FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA

SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY

DEPARTMENT OF INFORMATION AND MEDIA TECHNOLOGY

SECOND SEMESTER 2016/2017 EXAMINATION

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| **COURSE CODE:** | CIT 224 |
| **COURSE TITLE:** | DISCRETE MATHEMATICS |
| **CREDIT UNITS:** | 3 |
| **TIME ALLOWED:** | 2HRS 45MIN |
| **COURSE LECTURER(S):** | Mr. H. A. Zubairu and Mrs Stella O. Etuk |
| **NUMBER OF QUESTIONS:** | 4 |
| **NUMBER OF PAGES:** | 2 (INCLUDING THIS PAGE) |

**INSTRUCTIONS**

- Answer all questions
- **Do not** use red pen
- Please use a clear handwriting
- This exam is closed book, closed notes, closed laptop and closed cell phone
- Please use non-programmable calculators only

1. a) Show that $[p \to (q \wedge r)] \equiv [(p \to q) \wedge (p \to r)]$ 5mks

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \to q$ | $p \to r$ | $p \to (q \wedge r)$ | $(p \to q) \wedge (p \to r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | F | F | F |
| T | F | T | F | F | T | F | F |
| T | F | F | F | F | F | F | F |
| F | T | T | T | T | T | T | T |
| F | T | F | F | T | T | T | T |
| F | F | T | F | T | T | T | T |
| F | F | F | F | T | T | T | T |

b) Let p, q and r be the propositions:

p: You have the flu

q: You miss the final examination

r: You pass the course

Express the following compound propositions as English statements

i. $q \to \neg r$ ii. $(p \wedge \neg q) \to r$ iii $\neg q \leftrightarrow r$ 3mks

i. If you miss the final examination, then it is not the case that you pass the course.
ii. If you have the flu and you did not miss the final examination, then you pass the course
iii. If you have the flu and you miss the final examination and You pass the course

1x3= 3mks (max)

c) Use the technique of Mathematical Induction to prove that
$1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$, for all non-negative integers n. 7mks

**Solution.**

Basis step : P(n) = 0 is true, $2^0 = 1 = 2^1 - 1$. RHS = LHS 1 mk

Inductive Step: Let P(k) be true for any arbitrary nonnegative integer k.

Therefore, the equation becomes $1 + 2 + 2^2 + \ldots + 2^k = 2^{k+1} - 1$ ----------- (1) 0.5 mk

When p(k) is true, then p(k+1) must also be true,

Therefore, we show that $1 + 2 + 2^2 + \ldots + 2^k + 2^{k+1} = 2^{(k+1)+1} - 1 = 2^{k+2} - 1$ 0.5 mk

$1 + 2 + 2^2 + \ldots + 2^k + 2^{k+1} = (1 + 2 + 2^2 + \ldots + 2^k) + 2^{k+1}$ ------------------------(2) 1 mk

From (1) $1 + 2 + 2^2 + \ldots + 2^k = 2^{k+1} - 1$ 1 mk

Therefore, (2) $1 + 2 + 2^2 + \ldots + 2^k + 2^{k+1} = (2^{k+1} - 1) + 2^{k+1}$ 1 mk

$= 2 . 2^{k+1} - 1$ 1 mk

$= 2^{k+2} - 1$. As required. 1 mk

2. a) Let **A= {x. y}** and **B= {1, 2, 3}** be sets. Find the following

i. $A \times B$ ii. $|B|$ iii. $P(A)$ iv. $P(B)$ v. $A \cup B$ 5mks

3 a) Find the lexicographic ordering of these strings of lowercase English letters

i) quack, quacking, quick, quicksand, quicksilver 2mks

Ii) open, opened, opener, opera, operand 2mks

Iii) zero, zoo, zoological, zoology, zoom 2mks

b) Let $f$ and $g$ be functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 4$. Find:

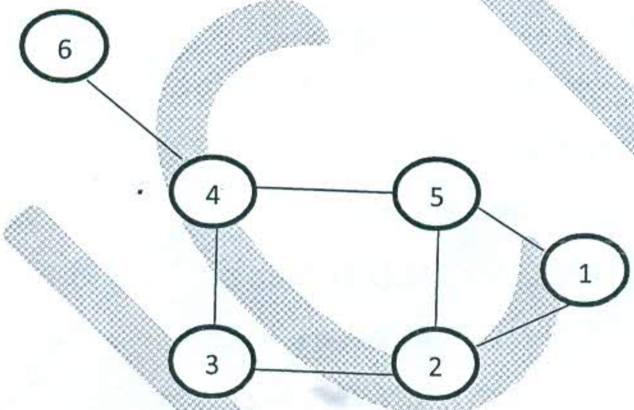
i. The composite $(fog)(x)$

$(fog)(x) = f(g(x)) = 2(3x + 4) + 3 = 6x + 11$

ii. The composite $(gof)\ (x)$

$(gof)\ (x) = g(f(x)) = 3(2x + 3) + 4 = 6x + 13$ 4mks

c) Draw the graph represented by the given adjacency matrix. Ordering the vertices as follows; *1, 2, 3, 4,5, 6.*




( 5 marks)

4 a) Study the Niche overlap graph represent in figure 2.

i) The degree of a vertex in a niche overlap graph is the number of species in the ecosystem that compete with the species represented by this vertex (2 mks)

ii) The mouse is the only species represented by a pendant vertex and there are no isolated vertices.

i) Mouse vertex and squirrel vertex are the least and maximum vertices respectively. (1mk)

b) Determine the degree of each vertex in the undirected graph below

Degree(a) = degree(b) = degree(c) = degree(d) = 5, (4 mks)

c) Study the graph in Figure 3.

i) *a, d, c, f, e* is a simple path of length 4, because $\{a, d\}$, $\{d, c\}$, $\{c, f\}$, and $\{f, e\}$ are all edges

ii) The path *a, b, e, d, a, b,* which is of length 5, is not simple because it contains the edge $\{a, b\}$ twice.

i. $AXB = \{(x,1),(x,2),(x,3),(y,1),(y,2),(y,3)\}$

ii. $|B| = 3$

iii. $p(A)=\{\{\ \},\ \{x\},\{y\},\{x,y\}\}$

iv. $p(B) = \{\{\ \},\{1\},\{2\},\ \{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$

v. $A \cup B = \{x,y,1,2,3\}$

b) Consider the "divides" relation on the set $\boldsymbol{A} = \{\mathbf{1,2,3,4\ ,5,6,7,8}\}$

i. Show that this relation is a partial order on $\boldsymbol{A}$.

ii. Draw a Hasse diagram for the "divides" relation.

iii. List the maximum elements, minimum elements, greatest element and least element

10mks

To show that "divides" is a partial order on set $A$, it suffices to show that the relation is Reflexive, Anti-symmetric and Transitive.

Let $R$ be the "divides" relation,

$$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),(1,8),(2,2),(2,4),(2,6),(2,8),(3,3),$$
$$(3,6),(4,4),(4,8),(5,5),(6,6),(7,7),(8,8)\}$$ 1mk

Reflexive property: $aRa,\ \forall a \in A, R = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8)\}$ 1mk

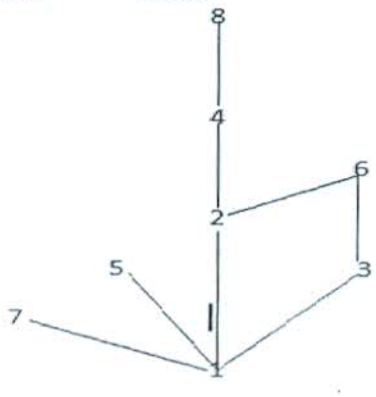
Anti-Symmetric property: $if\ aRb, then\ \nexists\ bRa\ \forall a \neq b$

$$R = \{(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),(1,8),(2,4),(2,6),(2,8),(3,6),(4,8)\}$$ 1mk

Transitive property: if $aRb\ and\ bRc, then\ aRc\ \forall a,b,c \in A$

$$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(1,7),(1,8),(2,2),(2,4),(2,6),(2,8),(3,3),$$
$$(3,6),(4,4),(4,8),(5,5),(6,6),(7,7),(8,8)\}$$ 1mk

ii.




Least element = 1, maximum element =5,6,7,8, Greatest element= nil, minimal element = nil 4mks

iii) $b, c, f, e, b$ is a circuit of length 4 because $\{b, c\}$, $\{c, f\}$, $\{f, e\}$, and $\{e, b\}$ are edges, and this path begins and ends at $b$.

iv) *4 and 5 for i and ii respectively (6 mks)*