General Sensitivity Analysis of Dynamic Properties of Metal Rubber backed Active Magnetic Bearing using Nastran

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Abstract

A two degree of freedom (2DOF) Mass-Damper-Stiffness model of rotor on Metal Rubber—Active Magnetic Bearing system is used to investigate the contributions of Metal Rubber damping to the Modal damping response of the system. The system modeling and analysis were carried out in Patran and Nastran. Patran is a computer aided design, processing and post processing software while Nastran is a FEA solver. This work further investigates the interactions between the stiffness and damping properties of Metal Rubber-Active Magnetic Bearing and the system's response of modal damping using General Sensitivity Analysis. The results i) affirms that with MR, the flexibility of ordinary AMB is increased; a favorable condition for support of light flexible rotors, ii) show that the frequency and damping of the rotor-support system is sensitive to the dynamic properties of the MR material.

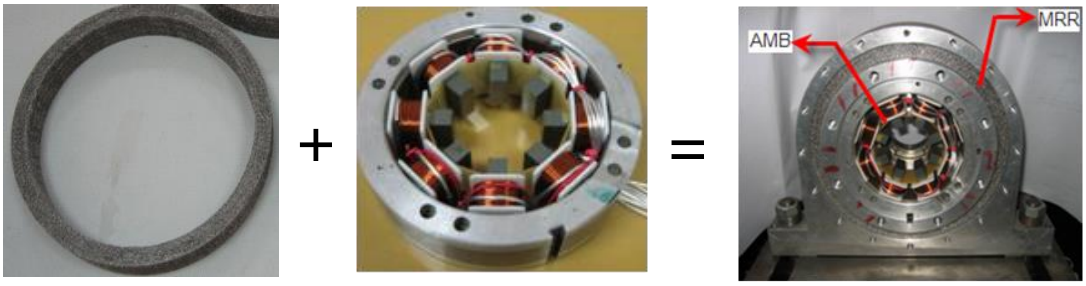
Keywords: Active Magnetic Bearing, Metal Rubber, Modal Natural frequency, Modal damping.

1. Introduction

In Active Magnetic Bearing (AMB) system technology, the rotor is suspended without contact by forces generated via electromagnetic action of current passing through the coils wound round some arranged poles of a metallic core. Full industrial application of AMB as sole bearing support for rotors is not yet realized because of the problem of low support damping especially for high-speed-flexible rotor (Schweitzer, 2009). Tackling this problem via Modern control strategies and unbalance compensation approaches had been widely explored (Fujiwara, Ebina, Ito, Takahashi, & Matsushita, 2002; Ito, Fujiwara, Okubo & Matsushito, 2000; Shuliang & Palazzolo, 2008). Away from these methods, Xie, Wang & Zhang (2009), further increased the flexibility of ordinary AMB using MR thus enhancing its suitability for support of high-speed-flexible rotor. Combine support had proven to be better off in vibration restraint than single support (Hamburg & Parkinson, 1962; Zeidan, San Andres &Vance, 1996). However, the functional relationship between the system's response properties and bearing component's dynamic properties of stiffness and damping need to be critically studied for proper design and optimization of such complex bearing unit. A 2DOF system is usually a starting point for such analysis as presented in this work and investigated using General Sensitivity Analysis.

2. Modeling and Equation of Motion

2.1 AMB-MR Bearing Unit



MR AMB AMB-MR BEARING

Figure 1: Bearing components: courtesy Xie's experimental lab, NUAA

Table1 Properties of AMB

Description	Specification
Bearing load	80.0N
Bore diameter	40.0mm
Bearing width	20.0mm
Number of turns	33.0turns
Bias current	2.5A
Clearance	0.25mm
Displacement Stiffness-coefficient	$-3.2\times10^{5}\,N/m$

The AMB-MR Bearing unit is a sandwich of MR ring unto the periphery of a conventional AMB. Figure1 shows the components of AMB-MR bearing units. MR is made of pressed stainless steel wire of diameter range 0.1-0.5mm. It has microstructures similar to that of rubber material, having combined quality of Metal: flexibility and high temperature resistance, and rubber: large damping and high elasticity (Gu, *et al*, 2010; Zhangi, DongXu, & Rui, 2012). MR dissipates vibration energy through friction, slippage and distortion between metal springs which occur repeatedly under vibration load (Zhangi *et al*., 2012), thus, acts as a damping material to attenuate the unwelcome vibration transmitted from the rotor. The model the complexed bearing support is as shown in Figure 2 below.

