A COMPARATIVE ANALYSIS OF GRILLAGE METHOD AND BEAM LINE ANALYSIS OF A REINFORCED CONCRETE WAFFLE BRIDGE DECK

Adamu, H. N.; Abbas B.A.; Abubakar, M.; Yusuf, A.; Kolo, D.N. & Shehu, M.
Department Of Civil Engineering, Federal University of Technology, Minna
Corresponding email: h.adamu@futminna.edu.ng

Abstract

The analysis of a reinforced concrete waffle bridge deck using Chanchaga Bridge as a case study was carried out with the aid of computer programme written in MATLAB. The bridge deck which is a beam bridge was idealized to be a waffle slab. A mathematical model of the bridge was developed using the method of grillages because very complex shapes of problem domain with prescribed conditions can be handled easily using the method. The bridge deck was modelled as interconnection of grid elements. The analysis was carried out using direct stiffness matrix method. The nodal displacements and the resulting static internal forces; shear forces, bending moments and twisting moments of each grid element were determined using the matrix. The results obtained using the method of grillages were then compared with beam line analysis and the former method gave a 10% decrease in forces which will result in the reduction of overall design and materials by 10%.

Keywords: Beam line analysis Computer-aided, Grillage analysis, MATLAB, waffle bridge deck.

INTRODUCTION

Waffle slabs are structural elements with a combination of top slab and a system of spaced longitudinal and transverse beams (Nithyambigai *et al.,* 2021). They are efficient in resisting lateral loads than flat slabs, and are suitable for large spans. They can withstand heavier load and cover large span as they exhibit higher stiffness and smaller deflections. The waffle slab system is an evolution of the solid slab that results from the elimination of concrete below the neutral axis that allows an economic increase in the total thickness of the slab with the inclusion of voids in a rhythmic arrangement. Waffle slabs are more advantageous as compared to other slabs such as flat slabs and RCC slabs, in terms of loading, large spans and aesthetic appearance (Khot *et al.,* 2016).

In recent years, there has been a sudden increase in the use of waffle slabs. That, however, makes it necessary to examine new ways in which it can be used in construction. Principally, static analysis of waffle slabs determines the amount and distribution of shear forces, bending moment and torsional moments acting on the structure (Chowdhury & Singh, 2012).

Over the years, researchers have analyzed waffle slabs substantially based on conventional methods; both analytical and numerical methods available in literature such as plate analogy by Timoshenko (1987), Rankine Grashoff theory (Hasan *et al.,* 2021), Finite Element Analysis and grillage analogy (Mallick and Bhushan 1983). However, it was clearly stated by Mallick and Bhushan, (1983) that when using grillage analysis, it should be substantiated by a detailed computer analysis. The direct stiffness gives more accurate results as concluded by Halkude and Mahamuni, (2014). However, research on the use of grillage analysis for waffle bridge decks has been rarely carried out.

Up until now, waffle slabs are found more in number in building construction than in bridge construction. An argument against this is that loads are distributed in two orthogonal directions in waffle slabs as against the one-way loading system in bridges. As a result, engineers deem it incompatible with bridges as loads are transferred in one way only in bridges. However, technical reasoning has shown that when loads are transferred to bridges in one way only, large twisting moments are produced, the orthogonal rib system in a waffle slab provides an efficient means of resisting these twisting moments by incorporating large bending moments in the two orthogonal directions (Kennedy and Bahkt, 1983).

The use of voided slab for a bridge deck was analysed by Rampariya and Choudhury, 2020 and concluded that they are more economical for greater spans of more than 15m. Also stated by Vaignan and Prashad, (2014), rectangular shaped cellular decks withstand more load than voided slabs when they analysed voided and cellular deck slab using MIDAS CIVIL.

For this purpose, serious attention needs to be given to the analysis of waffle slabs as bridge decks. Several methods have been used in the analysis of bridges. Each of the three dimensional structure is simplified based on assumptions on geometry, materials and relationship between components. The accuracy of analysis is dependent on the method used.

Bridge decks have been analyzed using several methods such as finite element (Halkude and Mahamuni, 2014), finite strip grillage analogy (Mallick and Bhushan, 1983) and orthotropic plate (Khot *et al.*, 2016).

Therefore, this research aims to analyse a solid slab bridge deck which is idealized as a waffle slab grillage analogy and then compare with conventional static beam line analysis of a bridge. The analysis of the waffle bridge deck using method of grillages was performed using direct stiffness method. MATLAB was used for writing the code as well as the analysis while excel program was used for beam line analysis.

