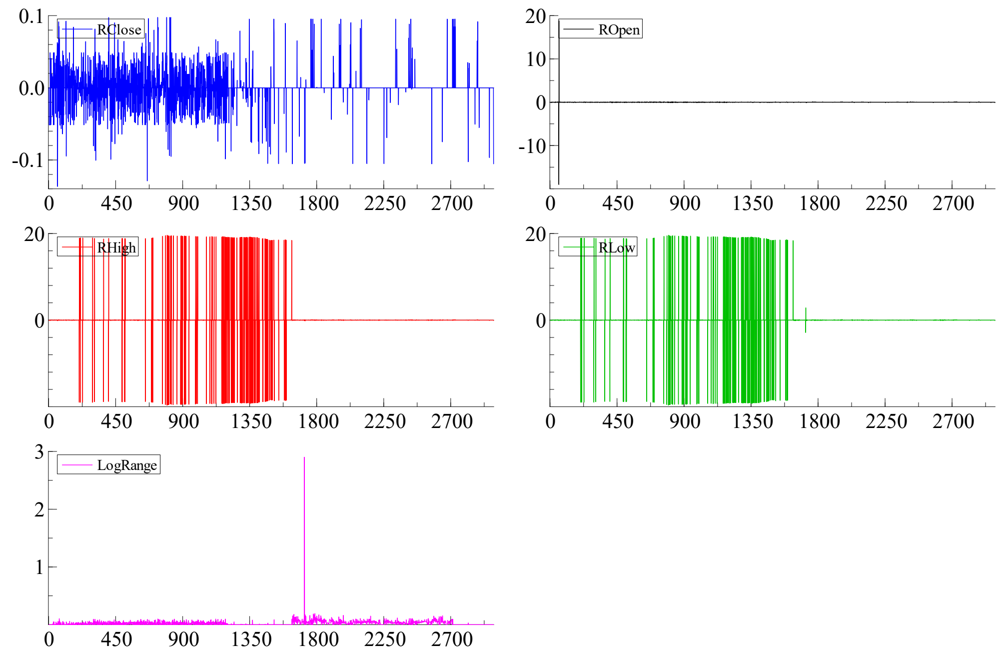
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)



(c) OANDO



(d) TOTAL

Figures 2(a, b, c & d): Time Plots of Chevron, Conoil, Oando and Total Stock Returns

Figures 2(a, b, c & d) show the time plot for Chevron, Conoil, Oando and Total Energies Stock Returns. The plots show the fluctuation of the stock returns which are indication of volatility clustering.

Received: August 09, 2025

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3.1 Stationarity and Pre-Diagnostic Tests

The results of the unit root test is presented in Table 2.

Table 2: Unit Root Test Results (ADF, PP)

Null Hypothesis: Logrange has a unit root						
Oil and Gas Stock			**Test critical values:**			
		Test Statistic	**1% level**	**5% level**	**10% level**	**Prob.***
Chevron	**ADF**	-4.73266	-3.43232	-2.8623	-2.56722	0.0001
	PP	-49.846	-3.43231	-2.86229	-2.56722	0.0001
Conoil	**ADF**	-7.23639	-3.4324	-2.86233	-2.56724	0.0000
	PP	-45.004	-3.43239	-2.86233	-2.56723	0.0001
Oando	**ADF**	-16.6256	-3.43212	-2.86221	-2.56717	0.0000
	PP	-57.496	-3.43212	-2.86221	-2.56717	0.0001
Total Energies	**ADF**	-13.6017	-3.43236	-2.86231	-2.56723	0.0000
	PP	-31.7958	-3.43235	-2.86231	-2.56722	0.0000
*MacKinnon (1996) one-sided p-values						

In Table 2, the unit root test results for the logrange series of Chevron, Conoil, Oando, and Total Energies using the Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) tests for all four stocks, the test statistics are less than the critical values at 1% significance level (for example, Chevron: ADF = –4.73266, PP = –49.846; Conoil: ADF = –7.23639, PP = –45.004; Oando: ADF = –16.6256, PP = –57.496; Total Energies: ADF = –13.6017, PP = –31.7958), with corresponding p-values close to zero. This indicated strong rejection of the null hypothesis of a unit root, confirming that the logrange series for each stock is stationary. The results implied that fluctuations in volatility are mean-reverting, supporting the suitability of applying range based-GARCH-family models for further volatility analysis.

