



Effect of Viscous Energy Dissipation on Transient Laminar Free Convective Flow of a Dusty Viscous Fluid through a Porous Medium

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ABSTRACT: A study on transient free convection flow of a dusty viscous fluid through a porous medium is important for improving the existing industrial processes and for developing new chemical and geothermal systems. This paper presents a mathematical model for transient laminar free convective flow of a dusty viscous fluid through a porous medium in the presence of viscous energy dissipation. The partial differential equations governing the phenomenon were non-dimensionalized using some dimensionless quantities. The dimensionless coupled non-linear partial differential equations were solved using harmonic solution technique. The result obtained were presented graphically and discussed. These results revealed that increase in Peclet number, Eckert number and Grashof number leads to increase in the velocity profile. Increase in the mass concentration of the dust particles, concentration resistance ratio, Eckert number and Peclet number leads to increase in the velocity profile of the dust particles. Increase in the Reynold number leads to a reduction in the velocity profile. Increase in Peclet number, Eckert number and Grashof number leads to increase in temperature profile. Similarly, increase in heat source parameter, coefficient of Grashof number and Reynold number lead to reduction in the temperature profile.

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Natural convection flow under the influence of a gravitational force have been studied most extensively because of their frequent occurrence in nature as well as in science and engineering applications. When a heated surface is in contact with the fluid, the result of temperature difference causes buoyancy force, which induces natural convection heat transfer. From a technological point of view, the study of convection heat transfer from a cone is of special interest and has wide range of practical applications. Mainly, these types of heat transfer problems deal with the design of spacecrafts, nuclear reactor, solar power collectors, power transformers, steam generators and others. Bapuji and Ekambavanan (2006) numerically studied the solutions of steady flow past plane and axi-

symmetrical shape bodies. A study on the problem of transient natural convection from a vertical cone with isothermal and non-isothermal surface temperature using an implicit finite difference method was also carried out (Bapuji *et al.*, 2008). Free-convection flows are now of practical importance in aeronautics because many aircraft propulsion systems contain components (such as gas turbines and helicopter ram jets) which rotate at high speeds and in which heat is being transferred. The method of free-convection cooling of gas-turbine rotor blades where the centrifugal forces create a free-convection flow of the coolant in the blade passages is an example of a practical application. Mollah *et al.* (2019) studied Bingham fluid flow through oscillatory porous plates with ion-slip and hall

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current using the explicit finite difference techniques. Khare (2008) investigated the flow of dusty viscous fluid through rectangular channel. Later, Khare and Singh (2010) studied Magneto-hydrodynamic (MHD) flow of dusty viscous incompressible fluid confined between two vertical walls with volume fraction of dust. Ghadikolaei *et al.* (2018) studied steady two-dimensional dusty nano fluid flow over radiating surfaces. The movement of oscillatory fluid and heat transfer through porous medium between parallel plates with inclined magnetic field, radiative heat flux and heat source was also investigated (Mehta *et al.*, 2020). Zaib *et al.* (2019) explored the aligned magnetic flow comprising of nano-liquid over a radially stretching surface with entropy generation. The convective magnetohydrodynamic two phase flow and heat transfer of a fluid in an inclined channel was investigated by Malashetty *et al.* (2001). Siddiqua *et al.* (2017) studied problems involving natural convection flow of a two-phase dusty non-Newtonian fluid along a vertical surface and solved it analytically and numerically to compute velocities and friction factors under influences of magnetic and porous medium resistances. Patil *et al.* (2017) obtained non-similar solutions of a mixed convective flow of a Newtonian fluid past a stretching surface in an exponential order. Attia (2006) investigated the time varying Couette flow with heat transfer of a dusty viscous incompressible, electrically conducting fluid under the influence of a constant pressure gradient without neglecting the Hall Effect. Krishna and Jyothi (2018) solved the unsteady rotating flow of a visco-elastic fluid through a porous medium over an infinite oscillating porous plate with heat source and chemical reaction using a regular perturbation method for small elastic parameter. Sandeep *et al.* (2013) analyses the laminar convective flow of a dusty viscous fluid through a porous medium of non-conducting walls in the presence of inclined magnetic field with volume fraction, heat source and considering porous

parameter. In this paper, viscous energy dissipation parameter is incorporated into the energy equation thereby extending the work of Sandeep (2013).

Model Formulation: The unsteady laminar flow of a dusty, incompressible, Newtonian, electrically conducting, viscous fluid through a porous medium of uniform cross section h is considered. One wall of the channel is fixed and the other is oscillating with time about a constant non-zero mean velocity. Initially at time $t \leq 0$, the channel wall as well as the fluid are assumed to be at the same temperature T_0 . At time $t > 0$, the temperature of the channel wall is instantaneously raised to T_w which oscillate with time and is thereafter maintained constant. The x-axis is taken along the fluid flow at the fixed wall and y-axis perpendicular to it. An inclined magnetic field is applied to the flow along y direction with the heat source.

The governing equations are written based on the following assumptions: The dust particles are solid, spherical, non-conducting, and equal in size and uniformly distributed in the flow region. The density of dust particles is constant and the temperature between the particles is uniform throughout the motion. The interaction and chemical reaction between the particles and liquid has not been considered to avoid multiple equations. The volume occupied by the particles per unit volume of the mixture, (that is, volume fraction of dust particles) and mass concentration have not been taken into consideration. The dust concentration is so small so that it is not disturbing the continuity and hydro magnetic effects. This means that the continuity equation is satisfied. The fluid flow is governed by the momentum and energy equation under the above assumptions is

