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Reliability Assessment Of Reinforced Concrete Beam Containing Iron Ore Tailings

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A B S T R A C T

This paper assesses the reliability of reinforced concrete beams incorporating IOT as a partial replacement for fine aggregates. An equation for the moment resistance capacity of IOT concrete beams was developed as well as the performance functions for the beam in flexure and shear conditions, providing a basis for evaluating structural performance under flexural loads. Reliability analysis was conducted using First Order Reliability Method to evaluate the safety indices of IOT concrete beam under flexure and shear conditions, based on stochastic modeling of load, material properties, and beam dimensions. Results showed that the reliability index (β) decreases with an increase in the load ratio, indicating higher failure risks under heavier loads. Sensitivity analysis also revealed that beam depth significantly influences reliability more than its length. The findings of the study can be used by engineers to optimize structural designs thus enhancing safety while addressing environmental concerns.

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INTRODUCTION

The increasing demand for sand in construction has led to a corresponding mining of this aggregate from rivers and flood plains. This in turn has led to severe environmental problems such as water pollution, deforestation, collapsing of river banks, destruction of landscape and reduction of farm and grazing lands [1] .

One of the fundamental issues worldwide is achieving a sustainable environment and eco-friendly community through effective utilization of resources and waste materials in the construction industries. In recent years, the construction industry has shifted focus toward recycling waste materials, as effective utilization of industrial and agricultural wastes offers significant environmental and economic benefits. Numerous researches have shown that by-products from industrial and agricultural activities can be utilized in the production of sustainable concretes [2].

The effective utilization of these industrial by-products in the production of concrete is not only cost efficient but also improved the mechanical properties such as reduced permeability and increased strength properties while mitigating environmental issues.

One of the most extensively researched and promising industrial by-products used as sand replacement in the production of a sustainable concrete is the iron ore tailings (IOT). Iron ore tailings are by-products of iron extraction processes and are usually disposed of in large quantities, posing significant environmental challenges. Globally, the annual production of IOT is estimated at over 2 billion tons [3], a figure that continues to rise due to increasing iron and steel demand. In countries like China, where iron production is extensive, tailings account for a significant portion of mining waste, often stored in large dams or disposal sites. Improper management of these tailings poses environmental risks, including land degradation, contamination of water resources, and harm to local ecosystems [4]. As a result, repurposing IOT in construction materials has emerged as a sustainable solution to mitigate these challenges.

Research has shown that replacing 30%–50% of natural aggregates with iron ore tailings (IOT) in concrete can lead to significant improvements in compressive strength, with reported gains ranging from 15% to 25%, depending on the specific mix design [1][4][5][6][7][8] and [9]. In addition to enhanced compressive strength, studies have demonstrated that concrete incorporating IOT exhibits lower permeability, increased resistance to chloride penetration, and improved durability, particularly in harsh environmental conditions [9][10] and [12]. These characteristics make IOT a promising alternative material for sustainable concrete production.

However, the majority of existing research has focused primarily on the static mechanical properties of concrete with IOT, such as compressive, tensile, and flexural strengths. While these properties provide valuable insights into the material's immediate performance, there is limited investigation into its long-term behavior, structural integrity, and reliability under real-world conditions.

The lack of comprehensive studies on creep, shrinkage, fatigue performance, and service life estimation raises concerns about its suitability for structural applications.

To bridge this research gap, this study aims to assess the long-term reliability and structural performance of reinforced concrete beams incorporating iron ore tailings. By evaluating their behavior under sustained loads, environmental exposure, and varying stress conditions, this research will provide crucial data on the feasibility of IOT-modified concrete for large-scale construction projects.

METHODS

Reliability analysis estimates the failure probability or safety associated with structural elements under uncertainties in loads and resistance. The concepts and mathematical formulations provided are essential in assessing reliability using probabilistic methods.

1. Basic Reliability Problem

Structural failure occurs when resistance, R is less than the applied load, S [13]. The probability of failure P_f can be expressed as Equation 1:

$$P_f = p[(R - S) < 0] \tag{1}$$

The failure region is where $G(x) = (R - S) < 0]$ (2)

G(X)=R−S<0, and the performance function G(X) is used to determine safety or failure:

i. G(X) < 0: Failure
ii. G(X) = 0: Limit state
iii. G(X) > 0: Safe

2. Probability of Failure

When R and S are independent random variables, the convolution integral determines P_f as per Equation 3.

$$P_f = \int_{-\infty}^{+\infty} f_R(x) F_S(x) dx \tag{3}$$

However, for normally distributed R and S, the safety margin M=R−S has a mean μ_M and variance σ_M^2 given by Equation 4;