Grillage method of analysis involves representing the bridge deck as a 2 by 2 system of interconnected beams intersecting each other. It is a numerical approach in analyzing bridge decks and also easy to use and comprehend (Shreedar and Kharde, 2013).

As structures become complex and large, several methods of simplifying their analysis have been developed among which use of computer aid. Computer aided analysis is a way of solving continuous system problems by dividing them into discrete elements thus simplifying analysis taking into consideration compatibility and boundary conditions. In the grillage analysis, the structure is represented by a plane grillage of discrete but interconnected beams. Almost any arrangement in plan is possible, so skew, curved, tapering or irregular decks can be analyzed. But the usual layout is sets of parallel beams in two directions by assuming the plane of the grillage to be horizontal whereas beam line analysis uses the moment distribution method in the analysis of loads; both static and moving to obtain the internal forces and settlements at the support.

METHODOLOGY

In a simple form of grillage analysis, each beam is assigned a torsional stiffness and flexural stiffness in the vertical plane. Vertical loads are applied only at the intersections of the beams. The matrix stiffness method of analysis is used by the existing software, to find the rotations about two horizontal axes and the vertical displacement at these nodes, and hence the bending and torsional moments and vertical shear forces in the beams at each intersection. Warping stresses and shear lag are neglected in the analysis.

Problem Formulation

A 125 m span simply supported right bridge deck of width 7.3m simply supported ends on two opposite sides and fixed ends on the other two sides. The thickness of the slab is assumed as 0.075m and the overall depth of the grid beam is assumed as 0. 375m.The width of the grid beam is assumed as 0.15m. The grade of concrete M30 and steel of grade

Fe 460 are assumed. The cracked moment of inertia is used to determine the rigidities of the deck. The dead loading considered is the self-weight and wearing course. The live load on the floor is HA loading as given in BS 5400-part 2 (1987) Clause 6.2.1. Load combination 1 of the BS 5400 part 2 is used. In this, eleven transverse members and five longitudinal members have been modeled with centre to centre spacing of ribs at 1.2m in both ways having same flexural and torsional rigidities.

Table I: Properties of Bridge Deck

Dimension	Actual Measurement
Width	11.8m
Length	123m
No of spans	10
Width of footpath	2m
Width of Notional Lanes	3.8m
Thickness of Slab Topping	0.075m
Depth of Bridge Deck	0.37m
Width of Grid Beam	0.15mm
Depth of Asphalt Overlay	0.05m
Grade of Concrete	C30
Grade of Steel	E460

Location of Grid Lines

a. Grid lines should be adopted along line of strength.
b. The longitudinal gridlines run in parallel direction to the edge of the deck that is free. For longitudinal direction, it may be along the longitudinal webs, centre line of girders or edge beams etc.
c. Where isolated bearings are present, the grid line may be along the line joining center of bearing.
d. For transverse direction, it should be considered as one of each end connecting the center of bearing and along the center line of transverse beam (Surana and Agrawal, 1998).

Number and Spacing of Grid Lines.

a. Where possible, odd numbers of gridlines should be chosen in both longitudinal and transverse directions.
b. The ratio of spacing of transverse grid line to those of longitudinal grids may be taken as 1 to 2.
c. As regards to the depth of slab, the minimum distance between longitudinal grid lines is limited to two to three times of the slab depth and the maximum separation of longitudinal members should not be more than one fourth of the effective span (Pandey and Maru, 2015).

A typical output gives the external reactions at each support. The bending and torsional moments will, in general, show a discontinuity at each joint. For an orthogonal grillage, each change in bending moment is equal to the change in torsional moment at that joint in the member at right angles to the one considered. Similarly, the change in torsional moment equals the change in bending moment in the perpendicular member.

Approximately one half of the local load can be distributed over the eight nodes of the vicinity to get correct results, even near the loaded point. An appropriate idealization for a continuous structure is carefully selected. Each T-section of the longitudinal and transverse sides of a waffle slab is represented by a grillage beam. The transverse grillage members should extend to the edge of the real slab and their ends should be attached to longitudinal grillage beams, even if the real slab has no significant edge stiffening.