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + g\beta^*(T - T_0) + \frac{1}{1-\phi} \left(\frac{KN_0}{\rho} (v - u) - \frac{\mu}{K_1} u - \frac{KN_0 \sigma \mu_c^2 H_0^2 \sin^2 \theta}{\rho} u \right) \quad (1)$$

$$\frac{\partial v}{\partial t} = \frac{\phi}{N_0 m} \left(-\frac{\partial P}{\partial x} + \mu \frac{\partial^2 v}{\partial y^2} + \rho g \beta^* (T - T_0) \right) + \frac{K}{m} (v - u) \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{Q_0}{\rho C_p} (T - T_0) + \frac{\nu}{C_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

With the initial and boundary conditions given as :

$$\left. \begin{aligned} u(y,0) &= 0, & u(0,t) &= Ue^{i\omega t}, & u(l,t) &= 0 \\ v(y,0) &= 0, & v(0,t) &= Ue^{i\omega t}, & v(l,t) &= 0 \\ T(y,0) &= T_0, & T(0,t) &= T_0 + (T_w - T_0)e^{2i\omega t}, & T(l,t) &= T_0 \end{aligned} \right\} \quad (4)$$

Where

$u(y,t)$ is the velocity of the fluid, $v(y,t)$ is the velocity of the dust particles, m is the mass of each dust particle, N_0 is the number density of dust particle, T is the temperature, T_0 is the initial temperature, T_w is the raised temperature, ϕ is the volume fraction of the dust particle, f is the mass concentration of dust particle, β^* is the volumetric

coefficient of the thermal expansion, K_1 is the porous parameter, K is the Stokes's resistance coefficient, σ is the electrical conductivity of the fluid, μ_c is the magnetic permeability, Q_0 is the heat source, H_0 is the magnetic field induction, C_p is the specific heat at constant pressure, k is the thermal conductivity.

Non-dimensionalisation: The model equations (1), (2), (3) and (4) are non-dimensionalized using the following dimensionless variables:

$$x' = \frac{x}{l}, \quad t' = \frac{Ut}{l}, \quad u' = \frac{u}{U}, \quad v' = \frac{v}{U}, \quad y' = \frac{y}{l}, \quad \theta = \frac{T - T_0}{T_w - T_0}, \quad P' = \frac{P}{\rho U^2} \quad (5)$$

For convenience, after non-dimensionalizing, equations (1), (2), (3) and (4) becomes

$$\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \frac{1}{R_e} \frac{\partial^2 u}{\partial y^2} + \frac{G_{r\theta}}{R_e} \theta + \varepsilon_1 v - \lambda u \quad (6)$$

$$\frac{\partial v}{\partial t} = \phi \left(-\frac{1}{f} \frac{\partial P}{\partial x} + \frac{1}{f R_e} \frac{\partial^2 v}{\partial y^2} + \frac{G_{r\theta}}{f R_e} \theta \right) + \beta(v - u) \quad (7)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_e} \frac{\partial^2 \theta}{\partial y^2} - S\theta + \frac{E_c}{R_e} \left(\frac{\partial u}{\partial y} \right)^2 \quad (8)$$

Where,

$$\left. \begin{aligned} \frac{Ul}{\nu} &= R_e, \quad G_{r\theta} = \frac{g\beta^* l^2 (T_w - T_0)\theta}{\nu U}, \quad f = \frac{mN_0}{\rho}, \quad \varepsilon_1 = \frac{f}{\sigma_1(1-\phi)}, \\ \varepsilon_2 &= \frac{1}{(1-\phi)}, \quad \varepsilon_3 = \frac{\mu l}{K_1 U (1-\phi)}, \quad \sigma_1 = \frac{mU}{Kl}, \quad \lambda = (\varepsilon_1 + \varepsilon_2 M + \varepsilon_3), \\ M &= M' \sin^2 \theta, \quad M' = \mu_c^2 H_0^2 \sigma, \quad \beta = \frac{f}{\sigma_1} = \frac{Kl}{mU}, \quad P_e = \frac{lU \rho C_p}{K}, \\ S &= \frac{Q_0 l}{U \rho C_p}, \quad E_c = \frac{U^2}{C_p (T_w - T_0)}, \quad R_e = \frac{lU \rho}{\mu} = \frac{lU}{\nu} \end{aligned} \right\} \quad (9)$$

With the dimensionless initial and boundary conditions as:

$$\left. \begin{aligned} u(y,0) &= 0, & u(0,t) &= e^{i\omega t}, & u(l,t) &= 0 \\ v(y,0) &= 0, & v(0,t) &= e^{i\omega t}, & v(l,t) &= 0 \\ \theta(y,0) &= 0, & \theta(0,t) &= e^{2i\omega t}, & \theta(l,t) &= 0 \end{aligned} \right\} \quad (10)$$

where $u(y,t)$ is the dimensionless velocity of the fluid, $v(y,t)$ is the dimensionless velocity of the dust particles, $\theta(y,t)$ is the dimensionless temperature of the fluid, R_e is Reynold number, $G_{r\theta}$ is the Grashof thermal number, $\lambda = \varepsilon_1 + \varepsilon_2 M + \varepsilon_3 = \text{constant}$, f is the mass concentration of dust particles, ϕ is the

volume fraction of the dust particle, β is the number, E_e is the Eckert number and S is the Heat Concentration resistance ratio, P_e is the Peclet source parameter.

