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Application of Modular Ratio in Predicting the Behavior of Externally Strengthened RE Beams

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Abstract

This study presents the development and validation of a theoretical based mathematical expression for predicting the flexural behaviour of externally reinforced concrete (RC) beams using steel plates and carbon fiber-reinforced polymer (CFRP) fabrics. The mathematical expression incorporates the modular ratio as an input variable and was validated against experimental results reported by John et al. (2022, 2022a). Six beam samples, including both control and retrofitted specimens, were analysed. The retrofitting techniques varied in terms of material type, bond thickness, and number of reinforcement layers. Comparative analysis between experimental and theoretical failure loads revealed a strong correlation, with percentage differences ranging from -1.08% to 6.37%. These results demonstrate that the model provides accurate and conservative estimates of load-carrying capacity, which is a desirable characteristic in structural design. Furthermore, Analysis of Variance (ANOVA) was conducted at a 95% confidence level to statistically evaluate the agreement between experimental and theoretical results. The ANOVA yielded a p-value of 0.673, indicating no significant difference between the two datasets. This confirms the model's predictive reliability and its potential for practical engineering applications. The study concludes that Equation (30) is an effective and efficient tool for assessing the flexural performance of retrofitted RC beams, offering an alternative to exhaustive experimental testing and supporting safer and more cost-effective structural retrofitting strategies.

1.0 INTRODUCTION

Reinforced concrete (RC) structures are widely used in modern construction due to their durability, strength, and adaptability to various forms and functions [1-2]. However, many RC structures around the world are experiencing deterioration due to aging, overloading, poor maintenance, or changes in usage requirements [3-4]. To extend the service life and improve the performance of such structures, various strengthening techniques have been developed, among which external strengthening using composite materials like fiber-reinforced polymers (FRP) or steel plates have proven to be effective and practical [5-7].

When externally bonded materials are used to strengthen RC beams, the structural system changes significantly [8-10]. The original assumptions used in conventional reinforced concrete design, particularly those relating to the stress distribution and strain compatibility, must be revisited [11-13]. One of the essential tools in analyzing composite cross-sections especially those involving materials with different elastic moduli is the modular ratio [14]. The modular ratio allows for the transformation of materials with different stiffnesses into an equivalent homogeneous section, enabling more straightforward stress and strain analysis [15-17].

Despite the extensive use of externally strengthened beams in both research and practice, there remains a need for accurate yet simplified analytical tools to predict their flexural behavior. While advanced numerical methods such as finite element analysis offer detailed awareness, they are often complex, computationally intensive, and may not be practical for routine design. In this context, the application of the modular ratio offers a more accessible analytical approach that can be used to approximate the behavior of strengthened RC beams with reasonable accuracy.

This study aims to explore and apply the concept of modular ratio in predicting the flexural behavior of RC beams that have been externally strengthened. By integrating material properties, cross-sectional geometry, and bond characteristics into a simplified model, the study seeks to bridge the gap between complex numerical techniques and practical engineering applications. The findings are expected to contribute to more efficient design procedures and better understanding of the structural behavior of strengthened RC elements.

2.0 METHODOLOGY

2.1 Development of theoretical model

Figure 1 illustrates the idealized stress and strain blocks in an externally strengthened RC beam, along with the equilibrium of forces at maximum load as presented in Wang *et al.* [18]. The forces in different parts of the RC beam cross-section are expressed in terms of the stresses in the internal reinforcement, concrete, and external strengthening material. Based on the assumptions outlined below, closed-form expressions for the load resistance of the strengthened beam cross-section are derived for various failure scenarios.

Assumptions

i. The transverse normal remains straight and normal to the neutral axis and undergoes no change in length during deformation.
ii. The ultimate practical concrete (ε_{cu}) strain is 0.0035.
iii. The external material for strengthening has a linear relationship to failure.
iv. Shear displacement within the epoxy matrix or adhesive layer is disregarded because the adhesive layer is relatively thin.
v. The design and analysis computations are dependent on the actual beam size, the arrangement of internal reinforcement, and mechanical characteristics of the concrete element under consideration.

