Analytical Investigation of Viscous and Non-Viscous Fluid Particles in a Restricted Region using Diffusion Magnetic Resonance Imaging Equation

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**Abstract.** Nuclear Magnetic Resonance (NMR) technology has been applied in several ways to provide vital information about petro-physical properties of reservoirs. However, due to the need to study the molecular behaviours of particles of the fluids in different restricted media, diffusion magnetic resonance equation evolved from Bloch’s fundamental equations of magnetic resonance is hereby applied in spherical coordinates and solved analytically using the method of separation of variables and solution of Legendre equation by Frobenius method. The viscous fluid considered in this research work is unused engine oil while the non-viscous fluid is water. Maple 17 was used in plotting the graphs. The results obtained show that the analytical solution of Diffusion MRI equation adopted in the research work has shown clearly the difference in the behaviours of the particles of unused engine oil (viscous fluid) with magnetization value of 1.466557018x1014 and water (non-viscous fluid) with magnetization value of 2.311919954x1014 as a result of their responses as simulated in the study.

# Introduction

Due to its non-invasiveness, nuclear magnetic resonance, NMR, techniques have proven to be a powerful and reliable tool in studying flow in different restricted geometries. It is particularly useful for studying diffusion because it can provide self-diffusion coefficient accurately for the individual components or multi-components systems at a very short time, where usual radioactive tracer techniques can take a very long time for each component and require isotropic substitution, Awojoyogbe *et al.,* 2011 [1]. The application of nuclear magnetic resonance measurement in petroleum industry gives better understanding of the interaction between fluids in the reservoirs and is one of the best tools for quantifying fluid properties, reservoir properties as well as determining the yields from the reservoir, Olaide *et al.,* 2020 [2]. In the same vein, Dada *et al.,* 2010 [3] used analytical technique in the form of a plane wave to transform the time dependent Bloch NMR flow equation to diffusion advection equation which was further used for the qualitative analysis of nuclear magnetization. The result obtained was used to study fluid flow in blood vessels under different bio-physico-geometrical conditions. This shows a significant application of diffusion advection equation in medical fields.

The nuclear in NMR was later dropped as it was believed to connote dangerous nuclear energy. Therefore, the name MRI – Magnetic Resonance Imaging was adopted. MRI is a very powerful tool and has application in various aspects of human life especially in the study of behaviour of fluids generally. It is a method that has been applied severally by scientists and has proven to be very effective in revealing the minute details of properties, activities and behaviour of particles of fluids under consideration. Several scientists have applied MRI in carrying out a lot of investigation of fluid in the field of medicine, physics, radiology, oil and gas and so on.

MRI has also been further applied in the area of imaging the nature of the materials that cause obstructions or blockage of fluids in a cylindrical pipe. This investigation was carried out by Yusuf *et al.,* 2019a[4]*.* In their research which was carried out by using magnetic resonance imaging to reveal and analyse the materials that are causing obstructions of fluids in a cylindrical pipe, it was found out that slurry and oil wax were parts of materials that could cause blockage to flow of fluids in a cylindrical pipe and such materials could be detected easily through the application of magnetic resonance imaging. Similarly, the analytical solution of magnetic resonance imaging (MRI) equation was used to study the general behaviour of physiological flow in human living tissues. The result of the research shows how relaxation times of any matter or material can be used to describe its behaviour or movement (flow) in a region or an environment, Yusuf *et al.,* 2010 [5]. Diffusion magnetic resonance imaging (DMRI) equation was further applied to study and analyse the discontinuities of flow of fluids in a symmetric cylindrical channel. In the work, it was shown that partial and total blockage could be determined using DMRI equation, Yusuf *et al.,* 2019b [6].

Yusuf *et al.,* 2015 [7] went further to analyse different types of blockages of unused engine oil in a radially symmetric cylindrical pipe using diffusion magnetic resonance equation. They examined free flow, partial blockage and total blockage of engine oil. The concluded that diffusion MRI can effectively detect, monitor and control the different levels of blockages.