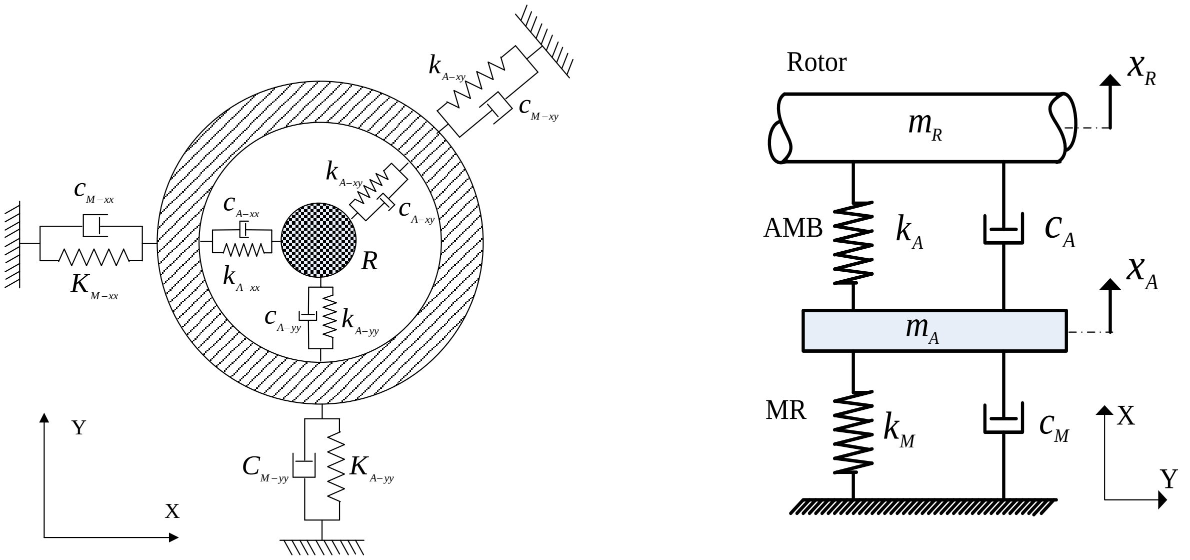


Figure 2: Complex Bearing Support Model

Using a similar method in Nicholas, Whalen, & Franklin (2002), to model our system, the first level of flexibility is the MR (subscript M) and the later, AMB (subscript A). Both are represented by 8 principal (xx, yy) and cross-couple (xy,yx) stiffnesses and damping coefficient. The rotor and AMB have masses m_R and m_A respectively. Bearing properties are assumed to be symmetrical and MR massless and rigidly fixed to the bearing frame. Adopting a linear 2 degrees of freedom (DOF) mass-spring-damper model for the unit, the stiffness and damping coefficients of the AMB and MR are k_A, c_A and k_m, c_m respectively. This single support mass with 2DOFcan further be simplified to 2 single degrees of freedom (SDOF) mass-spring-damper system in x and y directions. For demonstration purpose, only motion in only $x-$ direction is shown

2.2 Equation of Motion

The equation of motion of a linear 2DOF mass-spring-damper system can be described as

$$m_A \ddot{x}_A + (c_M + c_A)\dot{x}_A - c_A x_R + (k_M + k_A)x_A - k_A x_R = 0 \quad (1)$$

$$m_R \ddot{x}_R - c_A \dot{x}_A + c_A \dot{x}_R - k_A x_A + k_A x_R = f_R \quad (2)$$

or in matrix form as;

$$\underset{M}{\begin{pmatrix} m_A & 0 \\ 0 & m_R \end{pmatrix}} \underset{\ddot{X}}{\begin{bmatrix} \ddot{x}_A \\ \ddot{x}_R \end{bmatrix}} + \underset{C}{\begin{pmatrix} (c_A + c_M) & -c_A \\ -c_A & c_A \end{pmatrix}} \underset{\dot{X}}{\begin{bmatrix} \dot{x}_A \\ \dot{x}_R \end{bmatrix}} + \underset{K}{\begin{pmatrix} (k_M + k_A) & -k_A \\ -k_A & k_A \end{pmatrix}} \underset{X}{\begin{bmatrix} x_A \\ x_R \end{bmatrix}} = \underset{F}{\begin{bmatrix} 0 \\ f_R \end{bmatrix}} \quad (3)$$

In state form, the EOM can be rearranged as follows

$$\underset{\tilde{M}}{\begin{pmatrix} C & M \\ M & 0 \end{pmatrix}} \underset{\dot{\tilde{X}}}{\begin{bmatrix} \dot{X} \\ \ddot{X} \end{bmatrix}} + \underset{\tilde{K}}{\begin{pmatrix} K & 0 \\ 0 & -M \end{pmatrix}} \underset{\tilde{X}}{\begin{bmatrix} X \\ \dot{X} \end{bmatrix}} = \underset{F}{\begin{bmatrix} F \\ 0 \end{bmatrix}} \quad (4)$$

