FEDERAL UNIVERSITY OF TECHNOLOGY MINNA
SCHOOL OF INFORMATION & COMMUNICATION TECHNOLOGY
DEPARTMENT OF INFORMATION & MEDIA TECHNOLOGY
SECOND SEMESTER EXAMINATION 2012/2013 SESSION
ICT 224: DISCRETE MATHEMATICS

INSTRUCTION: ANSWER ANY FOUR (4) QUESTIONS
TIME ALLOWED: 2 $^{1/2}$ HRS

1. a) Let $A = \{a, b, c\}$and $\mathrm{B} = \{b, c, d, f, g\}$, Find the following:

   i) $(A \cup B) - (A \cap B)$ 2

   ii) $(A - B) \cup (B - A)$ 2

   iii) $A \oplus B$ 2

   iv)$A \times B$ 2

   v)$P(A)$ 2

   b) Let$\mathrm{A} = \{1, 2, 3, 4, 5, 6\}$, $\mathrm{A}_1 = \{1, 2\}$, $\mathrm{A}_2 = \{3, 4\}$, $\mathrm{A}_3 = \{5, 6\}$.

   Show that $\{\mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3\}$ is a partition of A.

   c)Determine whether each of the following statements is **True** or **False**.

   i) $x \in \{x\}$ ii) $\{x\} \subseteq \{x\}$ iii) $\{x\} \in \{x\}$ iv) $\{x\} \in \{\{x\}\}$

   v) $\emptyset \subseteq \{x\}$ vi) $\emptyset \in \{x\}$

2. a) Define the following terms:

   i) Function

   ii) Injective function

   iii) Surjective function

   iv) Inverse Relation

   v) Inverse Function

   b) Let $A = \{4, 5, 6\}$and$B = \{5, 6, 7\}$. Let R and S be two binary relations defined from A to B as follows:

   $$(x, y) \in A \times B, \; xRy \leftrightarrow 2/(x - y)$$

   $$(x, y) \in A \times B, (x, y) \in S \leftrightarrow x \geq y$$

   i) Find R ii) Find S iii) Find $R \cap S$

c) Indicate whether any of the relations R or S in (2b) above is a function. If any, state the type of function, if not give reasons.

3. a) Let Zbe the set of integers, show that congruence modulo 2 is an equivalent relation given that:

$x \equiv y \ (mod\ 2)$ ("x is congruent to y modulo 2") if and only if $x - y$ is even.

b) Let $\mathbf{Z}^+$be the set of non-negative integers and R the relationaRb if a divides b. Show that $\mathbf{Z}^+$ is a poset.

c) Let $\mathbf{A} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}and\mathbf{R} = \{(\mathbf{a}, \mathbf{a}), (\mathbf{b}, \mathbf{c}), (\mathbf{c}, \mathbf{b}), (\mathbf{d}, \mathbf{d})\}$.

i) Show that R is symmetric

ii) Show that R is not transitive.

4. a) Show that$(\mathbf{r} \vee \mathbf{p}) \wedge [(\overline{\mathbf{r}} \vee (\mathbf{p} \wedge \mathbf{q})) \wedge (\mathbf{r} \vee \mathbf{q})] \equiv \mathbf{p} \wedge \mathbf{q}$.

b) If p, q and r denote the following propositions:

p: Bats are blind

q: Goats eat grass

r: Ants have long teeth

Express the following compound propositions symbolically.

i) If bats are blind then goats don't eat grass.

ii) If and only if bats are blind or goats eat grass then ants don't have long teeth.

iii) Ants don't have long teeth and, if bats are blind then goats don't eat grass.

iv) Bats are blind or goats eat grass and, if goats don't eat grass, then ants don't have long teeth.

c) Indicate which of the following propositions is a tautology, a contradiction or a contingency.

i) $(p \wedge \neg q) \wedge (\neg p \vee q)$

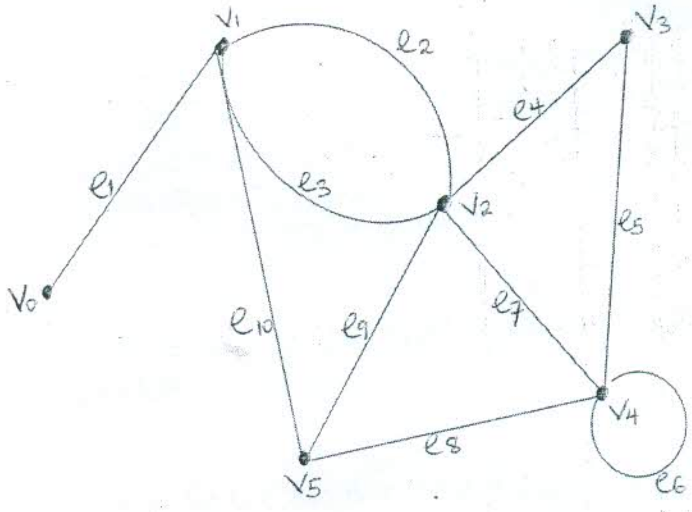
ii) $(p \wedge q) \vee \neg (p \wedge q)$

5. a) Show that $(\mathbf{Z}_2, +, \bullet, \bar{\ }, 0, 1)$ is a Boolean algebra.

b) Draw a combinatorial circuit for the Boolean expression $y = \overline{(x_1 . x_2) + x_3}$

c) Draw the logic table for the Boolean expression in (5b) above.

6. a) Consider the following graph G.



i) Find E_G, V_G, **card(V)** and **card (E)**

ii) List the isolated vertices.

iii) List the loops.

iv) List the parallel edges.

v) List the vertices adjacent to v_3.

vi) List the vertices adjacent to v_2.

vii) Find all edges incident on v_1.

viii) Find the degree of vertex v_4

b) In the graph above, determine whether the following sequences are paths, simple paths, circuits, or simple circuits

i. $v_0\, e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$.

ii. $v_1\, e_2 v_2 e_3 v_1$.

iii. $v_3\, e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$.

iv. $v_5\, e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4\, e_8 v_5$

c) Draw a complete graph K_5 and a complete bipartite graph $K_{2,3}$.

d) Define the following terms:

i. Simple graph.

ii. Complete graph.

iii. Pseudograph