3.2 ARCH-LM Test Results for Nigeria Oil and Gas Stock Returns

Table 3 presented the ARCH-LM tests results

Chevron			
F-statistic	621.0241	Prob. F(1,3035)	0.0000
Obs*R-squared	515.8746	Prob. Chi-Square(1)	0.0000
Conoil			
F-statistic	972.2288	Prob. F(1,2924)	0.0000
Obs*R-squared	730.1269	Prob. Chi-Square(1)	0.0000
Oando			

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F-statistic	25.84803	Prob. F(1,3340)	0.0000
Obs*R-squared	25.66489	Prob. Chi-Square(1)	0.0000
Total Energies			
F-statistic	992.9039	Prob. F(1,2984)	0.0000
Obs*R-squared	745.5073	Prob. Chi-Square(1)	0.0000

Table 3 show that for all the four stocks, both the F-statistic and the Obs*R-squared statistic were highly significant with associated p-values equal to 0.0000. This led to a rejection of the null hypothesis of no ARCH effects thereby indicating the presence of autoregressive conditional heteroskedasticity in the return series. These findings provided clear evidence of conditional heteroskedasticity across the sampled stocks, justifying the application of GARCH-family models to capture and model their time-varying volatility behaviour.

3.3 Estimation of Range based GARCH Models

Range based GARCH models including the RB-GARCH (1, 1) with Parkinson estimator, and RB-GARCH (1, 1) with Rogers-Satchel estimator were first estimated for comparative purposes. Their parameter estimates confirmed the persistence of volatility shocks. Table 4 show the estimated results of Range based GARCH Models.

Table 4: Estimation Results of Range based GARCH Models

Models	Oil and Gas Stock	Parameter	Estimate	Std. Error	z-Statistic	p-value
RB-GARCH with zero drift (Parkinson Estimator)	Chevron	ω	2.52E-08	4.21E-10	59.87924	0.0000
		α	0.060827	0.002324	26.16985	0.0000
		β	0.324126	0.004768	67.98615	0.0000
	Conoil	ω	0.000892	2.00E-05	44.55593	0.0000
		α	-0.001917	0.000136	-14.13183	0.0000
		β	0.098610	0.032557	3.028822	0.0025
	Oando	ω	2.75E-09	1.41E-09	1.952173	0.0509
		α	0.637893	0.142539	4.475205	0.0000
		β	0.899527	0.003398	264.7280	0.0000
	Total Energies	ω	8.74E-09	1.58E-10	55.40922	0.0000
		α	-8.24E-06	1.30E-07	-63.57167	0.0000
		β	0.997711	8.14E-06	122531.7	0.0000
RB-GARCH with non-zero drift (Rogers-Satchel's Estimator)	Chevron	ω	1.99E-05	5.15E-06	3.860927	0.0001
		α	-0.000824	0.000217	-3.792178	0.0001
		β	-0.070234	0.021287	-3.299435	0.0010

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	Conoil	ω	0.000105	3.84E-06	27.44001	0.0000
		α	-1.13E-07	4.08E-09	-27.71397	0.0000
		β	0.861471	0.004000	215.3734	0.0000
	Oando	ω	0.001044	7.38E-05	14.15181	0.0000
		α	-1.36E-06	9.44E-08	-14.45619	0.0000
		β	0.290746	0.044574	6.522805	0.0000
	Total Energies	ω	3.50E-08	2.25E-09	15.52987	0.0000
		α	-5.23E-10	2.62E-11	-19.95147	0.0000
		β	0.998279	3.14E-05	31795.78	0.0000
ARB-GARCH (adapt to zero and non-zero drift)	Chevron	ω	7.22E-05	3.63E-06	19.88685	0.0000
		α	0.032707	0.000924	35.38272	0.0000
		β	0.133098	0.031043	4.287532	0.0000
	Conoil	ω	0.000889	0.000692	1.284913	0.1988
		α	-0.000668	1.10E-05	-60.80876	0.0000
		β	0.103095	0.698999	0.147490	0.8827
	Oando	ω	6.43E-10	2.75E-10	2.341474	0.0192
		α	0.462709	0.206766	2.237839	0.0252
		β	0.875801	0.003388	258.5216	0.0000
	Total Energies	ω	9.56E-09	4.04E-10	23.69028	0.0000
		α	-4.21E-06	1.79E-07	-23.55065	0.0000
		β	0.997895	2.53E-05	39476.47	0.0000