The approach for calculating the moment capacity of externally strengthened rectangular RC beams is based on satisfying two key requirements: the static equilibrium of forces in the concrete, internal reinforcement, and external reinforcement; and the strain compatibility among the concrete, internal reinforcement, and external reinforcement. Considering Figure 1, the ultimate practical strain assumed for concrete is $\varepsilon = 0.0035$ [18].

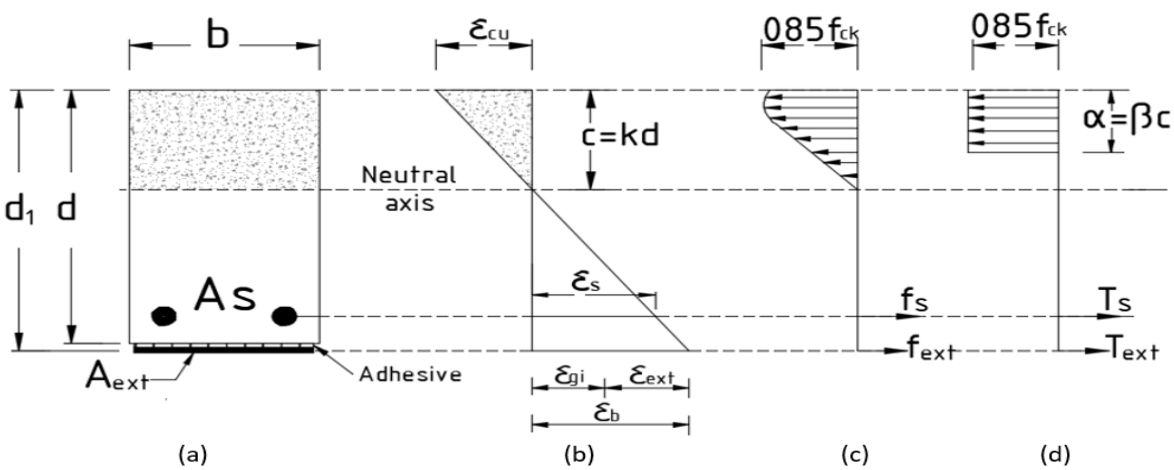

Figure 1: Stress, strain distribution and static equilibrium of forces for beam with external reinforcement: (a) beam cross-section, (b) strain block, (c) parabolic stress block, and (d) rectangular stress block.

where,
ε_s *is strain in* internal rebar
ε_{cu}, strains in concrete compression fibers ε_{cu}
ε_{gi} *is* strain in extreme tension fibers before strengthening
ε_{ext} *is* strain in extreme tension fibers after strengthening reinforcement- strengthening,
i denotes initial situations conforming to the nominal capacity of the rectangular beam with no external steel sheet/FRP while subscript g refers to the "glued side" of the beam. Also, the subscript e_{xt} refers to external reinforcement.

The ultimate strain expression in steel (ε_s), concrete (ε_{cu}), and at the tensile fibers where external reinforcement is applied (ε_b) are formulated as:

$$\varepsilon_b = \varepsilon_{gi} + \varepsilon_{ext} \quad (1)$$

Strain Compatibility: After bonding the external reinforcement, the following relationship is developed based on comparable triangles in Figure 1(b):

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_s}{d-c} = \frac{\varepsilon_b}{d_1-c} \quad (2)$$

Substituting $\varepsilon_b = \varepsilon_{gi} + \varepsilon_{ext}$ into Equation 2
Equation 2 can be expressed as:

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_s}{d-c} = \frac{\varepsilon_{ext} + \varepsilon_{gi}}{d_1-c} \quad (3)$$

