Homotopy Perturbation Analysis of the Spread and Control of Lassa Fever

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Abstract

Lassa fever, a viral infection transmitted by rodents, has emerged as a significant global health concern in recent times. It continues to garner significant attention daily basis owing to its rapid transmission and deadly nature. In this study, the Homotopy Perturbation Analysis was conducted to examine the spread and control of Lassa fever. The human population was categorized into susceptible, exposed, infected, and recovered compartments, while the rodent population was divided into susceptible and infected recovered compartments. By applying the Homotopy Perturbation Analysis to the nonlinear differential equations associated with these compartments, we were able to obtain the analytical solution for the spread and control of Lassa fever. The nonlinear differential equations were integrated into the Homotopy Perturbation framework and solved to form a power series solution. Finally, the final approximate solutions were obtained and simulation results were generated from the general solution graphically.

Keywords — Homotopy Perturbation Method, Lassa fever**,** Nonlinear Differential Equations

1. INTRODUCTION

In light of their rapid transmission and the severity of diseases such as HIV/AIDS, measles, tuberculosis, cholera, diarrhea, COVID-19, and Lassa fever Lassa fever infection continues to receive a lot of attention daily (Olumuyiwa *et al*., 2020; Agbata *et al.*, 2021). Lassa Fever is a zoonotic illness characterized by acute hemorrhagic symptoms, which is caused by the Lassa virus. This virus is primarily transmitted from animals to humans, with the Mastomys natalensis serving as its reservoir host. (Rodent) (Akinpelu and Akinwande, 2019; Anorue and Okeke, 2020). According to reports, one in every five infections leads to a severe case of the disease, wherein the virus impacts crucial organs like the liver, spleen, and kidneys (WHO, 2017). The virus can be transmitted to individuals by coming into contact with household items, food, water, or air that has been contaminated by the droppings or urine of infected multimammate rats (*Mastomyces natalensis*). Additionally, direct contact with infected rats or exposure to the virus blood, tissue, secretions, or excretions of a person with Lassa virus can lead to person–person transmission. Contaminated medical equipment, like reused needles, also poses a risk of transmission (CDC, 2019; Collins and Okeke, 2021; Anorue and Okeke, 2020; Bakare *et al.*, 2020).

The period of incubation for Lassa fever can vary between 6 to 21 days. The initial symptoms typically include fever, weakness, and malaise, followed by headache, sore throat, muscle pain, chest pain, nausea, vomiting, diarrhoea, cough, and abdominal pain. In severe cases, patients may experience facial swelling, fluid accumulation in the lungs, bleeding from various parts of the body, low blood pressure, presence of protein in the urine, shock, seizures, tremors, disorientation, and even coma in advanced stages (Sulaiman and Ibrahim, 2018; WHO, 2017). Fatal cases typically result in death within 14 days of onset. The disease becomes particularly severe in the late stages of pregnancy, with over 80% of cases experiencing maternal death and/or fetal loss during the third trimester (Sulaiman and Ibrahim, 2018; WHO, 2017).

Ribavirin appears to be an effective antiviral drug for treating for Lassa fever virus during the initial stages of the illness (WHO, 2017). In the event of a Lassa fever outbreak within a community, swift isolation of infected individuals, proper infection prevention and control measures, and thorough contact tracing are essential to halt the spread of the disease. It is crucial to promote "community hygiene" practices that deter rodents, such as securing food in rodents-proof containers, disposing of garbage away from homes, and maintaining clean living spaces. Additionally, family members should take precautions to avoid contact with blood and bodily fluids when caring take precautions to avoid contact and bodily fluids when caring for sick individuals, while healthcare and laboratory workers must adhere to strict infection control protocols to minimize the risk of exposure to contaminated materials (WHO, 2017; CDC, 2019; Adebayo *et al.*, 2015).

Dr. Ji Huan He, a Chinese researcher, introduced Homotopy Perturbation Method (HPM) in 1998 to solve both linear and nonlinear differential and integral equations. This method, which involves a series expansion, is particularly useful in tacking non-linear partial differential equations Jiya (2010). The HPM method utilizes a power series to convert the original non-linear differential equation into a series of linear differential equations Padma *et al.* (2021). This method combines the traditional perturbation and the homotopy method Anorue and Okeke (2020), providing a direct approach to obtaining analytical or approximatively solutions for a wide range of problems in various domains, by integrating topological homotopy with traditional perturbation techniques Otoo *et al,* this approach has proven successful in solving linear and nonlinear functional equations, yielding exact solutions and ensuring accurate quantitative predictions using the Homotopy Perturbation technique (HPM) Mechee and Al-Juaifri (2018). The accuracy of the Homotopy Perturbation Method (HPM) has led to its application in epidemic modeling.

2. Literature Review

In the study conducted by Mechee and Al-Juaifri (2018), they suggested utilizing the Homotopy Perturbation method approach for the SIR model with vital dynamics and constant population. The application of this approach yielded an effective and highly precise approximate solution. Padma *et al.* (2021) also employed the Homotopy Perturbation Method to solve the SIR infectious disease model by integrating vaccination. The (HPM) was utilized to derive an approximate solution for each compartment of the model. The resulting approximate solution was then utilized to visually represent the model, providing a better comprehension of the dynamics of the infectious disease. Furthermore, Ayoade *et al.* (2020) introduced Homotopy Perturbation Method to a SIR mumps model and the theoretical outcomes validated the effectiveness and suitability of HPM in solving epidemic models. Ojo *et al.* (2021) developed a deterministic model using systems of ordinary differential equations to investigate the transmission dynamic of Lassa fever in the population. The population was divided into human and rodent compartments. Their findings suggest that implementing control strategies and methods aimed at reducing rodent populations and minimizing transmission from rodents to humans would contribute to the effective management of Lassa fever in the population. In a separate study, Padma *et al*. (2021) utilized the modified Homotopy perturbation method to solve and analyze the transmission of the SIR model of this disease. The derived analytical expression of the population of the susceptible group S(t), the infected group I(t), and the recovered group R(t) at all-time values. The Homotopy Perturbation Method was then applied to the nonlinear differential equations representing the different compartments. By incorporating the nonlinear differential equations into Homotopy Perturbation constructor, they obtained the analytical solution for the transmission dynamics of Lassa fever in the form of a power series. Peter and Awoniran (2018) utilized the modified Homotopy perturbation method to solve and examine the transmission of the SIR model of

the particular disease. The analytical expression for the population of the susceptible group S(t), the infected group I(t), and the recovered group R(t) is derived for all time. Hence, this study aims to employ the Homotopy Perturbation Method to deduce the analytical solution of the spread and control of Lassa fever. The study also addresses the impact of different parameters, we conducted numerical simulation using MAPLE 17 and compared the results with our analytical findings. In this, article, we employ the homotopy perturbation analysis to investigate the spread and control of Lassa fever.

2.1. Basic Ideas of Homotopy Perturbation Method

The fundamental concept of HPM is demonstrated in this section.

