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Parametric □scillations in Electric □scillatory □ystem

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ABSTRACT
The paper presents the parametric oscillations generated in an electric oscillatory system. Parametric oscillations are oscillations that are periodically modulated with time. The modulation depth and the carrier frequency are investigated by MATLAB/Simulink Model developed from Mathieu's equation. With this Model, parametric oscillations are generated. The maximum and minimum amplitudes of oscillations for each characteristic number, a and the characteristic parameter, q is determined. The time taken for one oscillation (which is the period) for each characteristic number and characteristic parameter is determined. The relationship between the carrier frequency, the modulation depth and the characteristic number are established through graphical illustrations. These are approximate results of the solutions of Mathieu equation in electric oscillatory system.

Keywords: *Carrier frequency, characteristic number, characteristic parameter, Mathieu's equation, modulation depth, parametric oscillations.*

1 INTRODUCTION

T The second order differential homogeneous equation or alternatively known as Mathieu's equation in canonical form (Figueroa & Torrentí, 2017) is applied in this paper for the purpose of explaiining the parametric oscillation in electric oscillatory circuit. This equation is necessary for the description of the vibrating or oscillatory body. The Hill equation describes the vibration or the oscillation of electrical system or mechanical system**.** The generation of parametric oscillations in three-phase autonomous reluctance generator is discussed (Rabinovici, 2001). An autonomous induction generator generates self-oscillations (Hail, 1999). Electromechanical device develops parametric oscillatory motion (.Russel, and Pickup, 1978) and parametric transformer (Tez &. Smith, 1984) have the potential of generating parametric oscillations when excited. In this paper, the modulation depth and the carrier frequency are investigated by MATLAB/Simulink Model. In electrical system whose electric circuit consists of energy-storing elements such as the capacitor, inductor and the resistor, the component elements can be periodically modulated with time resulting in parametric oscillations using Mathieu's equation. Parametric oscillation is a stable phenomenon and they appear first before the appearance of the parametric resonance (the unstable phenomenon). The excitations of parametric oscillations generate the steady growth of amplitude of the oscillations with time resulting into unstable phenomenon known as parametric resonance.. At natural frequency of a system, the system oscillates or vibrates as it maintains its stable condition. A system becomes unstable if it operates twice its fundamental natural frequency (Mclachian, 1951).

2 METHODOLOGY

The Mathieu's equation in canonical form is expressed as:

$$\frac{d^2y}{dz^2} + (a + 2q\cos 2z)y = 0 \tag{1}$$

Equation (1) is in MATLAB/Simulink shown in Figure 1

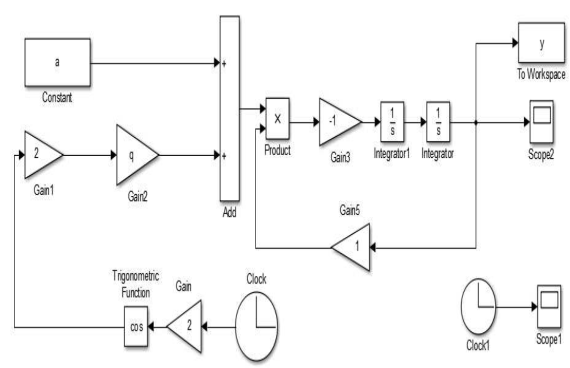

Figure 1: MATLAB/Simulink Model of Mathieu's equation

The time taken for one oscillation is called the period (in seconds). The period T is shown in Figure 2 when a=0.1 and q=0.1 and it is simulated for a 100 seconds. H_{max} and H_{min} are the maximum and the minimum amplitude of the oscillations generated. The difference between

H_{max} and H_{min} is the H_{diff} while the sum of H_{max} and H_{min} is denoted by H_{sum}. The parametric oscillations generated using Mathieu equation are investigated in Figure 3, 4, 5, 6, 7, 8 and Figure 9.