In another research, a simple and fast technique for solving the time dependent Bloch equations by using matrix operation method was derived by Murase & Tanki [8]. This method was validated in case of constant radio-frequency irradiation by comparing with the analytical solutions which indicates a good agreement between the methods.

Another useful application of MRI was carried out with T1 and T2 relaxation times from Bloch equations. These relaxation times were used to study age-related changes in white and grey matter in the human brain, Olaoye *et al.,* 2021 [9]. They agree with previous studies that white matter and grey matter develop throughout childhood and adolescence then reach peak values at early adulthood before they begin to decline at different rates.

The analytical solutions of the transient solid-state diffusion of the single-phase and two-phase in spherical and cylindrical geometries were considered. The modified differential equations were solved using error function method and solutions obtained used to analyse the diffusion interface position as a function of time and position in spheres and cylinders. The analytical solutions were validated with the results of a numerical approach called enthalpy method. The model was proved to be general, as far as, a semi-infinite solution for diffusion problems with phase change exist in the form of error function that enables them to derive the exact closed-form of analytical solutions by updating the root at each radial position of the moving boundary interface, Ferreira *et al.,* 2021 [10].

Datta, D. & Pal T. K, 2018 [11] studied one dimensional radial diffusion equation in spherical coordinate system using the Lattice Boltzmann scheme. The scheme was investigated and there was a great analogy between the simulation and the analytical solution. The result obtained showed that the scheme would be able to simulate the radial diffusion equation accurately.

Belyaev *et al.,* 2015 [12] investigated two dimensional variables of fluid particles of motion in a curved duct using numerical analysis. Navier-Stoke’s equation was used to model the phenomenon. Control Volume (CV) approach was applied to discretize the initial equations. The result showed that the trajectory of the moisture reduces the motion and its speed.

In the work of Fatumbi & Fenuga [13], reference was made to micropolar fluids which represent fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium, where particles deformation is ignored. These are group of fluids with non-symmetric stress tensor that are called polar fluids which constitute a substantial generalization of the Navier-Stoke’s model. These fluids offer a mathematical model for investigating the flow of complex and complicated fluids such as suspension solution, animal blood, liquid crystals, polymeric fluids and clouds with dust. Similarly, Isede *et al.*,2023[14] pioneered the classical unidirectional laminar flow problem of an incompressible and viscous electrically conductive fluid permeated by a non-varying magnetic field; applied transversely to the parallel walls of the channel. Popoola *et al.,* 2016 [15] examined the two-dimensional steady flow of heat and mass transfer in an incompressible magnetohydrodynamic viscous-elastic fluid pass a stretching sheet in the presence of thermal diffusion and chemical reaction. The similarity transformation method was used to convert the partial differential equations governing the flow of heat and mass transfer properties into the coupled ordinary differential equations.

From the research works so far reviewed, it is clear that though some scientists have worked on fluids flowing in some restricted regions, very few has been able to consider viscosity of the fluid through the application of magnetic resonance imaging. This research work, therefore, is aimed at using mathematical techniques to study and compare the behaviour of non-viscous fluid particles (water) and viscous fluid particles (unused engine oil) in spherical regions using diffusion magnetic resonance equation.

# Mathematical Formulation

The fundamental Bloch equations are as follows:

 (1)

 (2)

 (3)

where

Component of transverse magnetization along - axis

Component of transverse magnetization along - axis

Component of magnetization along the - axis

Equilibrium magnetization

Gyro-magnetic ratio of fluid spins -

 = Radio-frequency (RF) magnetic field - ()

Longitudinal or spin lattice relaxation time - ()

Transverse or spin-spin relaxation time - ()

The fluid velocity - (sec)

= time - (sec)

From the Bloch equations, the general equation for fluid flow was evolved by Awojoyogbe [16]:

(4)

where

, , and .

From equation (4), assuming that each of the terms extracted and listed in equation (5) is set to zero:

 (5)

Then, equation (4) reduces to:

 (6)

(7)

Let (, then equation (7) becomes:

 (8)

Hence, the parameter is called diffusion coefficient that is accurately defined in terms of MRI fluid flow which is an intrinsic part of the Bloch nuclear magnetic resonance equation. Its unit is The function is the forcing function.