Consider (1) that provides the differential equation

,

$$A(U)-f(r)=0,\ r\in\Omega \tag{1}$$

subject to the boundary condition (2).

$$B\left(U,\frac{\partial U}{\partial n}\right)=0,\quad r\in\Gamma \tag{2}$$

A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytical function and Γ represents the boundary of the domain Ω. The operator A can be split into linear (L) and nonlinear (N) components. Hence, equation (1) can be expressed as (3).

$$L(U)+N(U)-f(r)=0 \tag{3}$$

An artificial parameter p can be embedded in (3) as (4).

$$L(U)+p(U)-f(r)=0 \tag{4}$$

Where $P\in[0,1]$ is an embedding parameter (also called as an artificial parameter)

Using the homotopy technique, proposed by He (1999), we construct a homotopy;

$H:v(r,p):\Omega\times[0,1]\to R$ which satisfies (5)

$$H(V,\mathrm{P})=(1-P)\left[L(V)-L(U_0)\right]+P\left[A(V)-f(r)\right]=0 \tag{5}$$

And (6)

$$H(V,\mathrm{P})=L(V)-L(U_0)+PL(U_0)+p\left[N(V)-f(r)\right]=0 \tag{6}$$

At $p=0$

$$H(\mathrm{v},0)=L(\mathrm{v})-L(\mathrm{u}_0)=0 \tag{7}$$

And at $p=1$

$$H(\mathrm{v},1)=L(\mathrm{u})+N(\mathrm{u})-f(\mathrm{r})=0 \tag{8}$$

The transition process of p from zero to unity is just that of $v(r,\ p)$ from $u_0(r)$ to $u(r)$. In topology, this is referred to as deformation $L(\mathrm{v})-L(\mathrm{u}_0)$ and $L(\mathrm{u})+N(\mathrm{u})-f(\mathrm{r})$ is called homotopic. Following the HPM, we can introduce the embedding parameter p as a small parameter and express that the solutions of equations (7) and (8)can be written as a power series p as indicated in (9)

$$V=V_0+pV_1+p^2V_2+... \tag{9}$$

The results in the approximate solution of equation (1) may then also be obtained as (10)

$$U=\lim_{p\to 1} v=v_0+pv_1+p^2v_2+... \tag{10}$$

Which is the convergence series solution

3. Methodology

3.1 Disease Model

The nonlinear differential equations system is derived from the compartments model and incorporated in the Homotopy Perturbation Method. Subsequently, the equations were resolved to obtain analytical solutions for individual compartments. The spread of Lassa fever involves the interplay between human populations and rodent populations Usman and Adamu, (2018).

In this study, a six-compartmental model for the spread and control of Lassa fever is constructed using ordinary differential equations. The total human population at time t denoted by N is divided into four compartments namely; susceptible $S_h(t)$, exposed $E_h(t)$, infectious $I_h(t)$, and recovered $R_h(t)$. Thus, the total human population $N_h(t)$ is given as:

$$N_h(t)=S_h(t)+E_h(t)+I_h(t)+R_h(t) \tag{11}$$

Again, the total rodent population at a time t denoted by $N_r(t)$ which is divided into two compartments, namely: susceptible rodents $S_r(t)$ and infectious rodents $I_r(t)$. Thus, the total rodent population $N_r(t)$ is given as:

$$N_r(t)=S_r(t)+R_r(t) \tag{12}$$

3.2 Formulation of the Model

1)Susceptible Human (S_h)

This indicates the individuals within the entire human population that are susceptible to the disease. The population of susceptible humans S_h is populated by immigration or birth at a rate ϕ_h, and from recovered individuals due to their loss of immunity at the rate $\chi_h R_h$. The

susceptible human population is depopulated by infection following effective contact with infected individuals at the rates. The parameter ℓ_h represents the effective transmission probability of humans, which could be through direct contact with contaminated food by the urine or excretes of an infectious infected rodent, or laboratory transmissions that is sharing of medical equipment with infectious individuals without adequate sterilization Mayowa and Emile, (2022).We assume that all susceptible humans are further reduced by natural death at rate μ_h.

2) Exposed Human (E_h)

The Exposed humans are those that carry the bacteria but are not capable of infecting susceptible humans. The exposed human population is proven from an infection occurring from the susceptible population. This populace is reduced by natural death μ_h and the disease progression to the infectious population at the rate δ_h. It is imperative to note that, exposed humans are infected with the Lassa fever virus but are not showing symptoms yet. Following the disease incubation period which is between $6-21$ days (Sulaiman and Ibrahim, 2018; WHO, 2017). Such individuals progress to infectious population. This is the stage whereby they start showing symptoms of the disease.

3) Infected Human (I_h)

An infected human is any individual who has the pathogen and shows symptoms of the disease. The infectious human compartment is generated as a result of the rate from the exposed human population. The population is reduced by the recovery rate due to treatment at rate γ_h and disease-induced death (death caused by Lassa fever) at the rate ρ_h and natural death at the rate μ_h.

4) Recovered Human (R_h)

Following early treatment of individuals and diagnosed of Lassa fever disease, such individuals recover and progress to increase the recovered human population. However, since recovered individuals can be re-infected of the disease Mayowa and Emile, (2022) the recovered human populace is reduced by loss of immunity at rate γ_h and natural death at the rate μ_h.

5)Susceptible Rodent (S_r)

Susceptible rodents' population is established by the birth of rodents at a certain rate ϕ_r.This group is reduced by natural death at a specific rate μ_r, and is additionally decreased after being infected with Lassa virus from coming into contact with an infectious human or rodent at the rate. The parameters ℓ_r represents the effective transmission probability from human-to rodent and the effective transmission probability from rodent-to-rodent.

5) Infected Rodent (I_r)

The infectious rodent population is derived from infection occurring from the susceptible rodent population, while depopulated by natural death of rodents at rate μ_r.

3.3 *Nonlinear Equations of the Model*

The nonlinear system of differential equations is obtained by merging the equations for the various compartments.

$$\left.\begin{aligned}
\frac{dS_h}{dt} &= \phi_h + \chi_h R_h - \ell_h S_h - \mu_h S_h = 0 \\
\frac{dE_h}{dt} &= \ell_h S_h - (\delta_h + \mu_h) E_h = 0 \\
\frac{dI_h}{dt} &= \delta_h E_h - (\gamma_h + \rho_h + \mu_h) I_h = 0 \\
\frac{dR_h}{dt} &= \gamma_h I_h - (\mu_h + \chi_h) R_h = 0 \\
\frac{dS_r}{dt} &= \phi_r - \ell_r S_r - \mu_r S_r = 0 \\
\frac{dI_r}{dt} &= \ell_r S_r - \mu_r S_r = 0
\end{aligned}\right\} \tag{13}$$