Force of Equilibrium: The compressive force developed in concrete (F_c) is

$$F_c = 0.85 f_{ck} ab \quad (4)$$

The tensile force in steel (T_s) is

$$T_s = A_s f_s = A_s E_s \varepsilon_s \text{ (Before steel yielding, } \varepsilon_s < \varepsilon_y) \quad (5)$$

$$T_s = A_s f_y = A_s E_s \varepsilon_y \text{ (after steel yielding, } \varepsilon_s \geq \varepsilon_y) \quad (6)$$

The tensile force in external reinforcement (T_{ext}) before or at rupture $\left(\varepsilon_{ext} < \varepsilon_{ext,u}\right)$ is

$$T_{ext} = A_{ext} f_{ext} = A_{ext}(E_{ext}\varepsilon_{ext}) \quad (7)$$

2.1.1 Depth of Neutral Axis ($c = kd$) with and without External Reinforcement

The following expression, based on elastic analysis similar to that used for typical RC beams, indicates the depth of the neutral axis, kd, in a rectangular beam (Figure 2), both with and without external reinforcement. In this study, the factor k is referred to as the neutral axis factor.

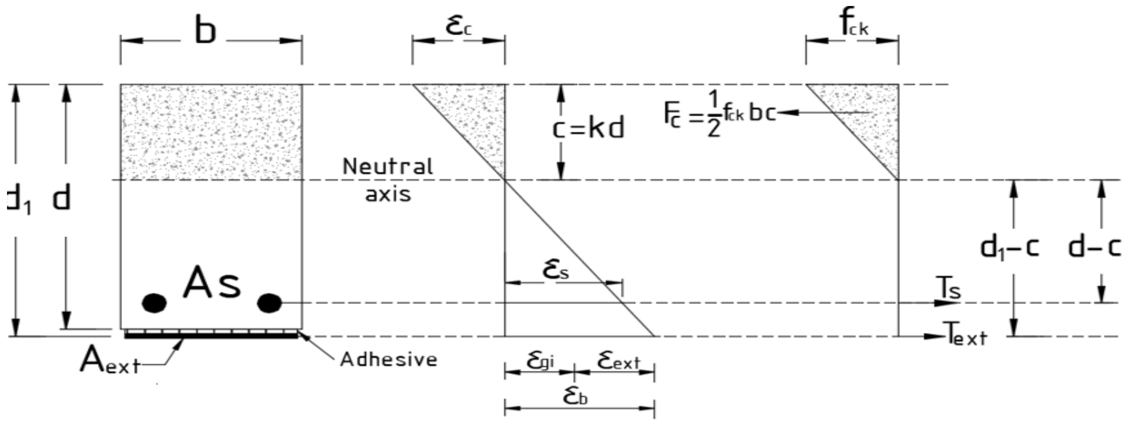

Figure 2: Strain and stress distribution of externally RC beam at service loads

2.1.2 Determination of the neutral axis factor (*k*) with external Reinforcement

The linear stress-strain relationship development

$$f_s = E_s\varepsilon_s, \qquad f_{ext} = E_{ext}\varepsilon_{ext}, \qquad f_c = E_c\varepsilon_{cu}) \quad (8)$$

Using the Eq. (8) stress values as a substitute for the force equilibrium in a cracked rectangular RC beam section with external reinforcement,

$$A_s f_s + A_{ext} f_{ext} = \frac{bc}{2} f_c \quad (9)$$

$$A_s(E_s\varepsilon_s) + A_{ext}(E_{ext}\varepsilon_{ext}) = \frac{bc}{2}(E_c\varepsilon_{cu}) \quad (10)$$

From Equation 2 and 3, we obtain, ε_{gi} is negligible after strengthening

$$\varepsilon_s = \varepsilon_{cu}\frac{d-c}{c} = \varepsilon_{cu}\left(\frac{d}{c} - 1\right), \quad \varepsilon_{ext} = \varepsilon_{cu}\frac{d_1 - c}{c} = \varepsilon_{cu}\left(\frac{d_1}{c} - 1\right)$$

