

**SIMULATION OF THE STABILITY CHARACTERISTICS OF NIGERIA
ASSEMBLED TRACTORS**

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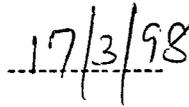
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CERTIFICATION

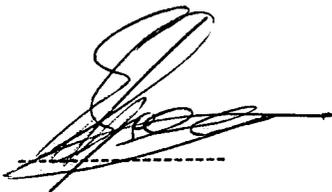
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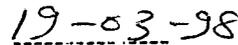
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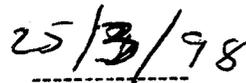
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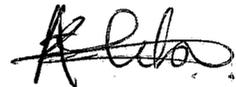


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DEDICATION

This work is dedicated to the memory of my late uncle, Mr. Benaiah Anajemba, whose ideology of life, even till death, can only be summarised in these words - Don't worry, be happy !!!

ELEBO ALEXANDER MYE



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"Learning is not attained by chance. It must be sought for with ardor, and attended to with diligence".

My greatest thanks goes to my creator, the Almighty Being and Universal truth, GOD, who has spared and kept me till this day and has always made my life challenges a testimony unto His name.

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ABSTRACT

The Prime purpose of this project is to simulate a tractor with adaptive technology that is suitable to our Nigerian conditions and also to study the prevailing conditions that will lead to instability and the parameters that also give rise to this instability of the tractor. Basic simplifying assumptions were made and the forces acting on the tractor which determines the performance and stability of the tractor were modelled.

Dynamic stability equations of the tractor in the three degrees of freedom considered (longitudinal, lateral, and yaw motions) were obtained. The impact of braking and steering inputs to these stability equations were also considered. These equations were then solved by numerical analysis using fourth order Runge-kutta grill method. The program language used to solve the equation is the C++ due to it's flexibility and closeness to the low level language (Machine language), limited experiments were then performance to validate the computer simulation.

CHAPTER 1

INTRODUCTION

" If an agricultural engineer who is specialising in farm power is to make his best contribution to the agricultural engineering profession, if as an agricultural engineer he is to earn and hold a recognised place among other engineers, he must have more than the garage mechanic's conception of the tractor. He must visualise the tractor as a unit and have a clear conception of all forces acting upon it. He must be informed concerning the fundamental laws of mathematics and physics which govern its kinematics and dynamic response to these forces. In no other way will he be able to make the tractor perform its maximum service. "

E. G. McKibben (1927)

Agricultural mechanisation in Nigeria, like any other developing country is needed to increase productivity of key food crops and to develop sustainable agricultural systems that can replace the old traditional farming systems. Since the introduction of the Engine Power Technology (EPT), the progress in the field of agriculture has being revolutionary. Tractor use in Nigeria today is synonymous with mechanisation. The tractor as the most common self-propelled machine in agriculture is now vastly being used all over the world to do different kinds of work both on and off the farm; it is used in combination with ploughs and harrows for tillage, used for harvesting and so many other uses. Nigerian farmers are now waking up to the realisation that the tractor is a reliable answer to their prayer for improved production on a large scale, less fatigue in their farm work, accessibility and ease of transportation of their goods and maximum utilisation of their "working time".

Though there still exist "bottlenecks" due to the land tenure system practised in Nigeria (which encourages fragmentation of land holdings), the farmers are now forming co-operatives, joining their lands and making mechanisation feasible. Through these co-

operatives, they jointly buy tractors (to help those among them that single handedly cannot afford tractors), hire out tractors and implements, and also help in other ways within their reach to improve agricultural production.

1.1 JUSTIFICATION

An accelerated use of the tractor has given rise to the development of a wide range of sophisticated implements and equipment's for carrying out essential agricultural operations better and cheaper, reducing drudgery, fatigue, improving quality and quantity of farm produce; hence better living standards for the farmers and even improved economy for the nation. Since the tractor is widely used for a whole lot of agricultural mechanisation work in Nigeria and beyond, there is need to ensure its relative stability both in static and dynamic conditions especially considering the fact that there still exist a low literacy level among the bulk of the Nigerian farmers.

Also, Nigeria is a tractor importing and / or assembling nation with major assembling plants at Bauchi (for STEYR PRODUCTS) and Kano (for FIAT PRODUCTS). The terrain conditions, topography, method of cultivation, soil characteristics peculiar to Nigeria are to some extent different from those obtainable in these nations where we import from. Hence, their stability specifications require investigation under Nigeria conditions. This project is part of such attempts to ascertain the stability limit of tractors under prevailing conditions.

1.2 OBJECTIVES

The prime objectives of this project are:

- To stimulate a tractor with adaptive technology that is suitable to our Nigerian conditions.
- To identify the parameters that affect the stability of these tractors.
- To study and analyse the prevailing conditions that will lead to instability of the tractor and the limits beyond which the tractor is considered unstable.

- To use the simulated data to ensure the stability of Nigerian assembled tractors given certain conditions.

It is hoped that this design will form a basic and foundation study for future analysis, design and manufacture of tractors in Nigeria.

CHAPTER 2

LITERATURE REVIEW

Even though a lot of resources have been expended in the last six decades on research into tractor stability in different countries, much hazard still surround the tractor especially in its dynamic state. These hazards are more pronounced in tractor - implement combinations. It is believed that if the tractor stability is ascertained, it will go a long way in forming foundation studies on the tractor - implement combination and stability.

Most researchers have paid considerable attention trying to find optimum solution to two major problems in the operating of tractors ⁽¹⁾. These are:

- a) Ensuring a greater safety in the operating of a tractor and also higher level of operator's comfort.
- b) A call for the optimisation of the performance of the tractor.

Solutions to these problems have formed the basis of most research works and has led to observation and analysis of tractor in it's static, dynamic (under numerous operating conditions) and also the behaviour of the tractor when mounted with implements.

2.1 PREVIOUS RESEARCH ON THE STATIC STABILITY OF THE TRACTOR

Much research has being done on the stability characteristics of vehicles in static conditions. The static stability of vehicles were usually measured on a tilt table but the tilt table is very expensive and only few of them are available. The Scottish centre of Agricultural Engineering improved on the tilt table by developing a method of predicting the stability of a wide range of vehicles using portable weigh pads ⁽²⁾. This weighpad has a great advantage over tilt table because it is very simple to use. This weighpad technique is based on measuring the weight transfer or the wheel to ground load exerted by vehicle wheels on moderately known slopes and then plotting the graph of these wheel to ground load against

there corresponding slope angles (measured with an inclinometer). It was found that by fitting a sinusoidal model through the data points, an equation for the angle at which the "uphill" wheel load becomes zero could be predicted. This angle is just the minimum slope angle at which the vehicle tips halfway towards over turning. Therefore it is just like a static stability index wherein a vehicle on a slope below it's predicted slope angle will be stable and if on a slope above it's predicted slope angle, it will be unstable. Hence vehicles with high recommended slope angles will be considered more stable than those with lower slope angles.

2.2 PREVIOUS RESEARCH ON DYNAMIC AND QUASI - STATIC CONDITIONS OF THE TRACTOR

The quasi - static and dynamic conditions of vehicles can be deduced directly from the weighpad method by slight adjustments to the static stability limits provided the initial effects on the vehicle remains constant. Quasi - static models of vehicles take into account constant speed turning and constant acceleration or deceleration and slope values at which zero wheel load is experienced can be predicted just like the case of static models. Dynamic models tend to predict the conditions that will lead to overturning. This is more difficult than predicting the zero wheel load condition because it has to take into consideration the motion of the vehicle while it lifts off the ground and passes through the point of balance. Two basic problems in obtaining a good dynamic model are:

- a) Simulating the interaction between the tyres and the ground.
- b) Predicting the behaviour of the whole vehicle in response to random ground inputs.

Due to the fact that the tractor tyres play a vital role in the description of performance, they have being given understandable attention by researchers. The cushioning effect and the ability of pneumatic tyres to envelope sharp obstacles have always being researched on for improvement. The tractor tyre constitute the only suspension for the tractor and it is through them that the tractor interacts with the surface on which it moves.

The force of the tyre normal to the ground is modelled by assuming the tyre could be represented by a spring and damper system which has a point contact with the ground⁽³⁾. Thompson, Liljedahl and Quinn (1970) in their analysis of the dynamic motion response of agricultural tyres showed that the irregular surfaces traversed by a pneumatic tyre can be considered smooth when the "spring plus damper" tyre model is used. Mathew, J. and Talamo⁽⁴⁾ analysed the comfort of tractor operators in riding the tractor. Their work was of the same view with that of Pershing and Yoerger⁽⁵⁾ on their simulation of tractors for transient response. Both Mathew and Pershing, and Yoerger measured agricultural tyre vertical spring and damping rates and found that in general tyre's dynamic vertical spring rate was higher than the spring rate obtained from a linearization of tyres static load - deflection relations.

Much work has also being done about the circumferential force properties of tractor tyres. The circumferential forces on the tractor tyres include: rolling resistance forces, traction forces, braking forces, and lateral forces. It is an evident fact that the lateral force developed by agricultural tyres is of paramount importance in the simulation of the handling behaviour of the tractor. The turning of the tractor or its deviation from the longitudinal axis are basically due to the generation of these lateral forces not just the kinematics involved in steering geometry. The lateral force is a function of radial force, camber angle, inflation pressure, tyre construction, slip angle and ground condition. For powered wheels, the lateral force varies with traction and braking forces (Krick, 1973). Horton and Crolla⁽⁶⁾ reviewed the handling behaviour of off road vehicles and made efforts in expressing lateral force by linear exponential or polynomial function of the radial force, traction force and slip angle for different tyre constructions and ground conditions.

The general formula for modelling lateral force is

$$L = \mu_c N, \text{ where}$$

$$L = \text{lateral force (N)}$$

$$\mu_c = \text{lateral force coefficient which is a function of slip angle.}$$

$$N = \text{normal force (N)}$$

The relation between the lateral force coefficient and slip angle is described by the following third order polynomial with constant coefficients ⁽⁷⁾

$$\mu_c = A + B\alpha + C\alpha^2 + D\alpha^3$$

where

μ_c = lateral force coefficient

α = slip angle

A, B, C, D are constants determined experimentally.

The normal force (radial force) N, has also being found as

$$N = C\dot{x} + Kx$$

where

c = damping coefficient of the tyre (KN s / m)

K = stiffness of the tyre

x = radial deflection of tyre (m)

\dot{x} = deflection rate (m / s)

The motion resistance force R, has always being predicted by the product of the motion resistance ratio and the normal load i.e.

$$R = \mu_R N \text{ where } R = \text{motion resistance (N).}$$

Also the traction force T, is determined by the product of the normal force N, and the coefficient of traction, μ_T

$$T = \mu_T N$$

Wismer and Luth ⁽⁸⁾ worked on off - road traction prediction and presented a method of simulating the tractive performance of agricultural tyres through the use of mathematical formulation to fit the form of many experimentally determined net traction coefficient ↔ drive wheel slip relations.

2.3 OVERTURNING OF A TRACTOR

When vehicles reach the limit of their static stability or dynamic stability or slide out of control, they may overturn. They are two categories of overturning:

(2) Control loss wherein the tractor slides bodily downhill before overturning.

An agricultural tractor overturns in three directions; rearward, sideways or forward. From previous analysis, sideways overturning occurs more frequently, rearwards overturning result in more fatalities than any other types of overturning and forward overturning has being very rare. Smith and Liljedahl⁽⁹⁾ developed a two part model to stimulate rearward overturning of agricultural tractors by first considering the tractor as rotating about its centre of gravity when all wheels are in contact with the ground and secondly considering the tractor frame as rotating about the back axle during the actual overturning. A more recent work by Liljedahl considered the influence of drawbar position on tractor rearward stability.

Smith et al⁽¹⁰⁾ based their simulation on the assumption that a conventional tractor tips sideways about two axes:

- (i) about the axis connecting the hinge point of the front axle to the contact point of the rear tyre on the ground during initial overturning,
- (ii) about the axis connecting the contact points of the front and rear tyres on the side of the tractor about which initial motion took place.

Based on the above assumptions, vector analysis was used to describe the three - dimensional motion of sideways overturning of farm tractors. Chilshohm⁽¹¹⁾ developed a mathematical model based on the force and displacement equations of equilibrium at each point where the tractor makes contact with the ground during overturning. His model incorporates tyre side forces relationships and, damping turns and also covers overturning phases. Kelly and Rehkugler⁽¹²⁾ determined the velocity at which a critically steered tractor will overturn on a given bank slope. The permissible bank slope would approach the static angle of tip as the velocity approaches zero. On a flat terrain, there will be a minimum velocity which will cause overturn.

Rehkugler⁽¹³⁾ conducted a full scale verification study to establish the level of confidence one may expect in the use of SIMTRAC (a computer simulation program) during high speed turns on level terrain. A small utility tractor was accelerated to maximum travel

speed in a straight line and then steered as rapidly as possible in one direction in a free wheeling mode with no braking or engine power being applied. The position of the tractor, as a function of time, was recorded for duplicate runs for the test on a flat surface. The same tests were then simulated with SIMTRAC to compare with the field test. Reasonable agreement was achieved.