Assuming a solution of $\tilde{X} = \psi\, e^{\tilde{\lambda} t}$ for the state variable, differentiating accordingly, then substituting into equation 4, gives

$$\left[\tilde{M}\tilde{\lambda} + \tilde{K}\right] \begin{bmatrix} \psi \\ \psi\tilde{\lambda} \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix} \quad (5)$$

For the normal mode vibration of the masses, the excitation forces F in equation (5) are zero, and then the non-trivial solution can be evaluated by equating the determinant of the matrix to zero (Ewins, 2000).

$$\left|\tilde{M}\tilde{\lambda} + \tilde{K}\right| = 0 \quad (6)$$

Using Experimental Modal Analysis, the initial parameters values are estimated as:

$k_A = 3.5519\times10^5\ N/m$, $k_M = 3.0\times10^6\ N/m$,

$c_A = 2.3159\times10^3\ Ns/m$, $c_M = 1.05\times10^3\ Ns/m$,

$M_R = 7.35kg$ $M_A = 2.095kg$ (Xie *et al.*, 2009).

Substituting these values in equation 6, the determinant evaluated using Matlab give solutions of α and β shown in Table 2 of the form $\lambda_i = -\alpha_i + j\beta_i \quad i = (1, 2, \ldots 4)$ of the eigenvalue problem from which the natural frequency f_i and damping D_i coefficients of the rotor-bearing system can be evaluated as

$$f = \frac{\beta}{2\pi} \quad ; D = \frac{2\alpha}{\beta} \quad (7)$$

3. Methodology

Coefficients k_A, c_A, k_m and c_m are considered input variables while the system's modal natural frequencies and damping values are the output features. Input parameter under investigation is set at its ***low*** and ***high*** values while the others are held at nominal value. Output feature values for this input are calculated using MSC/PATRAN and NASTRAN Finite Element Analysis Solver. The simulation run was done using Stat-Ease software to avoid bias. Table 3 shows the response of the system with the row configuration. General Sensitivity Analysis (GSA) is then calculated to give insight into the estimates of the derivatives these input-output pairs according to the simple finite differencing formula

$$\frac{\partial out}{\partial in} = \frac{Out_{Hi} - Out_{lo}}{In_{Hi} - In_{Lo}} \tag{8}$$

where *Out* and *In* refer to output features and input parameters (all normalized before the calculation to eliminate scaling issues), respectively, and *Hi* and *Lo* refer to the level at which the input parameter of interest is set.

4. Result and Discussion

Table2 Result of Eigen Value analysis

i	α	β	f	D
1	5295.20	0.00	0.00	0.00
2	-254.7	0.00	0.00	0.00
3	136.3	165.60	26.35	1.65
4	-136.3	-165.60	-26.35	-1.65

Table 2 show the solution of equation 6; the eigen values α and β as obtained using Matlab and the frequency (Hz) and damping factor calculated from equation 7. As can be seen, the solution contains conjugate pair mode; $i = 3,4$ of which $i = 4$ .is stable. This mode will be considered in the analyses.

Table 3 showed a decrease in the damping factor i.e., 2.56 to 2.28 for search below 5×10^6 N/m. That could possibly suggest on-set of instability. However, a decrease by 95% of the MR damping coefficient yielded a rise in damping factor by 11.39%. These observations suggest direction of optimum MR properties search. From the bar chart, it can be seen that Metal rubber parameters influences both the modal natural frequency and damping.

Table3 Result of Analysis

	Input Parameters(Specified)				Output	
	K_A	C_A	K_M	C_M	f	D
K_A Hi	3.5529	2.3159	3	1.05	26.36	1.65
K_A Lo	1.9014	2.3159	3	1.05	16.36	3.65
C_A Hi	2.7267	3.6025	3	1.05	24.75	3.21
C_A Lo	2.7267	1.0293	3	1.05	27.52	7.36
K_M Hi	2.7267	2.3159	5	1.05	20.01	2.56
K_M Lo	2.7267	2.3159	1	1.05	17.92	2.28
C_M Hi	2.7267	2.3159	3	2.00	19.04	2.37
C_M Lo	2.7267	2.3159	3	0.10	20.43	2.64

$K_A(\times 10^5$ N/m) $C_A(\times 10^3$ Ns/m) $K_M(\times 10^6$ N/m) $C_M(\times 10^4$ Ns/m)