Where,

ϕ_h	Recruitment rate of humans through birth or immigration
χ_h	Immunity waning rate of humans
δ_h	Disease progression rate from exposed to infectious human
γ_h	Recovery rate of infectious humans
μ_h	Natural death rate of humans
ρ_h	Disease induced death rate for humans
ℓ_h	Transmission probability from human-to-human and human-to-rodents
ℓ_r	Transmission probability from human-to-rodent and rodent-to-rodent
ϕ_r	Recruitment rate of rodents through birth
μ_r	Natural death rate of rodents

Let the initial conditions or approximate are as follows;

$$S_h(0) = s_o, \mathrm{E}_h(0) = e_o, \mathrm{I}_h(0) = i_o, \mathrm{R}_h(0) = \mathrm{r}_o, S_r(0) = k_o, \mathrm{I}_r(0) = r_o \tag{14}$$

3.4 Assumptions of Homotopy Perturbation Method

To ascertain the analytical solution of the model, the embedding parameter " p " from (5) is utilized as a small parameter based on the Homotopy perturbation approach. It assumed that the solution of the equations can be represented as a power series in the form of (15)

$$V = V_0 + pV_1 + p^2V_2 + \ldots \tag{15}$$

Setting $p = 1$ results in the approximate solution of equation (15)

$$U = \lim_{p \to 1} v = v_0 + pv_1 + p^2v_2 + \ldots \tag{16}$$

This method retains all the advantages of the traditional perturbation method while removing its limitations.

3.5 Application of the Homotopy Perturbation Method to the Compartmental Disease Model Equations

The Homotopy Perturbation Method is utilized to equations (13) by employing Homotopy constructor equation in order to obtain an approximate solution,

$$H(v, p) = (1 - p)\left[L(v) - L(v_0)\right] + p\left[L(v) + N(v) - f(r)\right] = 0 \tag{17}$$

In order to initiate the process of deriving the analytical or approximate solution of the model, we employ Homotopy Perturbation assumption stated in (9) as per the assumption;

$$S_h(t) = S_0 + pS_1 + p^2S_2 + \ldots \tag{18}$$

$$E_h(t) = E_0 + pE_1 + p^2E_2 + \ldots \tag{19}$$

$$I_h(t) = I_0 + pI_1 + p^2I_2 + \ldots \tag{20}$$

$$R_h(t) = R_0 + pR_1 + p^2R_2 + \ldots \tag{21}$$

$$S_r(t) = K_0 + pK_1 + K^2S_2 + \ldots \tag{22}$$

$$I_r(t) = F_0 + FI_1 + F^2I_2 + \ldots \tag{23}$$

With the initial conditions given by
$S_{hi}(0) = 0, \mathrm{E}_{hi}(0) = 0, \mathrm{I}_{hi}(0) = 0, \mathrm{R}_{hi}(0), S_{ri}(0) = 0, \mathrm{I}_{ri}(0) = 0 \forall_i = 1, 2, 3, \ldots\ldots$

Next, nonlinear differential (13) are substituted one after the other into the homotopy constructor in (17);

Firstly, substituting the first equation (13) into (17) gives

$$(1-p)\frac{dS}{dt} + p\left[\frac{dS}{dt} + \phi_h + \chi_h R_h - \ell_h S_h - \mu_h S_h\right] = 0 \tag{24}$$

$$\frac{dS}{dt} - p\frac{dS}{dt} + p\frac{dS}{dt} + p\phi_h + p\chi_h R_h - p\ell_h S_h - p\mu_h S_h = 0 \tag{25}$$

$$\frac{dS}{dt} + p\phi_h + p\chi_h R_h - p\ell_h S_h - p\mu_h S_h = 0 \tag{26}$$

Again, substituting (18) and (21) into (26)

$$\frac{d}{dt}\left(S_0 + pS_1 + p^2 S_2 + \ldots\right) + p\phi_h + \chi_h\left(R_0 + pR_1 + p^2 R_2 + \ldots\right) - p\ell_h\left(S_0 + pS_1 + p^2 S_2 + \ldots\right)$$
$$-p\mu_h\left(S_0 + pS_1 + p^2 S_2 + \ldots\right)$$

(27)

Then again, grouping the coefficient powers of p in (27)

$$p^0 : \frac{ds}{dt} = 0 \tag{28}$$

$$p^1 : \frac{ds_1}{dt} + \phi_h + \chi_h R_0 - \ell_h S_0 - \mu_h S_0 = 0 \tag{29}$$

$$p^2 : \frac{ds_2}{dt} + \chi_h R_1 - \ell_h S_1 - \mu_h S_1 = 0 \tag{30}$$

Also, by substituting second equation (13) into (17)

$$(1-p)\frac{dE}{dt} + p\left[\ell_h S_h - (\delta_h + \mu_h)E_h\right] = 0 \tag{31}$$

$$\frac{dE}{dt} - p\frac{dE}{dt} + p\frac{dE}{dt} + p\ell_h S_h - p(\delta_h + \mu_h)E_h = 0 \tag{32}$$

$$\frac{dE}{dt} + p\ell_h S_h - p(\delta_h + \mu_h)E_h = 0 \tag{33}$$

Again, substituting (18) and (19) into (33)

$$\frac{d}{dt}\left(E_0 + pE_1 + p^2 E_2 + \ldots\right) + p\ell_h\left(S_0 + pS_1 + p^2 S_2 + \ldots\right)$$
$$-p(\delta_h + \mu_h)\left(E_0 + pE_1 + p^2 E_2 + \ldots\right) = 0 \tag{34}$$

Then again, grouping the coefficient powers of p in (34)

$$p^0 : \frac{dE_0}{dt} = 0 \tag{35}$$

$$p^1 : \frac{dE_1}{dt} + \ell_h S_0 - (\delta_h + \mu_h) E_0 \tag{36}$$

$$p^2 : \frac{dE_2}{dt} + \ell_h S_1 - (\delta_h + \mu_h) E_1 \tag{37}$$

Also, by substituting third equation (13) into (17)

$$\frac{dI_h}{dt} = \delta_h E_h - (\gamma_h + \rho_h + \mu_h) I_h = 0 \tag{38}$$

$$(1-p)\frac{dI}{dt} + p\left(\delta_h E_h - (\gamma_h + \rho_h + \mu_h) I_h = 0\right) \tag{39}$$

$$\frac{dI}{dt} - p\frac{dI}{dt} + p\frac{dI}{dt} + P(\delta_h E_h) - p(\gamma_h + \rho_h + \mu_h) I_h = 0 \tag{40}$$

$$\frac{dI}{dt} + P(\delta_h E_h) - p(\gamma_h + \rho_h + \mu_h) I_h = 0 \tag{41}$$

Again, substituting (19) and (20) into (41)

$$\frac{d}{dt}\left(I_0 + pI_1 + p^2 I_2 + ...\right) + p\delta_h\left(E_0 + pE_1 + p^2 E_2 + ...\right) - p(\gamma_h + \rho_h + \mu_h)\left(I_0 + pI_1 + p^2 I_2 + ...\right) \tag{42}$$