The estimated results in Table 4 highlighted key differences in parameter behaviour across the RB-GARCH models with zero drift, non-zero drift, and the adaptive ARB-GARCH specification for the four oil and gas companies. For Chevron, the zero-drift model show significant and stable parameters with positive persistence ($\alpha = 0.0608$, $\beta = 0.3241$), whereas the non-zero drift model produced mixed results, including a negative β, suggesting potential instability. However, the ARB-GARCH improved the fit with all parameters highly significant, and captured both drift and no-drift dynamics more effectively. For Conoil, results were consistent and showed strong significance for ω and α across the zero- and non-zero-drift models, but β was weak in the adaptive model ($p = 0.8827$), which indicated limited persistence in volatility when drift adaptation was introduced. Oando exhibited robust persistence under both zero-drift ($\beta = 0.8995$) and adaptive models ($\beta = 0.8758$), though the adaptive model also yielded statistically significant α, suggesting that ARB-GARCH better balanced shock and persistence effects. Finally, for Total Energies, all three models displayed extremely high persistence ($\beta \approx 0.998$), but the adaptive model provided stronger significance and a better

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balance between shock response (α = -4.21E-06) and persistence, which implied superior adaptability to market drift conditions.

3.4 Estimation of the RB-GARCH-X Model

The proposed ARB-GARCH-X model was subsequently estimated via maximum likelihood using Student-t errors. In addition, RB-GARCH-X with Parkinson Estimator and RB-GARCH-X with Rogers-Satchel's Estimator were also estimated. The specification extended [33] range-based GARCH by integrating these estimators and incorporating exogenous factors. The results are shown in Table 5.

Table 5: Estimation Results of RB-GARCH-X Model

Models	Oil and Gas Stock	Parameter	Estimate	Std. Error	z-Statistic	p-value
RB-GARCH-X with zero drift (Parkinson Estimator)	Chevron	ω	-1.26E-06	3.38E-07	-3.736671	0.0002
		α	0.123579	0.006695	18.45824	0.0000
		β	0.714004	0.009008	79.26048	0.0000
		γ	2.00E-07	8.56E-09	23.32857	0.0000
	Conoil	ω	0.000702	0.000207	3.397746	0.0007
		α	-0.001614	0.000351	-4.603409	0.0000
		β	0.171348	0.235868	0.726458	0.4676
		γ	8.86E-07	7.58E-07	1.168283	0.2427
	Oando	ω	2.29E-05	3.52E-06	6.503755	0.0000
		α	0.209980	0.014221	14.76526	0.0000
		β	0.916636	0.004459	205.5803	0.0000
		γ	-2.97E-07	4.59E-08	-6.483027	0.0000
	Total Energies	ω	1.80E-05	5.14E-07	34.98306	0.0000
		α	-6.28E-05	4.06E-06	-15.46843	0.0000
		β	0.981121	0.000496	1979.108	0.0000
		γ	-1.66E-07	4.46E-09	-37.16952	0.0000
RB-GARCH-X with non-zero drift (Rogers-Satchel's Estimator)	Chevron	ω	-1.98E-05	7.98E-07	-24.77433	0.0000
		α	-0.000973	9.80E-05	-9.932248	0.0000
		β	0.072140	0.052908	1.363507	0.1727
		γ	9.99E-07	5.61E-08	17.81232	0.0000
	Conoil	ω	5.30E-05	6.28E-07	84.38199	0.0000
		α	-4.44E-08	6.43E-10	-69.09362	0.0000
		β	0.949245	0.001275	744.7083	0.0000
		γ	-1.86E-07	1.48E-08	-12.55417	0.0000
	Oando	ω	0.001463	0.000134	10.88143	0.0000
		α	-1.44E-06	9.03E-08	-15.91174	0.0000