2.3 Grillage Modeling

The slab is supported on orthogonal beam arrangement for the reason that each grillage member will represent a rib, and therefore the internal forces in the members can be taken directly for calculating load distribution. The spacing of ribs is the centre-to-centre spacing of 1.525 m. As a general rule, the spacing in either directions should be very similar. The explanations for this involves the application of load and distribution, and to keep almost the same mesh size, and therefore accuracy both ways. A spacing of 1.525 m for transverse grillage is chosen. Fig. 1 shows the modeling of the bridge deck.

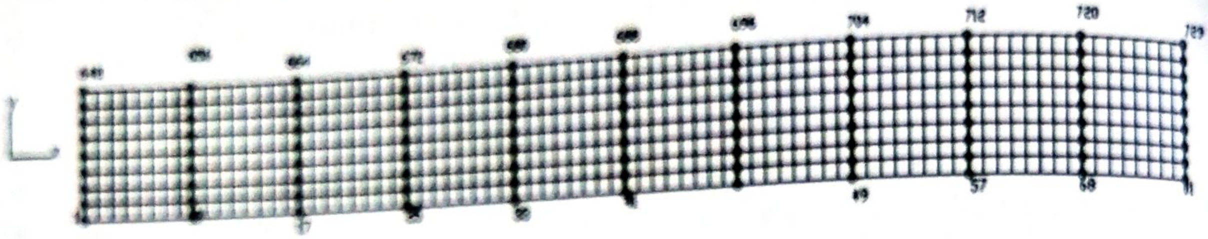


Fig. 1: Grillage Model of the Bridge Deck

The Analysis Model

Figure 1 shows a plan view of the analysis model of the idealized grillage. Full bending and torsion continuity is assumed at the nodes.

When establishing the data for idealization, it is most vital to ensure that the major axis of the elements is correctly oriented. In the case of the grillage shown in Fig. 1 the I_y values mentioned in Table 1 are the values of major axes.

They correspond to the local y axes of the members, which are in the global xy plane for the main beams and the transverse grillage members. These edge structures induce additional loading. These elements have an effect on the load distributions and exterior girder behaviour and were accounted for in modelling the bridge deck. Although the transverse grillages in this model were fairly basic, for some bridges and certain loading cases the transverse grillage members will become very important and will need modelling adjustments. For instance, when diaphragms are present in a bridge, the transverse grillage members model these elements.

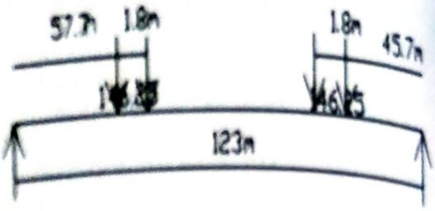


Fig. 2: Statical Diagram for Maximum Moment and Shear.

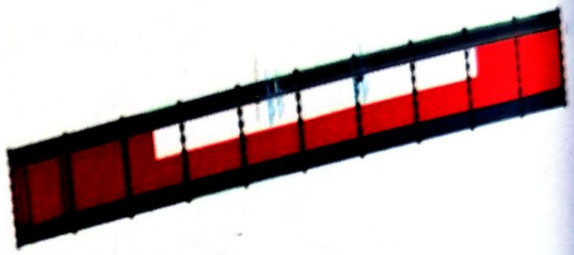
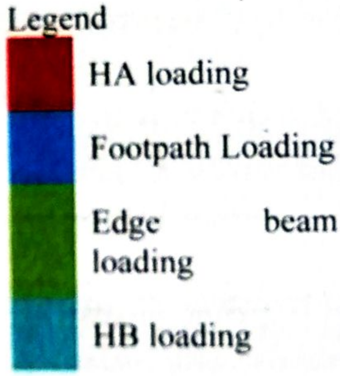


Fig. 3: Schematic Representation of Load Distribution



Dead load only

Analysis

1. Defining the nodal coordinates.
2. Numbering of numbers.

3. establishing the connectivity of elements
4. The length and angle of orientation
5. Material properties are modulus of elasticity and rigidities are defined.
6. For each element, the stiffness matrix computed the software.
7. The stiffness matrix for a grid member is a 6 by 6 matrix.
8. First the degrees of freedom at each node are identified and numbered; two perpendicular rotational displacement and one translational displacements$\Delta_1, \theta_2, \theta_3$.
9. The structures stiffness matrix for two nodes (one element) becomes;

   The global stiffness matrix is obtained by combining all the element stiffness matrices.
10. Assignments of boundary conditions.