Substituting the above values into Eq. 10

$$A_s E_s\left(\varepsilon_{cu}\frac{d-c}{c}\right) + A_{ext}E_{ext}\left(\varepsilon_{cu}\frac{d_1-c}{c}\right) = \frac{bc}{2}(E_c\varepsilon_{cu}) \quad (11)$$

Dividing Equation 11 throughout by ε_{cu} and simplifying,

$$A_s E_s\left(\frac{d-c}{c}\right) + A_{ext}E_{ext}\left(\frac{d_1-c}{c}\right) = \frac{bc}{2}(E_c) \quad (12)$$

$$A_s E_s(d-c) + A_{ext}E_{ext}(d_1-c) = \frac{bc^2}{2}(E_c) \quad (13)$$

$$A_s\frac{E_s}{E_c}(d-c) + A_{ext}\frac{E_{ext}}{E_c}(d_1-c) = \frac{bc^2}{2} \quad (14)$$

$(E_s/E_c = n_s)$ *is* modular ratio for internal *and* $(E_{ext}/E_c = n_{ext})$ is modular ratio for external reinforcement as stated by Bakalarz and Kossakowski [14].

$$A_s n_s(d-c) + A_{ext}n_{ext}(d_1-c) = \frac{bc^2}{2} \quad (15)$$

Dividing Equation 15 throughout by *bd* gives

$$2\frac{A_s}{bd}n_s(d-c) + 2\frac{A_{ext}}{bd}n_{ext}(d_1-c) = \frac{c^2}{d} \quad (16)$$

Equation 16 may be written to accommodate reinforcement ratios:

$(A_s/bd) = \rho_s$ and $(A_{ext}/bd_{ext}) = \rho_{ext}$:

$$2\rho_s n_s(d-c) + 2\rho_{ext}n_{ext}(d_1-c) = \frac{c^2}{d} \quad (17)$$

Eq. 17 can be rewriting as a quadratic form as;

$$\frac{c^2}{d} + 2(\rho_s n_s + \rho_{ext}n_{ext})c - 2(\rho_s n_s d + \rho_{ext}n_{ext}d_1) = 0 \quad (18)$$

Solving Eq.18 for c,

$$c = \frac{-2(\rho_s n_s + 2\rho_{ext} n_{ext}) + \sqrt{4(\rho_s n_s + \rho_{ext} n_{ext})^2 + (4)\left(\frac{2}{d}\right)(\rho_s n_s d + \rho_{ext} n_{ext} d_1)}}{\frac{2}{d}} \quad (19)$$

Which, when simplified, expressed

$$c = \left(\sqrt{(\rho_s n_s + \rho_{ext} n_{ext})^2 + 2(\rho_s n_s + \rho_{ext} n_{ext}\frac{d_1}{d})} - (\rho_s n_s + \rho_{ext} n_{ext})\right)d \quad (20)$$

$$\frac{c}{d} = \left(\sqrt{(\rho_s n_s + \rho_{ext} n_{ext})^2 + 2(\rho_s n_s + \rho_{ext} n_{ext}\frac{d_1}{d})} - (\rho_s n_s + \rho_{ext} n_{ext})\right)$$

Observing that $c = kd$, the expression Eq.20 can be used to predict the k (neutral axis factor) for a RC beam with internal and external reinforcement under service load conditions:

$$k = \sqrt{(\rho_s n_s + \rho_{ext} n_{ext})^2 + 2(\rho_s n_s + \rho_{ext} n_{ext}\frac{d_1}{d})} - (\rho_s n_s + \rho_{ext} n_{ext}) \quad (21)$$