Highway vehicles object simulation model (HVOSM) was modified for simulation of agricultural tractor dynamics. A battery powered scale model tractor was used to perform experimental runs for the partial validation of the computer model. Satisfactory agreement were obtained on pavement ⁽¹⁴⁾. Further refinements of the program and parameters is however deemed necessary before a satisfactory simulation is achieved, since agricultural tractors are operated on soil. Song, Huang and Bowen ⁽¹⁵⁾ recognised this necessity and further modified HVOSM and the associated computer programs to simulate the general dynamics of a tractor on soft ground by incorporating some soil parameters. The simulated results were verified for tractor overturns using a powered model tractor. The results showed that the modified HVOSM allowed close prediction of tractor dynamics.

Feng and Rehkugler ⁽¹⁶⁾ developed a mathematical model combining the three major factors (sloping grounds, bumps and turns) responsible for tractor overturns. They considered the situation where a tractor has three (3) degrees of freedom with respect to the ground (i.e. two translational, one rotational with three or four wheels in contact with the ground) and a situation where the tractor has four degrees of freedom (i.e. two translational and two rotational with only two wheels in contact with the ground). Newtonian mechanisms and Lagrange equations were applied in solving the first and second cases respectively.

Side slip phenomenon of agricultural tractors were studied by Machida ⁽¹⁷⁾ using a single wheel side slip analyser and a model tyre on a plywood. Side slips were found to be a function of slope angle and slip. Spencer and Crolla ⁽¹⁸⁾ have also presented a procedure for studying control of tractors on sloping ground and remote controlled tractors was used to verify the model they developed. Noh and Erbach ⁽¹⁹⁾ recently used the variational vector

approach that uses relatively generalised co-ordinates in Cartesian space to develop a semi - recursive dynamic algorithm which was evaluated by modelling an agricultural tractor. With the exception of this semi - recursive dynamic algorithm method, modelling of tractors has always been achieved either through vectorial mechanics (Newtonian approach) or through variational approach to mechanics (Lagrangian dynamics).

Many mathematical models for tractor dynamics and stability studies have been developed in the last three decades. A lot of instrumentation has also been developed to ascertain the accuracy of most models either by full - scale size tractors or small - scale models. Considerable research has been conducted on tractor overturning stability and dynamics on sloping ground, but since there still exist on unsatisfactory level of tractor accidents, research must therefore continue. Many researchers have recommended the working of tractors on safe operating slopes and the introduction of the roll over protective structures (ROPS) and other safety measures. This will no doubt reduce the number of fatalities, injuries and accidents with tractors but the avoidance of the accidents and overturns altogether is the optimum aim, hence research must still continue. If the tractor is considered stable and still performs maximum output work over a wide range of terrain, then the researchers, the equipment manufacturers and the operators will have rest for they have done their job well.

CHAPTER 3

METHODOLOGY

3.1 STABILITY OF A TRACTOR

It is of great importance to ensure the tractors stability over given conditions the stability of a tractor assumes that a tractor will follow a set path and change path under the operators control thus stability of a tractor can be regarded as "THE ABILITY OF A TRACTOR TO MAINTAIN STABLE ORIENTATION WHEN ACTED UPON BY EXTERNAL DISTURBANCES AND ITS ABILITY TO RETURN TO A BASIC ORIENTATION WHEN SUCH DISTURBANCES ARE REMOVED.

Since it is the forces acting on a tractor that basically determine its stability, motion, and performance , let us now analyse critically the forces acting on the agricultural tractor.

3.2 FORCES ACTING ON AGRICULTURAL TRACTORS

Performance characteristics of a tractor are the motions accomplished in acceleration, traction, cornering, braking e.t.c. This accomplished motion or performance is just a response to forces imposed on the tractor and a study of how and why these forces are generated will throw much light on tractor dynamics in general.

The dominant forces acting on the tractor are produced from the ground and the part of the tractor that has direct contact with the ground is the tyres. For optimum performance of the tractor, it is essential to understand the interaction between the tyres and the medium on which it operates → THE GROUND.

The tractor tyres (pneumatic types) are designed to perform and fulfil the following functions:

- (1) Support the weight of the tractor.
- (2) To cushion the tractor over surface irregularities

- (3) To provide sufficient traction for driving and braking
- (4) To provide adequate steering control and directional stability.

To describe the characteristics of a tyre and the forces and moments acting on it, it is necessary to define an axis system that serves as a reference for the definition of various parameters. One commonly used axis system recommended by the Society of Automotive Engineers (SAE) is simplified and shown below.

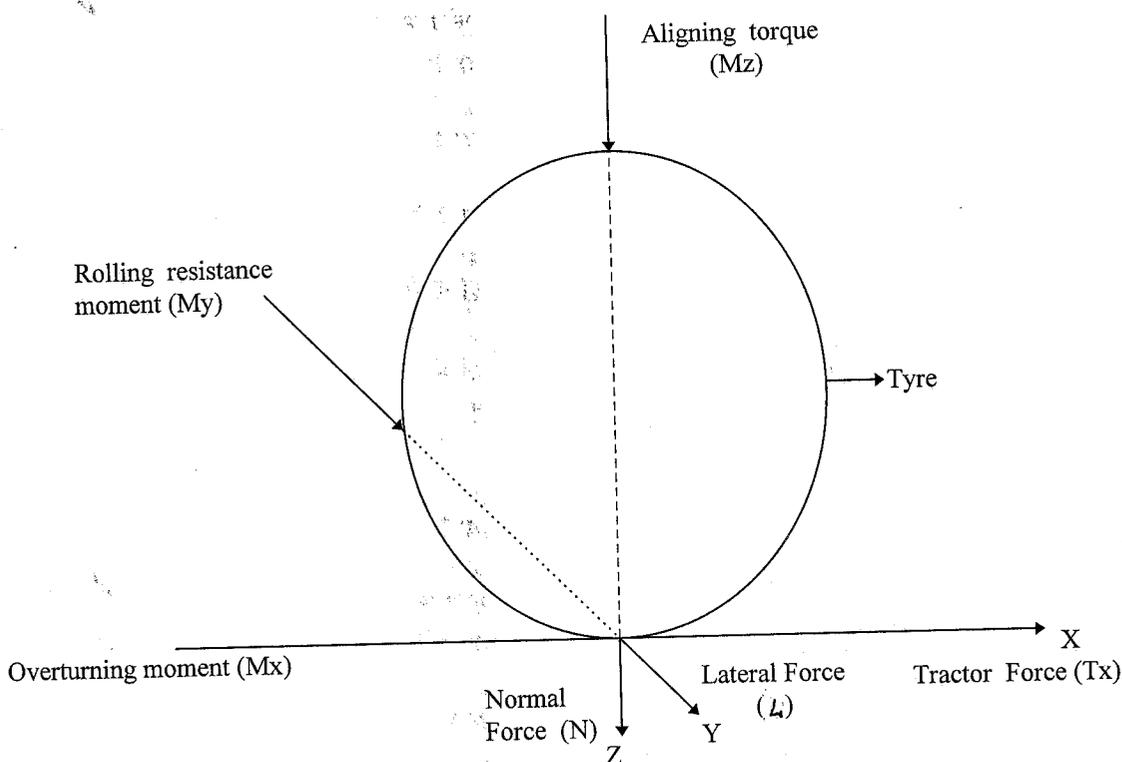


Figure 1. Tyre axis system and forces

The origin of this axis system is the centre of the tyre contact with the ground. The X - axis is along the intersection of the wheel plane and the ground plane. The Z - axis is perpendicular to the ground plane and the Y - axis is perpendicular to the wheel plane.

The tyre force is an external force acting on a wheel. The tyre has three mutually perpendicular components.

- i) The longitudinal force: which is the component of the force acting on the tyre by the ground plane in the ground and its direction is parallel to the intersection of the wheel plane and the ground plane. This force acts in X- axis direction.

ii) The lateral force: which is the component of the force acting on the tyre by the ground in the ground plane and normal to the intersection of the wheel plane with the ground plane.

It is the force component in Y - direction.

iii) The normal force (radial force): is the component of the force acting on the tyre by the ground which is normal to the plane of the ground. This force is in Z - direction.

The point of application of these forces is assumed to be the intersection of the wheel and ground plane, the ground plane directly under the wheel centre and the plane passing through the wheel centre, perpendicular to the ground and wheel plane.

The tractor frame has six (6) degrees of freedom, three (3) translational and three (3) rotational. These are:

a) The tractor can move forward and backward. This is longitudinal motion in X - axis.

b) It can move sideways (lateral motion in Y - axis)

c) It can bounce (vertical motion in Z - axis)

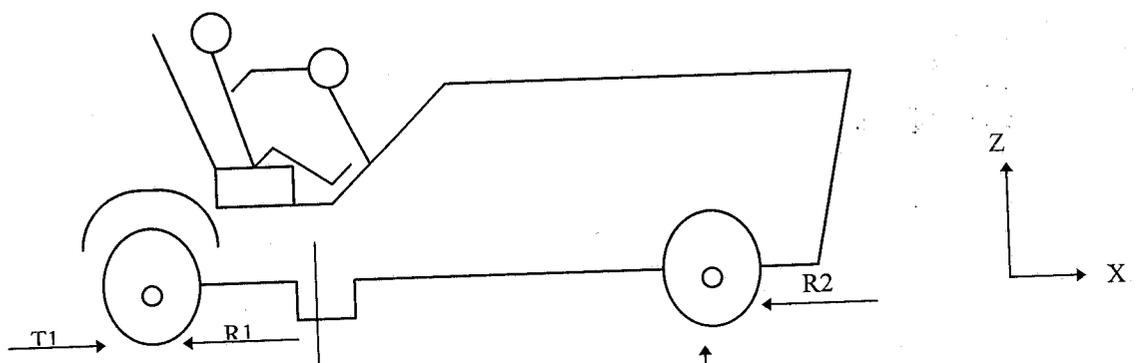
d) It can rotate about longitudinal axis. This is called roll motion.

e) It can rotate about lateral axis - Pitch motion.

f) It can rotate about vertical Z - axis - Yaw motion.

Analysing these forces critically and trying to understand the way the tractor achieves its motion in these directions let us split the 3 - dimensional X-Y-Z axis and consider separately the different 2 - translational dimensions of the pictorial projections of the tractor and showing the possible forces acting on the tractor in these directions.

3.3 Z - X TRANSLATIONAL PLANE OF TRACTOR



Considering the rear axle driven tractor above moving on a level ground, the forces acting on it from this view are:

- i) Traction force, "T" developed due to the axle torque.
- ii) The rolling resistance force " R_1 " and " R_2 " which try to oppose the movement of the tractor tyres.
- iii) The normal load or wheel to ground reactions " N_1 " and " N_2 " which tend to balance the weight of the tractor.
- iv) And the weight, W, of the tractor which acts downward from the centre of gravity of the tractor.

3.4 Z - Y TRANSLATIONAL PLANE OF THE TRACTOR

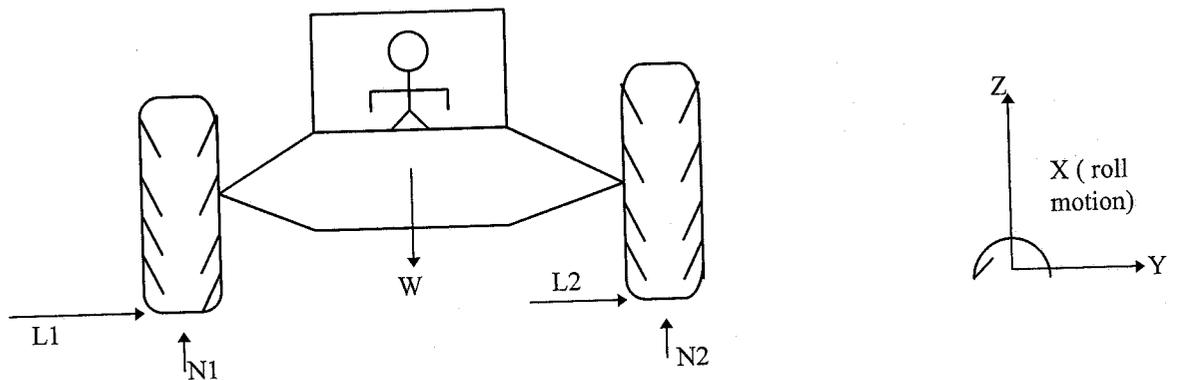


Figure 3 Rear view of tractor

Considering the Z - Y plane or model above the only visible force are

- i) Normal wheel to ground loads, " N_1 " and " N_2 "
- ii) The lateral force " L_1 " and " L_2 " with
- iii) Tractor weight, "W"