Method of solution: For a purely oscillating flow,

$$\frac{\partial P}{\partial x} = \sigma e^{i\omega t}, \quad u(y, t) = u(y) e^{i\omega t}, \quad v(y, t) = v(y) e^{i\omega t}, \quad \theta(y, t) = \theta(y) e^{i\omega t} \quad (11)$$

As used in (Mehta *et al.* 2020). So, substituting equation (11) into equations (6), (7), (8) and (10), the following ordinary differential equations and associated boundary conditions are obtained:

$$\frac{d^2 u}{dy^2} - a_1^2 u = \sigma R_e - G_{r\theta} e^{i\omega t} \theta - R_e \varepsilon_1 v \quad (12)$$

$$\frac{d^2 v}{dy^2} - a_2^2 v = \sigma R_e - e^{i\omega t} G_{r\theta} \theta + \frac{\beta f R_e}{\phi} u \quad (13)$$

$$\frac{d^2 \theta}{dy^2} - a_3^2 \theta = -\frac{P_e E_c}{R_e} \left(\frac{du}{dy} \right)^2 \quad (14)$$

where,

$$a_1 = \sqrt{R_e (\lambda + i\omega)} \quad (15)$$

$$a_2 = \sqrt{\frac{f R_e (i\omega - \beta)}{\phi}} \quad (16)$$

$$a_3 = \sqrt{P_e (S + 2i\omega)} \quad (17)$$

So, the corresponding dimensionless boundary conditions are:

$$\left. \begin{aligned} u(0) &= 1, & u(1) &= 0, \\ v(0) &= 1, & v(1) &= 0, \\ \theta(0) &= 1, & \theta(1) &= 0, \end{aligned} \right\} \quad (18)$$

Let,

$0 < G_{r\theta} \ll 1$, and $\varepsilon_1 = b G_{r\theta}$, such that

$$\left. \begin{aligned} u(y) &= u_0(y) + G_{r\theta} u_1(y) + \dots \\ v(y) &= v_0(y) + G_{r\theta} v_1(y) + \dots \\ \theta(y) &= \theta_0(y) + G_{r\theta} \theta_1(y) + \dots \end{aligned} \right\} \quad (19)$$

Put (19) in (12), (13) and (14) and equating corresponding coefficients on both sides, the following systems of differential equations are obtained.

For Order 0, That is $O(G_{r\theta}^0): 1$

$$\left. \begin{aligned} \frac{d^2 u_0}{dy^2} - a_1^2 u_0 &= \sigma R_e \\ u_0(0) &= 1, \quad u_0(1) = 0 \end{aligned} \right\} \quad (20)$$

$$\left. \begin{aligned} \frac{d^2 v_0}{dy^2} - a_2^2 v_0 &= \sigma R_e + \frac{\beta f R_e}{\phi} u_0 \\ v_0(0) &= 1, \quad v_0(1) = 0 \end{aligned} \right\} \quad (21)$$

$$\left. \begin{aligned} \frac{d^2 \theta_0}{dy^2} - a_3^2 \theta_0 &= -\frac{P_e E_c}{R_e} \left(\frac{du_0}{dy} \right)^2 \\ \theta_0(0) &= 1, \quad \theta_0(1) = 0 \end{aligned} \right\} \quad (22)$$

For Order 1, That is $O(G_{r\theta}^1): G_{r\theta}$

$$\left. \begin{aligned} \frac{d^2 u_1}{dy^2} - a_1^2 u_1 &= -e^{i\omega t} \theta_0 - R_e b v_0 \\ u_1(0) &= 0, \quad u_1(1) = 0 \end{aligned} \right\} \quad (23)$$

$$\left. \begin{aligned} \frac{d^2 v_1}{dy^2} - a_2^2 v_1 &= -e^{i\omega t} \theta_0 + \frac{\beta f R_e}{\phi} u_1 \\ v_1(0) &= 0, \quad v_1(1) = 0 \end{aligned} \right\} \quad (24)$$

$$\left. \begin{aligned} \frac{d^2 \theta_1}{dy^2} - a_3^2 \theta_1 &= -\frac{2 P_e E_c}{R_e} \left(\frac{du_0}{dy} \right) \left(\frac{du_1}{dy} \right) \\ \theta_1(0) &= 0, \quad \theta_1(1) = 0 \end{aligned} \right\} \quad (25)$$

Solving equation (20)- (25), the following results are obtained:

$$u_0(y) = A_1 e^{a_1 y} + A_2 e^{-a_1 y} + A_3 \quad (26)$$

$$v_0(y) = A_4 e^{a_2 y} + A_5 e^{-a_2 y} + A_6 + A_7 e^{a_1 y} + A_8 e^{-a_1 y} \quad (27)$$

$$\theta_0(y) = A_9 e^{a_3 y} + A_{10} e^{-a_3 y} + A_{11} e^{2a_1 y} + A_{12} + A_{13} e^{-2a_1 y}$$

(28)

$$u_1(y) = A_{14}e^{a_1y} + A_{15}e^{-a_1y} + A_{16}e^{a_3y} + A_{17}e^{-a_3y} + A_{18}e^{2a_1y} + A_{19} + A_{20}e^{-2a_1y} + A_{21}e^{a_2y} + A_{22}e^{-a_2y} + A_{23}ye^{a_1y} + A_{24}ye^{-a_1y} \quad (29)$$

$$v_1(y) = A_{25}e^{a_2y} + A_{26}e^{-a_2y} + A_{27}e^{a_3y} + A_{28}e^{-a_3y} + A_{29}e^{2a_1y} + A_{30}e^{-2a_1y} + A_{31} + A_{32}e^{a_2y}y + A_{33}e^{-a_2y}y + A_{34}e^{a_1y} + A_{35}e^{-a_1y} + b_1ye^{-a_1y} + b_2ye^{a_1y} \quad (30)$$

$$\theta_1(y) = A_{36}e^{a_3y} + A_{37}e^{-a_3y} + A_{38}e^{2a_1y} + A_{39} + A_{40}e^{(a_1+a_3)y} + A_{41}e^{(a_1-a_3)y} + A_{42}e^{3a_1y} + A_{43}e^{-a_1y} + A_{44}e^{(a_1+a_2)y} + A_{45}e^{(a_1-a_2)y} + A_{46}e^{-2a_1y} + A_{47}e^{(a_3-a_1)y} + A_{48}e^{-(a_1+a_3)y} + A_{49}e^{a_1y} + A_{50}e^{-3a_1y} + A_{51}e^{(a_2-a_1)y} + A_{52}e^{-(a_1+a_2)y} + A_{53}ye^{2a_1y} + A_{54}y + A_{55}ye^{-2a_1y} \quad (31)$$