Then again, grouping the coefficient powers of pin (42)

$$p^0 : \frac{dI_0}{dt} = 0 \tag{43}$$

$$p^1 : \frac{dI_1}{dt} + \delta_h E_0 - (\gamma_h + \rho_h + \mu_h) I_0 \tag{44}$$

$$p^2 : \frac{dI_2}{dt} + \delta_h E_1 - (\gamma_h + \rho_h + \mu_h) I_1 \tag{45}$$

Also, by substituting the fourth equation (13) into (17)

$$(1-p)\frac{dR}{dt} + p\left[\gamma_h I_h - (\mu_h + \chi_h) R_h\right] = 0 \tag{46}$$

$$\frac{dR}{dt} - p\frac{dR}{dt} + p\frac{dR}{dt} + p\gamma_h I_h - p(\mu_h + \chi_h) R_h \tag{47}$$

$$\frac{dR}{dt} + p\gamma_h I_h - p(\mu_h + \chi_h) R_h \tag{48}$$

Again, substituting (20) and (21) into (48)

$$\frac{d}{dt}\left(R_0 + pR_1 + p^2R_2 + ...\right) + p\gamma_h\left(I_0 + pI_1 + p^2I_2 + ...\right) - p\left(\mu_h + \chi_h\right)\left(R_0 + pR_1 + p^2R_2 + ...\right) \tag{49}$$

Then again, grouping the coefficient powers of p in equation (49)

$$p^0 : \frac{dR_0}{dt} = 0 \tag{50}$$

$$p^1 : \frac{dR_1}{dt} + \gamma_h I_0 - \left(\mu_h + \chi_h\right)R_0 = 0 \tag{51}$$

$$p^2 : \frac{dR_2}{dt} + \gamma_h I_1 - \left(\mu_h + \chi_h\right)R_1 = 0 \tag{52}$$

Also, by substituting fifth equation (13) into (17)

$$\left(1-p\right)\frac{dS}{dt} + p\left[\frac{dS}{dt} + \phi_r - \ell_r S_r - \mu_r S_r\right] = 0 \tag{53}$$

$$\frac{dS}{dt} - p\frac{dS}{dt} + p\frac{dS}{dt} + p\phi_r - p\ell_r S_r - p\mu_r S_r = 0 \tag{54}$$

$$\frac{dS}{dt} + p\phi_r - p\ell_r S_r - p\mu_r S_r = 0 \tag{55}$$

Again, substituting (22) into (55)

$$\begin{aligned}&\frac{d}{dt}\left(K_0 + pK_1 + p^2K_2 + ...\right) + p\phi_r - p\ell_r\left(K_0 + pK_1 + p^2K_2 + ...\right)\\ &-p\mu_h\left(K_0 + pK_1 + p^2K_2 + ...\right)\end{aligned} \tag{56}$$

Then again, grouping the coefficient powers of p in equation (3.292)

$$p^0 : \frac{dK}{dt} = 0 \tag{57}$$

$$p^1 : \frac{dK_1}{dt} + \phi_r - (\ell_r - \mu_r)K_0 = 0 \tag{58}$$

$$p^2 : \frac{dK_2}{dt} + \phi_r - (\ell_r - \mu_r)K_1 = 0 \tag{59}$$

Also, by substituting sixth equation (13) into (17)

$$(1-p)\frac{dI}{dt} + p\left[\ell_r S_r - \mu_r I_r\right] = 0 \tag{60}$$

$$\frac{dI}{dt} - p\frac{dI}{dt} + p\frac{dI}{dt} + p\ell_r S_r - p\mu_r I_r = 0 \tag{61}$$

$$\frac{dI}{dt} + p\ell_r S_r - p\mu_r I_r = 0 \tag{62}$$

Again, substituting (22) and (23) into (62)

$$\frac{d}{dt}\left(F_0 + pF_1 + p^2F_2 + ...\right) + p\ell_r\left(K_0 + pK_1 + p^2K_2 + ...\right)$$
$$-p\mu_r\left(K_0 + pK_1 + p^2K_2 + ...\right) = 0 \tag{63}$$

Then again, grouping the coefficient powers of p in equation (63)

$$p^0 : \frac{dF_0}{dt} = 0 \tag{64}$$

$$p^1 : \frac{dF_1}{dt} + \ell_r K_0 - \mu_r F_0 \tag{65}$$

$$p^2 : \frac{dF_2}{dt} + \ell_r K_1 - \mu_r F_1 \tag{66}$$

Firstly, the equations obtained by combining the coefficient powers of p are integrated with respect to time t. The equation related with powers p^o " are integrated first and from the initial conditions of homotopy perturbation;

Thus, integrating equation (28) that is $\int \frac{dS}{dt} = 0$

(67)

$$S_{ho}(t) = s_0 \tag{68}$$

Integrating equation (35) that is $\int \frac{dE}{dt} = 0$ (69)

$$E_{ho}(t) = e_0 \quad (70)$$

Integrating equation (43) that is $\int \frac{dI}{dt} = 0$

(71)

$$I_{ho}(t) = l_0 \quad (72)$$

Integrating equation (50) that is $\int \frac{dR}{dt} = 0$ (73)

$$R_{ho}(t) = r_0 \quad (74)$$

Integrating equation (57) that is $\int \frac{ds}{dt} = 0$

(75)

$$S_{ro}(t) = k_0 \quad (76)$$

Integrating equation (64) that is $\int \frac{dI}{dt} = 0$ (77)

$$I_{ro}(t) = f_0 \quad (78)$$

The procedure is continued by integrating the equations associated with powers s^1 with respect to time t; Thus, integrating (29) that is

$$\int \frac{ds_1}{dt} = \phi_h + \chi_h R_0 - \ell_h S_0 - \mu_h S_0 \quad (79)$$

$$\int ds_1 = \left[\phi_h + \chi_h R_0 - \ell_h S_0 - \mu_h S_0\right] dt \quad (80)$$

Substituting (68) and (74) into (80)

$$\int ds_1 = \left[\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right] dt \quad (81)$$

$$s_1 = \int \left[\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right] dt \quad (82)$$

$$s_1(t) = \left[\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right] t + c \quad (83)$$

At $(t) = 0; S(0) = 0;$ $\quad c = 0$

$$s_1(t) = \left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right) t \quad (84)$$

Integrating (36)

$$\int \frac{dE_1}{dt} = \ell_h S_0 - (\delta_h + \mu_h) E_0 \tag{85}$$

$$\int dE_1 = \left(\ell_h S_0 - (\delta_h + \mu_h) E_0\right) dt \tag{86}$$

Substituting (68) and (70) into (86)

$$E_1 = \int \left(\ell_h s_0 - (\delta_h + \mu_h) e_0\right) dt \tag{87}$$

$$E_1(t) = \left(\ell_h s_0 - (\delta_h + \mu_h) e_0\right) t + c \tag{88}$$

At $t = 0; E_1(0) = 0; c = 0$

$$E_1(t) = \left(\ell_h s_0 - (\delta_h + \mu_h) e_0\right) t \tag{89}$$

Integrating (43)

$$\int \frac{dI_1}{dt} = \delta_h E_0 - (\gamma_h + \rho_h + \mu_h) I_0 \tag{90}$$

$$\int dI_1 = \left(\delta_h E_0 - (\gamma_h + \rho_h + \mu_h) I_0\right) dt \tag{91}$$