3.5 Y - X TRANSLATIONAL PLANE OF THE TRACTOR

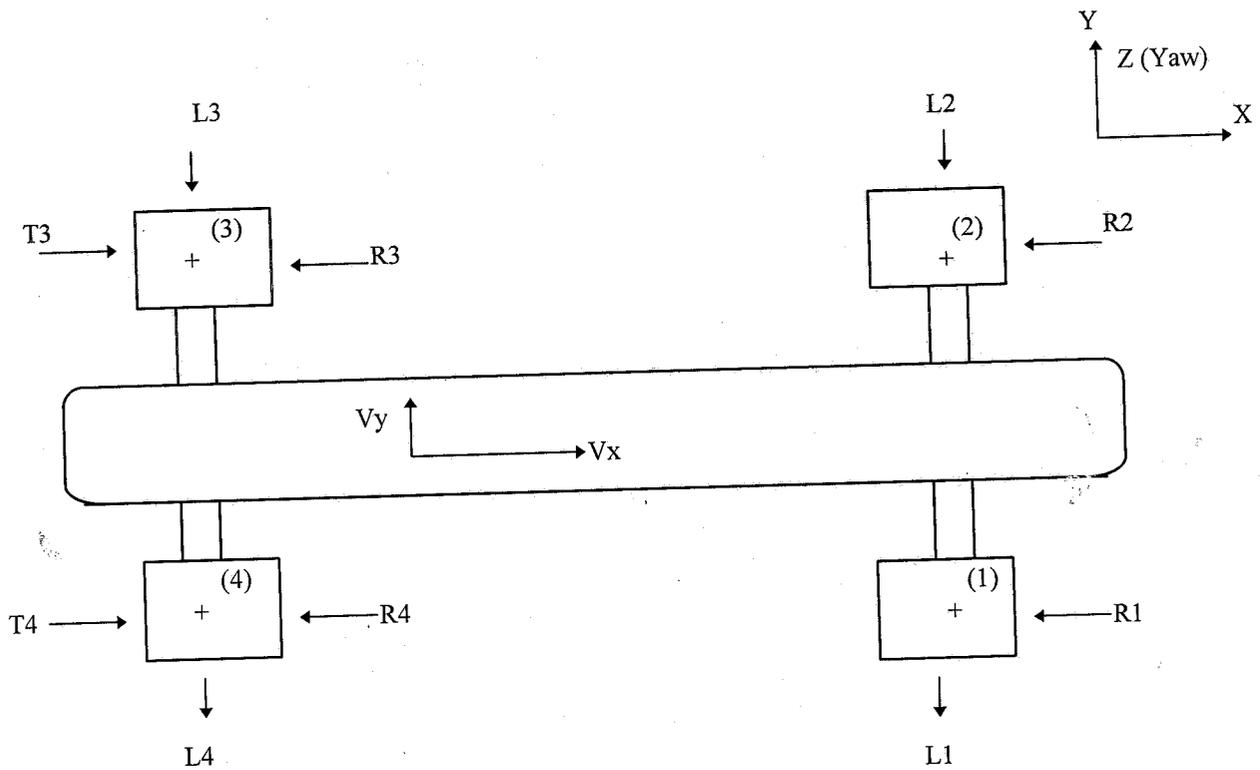


Figure 4 - Top view of tractor

The dominant forces on this model are:

- i) The lateral force, L_1, L_2, L_3, L_4
- ii) The rolling resistance forces, R_1, R_2, R_3, R_4
- iii) The traction forces T_3 and T_4 .

This project is particularly concerned with the analysis of the behaviour of the tractor under lateral inputs. Since the Y - X translational model above identifies the lateral interaction and forces on the tractor to a very large extent, it was used as the simulation model and all references, henceforth, are made to it.

3.6 BASIC ASSUMPTIONS

The following simplifying assumption apply to the motion of the rear - wheel driven tractor modelled above in fig IV.

Properties	
(1) Dynamic properties of the tractor	a) Tractor is a rigid body b) Tractor is symmetrical with respect to a plane perpendicular to the rear axle and passing through the mid - point. c) Dynamic properties are represented by the tractor mass, moments and products of inertia. d) Aerodynamic force are neglected.
(2) Tyre - Terrain intersection properties	e) Wheel ground reaction acts at a single point at each wheel. f) The tyre can be replaced by a spring and damper system.
3) Terrain properties	g) Terrain surface is planar and non - deformable.

3.7 MATHEMATICAL EQUATIONS FOR THE LATERAL MOTION OF A TRACTOR

The forces acting on a tractor either in longitudinal, lateral and vertical directions must balance themselves for the tractor to be in static, quasi - static or dynamic equilibrium. Considering a tractor in dynamic motion, the resolution of the forces acting on it gives mathematical equations of motion for its equilibrium state.

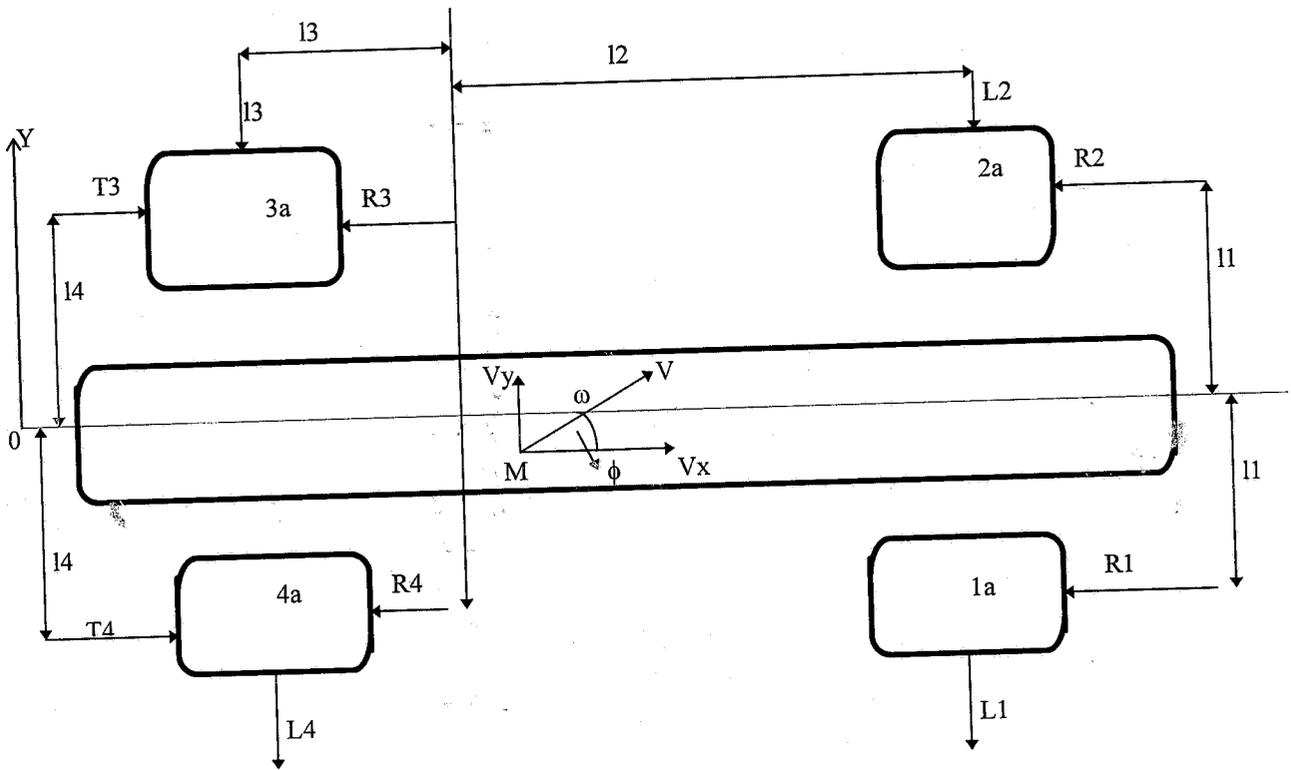


Figure 5 : Top view of tractor with forces acting on it

From the model above, "M" is the centre of mass of the tractor being concentrated at the centre of gravity of the tractor v_x and v_y are the component of the velocity " v " of the tractor at the centre of gravity along the x and y axis respectively. T_3 and T_4 are the tractive forces developed at the rear wheels. While L_i and R_i ($i=1, 2, 3, 4$) are forces (lateral and motion resistance forces respectively) acting either in lateral or horizontal directions of the tractor.

Resolving and summing forces in the x - direction,

force $f = \text{mass} \times \text{acceleration} \text{ (kg m/s}^2\text{)}$

$$\Sigma f_x = 0$$

$$M \dot{V}_x - M V_y \omega + T_3 + T_4 - R_1 - R_2 - R_4 = 0$$

$$M(\dot{V}_x - V_y \omega) + T_3 + T_4 - R_1 - R_2 - R_3 - R_4 = 0$$

(1)

where

$\omega = \text{angular velocity}$

$V_y \omega = \text{an opposing drag acceleration the forward acceleration } (\dot{V}_x)$

$\dot{V}_x = \text{acceleration in x- direction.}$

Also summing forces in the y - direction gives:

$$\Sigma f_y = 0$$

$$M(\dot{V}_y - V_x \omega) - L_1 - L_2 - L_3 - L_4 = 0 \quad (2)$$

Here,

$M\dot{V}_y$ = component of the force in Y - direction and

$V_x \omega$ = drag acceleration in the same direction as V_y .

Taking moment about the centre of gravity to get the rotational, yaw motion,

$$\Sigma M = 0$$

Moment = force x perpendicular distance of force. Hence

$$\begin{aligned} -I_{zz} \omega + l_4(T_3 - R_3) - l_4(T_4 - R_4) - l_3(L_3 + L_4) + l_2(L_1 + L_2) + l_1 R_1 - l_1 R_2 \\ I_{zz} \omega = l_4(T_3 - R_3) - l_4(T_4 - R_4) - l_3(L_3 + L_4) + l_2(L_1 + L_2) + l_1(R_1 - R_2) \end{aligned} \quad (3)$$

Equations 1, 2 and 3 are the differential equations of motion governing three (3) degrees of freedom of the tractor in longitudinal, lateral and yaw directions respectively.

Where

I_{zz} = Moment of inertia about Z - axis (kgm^2).

$\dot{\omega}$ = Angular acceleration ($1/\text{s}^2$).

T_3 and T_4 = Tractive forces on the rear wheel of the tractor (N).

R_1, R_2, R_3, R_4 = Rolling resistance forces on the wheels (N).

L_1, L_2, L_3, L_4 = Lateral forces acting on the wheels (N).

Let us now model each of these force acting on the tractor.

3.8 THE NORMAL FORCE (N)

The normal force is a vertical upward force that balances the weight of the tractor. This force determines the tractive effort obtainable, at the axle of the tractor, hence affecting acceleration, maximum speed, drawbar effort, etc. From the axis chosen on the tractor, this

force acts in the Z - axis direction and tends to balance any vertical downward force on the tractor.

In modelling this force, we have to consider the arbitrary forces that are significant in Z - X direction of the tractor, as shown below.

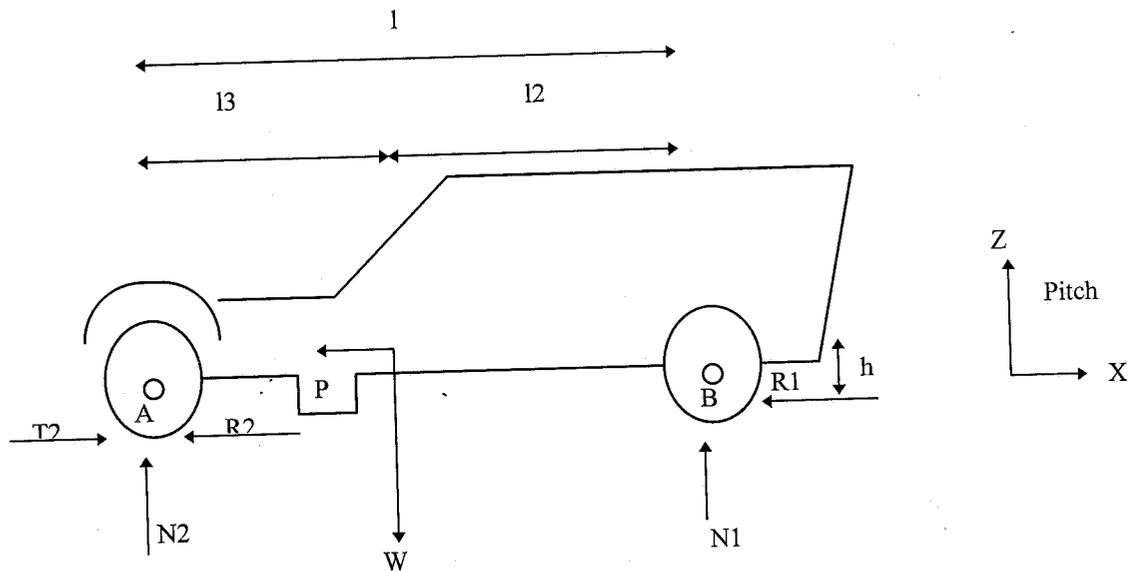


Figure 6 Side view of tractor with forces acting on it

W = weight of the tractor acting at the centre of gravity.

P = an inertia force (called d' Alembert's force) acting at the centre of gravity opposing the acceleration of the tractor.

The tractor force (T) and motion resistance R_1 and R_2 act in the ground plane in the tyre contact pitch.

N_1 and N_2 = the normal loads on the front and rear wheels of the tractor respectively.

From the figure, summing forces in the vertical direction

$$\Sigma f = 0$$

$$W = N_1 + N_2$$

The normal load on the front wheel can be found by summing moments about the point "A" under the rear tyre. Presuming that the tractor is accelerating in pitch, the sum of the torque at point "A" must be zero

$$\Sigma M_A = 0;$$

$$-h \cdot P + l_3 W - l \cdot N_1 = 0$$

$$-Ph + Wl_3 - N_1 l = 0$$

$$N_1 l = l_3 W - h \cdot P$$

$$N_1 = \frac{W \cdot l_3 - hP}{l}$$

but P, from Newton's second law is given as,

$$P = M \cdot a_x = (W/g) \cdot a_x$$

M = mass of the tractor (kg)

a_x = acceleration in the x - direction (m/s^2)

g = acceleration due to gravity (m/s^2)

$$= \frac{\left(Wl_3 - \frac{W}{g} a_x \cdot h \right)}{l}$$

$$N_1 = W \left(\frac{l_3}{l} - \frac{a_x \cdot h}{gl} \right)$$

h = height of centre of gravity from the surface.