Formulation of Stiffness Matrix

Governing differential equation

$$EI\frac{d^4v}{dx^4} = q \quad (1)$$

$$EI\frac{d^2v}{dx^2} = M \quad (2)$$

$$EI\frac{d^3v}{dx^3} = F \quad (3)$$

$$EI\frac{d^4v}{dx^4} = 0 \quad (4)$$

Integrating

$$EIv = a_0 + a_1x + a_2x^2 + a_3x^3 \quad (5)$$

The rotational degree of freedom

$$\frac{dv}{dx} = 0; \quad (6)$$

Applying boundary conditions

Solving for coefficients,

$$x = 0: \frac{dv}{dx} = 0;\ v = 1; \Rightarrow a_0 = 0\ and\ a_1 = 1 \quad (7)$$

$$x = L: \frac{dv}{dx} = 0;\ v = 0 \quad (8)$$

$$\frac{dv}{dx} = 0; \Rightarrow 2a_2 + 3a_3l \quad (9)$$

$$v = 1 + a_2l + a_3l^3$$

$$\Rightarrow a_2 = \frac{3}{l^2}\ and\ a_3 = \frac{2}{l^3} \quad (11)$$

Equation 5 becomes

$$v = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \quad (12)$$

$$-EI\frac{d^3v}{dx^3} = F \Rightarrow -EI\left(\frac{12}{L^3}\right) = \frac{12}{L^3}EI \quad (13)$$

$$EI\frac{d^2v}{dx^2} = M \Rightarrow M_{x=0} = -\left(\frac{6}{L^2}\right) \quad (14)$$

$$k_{11} = -F_{x=0} = EI\left(\frac{12}{L^3}\right) \quad (15)$$

$$k_{21} = -M_{x=0} = EI\left(\frac{6}{L^2}\right) \quad (16)$$

By imposing a twisting moment at node 1, gives a rotation θ and applying boundary conditions the constant of integration becomes;

$$T = \frac{GJ}{L}\theta \quad (17)$$

Therefore,

$$k_{33} = \frac{GJ}{L} \quad (18)$$

The remaining forces acting on the grid beam can be determined by applying unit displacement corresponding to translation and rotation at the two nodes of the beam.

$$\begin{bmatrix} k_{11}k_{12}k_{13}k_{14}k_{15}k_{16} \\ k_{21}k_{22}k_{23}k_{24}k_{25}k_{26} \\ k_{31}k_{32}k_{33}k_{34}k_{35}k_{36} \\ k_{41}k_{42}k_{43}k_{44}k_{45}k_{46} \\ k_{51}k_{52}k_{53}k_{54}k_{55}k_{56} \\ k_{61}k_{62}k_{63}k_{64}k_{65}k_{66} \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} & 0 \\ 0 & 0 & \frac{GJ}{L} & 0 & 0 & \frac{-GJ}{L} & 0 \\ \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{-GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 \end{bmatrix}$$

Code formulation procedure in MATLAB (2015a) software

i. Formation of global stiffness matrix:

function stiffness=formStiffnessGrid(GDof,...

```
numberElements,elementNodes,elementNode
s1,elementNodes2,elementNodes3,elementNo
des4,elementNodes5,xx,yy,E,I,G,J)
% function to form global stiffness for grid
element
% script file
E = 3.01e7;
I = 2.5e-5;
G = 1.31e7;
J = 5e-5;
%for edge beam
E1 = 3.01e7;
I1 = 0.0054;
G1 = 1.31e7;
J1= 0.0108;
%for footpath
E2 = 3.01e7;
I2 = 0.009 ;
G2 = 1.31e7;
J2= 0.018;
stiffness=zeros(GDof);
```

ii. Determination of forces in elements

```
% forces in elements
EF=zeros(6,numberElements);
```

iii. *Determination of displacements*

```
% displacements
disp('Displacements')
%displacements=displacements1;
jj=1:GDof; format
[jj' displacements]
% function to find solution in terms of global
displacements
activeDof=setdiff([1:GDof]',[prescribedDof])
;
U=stiffness(activeDof,activeDof)\force(active
Dof);
displacements=zeros(GDof,1);
displacements(activeDof)=U;
end
function outputDisplacementsReactions...
(displacements,stiffness,GDof,prescribedDof)
force=zeros(GDof,1);
force(1)=-10;
stiffness=formStiffnessGrid(GDof,numberEle
ments,...
elementNodes,xx,yy,E,I,G,J);
prescribedDof=[]';
displacements=solution(GDof,prescribedDof,
stiffness,force);
outputDisplacementsReactions(displacements
,stiffness,...
GDof,prescribedDof)
disp('forces in elements ')
EF=forcesInElementGrid(numberElements,el
ementNodes,...
xx,yy,E,I,G,J,displacements)
function
displacements=solution(GDof,prescribedDof,
stiffness,...
```

iv. Determination of reactions

```
F=stiffness*displacements;
reactions=F(prescribedDof);
disp('reactions')
[prescribedDof reactions]
End
```