2.1.3 Prediction of k without external reinforcement

Setting the variables relating to external reinforcement, i.e., $\rho_{ext}, and\ n_{ext}$ to zero in Equation 21 yields the function of neutral axis factor k, for RC beam without external reinforcement. Therefore,

$$k = \sqrt{(\rho_s n_s)^2 + 2\rho_s n_s} - (\rho_s n_s) \quad (22)$$

2.1.4 Nominal Flexural Capacity of a RC Beam

The tension and compression force equilibrium:

$$0.85 f_{ck} ab = A_s f_y + A_{ext}(E_{ext}\varepsilon_{ext,ru}) \quad (23)$$

In order to get the depth of the comparable RC beam stress block a, we must first solve for it.

$$a = \left(\frac{A_s f_y + A_{ext}(E_{ext}\varepsilon_{ext,ru})}{0.85 f_{ck} b}\right) \quad (24)$$

Also, from Figure1 and 3.2;

$$a = \beta c, \ c = kd, \quad (25)$$

From Equation 25 we obtain,

$$a = \beta kd, \ as\ reported\ by\ ACI\ 318\ [14] \quad (26)$$

Substituting Equation 21 into Equation 26, we have

$$a = \beta d\left[\sqrt{(\rho_s n_s + \rho_{ext} n_{ext})^2 + 2(\rho_s n_s + \rho_{ext} n_{ext}\frac{d_1}{d})} - (\rho_s n_s + \rho_{ext} n_{ext})\right] \quad (27)$$

But

$\beta = 1.09 - 0.008 f_{ck}$ **(**ACI 318, Section 10.2.7.3) (28)

The ultimate moment is computed by taking moments at the center of the depth of the stress block:

$$M_s = A_s f_y\left(d - \frac{a}{2}\right) + A_{ext}\left(E_{ext}\varepsilon_{ext,ru}\right)\left(d_1 - \frac{a}{2}\right) \quad (29)$$

For one-third point load application, the failure is express as

$$P = \frac{6}{L}\left(A_s f_y\left(d - \frac{a}{2}\right) + A_{ext}\left(E_{ext}\varepsilon_{ext,ru}\right)\left(d_1 - \frac{a}{2}\right)\right) \quad (30)$$

2.1.5. Beams investigated

The theoretical model developed with modular ratio as input variable was validated by comparing its theoretical results with experimental data reported by John et al. [19-20]. Referring to beams from John et al. [19-20], FA0 was used as the control beam and was tested without any CFRP fabric. Beam PFA-2 was strengthened with 1.0 mm-thick plates applied to the tension face, using a bond thickness of 2 mm.

Beam FA-2 was strengthened with 200 g/m² CFRP fabric strips measuring 100 mm in width and 1100 mm in length, bonded to the tension face with a 2 mm adhesive layer. Beam FC-2 was also strengthened using 300 g/m² CFRP fabric strips of the same dimensions (100 mm × 1100 mm), bonded to the tension face with a 2 mm bond thickness. Beam FR-2H was strengthened with 200 g/m² CFRP fabric, 100 mm wide and 1000 mm long (surface area of 0.1 m²), bonded at the tension face with a 2 mm adhesive layer. Beam FB-4/2 was strengthened with two layers of 200 g/m² CFRP fabric; the first layer had a 4 mm bond thickness, while the second layer had a 2 mm bond thickness. The material properties were also adopted from John et al., [20].