N_2 can also be solved from, hence from similar equations about point "B" we get

$$\Sigma M_B = 0$$

$$-h \cdot P + l_2 \cdot W + l \cdot N_2 = 0$$

$$N_2 = \frac{Wl_2 + Ph}{l} = \frac{\left(Wl_2 + \frac{W}{g} a_x \cdot h \right)}{l}$$

$$N_2 = W \left(\frac{l_2}{l} + \frac{a_x h}{gl} \right) \quad (5)$$

From the above equation, the vehicle is accelerating on a level ground at a low speed.

From equation A and equation B, we can deduct that during acceleration; load is transferred from the front wheels to the rear wheels in proportion to the acceleration and the ratio of the centre of gravity height to the wheel base.

3.8 ROLLING RESISTANCE FORCE

Representing the normal load on the wheel of the tractor by "N", the motion resistance force on the upward wheel can be predicted by the formula

$$R = \mu_R N$$

R = motion resistance force (N)

μ_R = motion resistance ratio or rolling resistance ratio

N = normal load (N)

Hence, the motion or rolling resistance ratio,

$$\mu_R = \frac{R}{N} = \frac{\text{Rolling resistance force}}{\text{Normal load}}$$

The rolling or motion resistance of a pneumatic tyre is dependent on many factors like the load on the tyre, its size and structure, the strength of the soil and also operating condition (surface condition of the ground, inflation pressure of the tyre, speed, temperature of the tyre etc.). For soils that are not too soft and for tyres that are operated at nominal tyre inflation pressure (with static tyre deflection and about 20% of the undeflected section height the rolling resistance ratio, μ_R is given as

$$\mu_R = \frac{R}{N} = \frac{1.2}{C_n} + 0.04 \quad (6)$$

where

$$C_n = \text{wheel numeric} = \frac{C I \cdot b \cdot d}{N}$$

CI = cone index and it is the average force per unit base area required to force a cone shaped probe into the soil at a steady rate.

b = Tyre section width

d = overall tyre depth

For the above equation 6, the safe condition for using the equation is when $(b/d) \cong 0.3$.

3.9 TRACTION FORCE

When a driving torque is applied to a pneumatic tyre, a tractive force is developed at the tyre - ground patch. Due to compression on the tyre caused by the normal load, the distance that the tyre travels when subjected to a driving torque will be less than that it will travel in free rolling. The reduction in travel speed of the wheel is caused by slippage. Hence, slip is a function of the tractive force developed by a tyre and this tractive force is also proportional to the applied wheel torque under steady - state conditions.

The slip of the tyre is usually defined as

$$S = \left(1 - \frac{V_a}{V_t}\right) = \left(1 - \frac{V}{r\omega}\right) = (1 - r_e/r)$$

where

V_a = actual travel speed or linear speed.

V_t = theoretical wheel speed.

r = rolling radius of wheel on hard surface or free rolling tyre.

r_e = effective rolling radius of tyre.

ω = angular velocity of the wheel.

The traction force on the powered wheel is given by

$$T = \mu_T N$$

where

μ_T = coefficient of gross traction

N = normal load on wheel.

This traction force "T" is just the force or gross traction supplied by the wheel rotation or torque. The pull or net tractive motion force, H, is the gross tractive force minus the overcome resistance force (R).

H = Gross traction force - motion resistance force

$$= T_i - R_i$$

and the pull coefficient = H/N

The variation of gross traction force with soil strength and slip have been incorporated into a relationship and the effect of wheel load and tyre size was also considered.

The gross traction force is given as, $T = \mu_g N$

Coefficient of gross traction, $\mu_T = T/N$

$$\mu_T = T/N = 0.75 (1 - e^{-0.3 C_n s}) \quad (7)$$

where

S = slip

e = base of natural logarithm

C_n = wheel numeric

The pull, $H = T - R$ and

pull coefficient = $H/N = (T-R)/N$

$$H/N = (T/N) - (R/N)$$

$$T/N = 0.75(1 - e^{-0.3 C_n s}) \quad \text{from equation 5}$$

$$R/N = (1.2/C_n + 0.04) \quad \text{from equation 4}$$

Pull coefficient H/N

$$H/N = 0.75(1 - e^{-0.3 C_n s}) - (1.2/C_n + 0.04) \quad (8)$$

Note: The practical restriction adhered to are

$$(1) \frac{\text{Tyre section width}}{\text{overall tyre diameter}} = \frac{b}{a} \cong 0.3$$

$$(2) \frac{\text{Tyre deflection}}{\text{section height}} = \frac{\sigma}{n} \cong 0.2$$

$$(3) \frac{\text{Tyre radius}}{\text{overall diameter}} = \frac{r}{d} \cong 0.475$$

3.10 THE LATERAL FORCE (L)

The lateral force is modelled as a function of slip angle and cornering coefficient of the vehicle. It is a function of the radial force, camber angle inflation pressure, tyre

$$L = \mu_c N$$

where

L = lateral force

μ_c = lateral force coefficient which is a function of the slip angle " α "

N = normal load

The lateral force coefficient can be got from the following third order polynomial with constant coefficients:

$$\mu_c = A + B\alpha + C\alpha^2 + D\alpha^3$$

where

μ_c = lateral or cornering coefficient

α = slip angle

A, B, C, D are constants determined experimentally

The slip angle " α " is the angular difference between the direction the tyre is headed (as determined by the dynamic forces acting on the tractor) and the direction the tyre is travelling or steered.

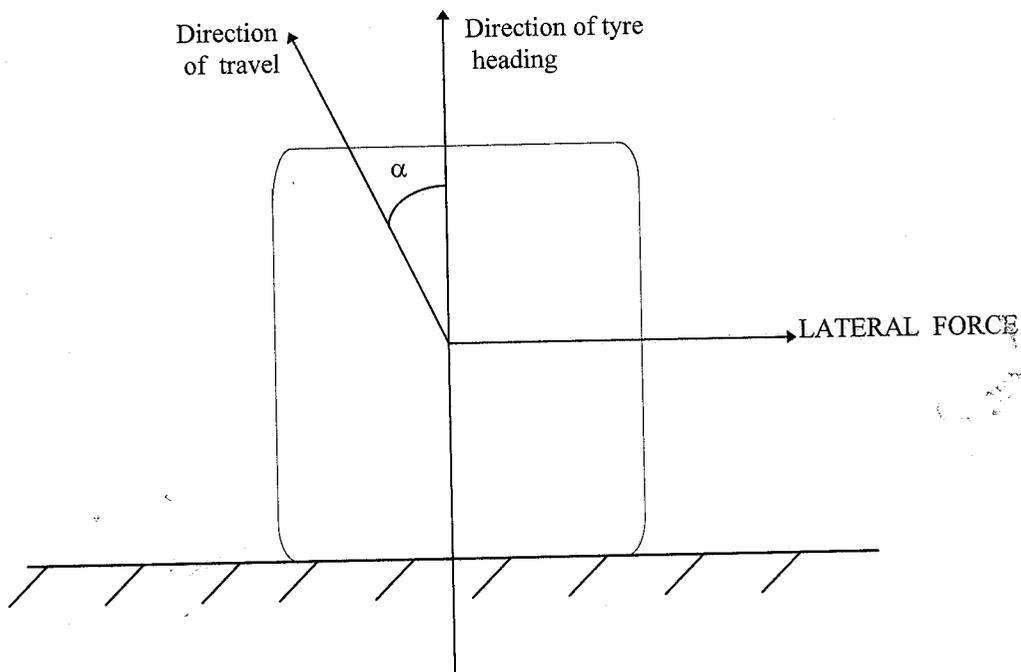


Figure 7 Tyre slip angle

$$\alpha = \tan^{-1} \frac{V_y}{V_x} - \delta \quad \text{where}$$

α = slip angle (rad)

V_y = lateral velocity (m/s)

V_x = longitudinal velocity (m/s)

$\tan^{-1} \frac{V_y}{V_x} - \delta$ defines the direction the tyre is heading while " δ " defines the direction the tyre is steered.

3.11 STABILITY EQUATIONS OF THE TRACTOR AT BRAKING INTERVALS

Braking forces are forces applied to the tractor wheel at intervals when deceleration is necessary. These forces act as opposing forces to the motion of the tractor. Regulation for agricultural tractors require the brakes to be fitted efficiently on at least two wheels of the tractor. The braking force are often activated from the brake pedal by the operator and this braking force depends on the sufficiency of the force exerted by the driver on the brake pedal. Considering the effect of these forces on the stability equation of the tractor, we have

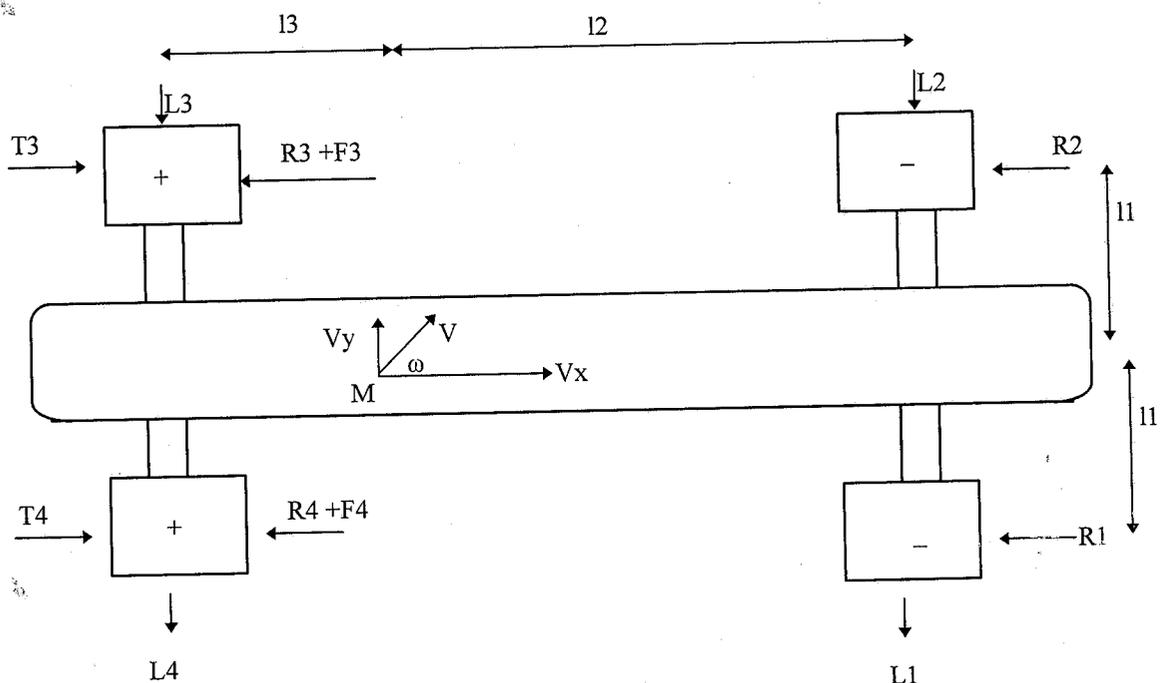


Figure 8 Braking forces and it's impact on tractor motion

The braking forces are only applied at the rear wheels as F_3 and F_4 . Resolving the forces in the horizontal direction gives

$$\Sigma f_x = 0$$

$$M(\dot{V}_x - V_y \omega) + T_3 + T_4 - R_1 - R_2 - (R_3 + F_3) - (R_4 + F_4) = 0$$

The forces on the lateral Y- direction will sum up to

$$\Sigma f_y = 0$$

$$M(V_y + V_x \omega) - L_1 - L_2 - L_3 - L_4 = 0 \quad (10)$$

Taking moment about the centre of gravity gives

$$\Sigma M = 0$$

$$\begin{aligned} -I_{zz} \omega + l_4 (T_3 - R_3 - F_3) - l_4 (T_4 - R_4 - F_4) - l_3 (L_3 + L_4) + l_2 (L_1 + L_2) + l_1 R_1 - l_1 R_2 &= 0 \\ I_{zz} \omega = l_4 (T_3 - R_3 - F_3 - T_4 + R_4 + F_4) - l_3 (L_3 + L_4) + l_2 (L_1 + L_2) + l_1 (R_1 - R_2) & \quad (11) \end{aligned}$$

Equations 9, 10, 11 are the differential equations of the motion governing the tractor at an instance when the brakes are applied. If the braking forces " F_3 " and " F_4 " are removed then the equations will return to equations 1, 2 and 3 respectively.

The braking forces are limited to lack of adhesion between the tyres and the ground.

Thus, Maximum $F_3 = \mu_f N_3$

$$F_4 = \mu_f N_4$$

where μ_f = coefficient of friction between the tyre and the ground.

3.12 STEERING INPUTS TO THE STABILITY EQUATIONS

Assuming that there is no steering inputs in the rear wheels (i.e. that only the front tyres are steered) and that the steer angles of the front wheels are δ_1 and δ_2 .