In 3-Dimension, equation (8) becomes:

 (9)

In spherical coordinates system defined as and using

, and  with ,

 and ,

equation (9) becomes:

 (10)

Assuming that, then  which implies that the particles of the fluid align in the same – direction upon introduction of magnetic field which makes the particles exhibit constant or uniform motion. Consequently, equation (10) reduces to:

. (11)

Applying the method of separation of variables and substituting.

The terms in the brackets on the right-hand side (RHS) of equation (11) are as follows:

, (12)

. (13)

Then, multiplying equation (13) by  , (14)

. (15)

Since the left-hand side of equation (15) depends only on and the right-hand side also depends only on  and using the method of separation of variables, then each side must be equal to a constant (say):

. (16)

So, the following equations are obtained from equation (16) as:

, (17)

. (18)

Now, solving equation (17) which can also be expressed as:

. (19)

Then, let the solution of equation (19) be of the form:

, (20)

, (21)

.. (22)

Substituting equations (20), (21) and (22) into equation (19) gives:

, (23)

. (24)

. (25)

This follows that if then

, (26)

or . (27)

Then, substituting equation (27) into equation (20), gives:

, (28)

where and are two arbitrary constants.

So, let

, (29)

and

. (30)

Then, substituting equations (29) and (30) into equation (28) gives:

, (31)

. (32)

Now, multiplying equation (27) by (28) and simplifying yields:

. (33)

Now, considering equation (18) and substituting equation (33) into it, gives:

, (34)

. (35)

To transform equation (35) to Legendre equation,

Let , then:

, (36)

. (37)

But then equation (37) becomes:

 (38)

Then, substituting equation (38) into equation (35) gives:

. (39)

Multiplying equation (39) by  we have:

. (40)

Recall that then equation (40) becomes:

. (41)

Now, let and  then equation (41) becomes:

, (42)

. (43)

Applying Frobenius method to solve equation (43) and letting the series solution of equation (43) be in the form:

, (44)

, (45)

. (46)

Then, substituting equations (44), (45) and (46) into equation (43) gives:

, (47)

, (48)

. (49)

Evaluating  at  and 

. (50)

Also equating the coefficients of  and  in equation (50) gives:

, if . (51)

Then,  or  {are called indicial roots}

. (52)

Then, for  is undefined and for .

Now, equation (50) is reduced to:

. (53)

Then, from the first term of equation (53) let  yields:

. (54)

Now, equating the coefficient ofin equation (54) yields:

. (55)

Considering the value of in equation (55), gives:

, (56)

, (57)

where

. (58)

Now, putting the values of into equation (58) the following are obtained:

For, , (59)

For, , (60)

For, , (61)

For, . (62)

The process continues in that order.

Also, substituting equations (59), (60), (61) and (62) into equation (44) and collecting the like terms, for  gives:

. (63)

Then, equation (63) can also be expressed as:

, (64)

where and are two arbitrary constants,

. (65)

But recall that  and, then equation (64) becomes:

, where . (66)

Thus, substituting equations (32) and (66) into equation (11), gives the general solution as:

, (67)

where the functions  and  are the Legendre functions of the first and second kinds and the function is the radio-frequency field applied to perturb the molecules of the fluid.

Since must be bounded at  and  either let choose  in equation (67), then the bounded solution is obtained as:

. (68)

Then, the boundary conditions to be imposed are:

(69)

andis bounded.