RESULTS AND DISCUSSIONS

Analysis of a waffle bridge deck was carried by using grillage analogy method by simulating full HA and HB loading. The displacements and bending moments are shown. The bending moments are estimated from the summation forces in members of adjacent to each other. The results obtained in the grillage analogy method were then compared with beam line analysis method. The results are shown in Figs. 4-6.

Comparison of Bending Moments

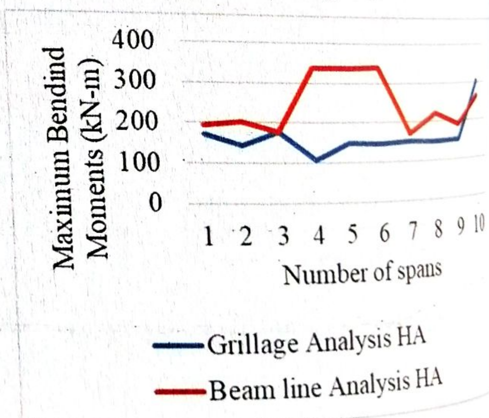


Fig. 4a: Bending Moments for Maximum Span Moments for HA loading.

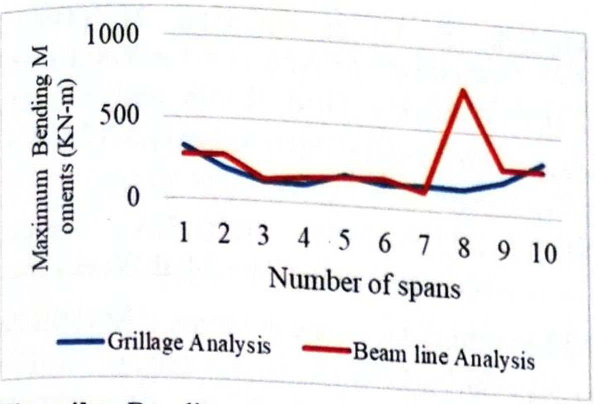


Fig. 4b: Bending Moments comparison for Maximum Span Moments for HB loading

From the graphical representations, the span moments obtained for span in the continuous beam analysed using the method of grillage gave lower bending moment values. The highest span moment occurred as a result of wheel loading on the span which was evenly distributed in grillage analysis to the nodes.

Comparison of Maximum Span Shear Forces

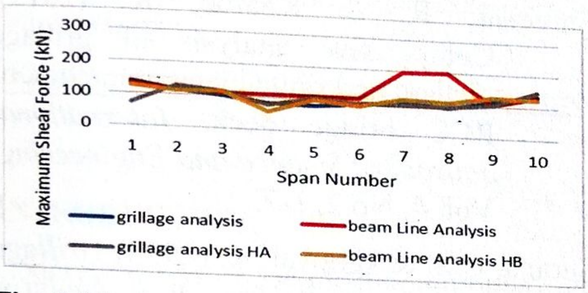


Fig. 5: Maximum Shear Force Comparison for HA and HB loading

The bending moment and shear forces obtained from grillage analysis in the spans showed very similar results to that of the beam line analysis models. Due to the close correlation of the results and the fact that two different methods will never give exactly the same results and that the values obtained from beam line analysis were slightly higher than that of grillage method, it is presumed that the grillage techniques and assumptions used in the moment modelling are appropriate.