Put (26), (27), (28), (29), (30) and (31) in (19) we have,

$$u(y) = u_0 + G_{r\theta}u_1 \quad (32)$$

$$v(y) = v_0 + G_{r\theta}v_1 \quad (33)$$

$$\theta(y) = \theta_0 + G_{r\theta}\theta_1 \quad (34)$$

Where,

$$\begin{aligned} A_1 &= 1 + \frac{\sigma R_e}{a_1^2} - \frac{\sigma R_e(1 - e^{a_1}) - a_1^2 e^{a_1}}{a_1^2(e^{-a_1} - e^{a_1})}, \quad A_2 = \frac{\sigma R_e(1 - e^{a_1}) - a_1^2 e^{a_1}}{a_1^2(e^{-a_1} - e^{a_1})}, \quad A_3 = -\frac{\sigma R_e}{a_1^2}, \\ A_4 &= \frac{e^{-a_2} - (A_6 + A_7 + A_8)e^{-a_2} + A_6 + A_7e^{a_1} + A_8e^{-a_1}}{(e^{-a_2} - e^{a_2})}, \\ A_5 &= \frac{(A_6 + A_7 + A_8)e^{a_2} - A_6 - A_7e^{a_1} - A_8e^{-a_1} - e^{a_2}}{(e^{-a_2} - e^{a_2})}, \quad A_6 = \frac{-(\sigma R_e\phi + \beta fR_eA_3)}{\phi a_2^2}, \quad A_7 = \frac{\beta fR_eA_1}{\phi(a_1^2 - a_2^2)}, \\ A_8 &= \frac{\beta fR_eA_2}{\phi(a_1^2 - a_2^2)} \\ A_9 &= \frac{e^{-a_3} - (A_{11} + A_{12} + A_{13})e^{-a_3} + A_{11}e^{2a_1} + A_{12} + A_{13}e^{-2a_1}}{(e^{-a_3} - e^{a_3})} \\ A_{10} &= \frac{(A_{11} + A_{12} + A_{13})e^{a_3} - A_{11}e^{2a_1} - A_{12} - A_{13}e^{-2a_1} - e^{a_3}}{(e^{-a_3} - e^{a_3})} \\ A_{11} &= -\frac{P_e E_c A_1^2 a_1^2}{R_e(4a_1^2 - a_3^2)} \\ A_{12} &= -\frac{2P_e E_c A_1 A_2 a_1^2}{R_e a_3^2} \\ A_{13} &= -\frac{P_e E_c A_2^2 a_1^2}{R_e(4a_1^2 - a_3^2)} \\ A_{14} &= \frac{\left(-(A_{16} + A_{17} + A_{18} + A_{19} + A_{20} + A_{21} + A_{22})e^{-a_1} + A_{16}e^{a_3} + A_{17}e^{-a_3} + A_{18}e^{2a_1} + A_{19} + A_{20}e^{-2a_1} + A_{21}e^{a_2} + A_{22}e^{-a_2} + A_{23}e^{a_1} + A_{24}e^{-a_1} \right)}{(e^{-a_1} - e^{a_1})} \end{aligned}$$