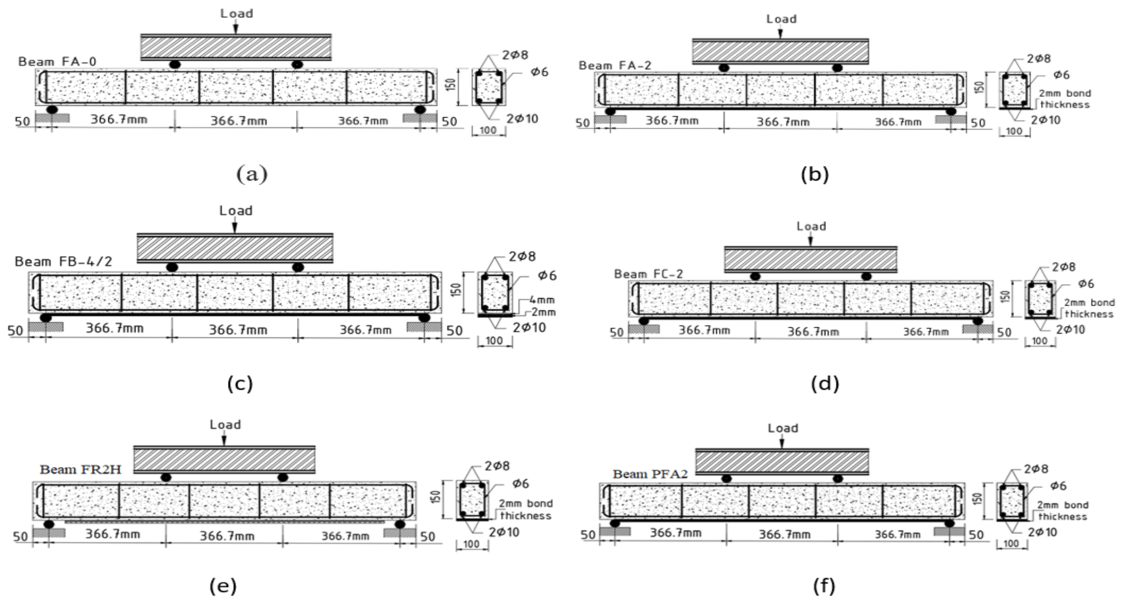

Figure 3. Details of beam to be validated

3.0 RESULTS AND DISCUSSION

Equation (30) was validated by comparing its predicted results with experimental data obtained from studies on externally reinforced concrete (RC) beams. These experimental results, reported by John et al. [19] and John et al. [20], provided a reliable benchmark to assess the accuracy and effectiveness of the proposed equation.

Table 1 provides a comparison between the experimental failure loads of various externally reinforced RC beam samples (as reported by John et al., [19-20]) and the theoretical failure loads predicted using Equation (30). The percentage difference between the experimental and theoretical values serves as an indicator of the equation's predictive accuracy. An evaluation of Table 1 reveals that the mathematical model accurately predicted the load-carrying capacity for all beams considered in this study. The results also indicate that Equation (30) generally provides conservative estimates of the load-carrying capacity.

Table 1: Comparison of Theoretical Versus Experimental Failure Loads

Sample ID	**Experimental failure load kN John et al. (2022) and (2022a)**	**Theoretical Failure Load kN (Equation 30)**	**Percentage Difference (%)**
FA-0	93.3	92.3	-1.08
FA-2	124	132	6.06
FB-4/2	144	153	5.88
FC-2	147	157	6.37
FR-2-H	125	132	5.30
PFA-2	122	123	0.81

3.1 Predictive Strength of the Equation

The predicted failure loads from Equation (30) show good alignment with the experimental results, with percentage differences ranging from -1.08% to 6.37%, indicating a strong correlation and reliability. The lowest deviation occurs with sample PFA-2 (0.81%), and even the highest deviation (6.37% for FC-2) remains within an acceptable error margin for engineering applications. This close agreement further validates the modelling assumptions. As stated in Section 2.1, the present model assumes a perfect bond between the plate and the concrete surface and neglects shear deformation, consistent with several established analytical and numerical studies [21-23]. The good correlation between predicted and experimental results confirms that this assumption is reasonable for the present study, given that the specimens employed a thin adhesive layer (≤ 2 mm) and adequate anchorage length beyond the effective bond length, ensuring minimal relative slip at the CFRP to concrete interface.

3.2 Performance of the Developed Equation

Equation (30) demonstrates a consistent and reasonable ability to predict the failure load of externally reinforced RC beams across various configurations. Most predictions slightly overestimate the failure load, which is typically conservative in structural design. This trend suggests the model errs on the side of safety, an essential trait in structural engineering. The small differences validate the robustness of the equation and affirm its utility as a predictive tool.