Let us consider the modification on the equation of the tractor motion

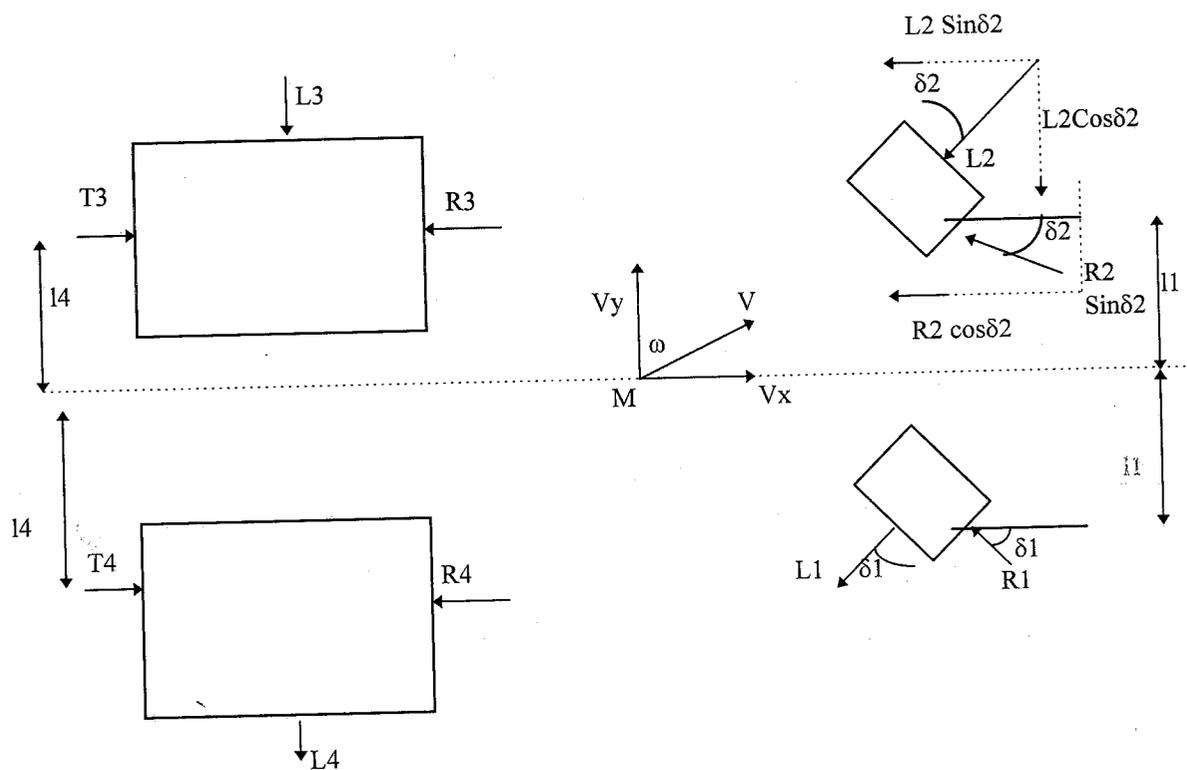


Figure 9 . Steering inputs

The equation of the plane motion of the tractor model above will be:

for x - axis,

$$\Sigma f_x = 0$$

$$M(\dot{V}_x - V_y \omega) = L_2 \sin \delta_2 + R_2 \cos \delta_2 + L_1 \sin \delta_1 - R_1 \cos \delta_1 - (T_3 - R_3) - T_4 - R_4 = 0 \quad (12)$$

For y - axis,

$$\Sigma f_y = 0$$

$$M(\dot{V}_y + V_x \omega) - L_3 - L_4 - L_2 \cos \delta_2 + R_2 \sin \delta_2 - L_1 \cos \delta_1 + R_1 \sin \delta_1 = 0$$

$$M(\dot{V}_y + V_x \omega) = L_3 + L_4 + L_2 \cos \delta_2 - R_2 \sin \delta_2 + L_1 \cos \delta_1 - R_1 \sin \delta_1 \quad (13)$$

Taking moment about the centre of gravity gives

$$\Sigma M = 0$$

$$\begin{aligned}
& -I_{zz} \dot{\omega} + l_4(T_3 - R_3) - l_4(T_4 - R_4) - l_3(L_3 + L_4) - l_1(L_2 \sin \delta_2) + l_2(L_2 \cos \delta_2) \\
& - l_2(R_2 \sin \delta_2) - l_1(R_2 \cos \delta_2) + l_1(R_1 \cos \delta_1) - l_2(R_1 \sin \delta_1) + l_1(L_1 \sin \delta_1) + l_2(L_1 \cos \delta_1) = 0
\end{aligned}$$

(14)

Equations 12, 13 and 14 are the differential equation of motion governing the three degrees of freedom of the tractor in longitudinal lateral and yaw motion respectively.

3.13 EFFECT OF BRAKING DURING CORNERING

As has been seen previously, braking forces applied at specific intervals during motion, increase the resistance to motion of the wheels of the tractor. Thus, the braking force, coupled with the motion resistance force acting on the wheel, tends to be greater than the tractive force on the wheel, forcing a deceleration.

When the tractor is negotiating a bend (with steer angles δ_1 and δ_2) and the operator has cause to decelerate (i.e. applying braking force), the resistance to motion "R" of the wheels with brakes will now be increased by the braking force applied. Hence the total resistance on the rear wheels (with brakes) will now be $(R_3 + F_3)$ and $(R_4 + F_4)$. Therefore, in the equilibrium equations of the tractor during a simultaneous cornering and application of braking force will then be a modification of equations 12, 13 and 14 by replacing or rather adding braking forces "F₃" and "F₄" to the rear wheel resistance "R₃" and "R₄" respectively. Thus,

$$\Sigma f_x = 0$$

$$\begin{aligned}
M(\dot{V}_x - V_y \omega) &= L_2 \sin \delta_2 + R_2 \cos \delta_2 + L_1 \sin \delta_2 \\
& - R_1 \cos \delta_1 - (T_3 - (R_3 + F_3)) - (T_4 - (R_4 + F_4)) = 0
\end{aligned} \tag{15}$$

$$\Sigma f_y = 0$$

$$\begin{aligned}
M(\dot{V}_y + V_x \omega) &= L_3 + L_4 + L_2 \cos \delta_2 - R_2 \sin \delta_2 + L_1 \cos \delta_1 \\
& - R_1 \sin \delta_1
\end{aligned}$$

$$M(\dot{V}_y + V_x \omega) = L_3 + L_4 + L_2 \cos \delta_2 - R_2 \sin \delta_2 + L_1 \cos \delta_1 - R_1 \sin \delta_1 \quad (16)$$

and

$$\begin{aligned} I_{zz} \dot{\omega} = & l_4(T_3 - (R_3 + F_3) - T_4 + (R_4 + F_4)) - l_3(L_3 + L_4) \\ & - l_1(L_2 \sin \delta_2 + R_2 \cos \delta_2 - R_1 \cos \delta_1 - L_1 \sin \delta_1) \\ & + l_2(L_2 \cos \delta_2 + L_1 \cos \delta_1 - R_2 \sin \delta_2 - R_1 \sin \delta_1) \end{aligned} \quad (17)$$

Equations 15, 16 and 17 are basically the equilibrium equations of motion of the tractor when it is decelerating at a bend with steer angles " δ_1 " and " δ_2 " if the braking force is removed, the traction force (T_i) will now be greater than the resistance force " R " on the powered wheels, hence the tractor will tend to accelerate. Thus, if $F_i = 0$, the equation of the motion of the tractor will be governed by equations 12, 13 and 14.

Also if there is no braking force applied and the tractor is moving in a straight longitudinal course (with steer angles - 0); the equation will return to equations 1, 2 and 3.

From equations 15, 16 and 17, making \dot{V}_x and \dot{V}_y the subject of the formulas respectively, we obtain:

$$\dot{V}_x = \frac{1}{M} \left(MV_y \omega + L_2 \sin \delta_2 + R_2 \cos \delta_2 + L_1 \sin \delta_1 - R_1 \cos \delta_1 - (T_3 - R_3 - F_3) - (T_4 - R_4 - F_4) \right) \quad (18)$$

$$\dot{V}_y = \frac{1}{M} \left(-MV_x \omega + L_3 + L_4 + L_2 \cos \delta_2 + L_1 \cos \delta_1 - R_2 \sin \delta_2 - R_1 \sin \delta_1 \right) \quad (19)$$

$$\dot{\omega} = \frac{1}{I_{zz}} \left[\begin{aligned} & l_4(T_3 - R_3 - F_3 - T_4 + R_4 + F_4) - l_3(L_3 + L_4) \\ & - l_1(L_2 \sin \delta_2 + R_2 \cos \delta_2 - R_1 \cos \delta_1 - L_1 \sin \delta_1) \\ & + l_2(L_2 \cos \delta_2 + L_1 \cos \delta_1 - R_2 \sin \delta_2 - R_1 \sin \delta_1) \end{aligned} \right] \quad (20)$$

Equations 18, 19 and 20 may be integrated to find the longitudinal velocity (V_x), the lateral velocity (V_y) and the yaw velocity (ω), resulting from a given time history of the front wheel steer angles δ_1 and δ_2 .

To find the trajectory of the centre of gravity of the tractor in the X, Y, Z space fixed system or global co-ordinate, we must first transform the velocity of the centre of gravity from the vehicle fixed system to the space fixed system.

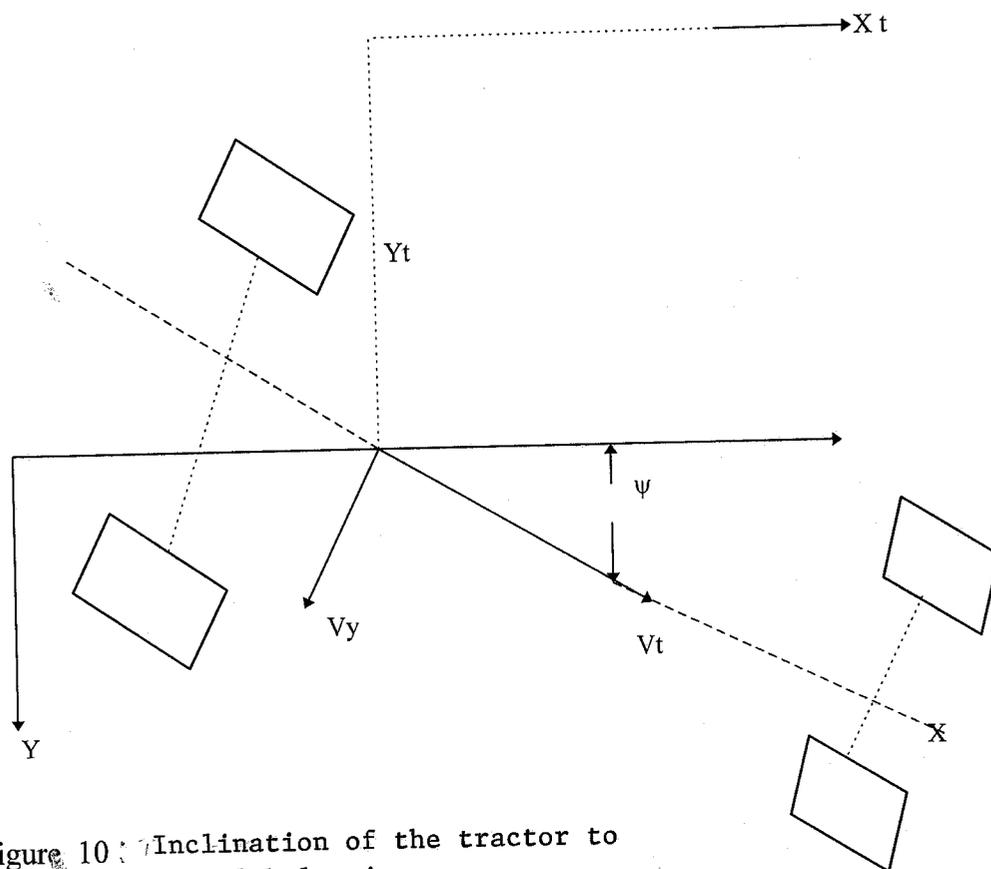


Figure 10: Inclination of the tractor to the global axis.

Assuming the tractors' x, y, system makes an angle Ψ with the global X, Y, Z system (see fig x), then the actual velocities in the global X, Y, Z directions will be:

$$\dot{X}_t = V_x \cos \psi - V_y \sin \psi \quad (21)$$

$$\dot{Y}_t = V_x \sin \psi + V_y \cos \psi \quad (22)$$

$$\dot{\psi} = \omega \quad (23)$$

Hence integration of the velocities \dot{X}_t , \dot{Y}_t and $\omega = \dot{\psi}$ determines the location of the centre of gravity of the tractor in the space fixed system and yaw angle (Ψ) of the tractor as a function of time.

These equations (equations 18 to 23) were then solved by integration using the computer and thus they formed the basic equation for the simulation methodology.

CHAPTER 4

SIMULATION

4.1 CONCEPT OF SIMULATION

In its literary term, simulation is a process whereby it is possible to model either functionally or mathematically, the behaviour of a real system. It is the operation of a model or simulator which is a representation of the system. This model should be amenable to manipulation which would be impossible, too expensive or impractical to perform on the entity it portrays. The operation of the model can be studied, and from it, properties concerning the behaviour of the actual system or its sub - system can be inferred.

4.2 RESEARCH METHODS AND TECHNIQUES INTO TRACTOR SAFETY

Better knowledge of the tractor dynamics and safety has always been achieved through the observation and analysis of the tractor under numerous operating conditions. Research into tractor stability and dynamics could be done through experimentation, computer simulation or computer simulation and experimentation. The use of experimental methods only is limited. These limitations arise based on the fact that such experiments would be slow and very expensive and even in some cases could be hazardous to both the life of the operator and the tractor itself. Also, it will be difficult to repeat such experiments under numerous operating conditions.