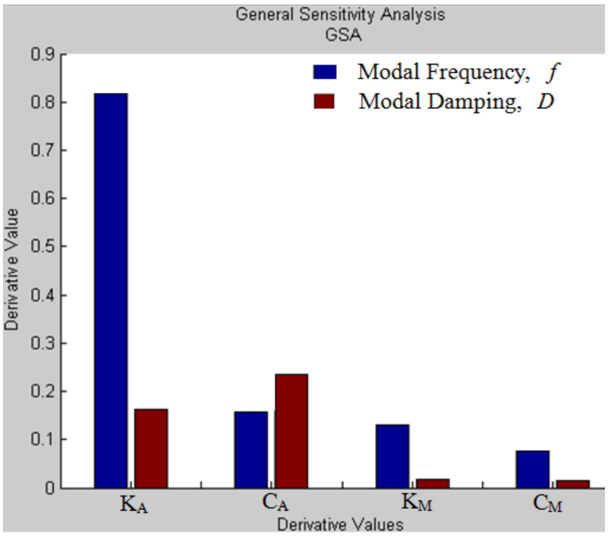


Figure 3: GSA Bar chart

Results of these finite difference calculations are shown below in the bar chart of Figure 3.

The findings agree with Xie *et al* (2009) experimental result, hence, can serve as an effective way of increasing modal damping and restraining the vibration further above the limit only AMB can. However, some modes are unstable. Such parameter range can be avoided in the design of MR.

5. Conclusion

General Sensitivity Analysis has been used to study the contributions of Metal Rubber material, used as part of MR- AMB support, to the modal natural frequency and damping of the rotor-bearing system. The result showed the contribution to be quite significant which is in agreement with Xie *et al* experimental finding, suggesting that the AMB system with MR is more stable. For better performance of the MR-AMB bearing, the stiffness has to be increased beyond set value of $3.0\times10^{6}\ N/m$. Also, search towards decreasing damping coefficient of $1.05\times10^{3}\ Ns/m$ promised good performance.

References

Ewins, D. (2000). Modal testing: theory practice and application. Letchworth Hertfordshire England: John Wiley.

Fitrial, A. (2007). System identification and control of Magnetic Bearing systems. [PhD Thesis]. University of Victoria, USA.

Fujiwara, H., Ebina, K., Ito, M., Takahashi, N., Matsushita, O. (2002). Control of flexible rotors supported by Active Magnetic Bearing. In Y. Okada & K. Nonami (Eds.), *Proceedins of Eight International symposium on magnetic bearing* (pp.145-150). Ibaraki, Japan: Department of Mechanical Engineering, Ibaraki University.

Gu, C., Chen, L., Wu X. (2010). Parameter fitting and finite element simulation of Metal Rubber. *International Conference on Mechanical and Electronics Engineering* (pp. 224-227). Kyoto, Japan：Institute of Electrical and Electronics Engineers, Incorporation.

Hamburg, G., Parkinson, J. (1962). Gas turbine shaft dynamics, SAE Trans., 70, 774-784

Ito, M., Fujiwara, H., Okubo, H., Matsushito, O. (2000). Unbalance vibration control for high order bending critical speeds of flexible rotor supported by Active Magnetic Bearings. In J.C. Han (Ed.), *Proceedings of the 8th International Symposium on*

Transport Phenomena and Dynamics of Rotating Machinery (pp. 968-973). Maui, Hawaii : Pacific Center of Thermal-Fluids Engineering.

.Nicholas, J., Whalen, J., Franklin, S. (2002). Improving critical speed calculation using flexible bearing support FRF compliance data. New York, USA : Dresser-Rand Wellsville.

Schweitzer, G. (2009). Applications and research topics for Active Magnetic Bearings. In K. Gupta, proceeding of IUTAM symposium on emerging trends in rotor dynamics, Volume 1011 (pp 263-273), Delhi, India: Springer-Verlag.

Shuliang, L., Palazzolo, A. (2008). Control of flexible rotor systems with Active Magnetic Bearing. *Journal of sound and vibration*, 314(1-2), 19-38. doi:10.1016/j.jsv.2007.12.028.

Xie, Z., Wang, T., Zhang, J. (2009). Dynamic characteristics of Active Magnetic Bearing system with Metal Rubber annuluses, Journal of Aerospace Power, 24(2), 378-383.

Zeidan, F., San Andres, L., Vance, M. (1996). Design and application of squeeze film dampers in rotating machinery. In J.C. Bailey & D.W. Childs, Proceeding of twenty-fifth turbomachinery symposium (pp.169-188). Texas, USA: Texas A&M University.

Zhangi, W., DongXu, L. Rui, X. (2012).Vibration isolation research on Metal Rubber damping rod of payload attach fitting. Advance material research, 391-392(2012), 467-473, doi: 10.4028/ AMR.391-393,467.