$$\begin{aligned}
A_{15} &= \frac{\left((A_{16} + A_{17} + A_{18} + A_{19} + A_{20} + A_{21} + A_{22})e^{a_1} - A_{16}e^{a_3} - A_{17}e^{-a_3} - \right. \\
&\quad \left. A_{18}e^{2a_1} - A_{19} - A_{20}e^{-2a_1} - A_{21}e^{a_2} - A_{22}e^{-a_2} - A_{23}e^{a_1} - A_{24}e^{-a_1} \right)}{(e^{-a_1} - e^{a_1})} \\
\left\{ \begin{array}{lll} A_{16} = -\frac{A_9 e^{i\omega t}}{a_3^2 - a_1^2} & A_{17} = -\frac{A_{10} e^{i\omega t}}{a_3^2 - a_1^2} & A_{18} = -\frac{A_{11} e^{i\omega t}}{3a_1^2} \\ A_{19} = \frac{A_{12} e^{i\omega t} + R_e b A_6}{a_1^2} & A_{20} = -\frac{A_{13} e^{i\omega t}}{3a_1^2} & A_{21} = -\frac{R_e b A_4}{a_2^2 - a_1^2} \\ A_{22} = -\frac{R_e b A_5}{a_2^2 - a_1^2} & A_{23} = -\frac{R_e b A_7}{2a_1} & A_{24} = \frac{R_e b A_8}{2a_1} \end{array} \right. \\
A_{25} &= \frac{\left(-(A_{27} + A_{28} + A_{29} + A_{30} + A_{31} + A_{34} + A_{35})e^{-a_2} + A_{27}e^{a_3} + A_{28}e^{-a_3} + \right. \\
&\quad \left. A_{29}e^{2a_1} + A_{30}e^{-2a_1} + A_{31} + A_{32}e^{a_2} + A_{34}e^{a_1} + A_{35}e^{-a_1} + b_1e^{-a_1} + b_2e^{a_1} \right)}{(e^{-a_2} - e^{a_2})} \\
A_{26} &= \frac{\left((A_{27} + A_{28} + A_{29} + A_{30} + A_{31} + A_{34} + A_{35})e^{a_2} - A_{27}e^{a_3} - A_{28}e^{-a_3} - \right. \\
&\quad \left. A_{29}e^{2a_1} - A_{30}e^{-2a_1} - A_{31} - A_{32}e^{a_2} - A_{33}e^{-a_2} - A_{34}e^{a_1} - A_{35}e^{-a_1} - b_1e^{-a_1} - b_2e^{a_1} \right)}{(e^{-a_2} - e^{a_2})} \\
A_{27} &= \frac{A_{16}\beta f R_e - \varphi A_9 e^{i\omega t}}{\varphi(a_3^2 - a_2^2)}, \quad A_{28} = \frac{A_{17}\beta f R_e - \varphi A_{10} e^{i\omega t}}{\varphi(a_3^2 - a_2^2)}, \quad A_{29} = \frac{A_{18}\beta f R_e - \varphi A_{11} e^{i\omega t}}{\varphi(4a_1^2 - a_2^2)}, \\
A_{30} &= \frac{A_{20}\beta f R_e - \varphi A_{13} e^{i\omega t}}{\varphi(4a_1^2 - a_2^2)}, \quad A_{31} = \frac{-(A_{19}\beta f R_e - \varphi A_{12} e^{i\omega t})}{\varphi a_2^2}, \quad A_{32} = \frac{A_{21}\beta f R_e}{2\varphi a_2^2}, \quad A_{33} = -\frac{A_{22}\beta f R_e}{2\varphi a_2^2} \\
A_{34} &= \left(\frac{A_{14}\beta f R_e - 2\varphi b_2 a_1}{\varphi(a_1^2 - a_2^2)} \right), \quad A_{35} = \left(\frac{A_{15}\beta f R_e + 2\varphi b_1 a_1}{\varphi(a_1^2 - a_2^2)} \right), \quad b_1 = \frac{A_{24}\beta f R_e}{\varphi(1 + a_1^2 - a_2^2)}, \\
b_2 &= \frac{A_{23}\beta f R_e}{\varphi(1 + a_1^2 - a_2^2)}, \\
A_{36} &= \frac{\left(-A_{56}e^{-a_3} + A_{38}e^{2a_1} + A_{39} + A_{40}e^{(a_1+a_3)} + A_{41}e^{(a_1-a_3)} + A_{42}e^{3a_1} + A_{43}e^{-a_1} + \right. \\
&\quad A_{44}e^{(a_1+a_2)} + A_{45}e^{(a_1-a_2)} + A_{46}e^{-2a_1} + A_{47}e^{(a_3-a_1)} + A_{48}e^{-(a_1+a_3)} + A_{49}e^{a_1} + \\
&\quad \left. A_{50}e^{-3a_1} + A_{51}e^{(a_2-a_1)} + A_{52}e^{-(a_1+a_2)} + A_{53}e^{2a_1} + A_{54} + A_{55}e^{-2a_1} \right)}{(e^{-a_3} - e^{a_3})},
\end{aligned}$$

$$A_{37} = \frac{\begin{pmatrix} A_{56}e^{a_3} - A_{38}e^{2a_1} - A_{39} - A_{40}e^{(a_1+a_3)} - A_{41}e^{(a_1-a_3)} - A_{42}e^{3a_1} - A_{43}e^{-a_1} - \\ A_{44}e^{(a_1+a_2)} - A_{45}e^{(a_1-a_2)} - A_{46}e^{-2a_1} - A_{47}e^{(a_3-a_1)} - A_{48}e^{-(a_1+a_3)} - A_{49}e^{a_1} - \\ A_{50}e^{-3a_1} - A_{51}e^{(a_2-a_1)} - A_{52}e^{-(a_1+a_2)} - A_{53}e^{2a_1} - A_{54} - A_{55}e^{-2a_1} \end{pmatrix}}{(e^{-a_3} - e^{a_3})}$$

$$A_{38} = -\left(\frac{2E_c P_e A_1 (A_{14}a_1^2 + A_{23}a_1) + 4a_1 R_e A_{53}}{R_e (4a_1^2 - a_3^2)}\right),$$

$$A_{39} = -\frac{2E_c P_e (a_1^2 (A_1 A_{15} + A_2 A_{14}) + a_1 (A_2 A_{23} - A_1 A_{24}))}{R_e a_3^2}, \quad A_{40} = \frac{-2E_c P_e A_1 A_{16} a_1 a_3}{R_e ((a_1 + a_3)^2 - a_3^2)},$$

$$A_{41} = \frac{2E_c P_e A_1 A_{17} a_1 a_3}{R_e ((a_1 - a_3)^2 - a_3^2)}, \quad A_{42} = \frac{-4E_c P_e A_1 A_{18} a_1^2}{R_e (9a_1^2 - a_3^2)}, \quad A_{43} = \frac{4E_c P_e A_1 A_{20} a_1^2}{R_e (a_1^2 - a_3^2)},$$

$$A_{44} = \frac{-2E_c P_e A_1 A_{21} a_1 a_2}{R_e ((a_1 + a_2)^2 - a_3^2)}, \quad A_{45} = \frac{2E_c P_e A_1 A_{22} a_1 a_2}{R_e ((a_1 - a_2)^2 - a_3^2)}, \quad A_{46} = \frac{2E_c P_e A_2 (A_{24}a_1 - A_{15}a_1^2) + 4R_e a_1}{R_e (4a_1^2 - a_3^2)}$$

$$A_{47} = \frac{2E_c P_e A_2 A_{16} a_1 a_3}{R_e ((a_3 - a_1)^2 - a_3^2)}, \quad A_{48} = \frac{-2E_c P_e A_2 A_{17} a_1 a_3}{R_e ((a_1 + a_3)^2 - a_3^2)}, \quad A_{49} = \frac{4E_c P_e A_2 A_{18} a_1^2}{R_e (a_1^2 - a_3^2)},$$

$$A_{50} = \frac{-4E_c P_e A_2 A_{20} a_1^2}{R_e (9a_1^2 - a_3^2)}, \quad A_{51} = \frac{2E_c P_e A_2 A_{21} a_1 a_2}{R_e ((a_2 - a_1)^2 - a_3^2)}, \quad A_{52} = \frac{-2E_c P_e A_2 A_{22} a_1 a_2}{R_e ((a_1 + a_2)^2 - a_3^2)},$$

$$A_{53} = \frac{-2E_c P_e A_1 A_{23} a_1^2}{R_e (4a_1^2 - a_3^2)}, \quad A_{54} = \frac{-2E_c P_e a_1^2 (A_1 A_{24} + A_2 A_{23})}{R_e a_3^2}, \quad A_{55} = \frac{-2E_c P_e a_1^2 A_2 A_{24}}{R_e (4a_1^2 - a_3^2)}$$