$\mu_M = \mu_R - \mu_S$ and variance $\sigma_M^2 = \sigma_R^2 + \sigma_S^2$ (4)

The reliability index β thus measures safety and is given in Equation 5.

$$\beta = \frac{\mu_M}{\sigma_M} \tag{5}$$

The probability of failure can then be computed using the standard normal distribution as per Equation 6.

$$P_f = \emptyset(-\beta) \tag{6}$$

3. Performance Functions

The performance functions describe failure or safety under specific conditions, formulated as

G(X) = R−S, where R and S depend on random variables like material properties and loads.
Linearization using the First Order Reliability Method (FORM) approximates G(X) as given in Equation 7.

$$G(X) \approx G(\mu_X) + \sum_{i=1}^{n} \frac{\delta G}{\delta X_i}(X_i - \mu_{Xi}) \tag{7}$$

The mean μG and standard deviation σG of G(X) are given by Equation 8.

$$\mu_G = \sqrt{\sum_{i=1}^{n} (\frac{\delta G}{\delta X_i} \sigma X_i)^2} \tag{8}$$

4. Derivation of the Performance Functions

Failure modes due to flexure, shear and deflection were considered based on BS 8110 design guidelines, the performance functions are expressed as follows;

A. Moment capacity of IOT concrete

Simply reinforced section is the steel reinforcement that is provided only at the tension zone. The form of section, stress distribution and forces on a section are as shown in Figure 1:

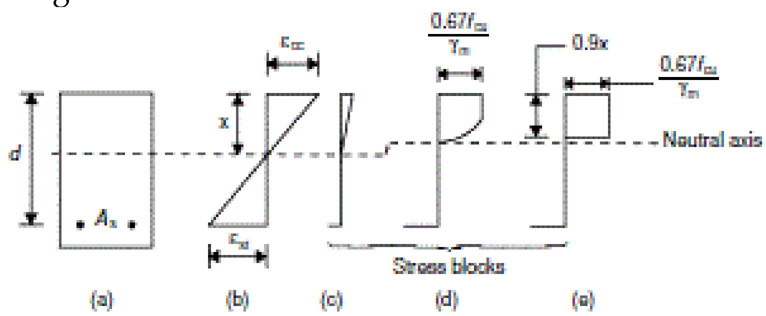

Figure 1: Stress-Starin distribution at a section of the beam a) section b) strain c) triangular (low strain) d) rectangular parabolic (large strain) e) equivalent rectangular [14]

For the section to be in a balance state, the sum of the forces must be equal to zero, thus

Fcc = Fst

$$0.87 f_y A_s = 0.405 f_{cu} b x \tag{9}$$

$$x = \frac{0.87 f_y A_s}{0.405 f_{cu} b} \tag{10}$$

Taking moment about Fcc or Fst, the moment of resistance of the section can be calculated thus

$$M = F_{st} Z$$

$$\left(0.87 f_y A_s\right)(d - 0.45x) \tag{11}$$

Or

$$M = F_{cc} Z$$

$$(0.405 f_{cu} b x)(d - 0.45x)$$

$$(0.405 \frac{x}{d})(1 - \frac{0.45x}{d})(f_{cu} b d^2) \tag{12}$$

The distance of the maximum moment of resistance of the IOT concrete beam section can be obtained from

$$\frac{\varepsilon_{st}}{(d-x)} = \frac{\varepsilon_{cc}}{x} \quad (13a)$$

Where ε_{cc} = 0.0026 for concrete with 15% IOT replacement [15]
ε_{st} =*0.00219* (standard for high yield steel of 460 MPa strength)
Thus;

$$x = 0.54d \quad (13b)$$

by substituting Equations 13b into 13a, Equation 12 can be re-written as

$$M_{ult} = 0.169 f_{cu} bd^2 \quad (14)$$

Equation 14 is thus the ultimate moment carrying capacity of a reinforced concrete beam section incorporating 15% IOT