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		β	0.281380	0.043899	6.409697	0.0000
		γ	-5.54E-06	1.13E-06	-4.894017	0.0000
	Total Energies	ω	1.10E-05	3.64E-07	30.34084	0.0000
		α	-2.77E-09	1.43E-10	-19.29746	0.0000
		β	0.991056	0.000253	3915.005	0.0000
		γ	-1.12E-07	3.47E-09	-32.29546	0.0000
ARB-GARCH–X (adapt to zero and non-zero drift)	Chevron	ω	6.08E-05	3.72E-06	16.36723	0.0000
		α	0.038117	0.000808	47.18410	0.0000
		β	0.174190	0.021248	8.198046	0.0000
		γ	2.55E-07	4.29E-08	5.934988	0.0000
	Conoil	ω	0.000795	0.000575	1.383591	0.1665
		α	-0.000616	3.49E-05	-17.63133	0.0000
		β	0.171291	0.599084	0.285922	0.7749
		γ	-7.59E-07	8.48E-07	-0.894652	0.3710
	Oando	ω	3.63E-06	2.43E-08	149.6590	0.0000
		α	0.088207	0.006639	13.28633	0.0000
		β	0.910235	0.004235	214.9561	0.0000
		γ	-4.72E-08	4.86E-10	-97.19640	0.0000
	Total Energies	ω	1.36E-05	4.14E-07	32.71878	0.0000
		α	-2.60E-05	6.84E-07	-37.92745	0.0000
		β	0.981522	0.000523	1878.011	0.0000
		γ	-1.25E-07	3.62E-09	-34.39739	0.0000

Table 5 revealed important differences in how the RB-GARCH-X models (zero drift, non-zero drift, and adaptive) captured volatility dynamics for the selected oil and gas stocks. For Chevron, the zero-drift specification showed strong persistence ($\beta = 0.7140$, $p < 0.01$) with significant shock effects ($\alpha = 0.1236$), while the non-zero drift model produced an insignificant β ($p = 0.1727$), suggesting weaker persistence. The adaptive ARB-GARCH-X improved overall balance with α (0.0381) and β (0.1742) both significant, alongside a robust γ term, indicating that adaptability enhanced volatility modelling. For Conoil, the non-zero drift model performed best with highly significant persistence ($\beta = 0.9492$, $p < 0.01$), while both the zero-drift and adaptive specifications produced weak or insignificant β (0.1713, $p = 0.7749$), suggesting that drift-adjusted estimation captured Conoil's volatility structure more effectively. In Oando, the zero-drift and adaptive models both demonstrated strong persistence ($\beta \approx 0.91$, $p < 0.01$) and significant α, indicated that either specification captured Oando's volatility clustering well, while the non-zero drift model showed lower persistence ($\beta = 0.2814$). Finally, for Total Energies, all three models revealed very high persistence ($\beta \approx 0.98$–0.99, $p < 0.01$), but the adaptive model balances drift and no-drift conditions with consistent significance across parameters.

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3.5 Post-Diagnostic Tests

3.5.1 Model Selection Criteria

Competing models were assessed using the log-likelihood, Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), and Hannan–Quinn Criterion (HQC).

Table 6: Model Comparison of RB-GARCH-X Using Information Criteria

Oil and Gas Stock	**Model**	**AIC**	**SIC**	**HQC**	Log likelihood
Chevron	RB-GARCH-X with zero drift (Parkinson Estimator)	-5.669755	-5.657864	-5.665481	8615.523
	RB-GARCH-X with non-zero drift (Rogers-Satchel's Estimator)	-5.775598	-5.763707	-5.771323	8776.245
	ARB-GARCH-X (adapt to zero and non-zero drift)	-5.587733	-5.575842	-5.583458	8490.972
Conoil	RB-GARCH-X with zero drift (Parkinson Estimator)	-4.183535	-4.171270	-4.179118	6126.512
	RB-GARCH-X with non-zero drift (Rogers-Satchel's Estimator	-4.404179	-4.391914	-4.399762	6449.314
	ARB-GARCH-X (adapt to zero and non-zero drift)	-4.184654	-4.172388	-4.180236	6128.148
Oando	RB-GARCH-X with zero drift (Parkinson Estimator)	-3.843228	-3.832250	-3.839301	6428.034
	RB-GARCH-X with non-zero drift (Rogers-Satchel's Estimator	-3.676893	-3.665916	-3.672966	6150.088
	ARB-GARCH-X (adapt to zero and non-zero drift)	-3.896253	-3.885276	-3.892326	6516.639
Total Energies	RB-GARCH-X with zero drift (Parkinson Estimator)	-5.021226	-5.009166	-5.016887	7502.690
	RB-GARCH-X with non-zero drift (Rogers-Satchel's Estimator	-5.132054	-5.119994	-5.127715	7668.156
	ARB-GARCH-X (adapt to zero and non-zero drift)	-5.102854	-5.090794	-5.098515	7624.561