Therefore, the general solutions of the problem (1), (2), (3) and (4) are respectively
 $u(y, t) = u(y)e^{i\omega t}$, $v(y, t) = v(y)e^{i\omega t}$, $\theta(y, t) = \theta(y)e^{2i\omega t}$ (35)

RESULTS AND DISCUSSION

The impact of Peclet number (P_e), Mass concentration of dust particles (f), Concentration resistance ratio (β), Heat source parameter (S), Eckert number (E_c), Coefficient of Grashof Thermal (b), Volume fraction of the dust particle (ϕ), Reynolds number (R_e) and Grashof thermal number ($G_{r\theta}$) on the temperature $\theta(y, t)$ and velocity $u(y, t)$ of the fluid and velocity of the dust particles $v(y, t)$ are investigated. The results obtained in equation (35) are shown graphically in the Figures below with the aid of MAPLE 17 software as follows.

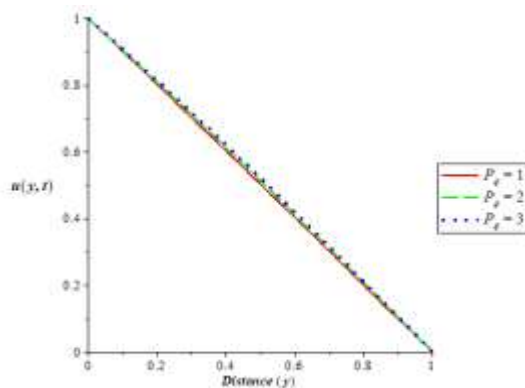


Fig 1: Effect of change in Peclet number (P_e) on the Velocity of the fluid $u(y, t)$

Figure 1 shows the effect of Peclet number (P_e) on the velocity of the fluid $u(y,t)$. It is observed that velocity of the fluid increases with an increase in the Peclet number (P_e). Figure 2 displays the effect of Peclet number (P_e) on the velocity of dust particles. It is observed that the velocity of the dust particles $v(y,t)$ increases with increase in the Peclet number (P_e). In Figure 3, the effect of Peclet number (P_e) on the temperature of the fluid $\theta(y,t)$ is presented. The graph shows that the temperature of the fluid $\theta(y,t)$ increases with an increase in Peclet number (P_e).

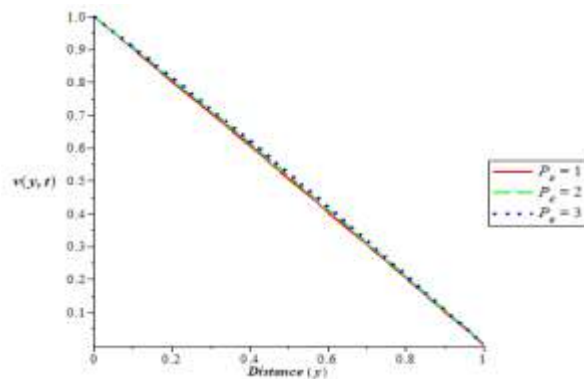


Fig 2: Variation of Peclet number (P_e) on Velocity of the dust particles $v(y,t)$

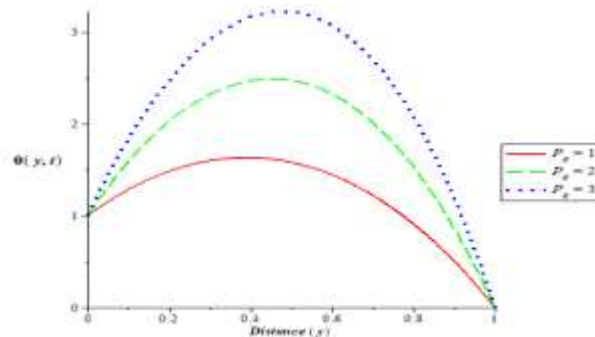


Fig 3: Variation of Peclet number (P_e) on Temperature of the fluid $\theta(y,t)$.

The effect of Mass concentration of dust particles (f) on the velocity of the fluid $u(y,t)$ is presented in Figure 4. The graph shows that the fluid does not change much with an increase in the Mass

concentration of dust particles (f) at steady time. Figure 5 shows the effect of Mass concentration of dust particles (f) on the velocity of the dust particles $v(y,t)$. The graph shows that the velocity of the dust particles increases with an increase in the Mass concentration of dust particles (f).