Since  is bounded at choosing in equation (68) it gives:

. (70)

Now, replacingbyand applying superposition principle in equation (70) yields:

, (71)

(72)

where . Then, from equation (72) considering:

. (73)

So, when then equation (73) becomes:

. (74)

Now, by the orthogonality of Legendre polynomials and multiplying both sides by  and integrating from

(-1, 1) then equation (74) becomes:

. (75)

Then, if  and  equation (75) becomes:

. (76)

Recall from the generating functions of Legendre polynomials given by

, (77)

. (78)

Then, multiplying equation (77) by (78) gives:

(79)

Now, if and integrating both sides from  with respect to gives:

(80)

(81)

(82)

(83)

(84)

(85)

Recall that;

. (86)

Then equation (85) becomes:

, (87)

, (88)

 , (89)

 • (90)

Then, equating the coefficients of gives

. (91)

Now, substituting equation (91) into equation (76):

. (92)

Then, making  the subject in equation (92) gives:

. (93)

Applying boundary conditions to equation (93) gives:

, (94)

. (95)

Now, putting into equation (95) yields the following:

For  (96)

For  (97)

For  (98)

For  (99)

For  (100)

For  (101)

The process continues in that order.

Now, substituting equations (96), (97), (98), (99), (100) and (101) into equations (72) gives:

. (103)

Now, solving the integral in equation (103) as follows:

, (104)

(105)

(106)

A radio-frequency pulse is required and introduced to transmit energy to the sample of fluid considered in the sphere in order to activate the nuclei so that they emit signal. This is the underlying principle of MRI.

Recall from equation (4) that:

and .

Then, equation (106) becomes:

, (107)

(108)

Therefore,

(109)

From the solution of the equation and using Maple 17 software, the graphical representations of the viscous fluid (unused engine oil) and the non-viscous fluid (water) under consideration are presented and compared. The plotting was done in 3-dimensions with the Magnetization (), plotted against angle of inclination and radial adjustment (). Values of respective relaxation times and coefficient of diffusion of each of the fluids were also considered in plotting the graphs in Figures 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23 – 25 for water and Figures 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 and 22 for unused oil.

|  |  |
| --- | --- |
| **FIGURE 1.** Plot of  against (θ = 0..2π) and  (r = 0..60) for water | **FIGURE 2.** Plot of  against (θ = 0..2π) and  (r = 0..60) for unused engine oil |
| **FIGURE 3.** Plot of  against (θ = 0..2π) and  (r = 0..55) for water | **FIGURE 4.** Plot of  against (θ = 0..2π) and  (r = 0..55) for unused engine oil |
| **FIGURE 5.** Plot of  against (θ = 0..2π) and  (r = 0..50) for water | **FIGURE 6.** Plot of  against (θ = 0..2π) and  (r = 0..50) for unused engine oil |
| **FIGURE 7.** Plot of  against (θ = 0..2π) and  (r = 0..45) for water | **FIGURE 8.** Plot of  against (θ = 0..2π) and  (r = 0..45) for unused engine oil |
| **FIGURE 9.** Plot of  against (θ = 0..2π) and  (r = 0..40) for water | **FIGURE 10.** Plot of  against (θ = 0..2π) and  (r = 0..40) for unused engine oil |
| **FIGURE 11.** Plot of  against (θ = 0..2π) and  (r = 0..30) for water | **FIGURE 12.** Plot of  against (θ = 0..2π) and  (r = 0..30) for unused engine oil |
| **FIGURE 13.** Plot of  against (θ = 0..2π) and  (r = 0..25) for water | **FIGURE 14.** Plot of  against (θ = 0..2π) and  (r = 0..25) for unused engine oil |
| **FIGURE 15.** Plot of  against (θ = 0..2π) and  (r = 0..20) for water | **FIGURE 16.** Plot of  against (θ = 0..2π) and  (r = 0..20) for unused engine oil |
| **FIGURE 17.** Plot of  against (θ = 0..2π) and  (r = 0..15) for water | **FIGURE 18.** Plot of  against (θ = 0..2π) and  (r = 0..15) for unused engine oil |
| **FIGURE 19.** Plot of  against (θ = 0..2π) and  (r = 0..10) for water | **FIGURE 20.** Plot of  against (θ = 0..2π) and  (r = 0..10) for unused engine oil |
| **FIGURE 21.** Plot of  against (θ = 0..2π) and  (r = 0..5) for water | **FIGURE 22.** Plot of  against (θ = 0..2π) and  (r = 0..5) for unused engine oil |
| **FIGURE 23.** Plot of  against (θ = 0..2π) and  (r = 0..4) for water | Point of Relaxation Reached |
| **FIGURE 24.** Plot of  against (θ = 0..2π) and  (r = 0..3) for water | Point of Relaxation Reached |
| **FIGURE 25.** Plot of  against (θ = 0..2π) and  (r = 0..2) for water | Point of Relaxation Reached |