For this reason, computer simulation is considered a better and safer method of studying the tractor dynamics and stability. Through computer simulation, response of tractors to external disturbances and given terrain conditions are easily predicted. But simulation only, though unique, cannot ascertain the validity of the simulated model to true life occurrences. Hence, computer simulation and experimentation is adjudged one of the most reliable methods for the study of tractor stability and dynamics. Therefore, in validating

the computer simulated models of the tractor, limited experiments using scale model or full - size tractors are always conducted.

4.3 FORMULATION OF COMPUTER PROGRAM

The formulation of computer program for the purpose of conducting computer simulation and experiments requires that special consideration be given to the following activities:

- (a) Draw a flowchart outlining the logical sequence of events to be carried out by the computer.
- (b) Choose a computer code that will be used to run the experiments on a computer. For the purpose of this project, C++ language was used.
- (c) The computer language compiler should be that which provides a powerful error checking technique.
- (d) Input data and starting conditions should be assigned values since computer programs are by nature dynamic experiments.

4.4 STABILITY PARAMETERS FOR SIMULATION

The tractor and tyre parameters are variables identified to influence tractor stability and hence can lead to overturning. These parameters were grouped into five types and tabulated as shown below. The value for these parameters were either determined or adopted from literature.

Parameter affecting tractor stability

	(Type	Parameter	Symbol	Value
	Dimension			
A	STATIC	(1) Tractor mass	M	Kg
		(2) Centre of mass height	h	m
		(3) Wheel track	2 l ₄	m
		(4) Wheel base	l	m
B	DYNAMICS	(5) Inertia	I _{zz}	kgm ²
		(6) Effective rolling radius	r _e	m
		(7) Tyre width	b	m
		(8) rolling radius on hard surface	r	m
C	INITIAL CONDITIONS	(9) Initial velocity of centre of mass	V _o	m/s
		(10) Angular velocity	ω	rad/s
		(11) Longitudinal acceleration of centre of mass	a ₂	m/s ²
		(12) Angular acceleration	ω̇	rad/s ²
D	DRIVEN CONTROLLED	(13) Lateral forces		N
		(14) Steering angles	δ ₁ , δ ₂	rad
		(15) Braking forces	f	N
		(16) Traction forces	T	N
E	TERRAIN PROPERTIES	(17) Gross coefficient of traction	μ _t	
		(18) Lateral force coefficient	μ _c	
		(19) Tyre slip angle	α	rad
		(20) Rolling resistance	R	N
		(21) Cone index	CI	N/m ²

4.5 SIMULATION OF THE STABILITY CHARACTERISTICS OF THE TRACTOR

The mathematical model derived in Chapter 3 (equations 18 to 23) were translated into a computer program using C++ - language and a number of simulations were conducted. The programme performs, integration of the differential equations using a fourth order Runge - Kutta Grill method (RKGM).

The basic formulae from this RKGM are:

$$K_1 = h * f(t_0, V[0])$$

$$K_2 = h * f(t_0 + h/2, V[0] + K_1/2)$$

$$K_3 = h * f(t_0 + h/2, V[0] + K_2/2)$$

$$K_4 = h * f(t_0 + h, V[0] + K_3)$$

$$\Delta V_0 = (K_1 + 2K_2 + 2K_3 + K_4)/6$$

$$V(i) = V_0 + \Delta V_0$$

$V[0]$ = initial value of velocity and distance

t_0 = initial value of time

h = increment in time

ΔV_0 = increment in the initial value of the velocities and distances

$V(i)$ = new values of velocities and distances.

4.6 THE PROGRAM

Before the program was formulated, many intermediate steps were taken which included:

- (i) Identification of the problem: this is just what the program is expected to do. The computer program is expected to calculate the velocities and distances moved by a tractor in its longitudinal lateral and yaw motions with respect to time.

(ii) Analysis of the problem. The tractor modelled is a rear wheel driven tractor similar to those assembled in Nigeria. Initial velocities and distances are set on the stability equations and the tractor parameters inputted into the equations. Runge Kutta Grill method was then used to solve the differential equations of the acceleration and velocities to get the respective velocities and distances in the 3 - degrees of freedom studied.

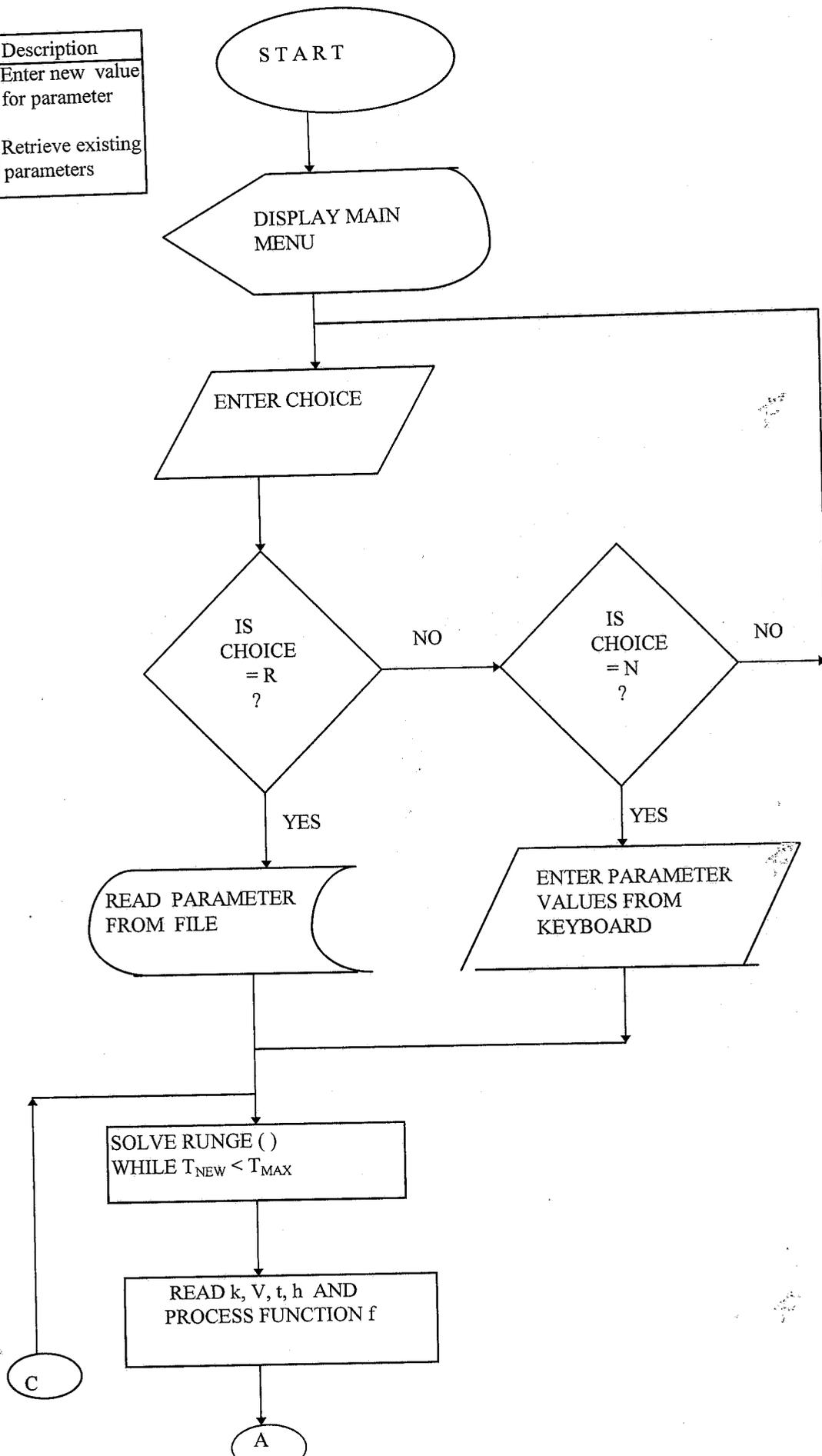
(iii) Description of the program

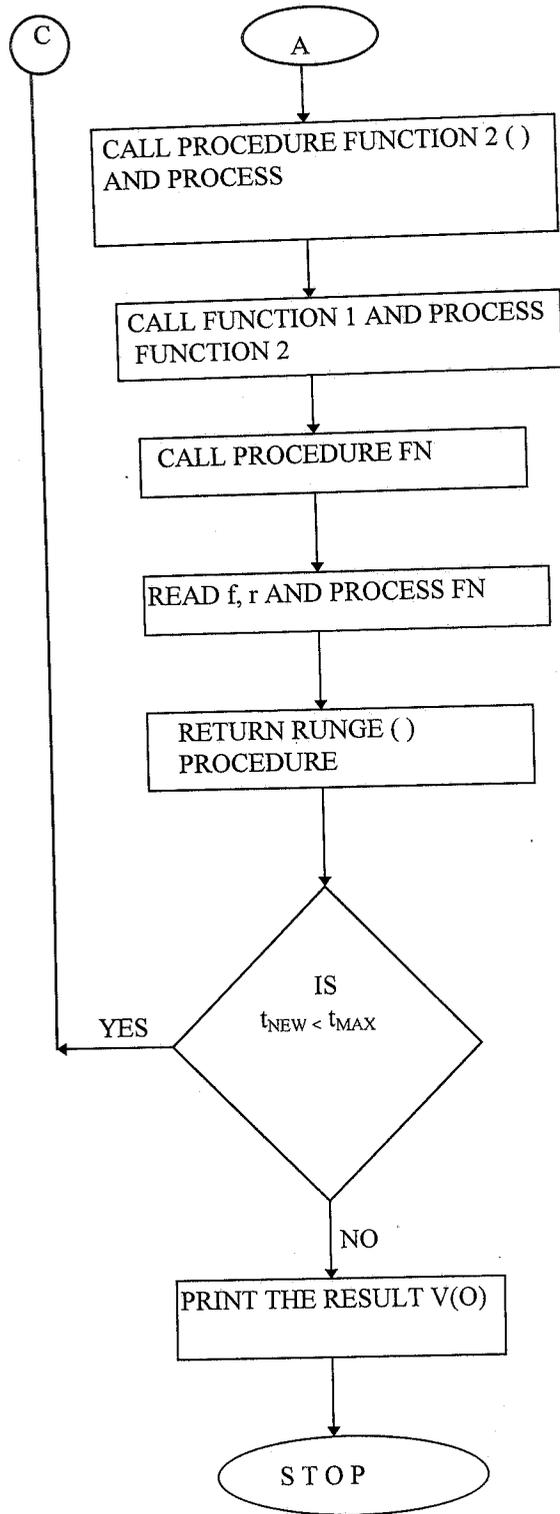
Algorithm

- Input tractor parameters
 - Initialise velocities and distances
 - Calculate normal loads, N_f and N_r (function f_N)
 - Calculate slip angles for different tyres (function f_q)
 - Use the slip angle to calculate the lateral force coefficients and hence lateral forces
 - Calculate wheel numeric for front and rear wheels
 - Solve for wheel slip, μ_T, μ_R, T, R
 - Solve for constants A, B, C (function 2)
 - Use Runge to integrate the six (6) equations
 - Print results
- Function 1

4.7 FLOWCHART OF THE PROGRAM

Choice	Description
N	Enter new value for parameter
R	Retrieve existing parameters





4.8 RESULTS

For $f_r = 0$, $\delta_1 = 0$, $\delta_2 = 0$

t (s)	V_x (m/s)	x (m)
0.2	0.00426	0.00213
0.4	0.0085	0.00852
0.6	0.0128	0.0192
0.8	0.0171	0.0341
1.0	0.024	0.0533
1.2	0.257	0.0768
1.4	0.0303	0.105
1.6	0.0343	0.137
1.8	0.0387	0.173

For $f_r = 0$, $\delta_1 = 0.175$ rad, $\delta_2 = 0.157$ rad

t (s)	V_x (m/s)	x (m)
0.0	0.00	0.00
0.2	0.00391	0.00199
0.4	0.00774	0.00779
0.6	0.0117	0.0174
0.8	0.0159	0.0310
1.0	0.0204	0.0488
1.2	0.252	0.0707
1.4	0.0305	0.0971
1.6	0.0361	0.128
1.8	0.0425	0.165

For $f_r = 650 \text{ N}$, $\delta_1 = 0$, $\delta_2 = 0$

t (s)	V_x (m/s)	x (m)
0.0	2.00	2.00
0.2	2.70	2.08
0.4	3.02	2.21
0.6	2.82	5.11
0.8	2.19	7.65
1.0	1.23	9.36
1.2	0.11	10.30
1.4	-1.01	10.90

For $f_r = 30 \text{ N}$, $\delta_1 = 0$, $\delta_2 = 0 \text{ rad}$

t (s)	V_x (m/s)	x (m)
0.0	0.00	0.00
0.2	0.0166	0.00823
0.4	0.0331	0.0331
0.6	0.0497	0.0745
0.8	0.0662	0.132
1.0	0.0828	0.207
1.2	0.0993	0.298
1.4	0.1160	0.405
1.6	0.1320	0.530
1.8	0.1490	0.670