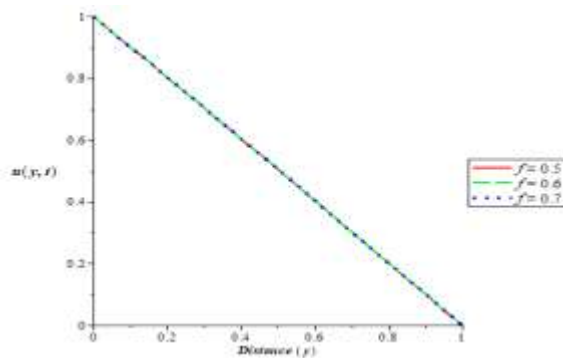


Fig 4: Variation of Mass concentration of dust particles (f) on

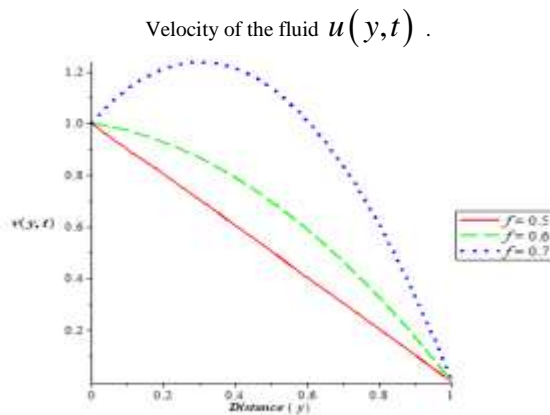


Fig 5: Variation of Mass concentration of dust particles (f) on

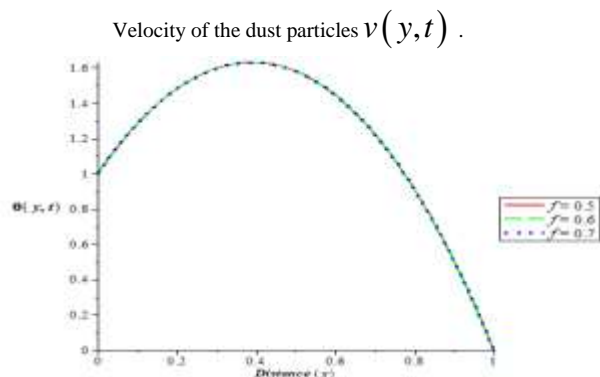


Fig 6: Variation of Mass concentration of dust particles (f) on Temperature of the fluid $\theta(y,t)$.

Figure 6 shows the effect of Mass concentration of dust particles (f) on the temperature of the fluid $\theta(y, t)$. The graph shows that the temperature of the fluid does not change much with an increase in the Mass concentration of dust particles (f).

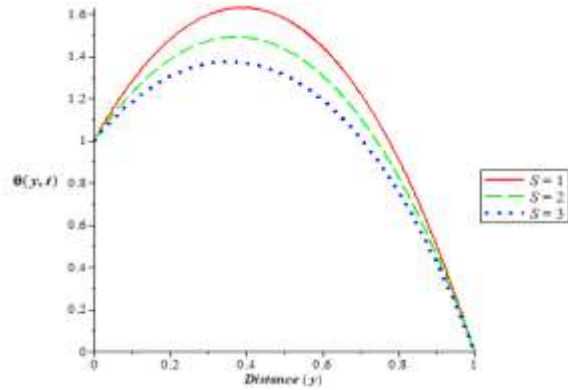


Fig 7: Variation of Heat source parameter (S) on Temperature of the fluid $\theta(y, t)$

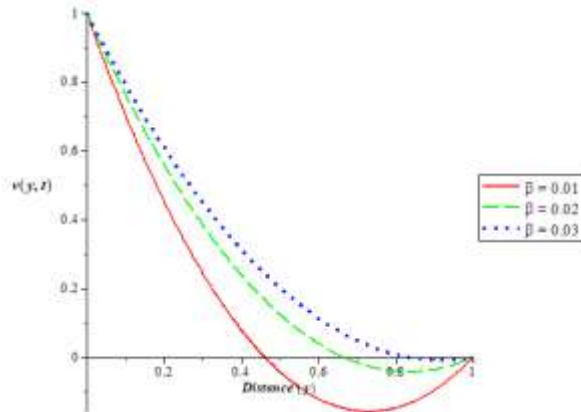


Fig 8: Variation of Concentration resistance ratio (β) on Velocity of the dust particles $v(y, t)$

Figure 7 shows the effect of Heat source parameter (S) on the temperature of the fluid $\theta(y, t)$. The graph shows that the temperature of the fluid reduces with an increase in the Heat source parameter (S). In Figure 8, the effect of Concentration resistance ratio (β) on the velocity of the dust particles $v(y, t)$ is presented. It is observed that velocity of the dust particles increases with an increase in the Concentration resistance ratio (β). The effect of Eckert number (E_c) on the velocity of the fluid

$u(y, t)$ is presented in Figure 9. It is observed that velocity of the fluid increases with an increase in Eckert number (E_c).

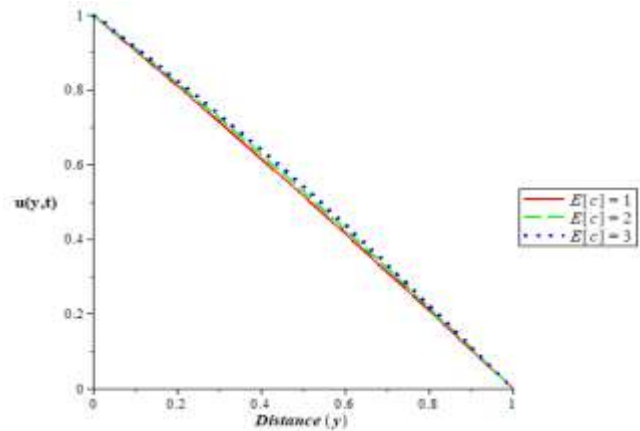


Fig 9: Variation of Eckert number (E_c) on Velocity of the fluid

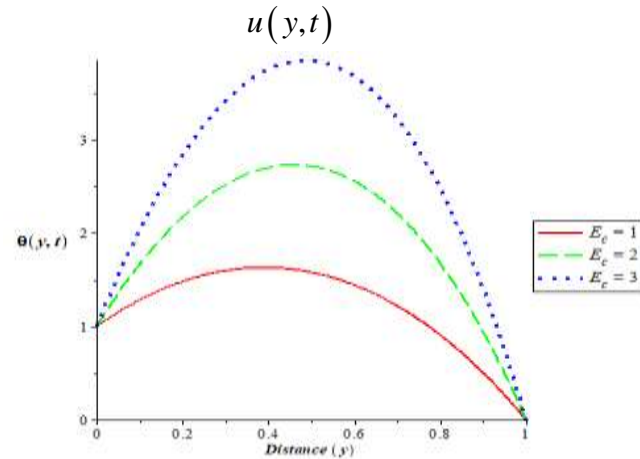


Fig 10: Variation of Eckert number (E_c) on Temperature of the fluid $\theta(y, t)$

Similarly, the effect Eckert number (E_c) on the temperature of the fluid $\theta(y, t)$ is presented in Figure 10. The graph shows that the temperature of the fluid increases with an increase in Eckert number (E_c). Figure 11 shows the effect of Reynold number (R_e) on the velocity of the fluid $u(y, t)$. It is observed that velocity of the fluid reduces with an increase in Reynold number (R_e).