# Results and Discussion

It is clear that from Figure 1 that the value of magnetization registered ranged from 2.31191995415 x 1014 to 2.31191995385 x 1014. It reduced in Figure 3 from 2.31191995408 x 1014 to 2.31191995392 x 1014. This further reduced from 2.311919955406 x 1014 to 2.31191995394 x 1014 in Figure 5. This reduction continues until total relaxation is attained at *2.3* x 1014 and radial adjustment at 2cm.

Water being the non-viscus fluid, did not begin to show appreciable difference under the interplay of magnetic resonance until when the magnetization was *2.311919954x1014* and radial adjustment, 10cm. The significance of this is that it is in conformity with Free Induction Decay (FID) an important phenomenon in MRI. This implies that as the radio frequency field used to perturb the fluid dynamics is removed, the fluid begins to relax gradually until it attains total relaxation. This is also in agreement with previous results obtained by Yusuf *et al.* [11].

However, in Figure 2, the unused engine oil has its magnetization ranging from 1.46655701800015 x 1014 to 1.46655701799985 x 1014. In Figure 4, the value of magnetization reduced from 1.4665570180001 x 1014 to 1.4665570179999 x 1014. It further reduced from 1.466557018000077 x 1014 to 1.466557017999923 x 1014 in Figure 6 and the process continued until total relaxation was attained. The significance of this is that this also conforms with Free Induction Decay (FID) as earlier explained in the case of water in Figures 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23 – 25. The radio frequency field used to perturb the fluid dynamics got deactivated and the fluid begins to relax gradually until it attains total relaxation.

However, a careful consideration of the Figures showed that while unused engine oil relaxes faster at magnetization value of 1.48x 1014 (Figure 22), water did not relax until when the magnetization value was *2.3* x 1014 and radial adjustment at 5cm.

The reason for this is because unused engine oil is viscous and relaxes faster while water is non-viscous and requires more rigorous collision which could be traced to additional graphs in Figures 23 – 25 and the reading from the MRI machine which registered the signals at a magnetization that is relatively higher than that of viscous fluid. The essence of radial adjustment is to create an avenue for the particles of the fluid to experience more vigorous collision that will enable the MRI machine register the signals with a view to studying the behaviour of the particles that made up the fluid.

By and large, it is clear from the results obtained from the solution of the mathematical model equations as well as the various graphs plotted and interpreted that the principle of magnetic resonance can clearly be adopted to study the motion of fluids and the behaviours of the minute particles that form the fluid. Hence, magnetic resonance imaging is a phenomenon that is still emerging and has immense applications not only in the area of medical physics and human health but also in general fluid dynamics.

# Conclusion

The general MRI flow equation has been solved in a spherical region. The fluids considered in this research work were unused engine oil (viscous fluid) and water (non-viscous fluid). The method of separation of variables adopted in spherical region led to Legendre equation of the first and second kinds. The forcing function added to the equation represents the radio-frequency field introduced to perturb the particles of the fluids. This perturbation is the principle underlying the application of magnetic resonance imaging. As the fluid is introduced to the static magnetic field, the particles immediately align in one direction (), hence the motion of the particles on this axis is constant or uniform. When they are perturbed, they undergo precession and later relax at their unique relaxation times. During this process, they evolve echo which are read as signals. The signals which are interpreted by machines contain useful information on the particles being considered non-invasively. This is the magnetization value of the body or its response under MRI. It can be recalled that MRI registers its signals in pico- or centi-seconds!

From the results obtained, it can be concluded that the analytical solution of Diffusion MRI equation adopted in the research work has shown clearly the difference in the behaviours of the particles of unused engine oil (viscous fluid) and water (non-viscous fluid) in a restricted region as a result of their responses as simulated in the study.

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