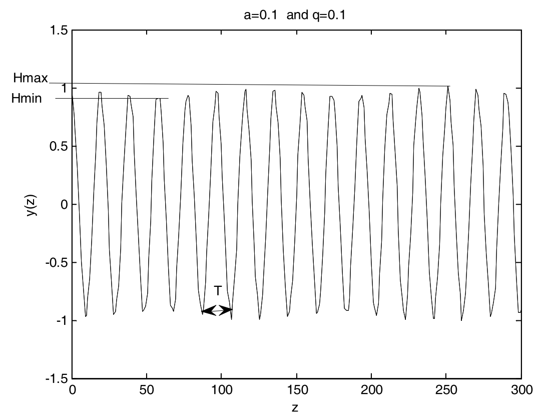

Figure 2: a=0.1 and q=0.1

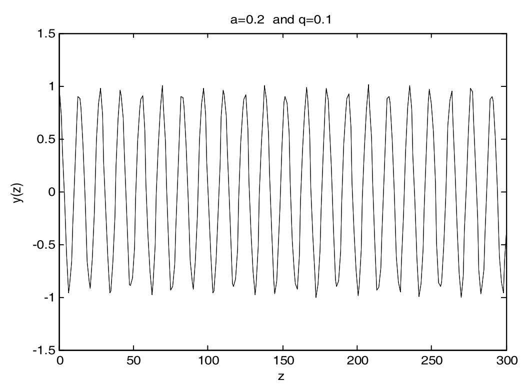

Figure 3: a=0.2 and q=0.1

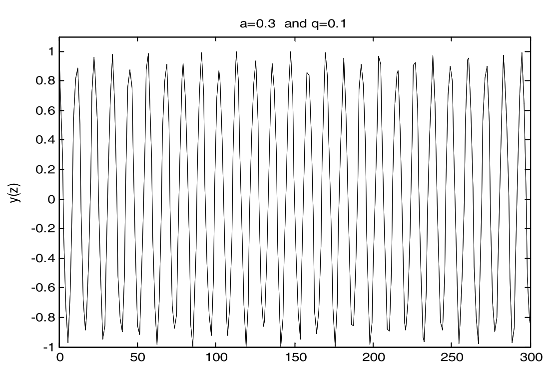

Figure 4: a=0.3 and q=0.1

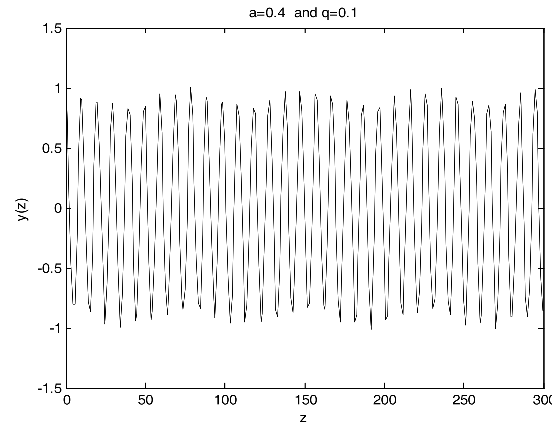

Figure 5: a=0.4 and q=0.1

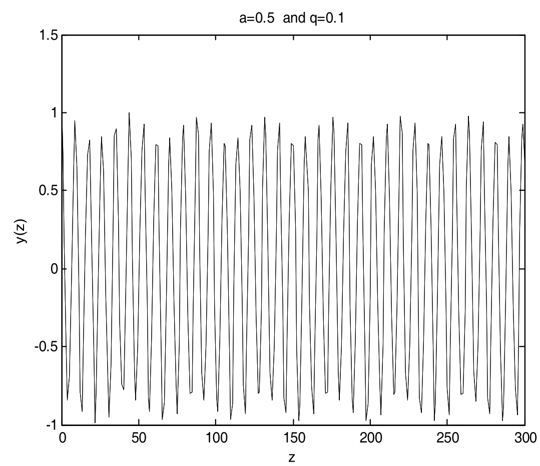

Figure 6: a=0.5 and q=0.1

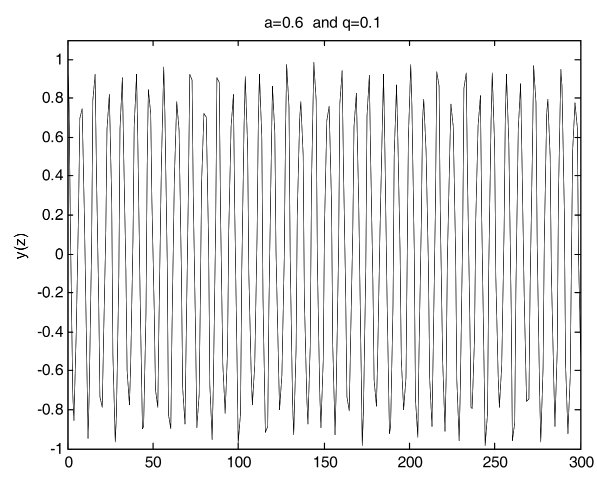

Figure 7: a=0.4 and q=0.1

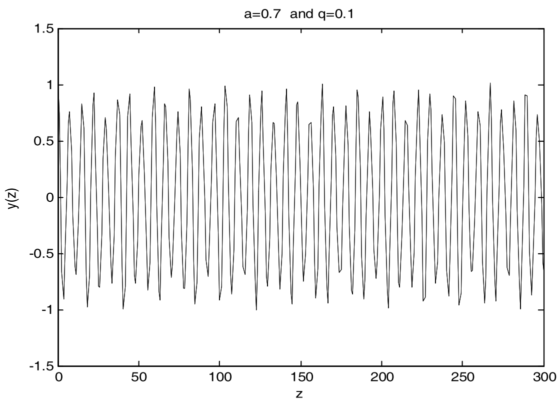

Figure 8: a=0.7 and q=0.1

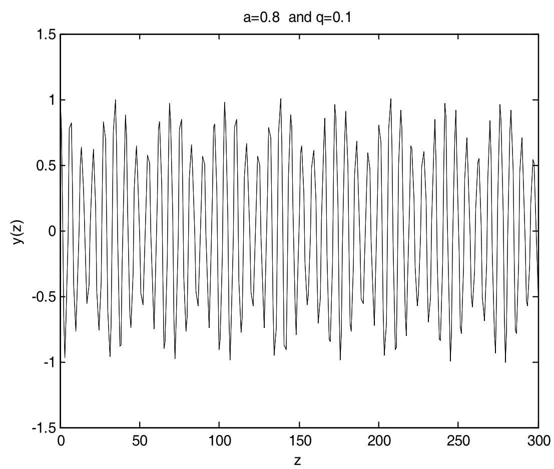

Figure 9: a=0.8 and q=0.1

2.1 RESULTS AND DISCUSSION

The results of the investigation are shown in Table 1

Table 1: Characteristic number and Modulation depth

a	H_{max}	H_{min}	H_{diff}	H_{sum}	$m = \frac{H_{diff}}{H_{sum}}$
0.1	1.017	0.9088	0.1082	1.9258	0.0562
0.2	1.011	0.8967	0.1143	1.9077	0.0599
0.3	0.998	0.8552	0.1428	1.8532	0.0771
0.4	1.003	0.8309	0.1721	1.8339	0.0938
0.5	1.002	0.7873	0.2147	1.7893	0.1200
0.6	0.9884	0.7482	0.2402	1.7366	0.1383
0.7	1.016	0.6538	0.3622	1.6698	0.2169
0.8	1.004	0.5377	0.4663	1.5417	0.3025