Substituting (70) and (72) into (91)

$$I_1 = \int \left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h) l_0\right) dt \tag{92}$$

$$I_1(t) = \left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h) l_0\right) t + c \tag{93}$$

$t = 0; \mathrm{I}_1(0) = 0; c = 0$

$$I_1(t) = \left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h) l_0\right) t \tag{94}$$

Integrating (51)

$$\int \frac{dR_1}{dt} = \gamma_h I_0 - (\mu_h + \chi_h) R_0 \tag{95}$$

$$\int dR_1 = \left(\gamma_h I_0 - (\mu_h + \chi_h) R_0\right) dt \tag{96}$$

Substituting (72) and (74) into (96)

$$R_1 = \int \left(\gamma_h l_0 - (\mu_h + \chi_h) r_0\right) dt \tag{97}$$

$$R_1(t) = \left(\gamma_h l_0 - (\mu_h + \chi_h) r_0\right) t + c \tag{98}$$

$t = 0; \mathrm{R}_1(0) = 0; c = 0$

$$R_1(t) = \left(\gamma_h l_0 - \left(\mu_h + \chi_h\right) r_0\right) t \tag{99}$$

Integrating (58) that is

$$\int \frac{dk_1}{dt} = \phi_r - \ell_r K_0 - \mu_r K_0 \tag{100}$$

$$\int dk_1 = \left[\phi_r - \ell_r K_0 - \mu_r K_0\right] dt \tag{101}$$

Substituting (76) into (101)

$$k_1 = \int \left[\phi_r - \ell_r k_0 - \mu_r k_0\right] dt \tag{102}$$

$$k_1(t) = \left[\phi_r - \ell_r k_0 - \mu_r k_0\right] t + c \tag{103}$$

At $(t) = 0; \mathrm{K}_1(0) = 0; \quad \mathrm{c} = 0$

$$k_1(t) = \left(\phi_r - \ell_r k_0 - \mu_r k_0\right) t \tag{104}$$

Integrating (78)

$$\int \frac{df_1}{dt} = \ell_r K_0 - \mu_r f_0 \tag{105}$$

$$\int df_1 = \left(\ell_r K_0 - \mu_r F_0\right) dt \tag{106}$$

Substituting (76) and (78) into (106)

$$F_1 = \int \left(\ell_r k_0 - \mu_r f_0\right) dt \tag{107}$$

$$F_1(t) = \left(\ell_r k_0 - \mu_r f_0\right) t + c \tag{108}$$

At $t = 0; \mathrm{F}_1(0) = 0; c = 0$

$$F_1(t) = \left(\ell_r k_0 - \mu_r f_0\right) t \tag{109}$$

Lastly, the coefficients with power p^2 are also integrated with respect to t.

Integrating (30)

$$\int \frac{ds_2}{dt} + \chi_h R_1 - \ell_1 S_1 - \mu_h S_1 = 0 \tag{110}$$

$$\int \frac{ds_2}{dt} = -\chi_h R_1 + \ell_1 S_1 + \mu_h S_1 \tag{111}$$

Substituting $R_1(t); \mathrm{S}_1(\mathrm{t})$ into (111)

$$ds_2 = \int -\chi_h\left(\gamma_h l_0 - \left(\mu_h + \chi_h\right) r_0\right)t + \ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)t + \mu_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)t \tag{112}$$

$$ds_2 = \int\left(-\chi_h\left(\gamma_h l_0 - \left(\mu_h + \chi_h\right) r_0\right) + \ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right) + \mu_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)\right)tdt \tag{113}$$

$$ds_2 = \frac{1}{2}t^2\left(-\chi_h\left(\gamma_h l_0 - \left(\mu_h + \chi_h\right) r_0\right) + \ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right) + \mu_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)\right) + c \tag{114}$$

At; $t = 0,\ S_2(0) = 0;\ c = 0$

$$ds_2 = \frac{1}{2}t^2\left(-\chi_h\left(\gamma_h l_0 - \left(\mu_h + \chi_h\right) r_0\right) + \ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right) + \mu_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)\right) \tag{115}$$

From (37);

$$\frac{dE_2}{dt} = -\ell_h S_1 + \left(\delta_h + \mu_h\right) E_1 \tag{116}$$

Substituting $E_1(t)$ and $S_1(t)$ into (116)

$$\frac{dE_2}{dt} = -\ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)t - \left(\delta_h + \mu_h\right)\left(\ell_h s_0 - \left(\delta_h + \mu_h\right)e_0\right)t \tag{117}$$

$$\int \frac{dE_2}{dt} = \int\left[-\ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)t - \left(\delta_h + \mu_h\right)\left(\ell_h s_0 - \left(\delta_h + \mu_h\right)e_0\right)\right]tdt \tag{118}$$

$$\frac{dE_2}{dt} = -\frac{1}{2}t^2\left[-\ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)t - \left(\delta_h + \mu_h\right)\left(\ell_h s_0 - \left(\delta_h + \mu_h\right)e_0\right)\right] + c \tag{119}$$

At; $t = 0; E_2(0) = 0; \mathrm{C} = 0$

$$\frac{dE_2}{dt} = -\frac{1}{2}t^2\left[-\ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)t - \left(\delta_h + \mu_h\right)\left(\ell_h s_0 - \left(\delta_h + \mu_h\right)e_0\right)\right] \tag{120}$$

From (45)

$$\frac{dI_2}{dt} = -\delta_h E_1 + \left(\gamma_h + \rho_h + \mu_h\right) I_1 \tag{121}$$

Substituting $I_1(t)$ and $E_1(t)$ into (121)

$$\frac{dI_2}{dt} = -\delta_h\left(\ell_h s_0 - \left(\delta_h + \mu_h\right)s_0\right)t + \left(\gamma_h + \rho_h + \mu_h\right)\left(\delta_h e_0 - \left(\gamma_h + \rho_h + \mu_h\right)l_0\right)t \tag{122}$$

$$\int dI_2 = \int\left[-\delta_h\left(\ell_h s_0 - \left(\delta_h + \mu_h\right)s_0\right) + \left(\gamma_h + \rho_h + \mu_h\right)\left(\delta_h e_0 - \left(\gamma_h + \rho_h + \mu_h\right)l_0\right)t\right]tdt \tag{123}$$

$$I_2(t) = -\frac{1}{2}t^2\left[\delta_h\left(\ell_h s_0 - (\delta_h + \mu_h)s_0\right) + (\gamma_h + \rho_h + \mu_h)\left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h)l_0\right)\right] + C \quad (124)$$