Graph 1

scale

1cm to 10 units - y axis
1cm to 0-1 units - x - axis

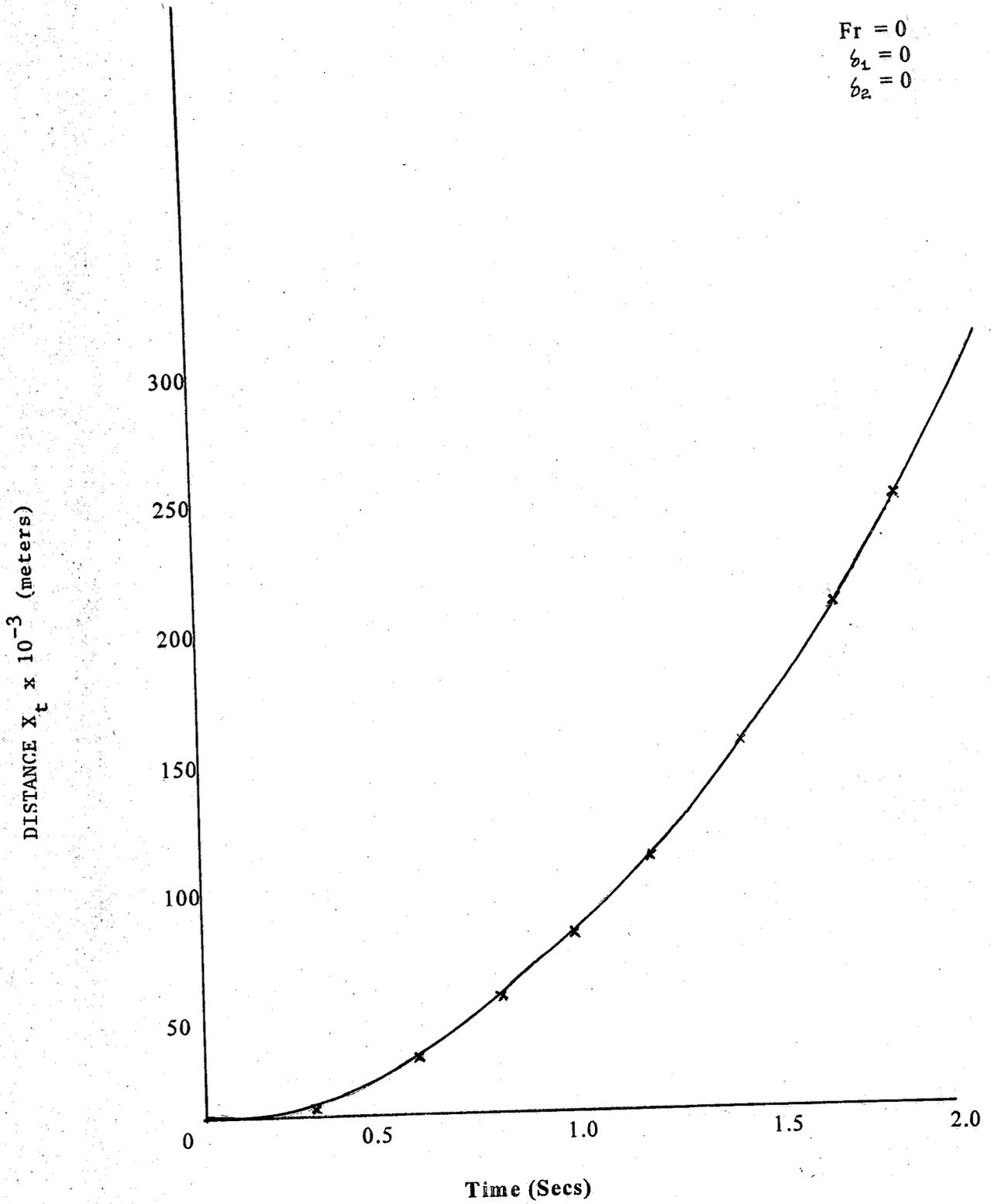


Fig 11: distance - time graph. (No steering or braking input).

1cm to 2 units - y - axis
1cm to 0.1 unit - x - axis

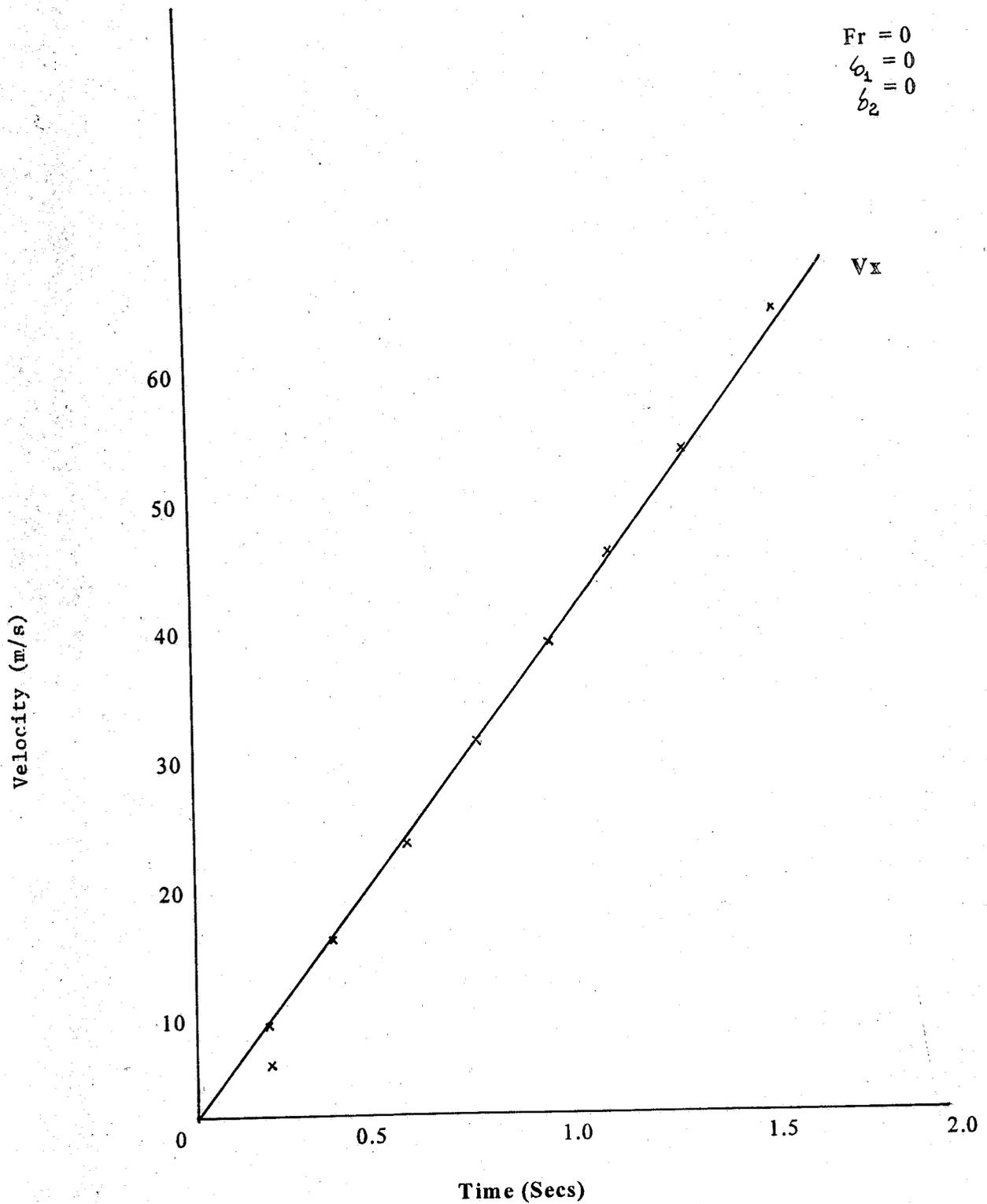


Fig 12: Velocity - Time Graph (No steering or breaking input).