B. Flexural failure

Condition of failure due to flexure in rectangular reinforced concrete beam is given as:

$$M_R - M_A \leq 0$$

Hence, the performance function is given by Equation 15;

$$G(X) = M_R - M_A \quad (15)$$

where:
Moment of resistance $M_R = 0.169 f_{ck} bd^2$
Applied moment $(M_A) = \frac{W_u l^2}{8} = (1.4G_k + 1.6Q_k)\frac{l^2}{8} = (1.4\frac{G_k}{Q_k} + 1.6)\frac{Q_k l^2}{8}$
Let $\frac{G_k}{Q_k} = \alpha$ = load ratio
$= 0.2Q_k l^2(0.875\alpha + 1)$
Hence, the performance function for the beam under pure bending is given by Equation 16;

$$G(X) = 0.169 f_{ck} bd^2 - 0.2Q_k l^2(0.875\alpha + 1) \quad (16)$$

C. Shear Failure

The design shear force must be less than the concrete shear resistance and steel shear resistance
as given by Equation 17;

$$V_d \leq V_{conc} + V_{steel} \quad (17)$$

$$V_d = \vartheta_c bd + \frac{d}{S_v} A_{sv} 0.87 f_{yv} \quad (18)$$

For simply supported rectangular beam, Equation 18 can be written as

$$0.5wl = \vartheta_c bd + \frac{d}{S_v} A_{sv} 0.87 f_{yv} \quad (19)$$

Thus, the limit state function is given by Equation 20;

$$G(X) = v_{E,d} - v_{Rd,c} \quad (20)$$

Thus, the limit state function for the beam in shear is given by Equation 21;

$$G(X) = \vartheta_c bd + \frac{d}{S_v} A_{sv} 0.87 f_{yv} - 0.8(0.875\alpha + 1)Q_k l \quad (21)$$

RESULTS AND DISCUSSION

Random variables and their statistical characteristics

Table 1. Statistical Characteristics of Random Variables for Stochastic Modelling of IOT Beam in Flexure, Shear and Deflection

X(i)	Physical meaning	Distribution type	EX(i)	SX(i)	COV
1	Characteristic strength f_{ck}	Normal	$30.56N/mm^2$	$4.6N/mm^2$	0.15
2	Width of beam, b	Normal	230mm	11.5mm	0.05
3	Depth of beam, d	Normal	420mm	21mm	0.05
4	Live load, Q_k	Log-Normal	$1.5N/mm^2$	$0.45N/mm^2$	0.3
5	Length of beam, L	Normal	3000mm	150mm	0.05
6	Conc. Shear stress, vc	Normal	$0.715\ N/mm^2$	$0.04N/mm^2$	0.05
7	yield strength, f_{yk}	Log-Normal	$250N/mm^2$	$37.5N/mm^2$	0.15

Table 1 presents the stochastic parameters used in the reliability analysis of iron ore tailings (IOT) beams subjected to flexure, shear, and deflection. These parameters account for the inherent variability in both material properties and applied loads, ensuring a more realistic assessment of structural performance. The table includes physical properties such as compressive strength, modulus of elasticity, and density, along with load parameters like dead load, live load, and environmental loads. Each parameter is characterized by a probability distribution type (e.g., normal, log-normal, or uniform) and an associated coefficient of variation (COV) to capture the degree of uncertainty in its behavior.

1. Reliability Analysis Results for Flexural Performance

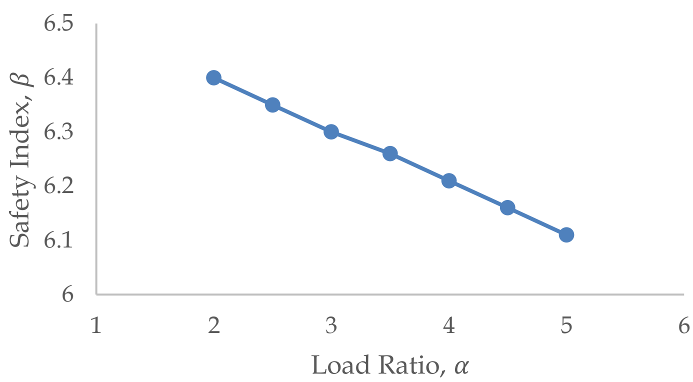

Figure 2. Variation of Safety Index (β) with Load Ratio (α) for IOT Concrete Beams under Flexure

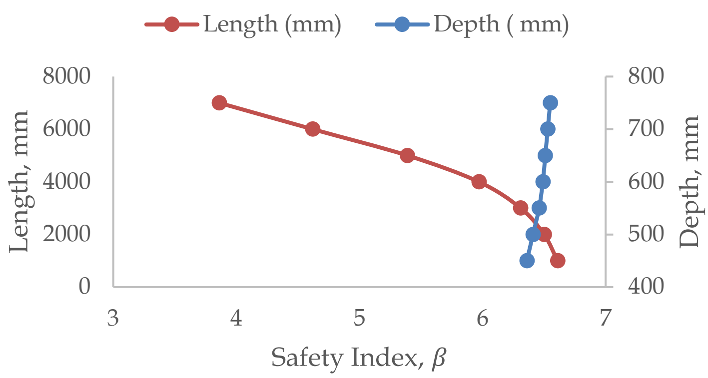

Figure 3. Sensitivity Analysis of Safety Index (β) to Beam Depth and Length under Flexure