3.3 Real-Life Application

In practical scenarios, accurate prediction of failure loads is crucial for the safety and cost-effectiveness of structural retrofitting and reinforcement strategies. The results suggest that Equation (30) can be confidently applied in the design and assessment of externally reinforced RC beams in real-world projects. Engineers can use this equation to estimate the structural performance of retrofitted elements, potentially reducing the need for extensive physical testing while maintaining reliability and compliance with safety standards. The comparative analysis confirms that Equation (30) is a reliable, practical, and efficient tool for predicting the flexural failure loads of externally reinforced RC beams, supporting both academic research and engineering practice.

3.4 Data analysis

A linear regression-based Analysis of Variance (ANOVA) was performed to check if there is a meaningful difference between the experimental results and the theoretical predictions, using a 95% confidence level ($\alpha = 0.05$).

Referring to Table 2, the ANOVA analysis at a 95% confidence level shows no statistically significant difference between the experimental and theoretical failure loads predicted by Equation (30). With an F-value of 0.189 and a p-value of 0.673 (greater than the 0.05 threshold), the results indicate that the variation between the two sets of data is not significant. This suggests a strong correlation and confirms that the mathematical model provides reliable and consistent predictions of the load-carrying capacity for the externally reinforced RC beams. Therefore, Equation (30) can be considered accurate and suitable for practical structural design applications.

The paired t-test revealed no statistically significant difference between the theoretical and experimental failure loads, with a mean difference of 5.37 kN (SD = 4.32 kN, 95% CI = 0.84–9.90 kN,). However, the theoretical model (Equation 30) generally overpredicts the experimental results, but the magnitude of this deviation remains within an acceptable engineering range. The error analysis further supports the model's reliability, with a root mean square error (RMSE) of 6.66 kN, a mean absolute percentage error (MAPE) of 4.31%, and a mean bias of 5.37 kN, all suggesting strong agreement between predicted and observed values. Overall, the relatively low MAPE and RMSE demonstrate that the theoretical model provides a consistent and accurate estimation of the experimental failure loads, with deviations that are minor and statistically acceptable for practical engineering applications.

Table 2. ANOVA Table (95% Confidence Level)

Source	Sum of Squares	df	F-value	p-value
Type	86.5	1	0.19	0.67
Residual	4578	10		

Referring Figure 4, the high R^2 value of 0.9911 suggests that the theoretical model explains about 99% of the variation in the experimental results, indicating good predictive accuracy. The relatively low Root Mean Square Error (RMSE) of 6.69 kN shows that the average deviation between theoretical and experimental loads is small. However, a slight overprediction is noticeable at higher load levels, implying that the model tends to slightly overestimate the beam strength, a conservative and acceptable trend in structural design.

4.0 Conclusion

This study developed a theoretical model to predict externally strengthened RC beams. The model closely matched the experimental results and showed that adding steel plates/FRP significantly improved both the load-carrying capacity and stiffness of the beams. Here are four concise conclusions based on the analysis of the predictive performance of Equation (30):

i. Equation (30) demonstrated strong predictive capability, with percentage differences between experimental and theoretical failure loads ranging from -1.08% to 6.37%, indicating reliable performance across different beam samples.
ii. The equation consistently provided slightly higher failure load predictions, which is beneficial in structural engineering and safer design approach.
iii. The model performed well across various reinforcement configurations, showing its versatility and robustness in predicting the flexural behaviour of externally reinforced RC beams.