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The model comparison results in Table 6 indicated clear differences in the performance of the RB-GARCH-X models across the four oil and gas companies. For Chevron, the non-zero drift specification (Rogers-Satchell estimator) provided the best fit, as reflected in the lowest information criteria values (AIC = -5.7756, SIC = -5.7637, HQC = -5.7713) and the highest log-likelihood (8776.245), suggesting that incorporating drift captured Chevron's volatility structure more effectively than the zero-drift or adaptive alternatives. Similarly, for Conoil, the non-zero drift model also dominated with markedly lower AIC, SIC, and HQC values (-4.4042, -4.3919, -4.3998) and a substantially higher log-likelihood (6449.314), confirming that drift-adjusted volatility dynamics are more suitable. In the case of Oando, however, the adaptive ARB-GARCH-X outperformed both zero and non-zero drift models, with the lowest information criteria (AIC = -3.8963, SIC = -3.8853, HQC = -3.8923) and the highest log-likelihood (6516.639), highlighting the benefit of adaptability in capturing its complex volatility process. For Total Energies, the non-zero drift specification again provided the best performance (AIC = -5.1321, SIC = -5.1200, HQC = -5.1277, log-likelihood = 7668.156), though the adaptive model also performed strongly, outperforming the zero-drift alternative. In general, the results suggested that the non-zero drift RB-GARCH-X model generally provided superior fit for Chevron, Conoil, and Total Energies, while the adaptive ARB-GARCH-X was most effective for Oando, emphasizing that the optimal volatility specification depends on the underlying stocks return dynamics.

3.5.2 Out-of-Sample Forecast Evaluation

Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Theil's U-statistic were used to evaluate the out-of-sample forecast performance. The results are shown in Table 7.

Table 7: Forecast Accuracy Measures for Competing Models

Oil and Gas Stocks	Best fitted Model	RMSE	MAE	MAPE	Theil Inequality
Chevron	RB-GARCH-X with non-zero drift (Rogers-Satchel's Estimator	0.020753	0.005128	7.709918	0.998915
Conoil	RB-GARCH-X with non-zero drift (Rogers-Satchel's Estimator	0.031278	0.014748	31.79043	0.998683
Oando	ARB-GARCH-X (adapt to zero and non-zero drift)	0.039162	0.026428	81.48314	0.999315
Total Energies	RB-GARCH-X with non-zero drift (Rogers-Satchel's Estimator	0.023480	0.011671	61.86186	0.957551

The forecast accuracy results in Table 7 indicated that the RB-GARCH-X with non-zero drift (Rogers-Satchell's estimator) provided the best predictive performance for Chevron,