Comparison of Deflections

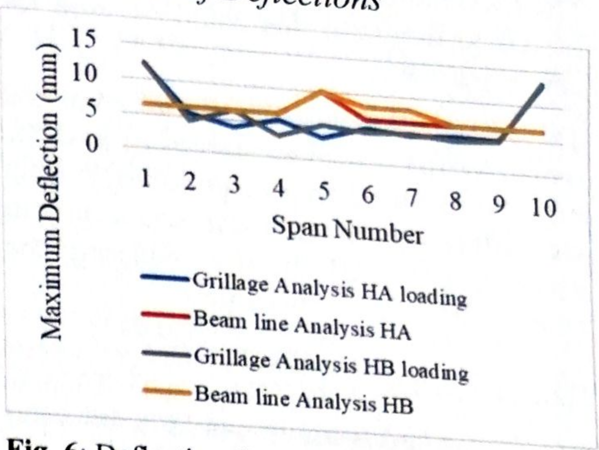


Fig. 6: Deflection Comparison for HA and HB Loading

The deflection results were investigated for critical points for two different sections with different loading conditions. The comparison was made to ensure that the variability in their results as compared to the manual method is not excessive. It was found that the deflection results from the grillage analysis are within 10% of the manual method and the highest deflection occurred at the point of application of wheel load. This shows that the grillage method gives lower deflection values as compared to the other method.

CONCLUSION

The ten span reinforced concrete waffle slab bridge deck (a case study of the Chanchaga Bridge, Minna, Niger State, Nigeria) was modeled using grillage analogy approach. The HA and HB loading of the BS 5400-2 (2000) was applied to the deck. Nodal displacements, reactions and member forces were obtained from the analysis. The values obtained from the grillage analysis were compared with manual calculations (beam line analysis). The following conclusions were arrived at:

The beam line method of analysis used for comparison yielded higher values of bending moments and shear forces, because the models results in a higher stiffness values compared to the manual method.

The maximum moment occurred as a result of HB loading and this could be used for design purposes.

This research work has developed a method (code) for the structural analysis of a waffle bridge deck based on grillage analogy using the stiffness matrix approach and a written code in MATLAB thereby rendering the approach computer amenable

REFERENCES

British Standard Institution (1987). BS 5400 Part 2. *Code of practice for the design of steel, concrete and composite bridges*. British Standard Institution

Chowdhury, I. & Singh, J.P. (2012). Analysis and Design of Waffle Slab with different Boundary Conditions. *The Indian concrete journal*, Vol. 2, No.1, 1-10.

Halkude S. A. & Mahamuni S.V. (2014). Comparison of various methods of analysis of grid floor frame.*International Journal of Engineering Science Invention*, 3(2), Vol. 2, No.1, 01-07.

Hasan, K., Fayadh, O. & Hasan, Q.F. (2021). Design and analysis of flat and grid slab system with conventional slab comparative approach. *Design Engineering*, Vol. 1, No. 7, 1628-169.

Kennedy J.B & Bakht B. (1983). *Feasibility of waffle slabs for bridges.* Edmonton, Alberta: Paper presented at The Annual Conference of the Canadian Society for Civil Engineering.

Khot, S.R., Mahajan, H.V., Purval, D.S., Jadhav, V.V., Siddhartha V.T. & Bharekar, K.T. (2016). Comparative Study of Waffle Slabs with Flat Slabs and Conventional RCC Slabs. *International Journal of Engineering Research & Technology (IJERT)*, Vol. 5(04) Vol 2, No.1, 1-10.

Mallick, S. K. & Bhushan. N. (1983). Methods of Analysis for Reinforced Concrete Grid Roofs and Flo.ors. *Indian Concrete Journal*, 57(3), Vol. 57, No.3, 76-81.

MATLAB. (2015). (R2015a). Natick, Massachusetts: The MathWorks Inc.

Nithyambigi, G, Rameshwaran, P.M &Stella M.F. (2021). Behaviour of waffle slab. Article in press. *Engineering Materials Toady: Proceeding*. doi: 10.1016/j.matpr.2021.02.016.

Pandey, K. K & Maru, S. (2015) Modelling of Skew Bridge Deck Slab by Grillage. *International Journal of Engineering Research and General Science*, Vol. 3, No.4, 1-5.

Rampariya, R.P & Choudhary, R.K. (2020). Use of voided slab for bridge deck. *AEGAEUM journal*. Vol. 8, No.7, 1-10.

Shreedar, R. & Kharde, R. (2013). Comparative analysis of grillage method and finite element method of RCC bridge deck. *International Journal of Science and Engineering*, Vol. 4, No.2, 1-7.

Surana, C.S. &Agrawal, R. (1998) *Grillage Analogy in Bridge Deck Analysis.* India: Narosa Publishing House 16-17.

Timoshenko, S. (1987). *Theory of plates and Shells*. McGraw Hill, New York.

Vaignan, B. & Prasad, S.R.K. (2014). Analysis of voided slab deck using MIDAS CIVIL. *International journal of engineering research and technology,* Vol.3, No 9, 1277-1288.