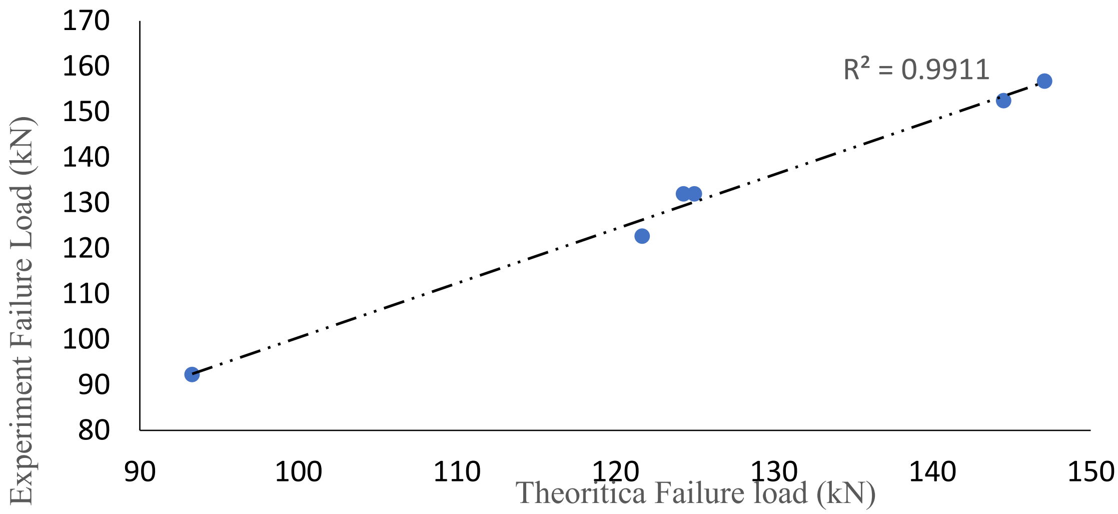

Figure 4. Experimental vs Theoretical Failure Loads

iv. Given its accuracy and reliability, Equation (30) is suitable for real-world structural design and assessment, reducing dependence on exhaustive experimental testing and facilitating more efficient retrofitting strategies.

Notations/Nomenclature

ε_{cu} strains in concrete compression fibers
ε_s strain in internal rebar
ε_{gi} strain in extreme tension fibers before strengthening
ε_{ext} strain in extreme tension fibers after strengthening
i initial situations conforming to the nominal capacity of the rectangular beam with no external steel sheet/FRP
e_{xt} external reinforcement
ε_b Strain in concrete substrate developed by a given bending moment (tension in positive)
c Distance from extreme compression fiber to the neutral axis
d Distance from extreme compression fiber to centroid of tension reinforcement
f_c Compressive stress in concrete
b Width of compression face of member
A_s Area of non-prestressed steel reinforcement
f_y yield strength of non-prestressed steel reinforcement
ε_s Strain in non-prestressed steel reinforcement
ε_y Yield Strain in non-prestressed steel reinforcement
k Experimental constant describing the gradient of the effective bond length
f_c Compressive stress in concrete
n_{ext} Modular ratio of external reinforcement
E_{ext} Modulus of elasticity of external reinforcement
n_s Modular ratio of elasticity between steel and concrete
A_{ext} Area of non-prestressed steel external reinforcement
ρ_s Ratio of non-prestressed reinforcement
ρ_{ext} External reinforcement ratio
β Ratio of depth of equivalent rectangular stress block to depth of the neutral axis
(T_s) The tensile force in steel
(T_{ext}) The tensile force in external reinforcement
A_f Area of FRP external reinforcement
E_f Modulus of elasticity of steel
ε_{ext} Strain in external reinforcement
M_s Ultimate moment

Statements and Declarations

Conflicts of interest

The authors declare no conflicts of interest.

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This study did not receive any funding.

Data availability

The datasets generated and analyzed in this study are available from the corresponding author upon valid request.

Author contributions

FRANCIS, E. W: Conceptualization, Investigation, Review, Methodology, and Validation. AUTA: Conceptualization, Investigation and Validation. [1]AGUWA. and ABDULLAHI, A: Data analysis, editing, and Validation

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