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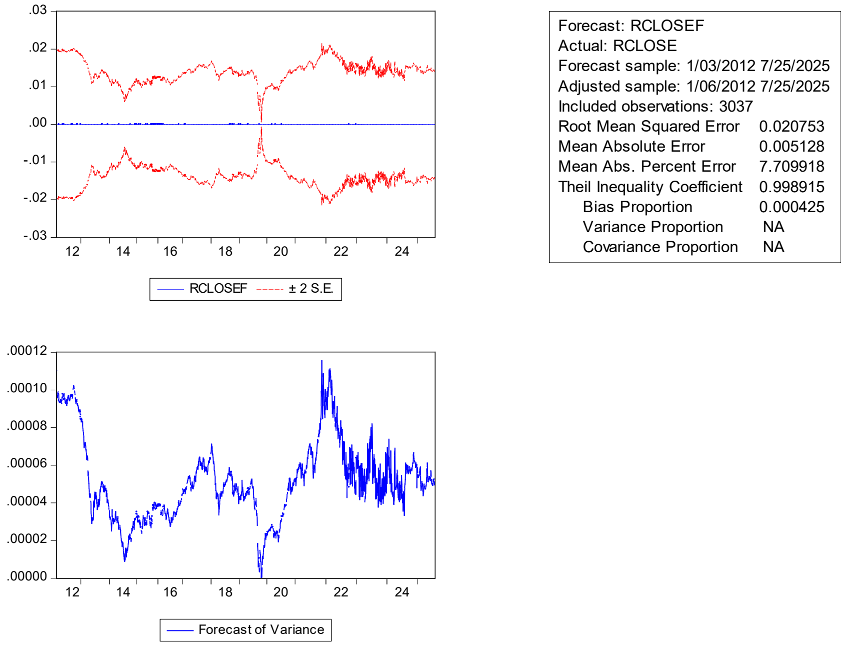
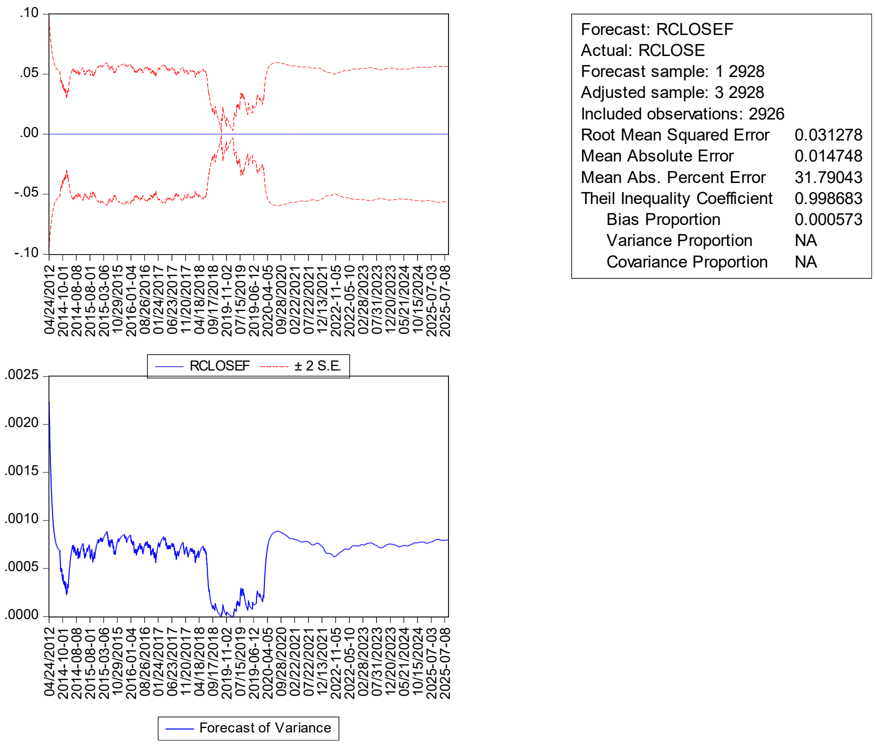
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)

Conoil, and Total Energies, with relatively low RMSE, MAE, and strong efficiency as shown by Theil's index. In contrast, Oando was better captured by the adaptive ARB-GARCH-X model, reflecting its more volatile dynamics despite higher forecast errors. Overall, the findings suggested that the non-zero drift specification is generally superior, while the adaptive model is more effective when market conditions deviate sharply from drift assumptions. Figures 3(a, b, c & d) are the forecast plot for Chevron, Conoil, Oando and Total Energies stock returns respectively for the study periods.



(a) Chevron



(b) Conoil

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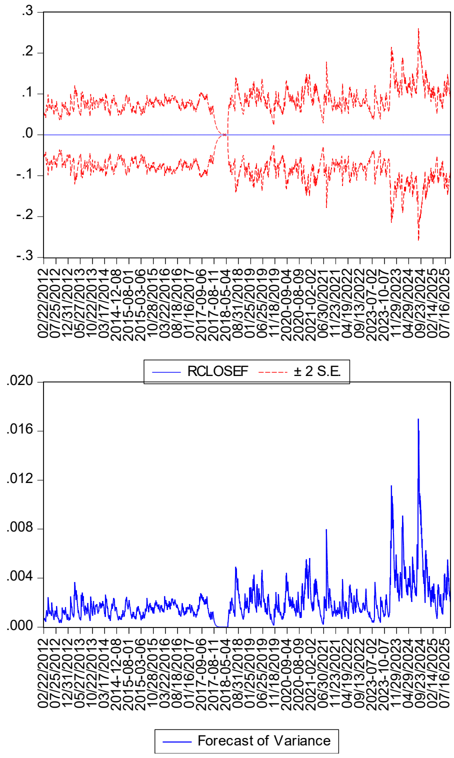
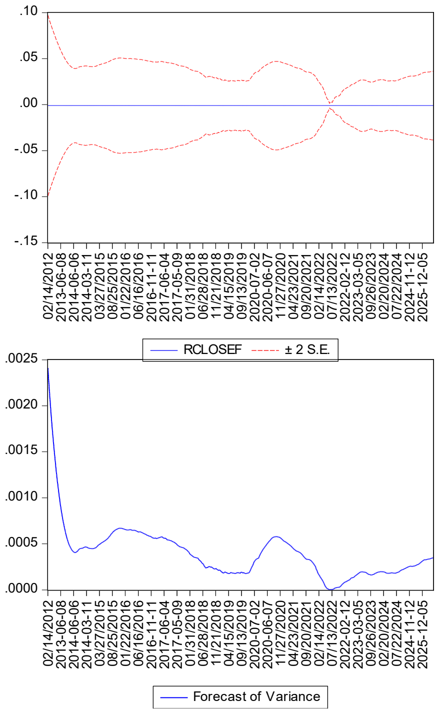