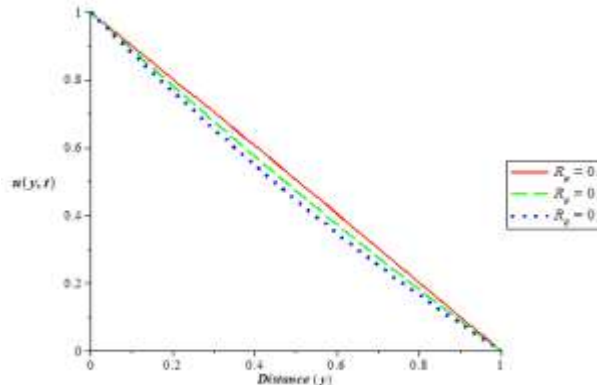


Fig 11: Variation of Reynold number (R_e) on Velocity of the fluid $u(y,t)$

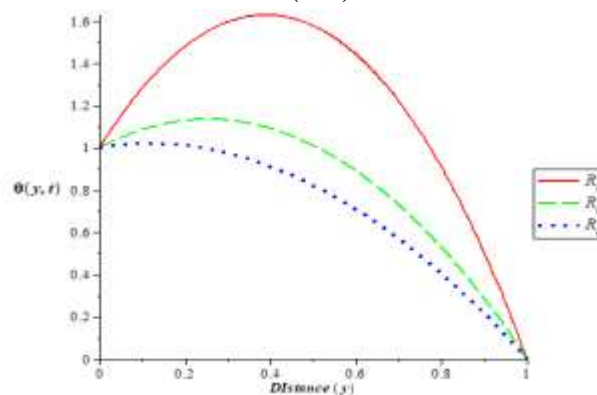


Fig 12: Variation of Reynold number (R_e) on Temperature of the fluid $\theta(y,t)$

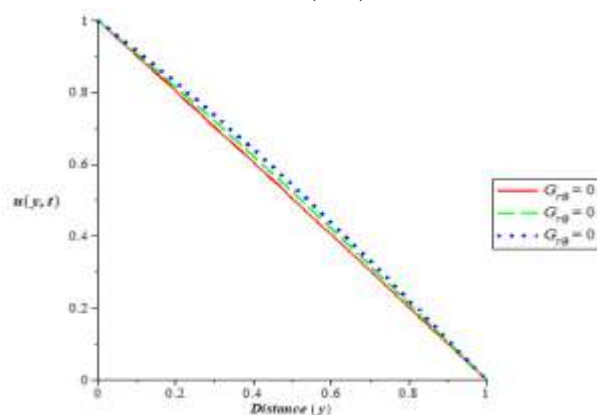


Fig 13: Variation of Grashof thermal number ($G_{r\theta}$) on Velocity of the fluid $u(y,t)$

The effect of Reynold number (R_e) on the temperature of the fluid $\theta(y,t)$ is presented in Figure 12. The graph shows that the temperature of the fluid reduces with an increase in Reynold number

(R_e). Figure 13 show the effect of Grashof thermal number ($G_{r\theta}$) on the velocity of the fluid $u(y,t)$. It is observed that the velocity of the fluid increases with an increase in the Grashof thermal number ($G_{r\theta}$). Lastly, Figure 14 show the effect of Grashof thermal number ($G_{r\theta}$) on the temperature of the fluid $\theta(y,t)$. It is observed that the temperature of the fluid increases with an increase in the Grashof thermal number ($G_{r\theta}$).

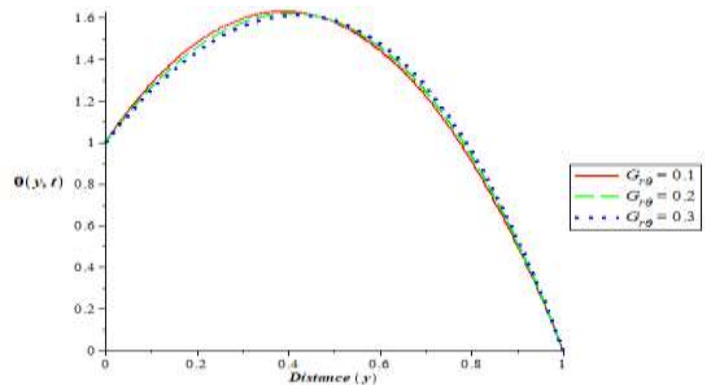


Fig 14: Variation of Grashof thermal number ($G_{r\theta}$) on Temperature of the fluid $\theta(y,t)$

Conclusion: A mathematical analysis has been carried out on the transient laminar free convective flow of a dusty viscous fluid through a porous medium in the incorporating viscous energy dissipation. The dimensionless governing coupled non-linear partial differential equations for this investigation were solved analytically using harmonic solution technique. The effects of the dimensionless parameters as shown on the graph were analyzed. The results revealed that changes in the flow parameters- Peclet energy number, Eckert number, Grashof thermal number, Mass concentration of dust particles and Reynold number affect the velocity of the fluid particles, dust particles and the fluid temperature as discussed above.

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