At; $t = 0; \mathrm{I}_2(0) = 0; C = 0$

$$I_2(t) = -\frac{1}{2}t^2\left[\delta_h\left(\ell_h s_0 - (\delta_h + \mu_h)s_0\right) + (\gamma_h + \rho_h + \mu_h)\left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h)l_0\right)\right] \quad (125)$$

From equation (52);

$$\frac{dR_2}{dt} = -\gamma_h I_1 + (\mu_h + \chi_h)R_1 \quad (126)$$

Substituting $I_1(t)$ and $R_1(t)$ into (126)

$$\frac{dR_2}{dt} = -\gamma_h\left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h)l_0\right)t + (\mu_h + \chi_h)\left(\gamma_h l_0 - (\mu_h + \chi_h)r_0\right)t \quad (127)$$

$$\int dR_2 = \int\left[\left(-\gamma_h\left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h)l_0\right)\right) + (\mu_h + \chi_h)\left(\gamma_h l_0 - (\mu_h + \chi_h)r_0\right)\right]tdt \quad (128)$$

$$R_2 = \frac{1}{2}\left(-\gamma_h\left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h)l_0\right)\right) + (\mu_h + \chi_h)\left(\gamma_h l_0 - (\mu_h + \chi_h)r_0\right) + C \quad (129)$$

At; $t = 0; \mathrm{R}_2(0) = 0; C = 0$

$$R_2 = \frac{1}{2}\left(-\gamma_h\left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h)l_0\right)\right) + (\mu_h + \chi_h)\left(\gamma_h l_0 - (\mu_h + \chi_h)r_0\right) \quad (130)$$

From equation (59);

$$\frac{dk_2}{dt} + \phi_r - \ell_r K_1 - \mu_r K_1 = 0 \quad (131)$$

Substituting $K_1(t)$ into equation (131)

$$dk_2 = -\phi_r + \ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right)t + \mu_h\left(\phi_r - \ell_r k_0 - \mu_r k_0\right)t \quad (132)$$

$$\int dk_2 = \int\left[-\phi_r + \ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right) + \mu_h\left(\phi_r - \ell_r k_0 - \mu_r k_0\right)\right]tdt \quad (133)$$

$$dk_2 = -\frac{1}{2}t^2\left[\phi_r + \ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right) + \mu_h\left(\phi_r - \ell_r k_0 - \mu_r k_0\right)\right] + c \quad (134)$$

At; $t = 0,\ \mathrm{k}_2(0) = 0;\ c = 0$

$$dk_2 = -\frac{1}{2}t^2\left[\phi_r + \ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right) + \mu_h\left(\phi_r - \ell_r k_0 - \mu_r k_0\right)\right] \quad (135)$$

From equation (66);

$$\frac{dF_2}{dt} + \ell K_1 - \mu_r F_1 \tag{136}$$

Substituting $F_1(t)$ and $K_1(t)$ into equation (136)

$$\frac{dF_2}{dt} = -\ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right)t + \mu_r\left(\ell_r k_0 - \mu_r f_0\right)t \tag{137}$$

$$\int dF_2 = \int\left(-\ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right) + \mu_r\left(\ell_r k_0 - \mu_r f_0\right)\right)t\,dt \tag{138}$$

$$dF_2 = -\frac{1}{2}t^2\left(\ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right) + \mu_r\left(\ell_r k_0 - \mu_r f_0\right)\right) + c \tag{139}$$

At; $t = 0; \mathrm{F}_2(0) = 0; \mathrm{C} = 0$

$$dF_2 = -\frac{1}{2}t^2\left(\ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right) + \mu_r\left(\ell_r k_0 - \mu_r f_0\right)\right) \tag{140}$$

From (18) to (23), the approximate solution at $P = 1$ is written as;

$$S_h(t) = S_0 + S_1(t) + S_2(t) + \ldots \tag{141}$$

$$E_h(t) = E_0(t) + E_1(t) + E_2(t) + \ldots \tag{142}$$

$$I_h(t) = I_0(t) + I_1(t) + I_2(t) + \ldots \tag{143}$$

$$R_h(t) = R_0(t) + R_1(t) + R_2(t) + \ldots \tag{144}$$

$$S_r(t) = K_0(t) + K_1(t) + K_2(\mathrm{t}) + \ldots \tag{145}$$

$$I_r(t) = F_0(t) + F_1(t) + F_2(t) + \ldots \tag{146}$$

Hence, the final approximate solutions of (141) to (146) are obtained as follows;

$$S_h(t) = s_0 + \left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)t - \frac{1}{2}\left(\chi_h\left(\gamma_h l_0 - \left(\mu_h + \chi_h\right)r_0\right) + \ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right) + \mu_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)\right)t^2 \tag{147}$$

$$E_h(t) = e_0 + \left(\ell_h s_0 - \left(\delta_h + \mu_h\right)s_0\right)t - \frac{1}{2}\left[\ell_h\left(\phi_h + \chi_h r_0 - \ell_h s_0 - \mu_h s_0\right)t - \left(\delta_h + \mu_h\right)\left(\ell_h s_0 - \left(\delta_h + \mu_h\right)e_0\right)\right]t^2 \tag{148}$$

$$I_h(t) = l_0 + \left(\delta_h e_0 - \left(\gamma_h + \rho_h + \mu_h\right)l_0\right)t - \frac{1}{2}\left[\delta_h\left(\ell_h s_0 - \left(\delta_h + \mu_h\right)s_0\right) + \left(\gamma_h + \rho_h + \mu_h\right)\left(\delta_h e_0 - \left(\gamma_h + \rho_h + \mu_h\right)l_0\right)\right]t^2 \tag{149}$$

$$R_h(t) = r_0 + \left(\gamma_h l_0 - (\mu_h + \chi_h) r_0\right) t - \frac{1}{2}\left(\gamma_h \left(\delta_h e_0 - (\gamma_h + \rho_h + \mu_h) l_0\right)\right) + (\mu_h + \chi_h)\left(\gamma_h l_0 - (\mu_h + \chi_h) r_0\right) t^2 \quad (150)$$

$$S_r(t) = k_0 + \left(\phi_r - \ell_r k_0 - \mu_r k_0\right) t - \frac{1}{2}\left[\phi_r + \ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right) + \mu_h\left(\phi_r - \ell_r k_0 - \mu_r k_0\right)\right] t^2 \quad (151)$$

$$I_r(t) = f_0 + \left(\ell_r k_0 - \mu_r f_0\right) t - \frac{1}{2}\left(\ell_r\left(\phi_r - \ell_r k_0 - \mu_r k_0\right) + \mu_r\left(\ell_r k_0 - \mu_r f_0\right)\right) t^2 \quad (152)$$

Therefore, equations (147) through (152) represent the analytical solution for the different compartments. These equations are structured as a series of solutions, each representing specific compartments considered analyzed in the study. By simulating various parameters values based on these equations, one can ascertain their impact on the population being studied.

4. Results and Discussion

This section shows, the parameter values, graphs generated from the general solution (147) to (152) and discussion of the results.