Forecast: RCLOSEF	
Actual: RCLOSE	
Forecast sample: 1 3344	
Adjusted sample: 3 3344	
Included observations: 3342	
Root Mean Squared Error	0.039162
Mean Absolute Error	0.026428
Mean Abs. Percent Error	81.48314
Theil Inequality Coefficient	0.999315
Bias Proportion	0.000063
Variance Proportion	NA
Covariance Proportion	NA

(c) Oando



Forecast: RCLOSEF	
Actual: RCLOSE	
Forecast sample: 1 2988	
Adjusted sample: 3 2988	
Included observations: 2986	
Root Mean Squared Error	0.023480
Mean Absolute Error	0.011671
Mean Abs. Percent Error	61.86186
Theil Inequality Coefficient	0.957551
Bias Proportion	0.003837
Variance Proportion	NA
Covariance Proportion	NA

(d) Total

Figures 3(a, b, c & d): Forecast Plots.

Figures 3(a, b, c & d) presented the forecast plots for Chevron, Conoil, Oando and Total stock returns respectively. The plots showed the actual and forecast values of the stock returns within a standard error band as well as the forecast of variance over the same time frame.

4. Findings

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This paper revealed that the proposed Range-Based GARCH-X (RB-GARCH-X) models effectively captured volatility dynamics in Nigeria's oil and gas sector, with the non-zero drift specification (Rogers–Satchell estimator) providing the best fit and forecast accuracy for Chevron, Conoil, and Total Energies, while the Adaptive RB-GARCH-X (ARB-GARCH-X) proved superior for Oando, highlighting its flexibility in highly volatile markets. Parameter estimates confirmed strong volatility persistence ($\beta > 0.8$) with the adaptive model balancing short- and long-term effects more effectively. Forecast evaluation revealed that non-zero drift models generally offered the best predictive accuracy, except for Oando, where the adaptive model excelled. Incorporating exogenous variables improved the models' responsiveness to structural and external shocks, reflecting the realities of Nigeria's oil-dependent economy.

5. Conclusion

The proposed models provided a robust, flexible, and context-specific framework for volatility modelling, with practical implications for forecasting, risk management, and policy formulation. Nigeria oil and gas sector must integrate advanced volatility models into investment, regulatory, and policy frameworks to enhance stability, attract investment, and reduce vulnerability to external shocks. Policymakers, regulators, and industry stakeholders in Nigeria's oil and gas sector should prioritize the integration of advanced range-based volatility models, specifically the Range-Based GARCH-X (RB-GARCH-X) with non-zero drift (Rogers–Satchell estimator) for most firms and the Adaptive RB-GARCH-X (ARB-GARCH-X) for highly volatile stocks such as Oando; into risk management frameworks, investment decision processes, and regulatory stress-testing protocols. Finally, the Nigerian Upstream Petroleum Regulatory Commission is urged to encourage or mandate the use of these context-specific, high-frequency-capable volatility models in capital adequacy assessments, margin requirements, and scenario analysis to reduce systemic risk and foster greater investor confidence in Nigeria's oil-dependent economy.

Acknowledgement

I am sincerely grateful to the Government of Nigeria and the Petroleum Technology Development Fund (PTDF) for their support through the scholarship that made this study possible.

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