Table 2 shows the output data of period and carrier frequency

Table 2: Characteristic number, period and the carrier frequency

a	T_1 (sec)	T_2 (sec)	T (sec)	$w = \frac{2\pi}{T}$ (rad/s)
0.1	87.21	106.6	19.3900	0.3240
0.2	90.53	103.3	12.7700	0.4920
0.3	96.85	107.7	10.8500	0.5791
0.4	93.98	103.1	9.1200	0.6889
0.5	172.3	180.3	8.00	0.7854
0.6	140.9	148.4	7.5000	0.8378
0.7	181.4	188.8	7.4000	0.8491
0.8	99.99	106.9	6.9100	0.9093

Table 3 shows the characteristic number and the number of oscillations generated

Table 3: Characteristic number and the number of oscillations

a	Number of oscillations, (Nso)
0.1	16
0.2	22
0.3	27
0.4	31
0.5	34
0.6	38
0.7	41
0.8	44

Figure 10 shows the characteristic number versus the period of parametric oscillations. The characteristic number increases with the decrease in the period of oscillations.

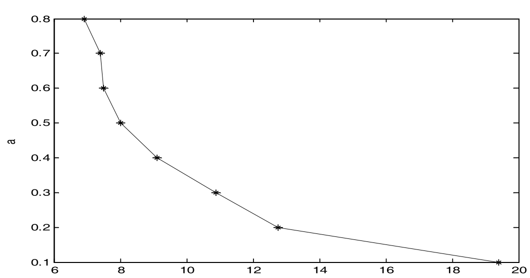

Figure 10: Characteristic number (a) versus period (T)

Figure 11 shows the modulation depth versus the carrier frequency. The carrier frequency increases as the modulation depth decreases

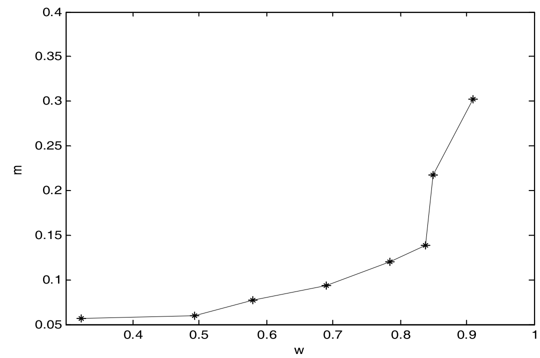

Figure 11: Modulation depth versus carrier frequency

Figure 12 shows the characteristic of the characteristic number versus the modulation depth and the carrier frequency. In Figure 12 (a), the modulation depth increases as the characteristic number increases. The carrier frequency in Figure 12 (b) increases gradually with the characteristic number

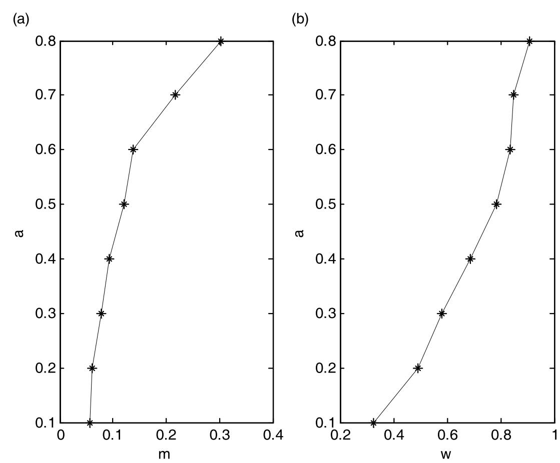

Figure 12: Characteristic number versus modulation depth and carrier frequency

CONCLUSION

The solutions to Mathieu equation in electric oscillatory system are in the form of amplitude modulated oscillations. MATLAB/Simulink is developed for the generation of parametric oscillations. The modulation depth and carrier frequency are computed experimentally. The characteristic number versus period, modulation depth versus the carrier frequency are plotted graphically for various combinations of the coefficient of a and q. The graphical results show the approximate solution of Mathieu equation in electric oscillatory circuit using energy conversion system.

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