Table 1. Parameter values for the series solutions Variables/ Parameters

Parameters	Value	Reference
ϕ_h	1.20	(Peter et al., 2020a)
χ_h	Assumed	
δ_h	0.00385	(Peter et al., 2020a)
γ_h	Assumed	
μ_h	0.003465	(Lakshmikantham et al., 1989
ρ_h	0.00019231	(White et al., 1996)
ℓ_h	0.025	(Abdulraheem, 2002)
ℓ_r	0.0182	(Peter et al., 2020a)
ϕ_r	0.00001	(Abdulraheem, 2002)
μ_r	0.0038	(Lakshmikantham et al., 1989)
$\mathbf{S}_h(0)$	100	(Peter et al., 2020b)
$E_h(0)$	20	(Peter et al., 2020b)
$I_h(0)$	10	(Peter et al., 2020b)
$R_h(0)$	5	(Peter et al., 2020b)
$\mathbf{S}_r(0)$	1000	(Peter et al., 2020b)
$I_r(0)$	20	(Peter et al., 2020b)

4.1 Results

This section shows graphs generated from the general solution of our equation (147) to (152) using MAPLE.

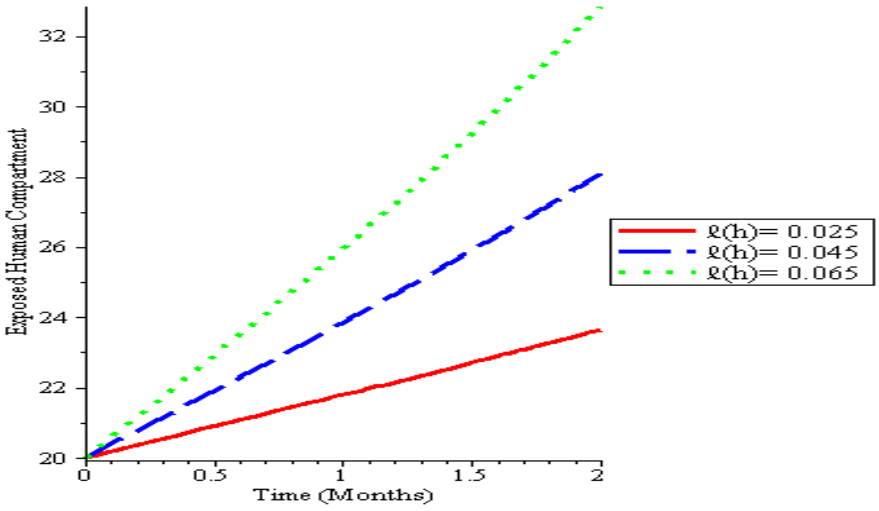

Fig. 1: Simulations of result show the relationship between the Exposed Human Compartment and time (t) for various values of $\ell_h = 0.025, \ell_h = 0.045$ and $\ell_h = 0.065$.

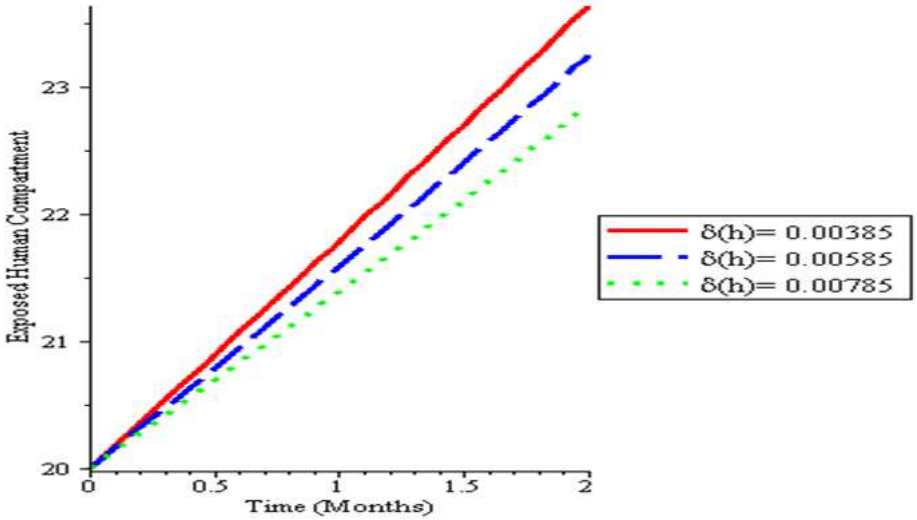

Fig. 2: Presents a graphical representation of the Exposed Human Compartment over time (t) for various values of $\delta_h = 0.00385$, $\delta_h = 0.00585$ and $\delta_h = 0.00785$.

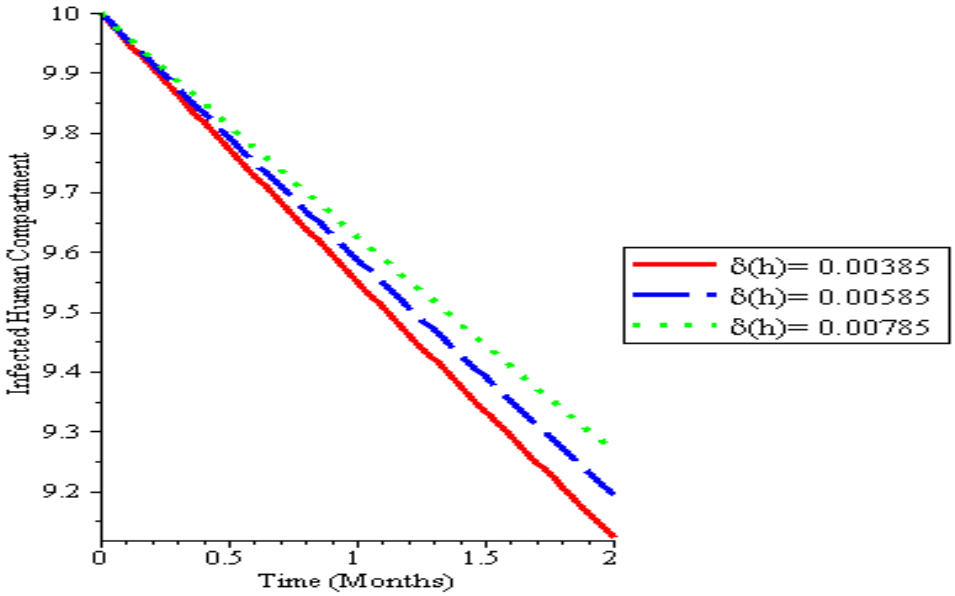

Fig. 3: Represents the graph of the Infected Human Compartment over time (t) for various values of $\delta_h = 0.00385$, $\delta_h = 0.00585$ and $\delta_h = 0.00785$.