Figure 2 illustrates the relationship between the safety index (β) and the load ratio (α). As the load ratio increases, the safety index decreases. This is expected, as a higher load ratio implies a greater risk of failure. The decreasing trend indicates that the structure becomes less reliable as the applied load increases. While Figure 3 presents a sensitivity analysis, showing the impact of variations in length and depth on the safety index. The graph indicates that the safety index is more sensitive to changes in depth than length. This means that increasing the depth of the beam has a more significant effect on improving its reliability compared to increasing its length. Similar trends have been observed in previous studies on the reliability analysis of reinforced concrete structures [13].

2. Reliability Analysis Results for Shear Performance

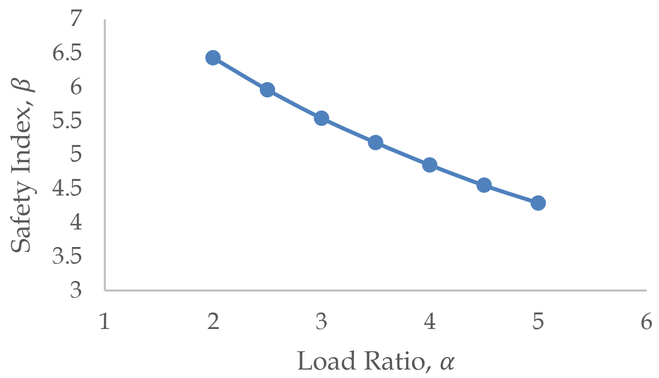

Figure 4. Variation of Safety Index (β) with Load Ratio (α) for IOT Concrete Beams under Shear

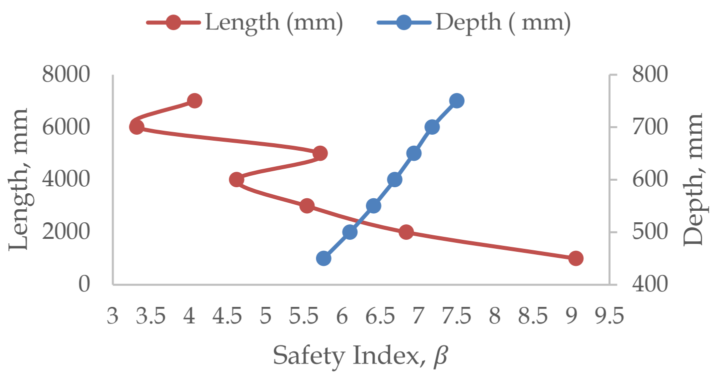

Figure 5. Sensitivity Analysis of Safety Index (β) to Beam Depth and Length under Shear

Figure 4 illustrates the relationship between the safety index (β) and the load ratio (α). As the load ratio increases, the safety index decreases. This is expected, as a higher load ratio implies a greater risk of failure. The decreasing trend indicates that the structure becomes less reliable as the applied load increases. Figure 5 presents a sensitivity analysis, showing the impact of variations in length and depth on the safety index. The graph indicates that the safety index is more sensitive to changes in depth than length. This means that increasing the depth of the beam has a more significant effect on improving its reliability compared to increasing its length. However, the staircase-like shape in the sensitivity analysis graph arises from the optimization process. It indicates that the optimization algorithm may encounter local minima, leading to non-smooth variations in the sensitivity curves.

CONCLUSION

This study demonstrates the reliability and structural performance of singly reinforced concrete beams incorporating iron ore tailings (IOT) as a partial replacement for fine aggregates. The developed equation for the moment resistance capacity of IOT concrete provides a practical tool for evaluating flexural performance. Reliability analysis under flexure and shear conditions confirmed that the reliability index (β) decreases with increasing load ratios, indicating a direct relationship between applied loads and structural safety risks. Sensitivity analysis highlighted the critical role of beam depth in improving reliability compared to other parameters. The results suggest that IOT is a viable alternative to natural aggregates, offering both environmental and economic advantages. Incorporating IOT in reinforced concrete beams not only reduces waste but also enhances durability and structural efficiency. Based on these findings, the study recommends optimizing beam depth in structural designs to improve safety margins. These conclusions provide a strong foundation for the broader adoption of IOT in structural applications.

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