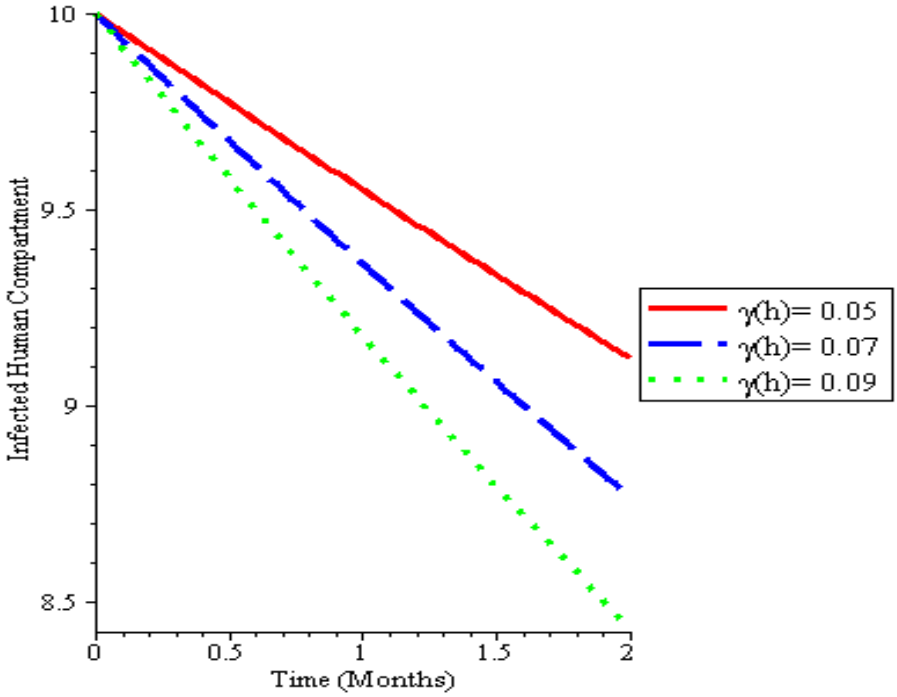

Fig. 4 Displays the graph of the Infected Human Compartment over time (t) for various values of $\gamma_h = 0.05$, $\gamma_h = 0.07$ and $\gamma_h = 0.09$.

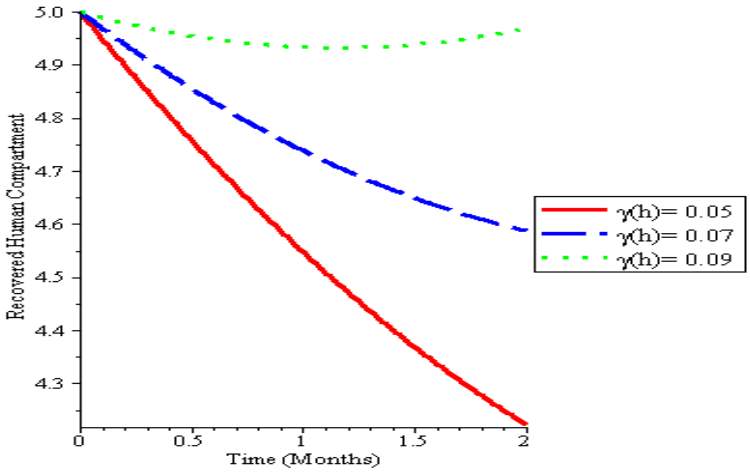

Fig. 5: Present the graph illustrating the Infected Human Compartment against time (t) for varying values of $\gamma_h = 0.05$, $\gamma_h = 0.07$ and $\gamma_h = 0.09$.

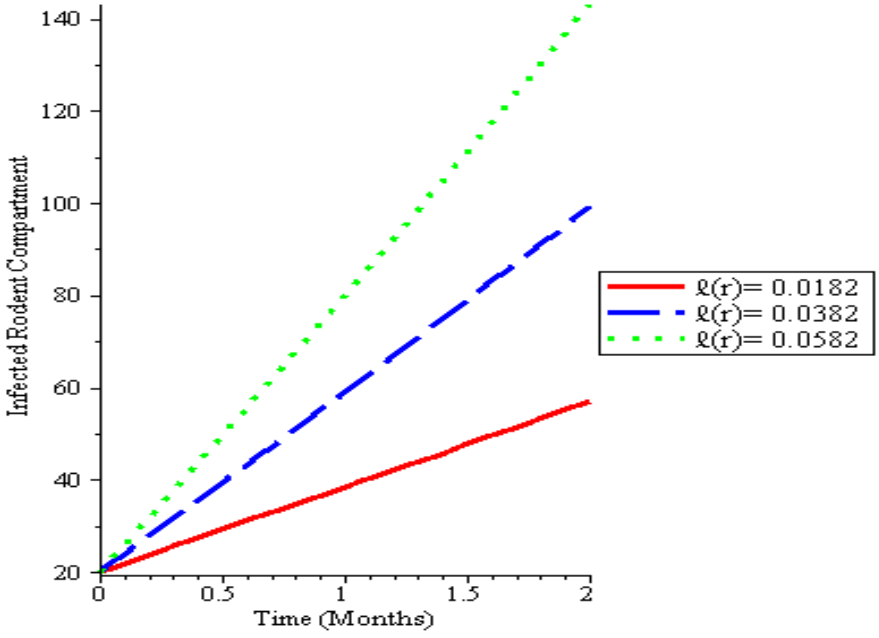

Fig. 6: Illustrates the graph of the Infected Rodent Compartment over time (t) for various values of $\ell_r = 0.0182$, $\ell_r = 0.0382$ and $\ell_r = 0.0582$.

4.2 Discussion of the Results

Figure 1. The graph distinctly demonstrates that an increase in ℓ_h leads to a higher population in the exposed human compartment, subsequently reducing the susceptible human class. This reduction is attributed to contact, which may occur through direct exposure to contaminated food via urine or excretes of an infectious rodent, as well as through laboratory transmissions involving the sharing of medical equipment with infected individuals without proper sterilization.

Figure 2. The graph conspicuously indicates that with an increase in δ_h, the population of the exposed human compartment decreases, subsequently resulting in an increase in the infected human compartment due to the progression of the disease within the population.

Figure 3: The graph clearly indicates a decrease in the infected human compartment with time (t), but experiences an increment when increases δ_h. This is a consequence of the disease progressing from the human exposed class

Figure 4. The graph illustrates a reduction in the infected human compartment over time, and it further diminishes as the recovery rate increases. This highlights the effectiveness and efficiency of the recovery rate.

Figure 5. The graph illustrates that the number of individuals in the recovered human compartment decreases over time. However, it increases as the recovery rate rises, showcasing the effectiveness and efficiency of the recovery rate.

Figure 6.The graph distinctly shows that as ℓ_r increases, the infected rodent compartment becomes populated, leading to a reduction in the susceptible rodent class. This occurs due to the effective transmission probability from human to rodent and the effective transmission probability from rodent to rodent.

5. Conclusion

In this paper, the equations representing the different compartments are transformed into first-order non-linear differential equations. The system of non-linear equations is then solved using the homotopy perturbation method, the final approximate solutions are derived and the simulation results are compared, revealing a satisfactory agreement,

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