Graph 3

scale

1cm to 10 units - y - axis

1cm to 0.1 unit - x - axis

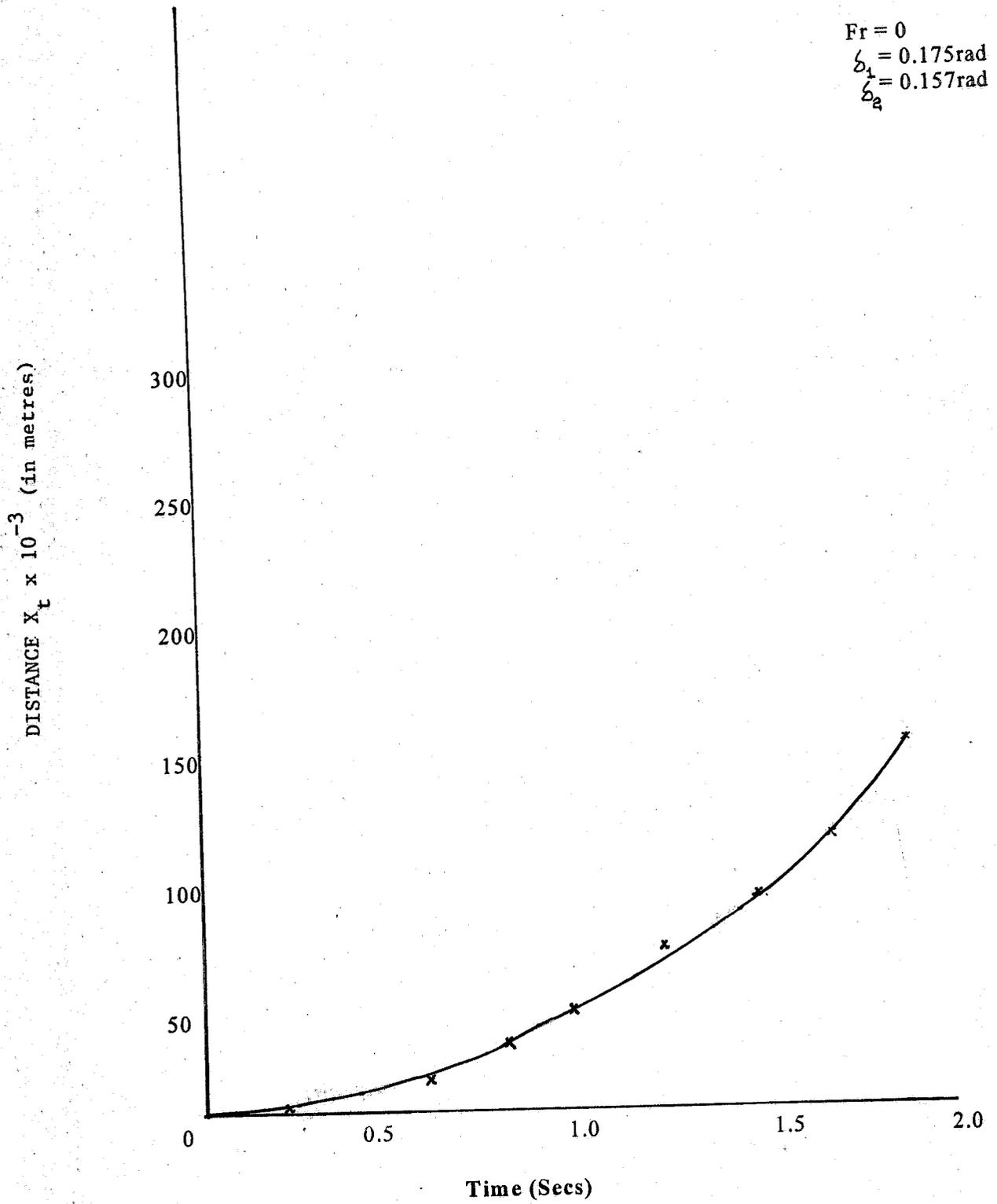


Fig 13: Effect of steering on tractor velocity

SCALE 1 CM TO 2 UNITS - $\dot{\gamma}$ AXIS
 1 CM TO 0.1 UNITS - x AXIS

$f = 0$
 $\delta_1 = 0.175 \text{ rad.}$
 $\delta_2 = 0.154 \text{ rad.}$

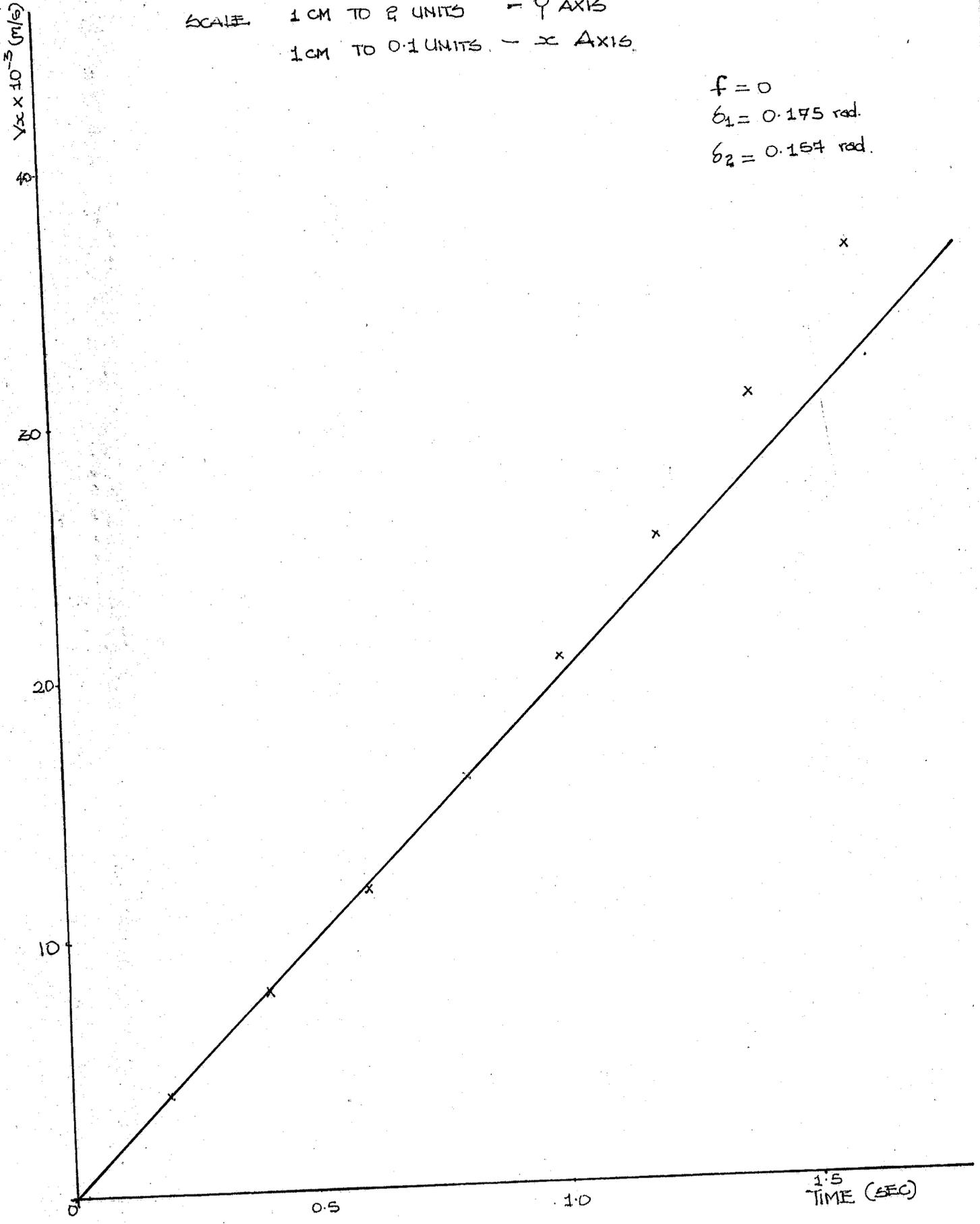


Fig 14: Velocity - Time Graph (small steering input)

C: graph of distance against time (m/s)
D: graph of velocity against time (m/s²)

scale
1cm to 20 units - y axis
1cm to 0.1 units - x-axis

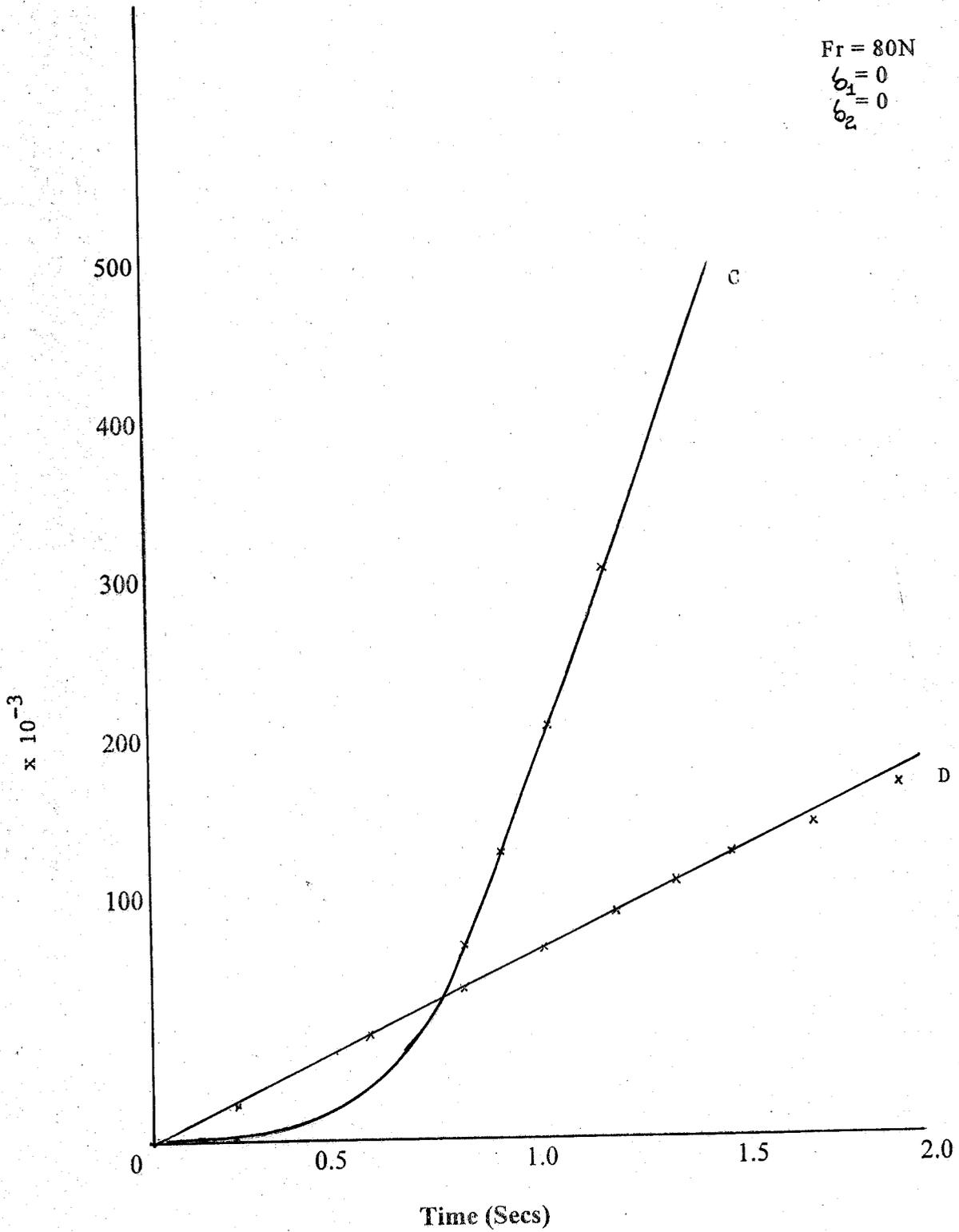


Fig 15: Low Breking force effect on the dynamic behaviour of the tractor

scale

A: graph of Distance against time(m/s)

1cm to 0.5 units - y-axis

B: Graph of velocity against time(m/s²)

1cm to 0.1 units - x-axis

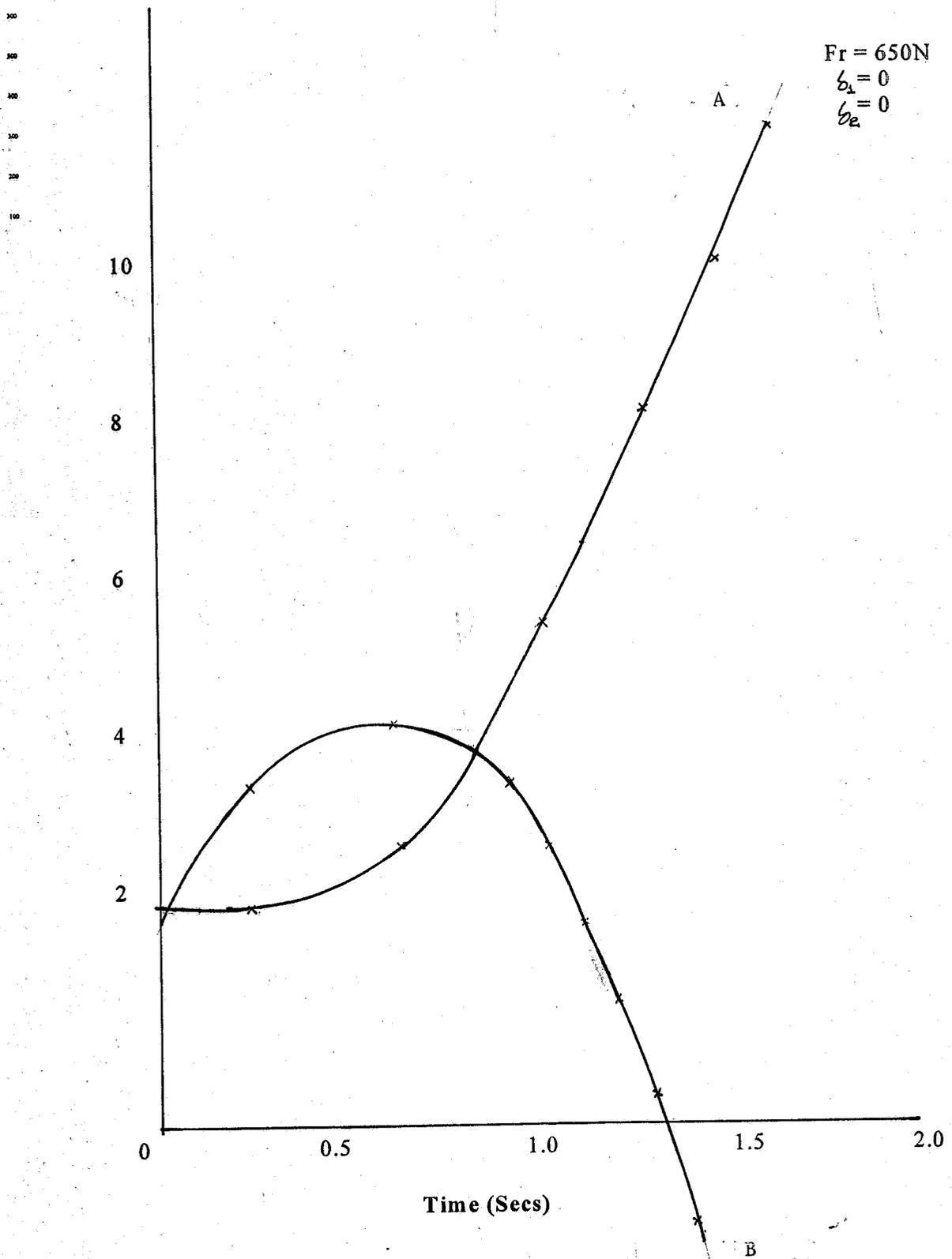


Fig 16: Velocity and acceleration behaviour of the tractor.

4.9 DISCUSSION OF RESULTS

From the results, we could see that when there is no braking and steering inputs (graph 1), the tractor speed increases gradually with time (slope of the distance time graph). This speed is very slow initially but smoothly increases with time. The safe region is taken to be the points below the curve and this safe region reduces as the speed increases. The tractor is seen to move with constant acceleration (graph 2). At small steer angles (graph 3), the relative velocity of the tractor does not change much and the stability region reduces slightly due to the lateral input of the steer angle. It was also seen that stability region reduces with increasing steer angle.

When a small braking force was applied (graph 5), it can be seen from the graph that it was not enough to induce a deceleration but the safety region of the tractor increases. The slope of the velocity time graph reduce though there is still constant acceleration. When a high braking force was introduced during the course of motion of the tractor, it was found that the braking impact did not cause an instant deceleration. This can be attributed to the resistance offered by the force of inertia. When this was overcome, the tractor started decreasing in speed (graph 6).

The above results represent a set of tractor parameters only. The area of the safety region is expected to change with variations in these parameters e.g. an increase in the tractor mass is expected to cause a remarkable change in the overall stability region and also on the velocity of the tractor.

CHAPTER 5

EXPERIMENTATION

Limited experiments on the braking and cornering effects on the tractor stability were conducted to validate the computer simulation. The tractor used in the experiment is a rear-wheel driven type (model 666DT manufactured by FIAT Company Ltd).

The weights and dimensions of the tractor used in the investigations were either measured or got from the tractor manual (literature) and are given in the table below where

Weight and dimensions of tractor

W	l_1	l_2	l_3	l_4	l	h
(kg)	in metres					
2580	0.7	1.54	0.77	0.8	2.208	0.7

Weight of tractor operator = 67 kg.

5.1 EXPERIMENTAL PROCEDURE AND RESULTS

Braking test

The braking test was carried out on :

- (i) Tarmac surfaces (tarred) similar to that of many public roads.
- (ii) Untarred paved road with known cone index.

A reference line was marked out on the road where the tractor will pass - bye. The tractor approaching this line was forced to decelerate immediately the tyres touched the marked line (by the actuation of the brake pedal force). The distance the tractor moved before coming to rest and the time taken from the application of the brakes to the stopping of the

tractor were also recorded using a stop watch. The decelerating speed was calculated by dividing the braking distance with the time.

The experiment was then repeated many times and the results are tabulated below.

5.2 EXPERIMENTAL RESULTS

S/NO	Tarmac		Untarred paved road	
	Braking distance (m)	Time (sec)	Braking distance (m)	Time (sec)
1	1.61	1.78	0.457	0.28
2	2.36	1.84	1.219	0.81
3	4.27	2.13	2.286	1.59
4	3.83	2.54	5.486	1.79
5	7.23	2.16	8.230	2.46
6	5.67	2.18	8.077	2.50
7	4.70	1.50	10.973	1.97
8	3.76	1.69		

5.3 DISCUSSION

The distance - time graph of both experiments gives a second - order curve with variable gradient along the curve. They also show a marked but tolerable deviation from an ideal distance - time relationship obtained from the simulation in chapter 4. It can be seen that the velocity or the slope of the graph in the tarmac condition is steeper than that obtained when the untarred road is considered. This can be attributed to the difference in their cone index. The tarmac surface has a higher cone index value, thus, its resistance to motion is lower than that offered by the untarred surface.

untarred paved road

scale

1cm to 0.5 units - y - axis

1cm to 0.2 units - x - axis

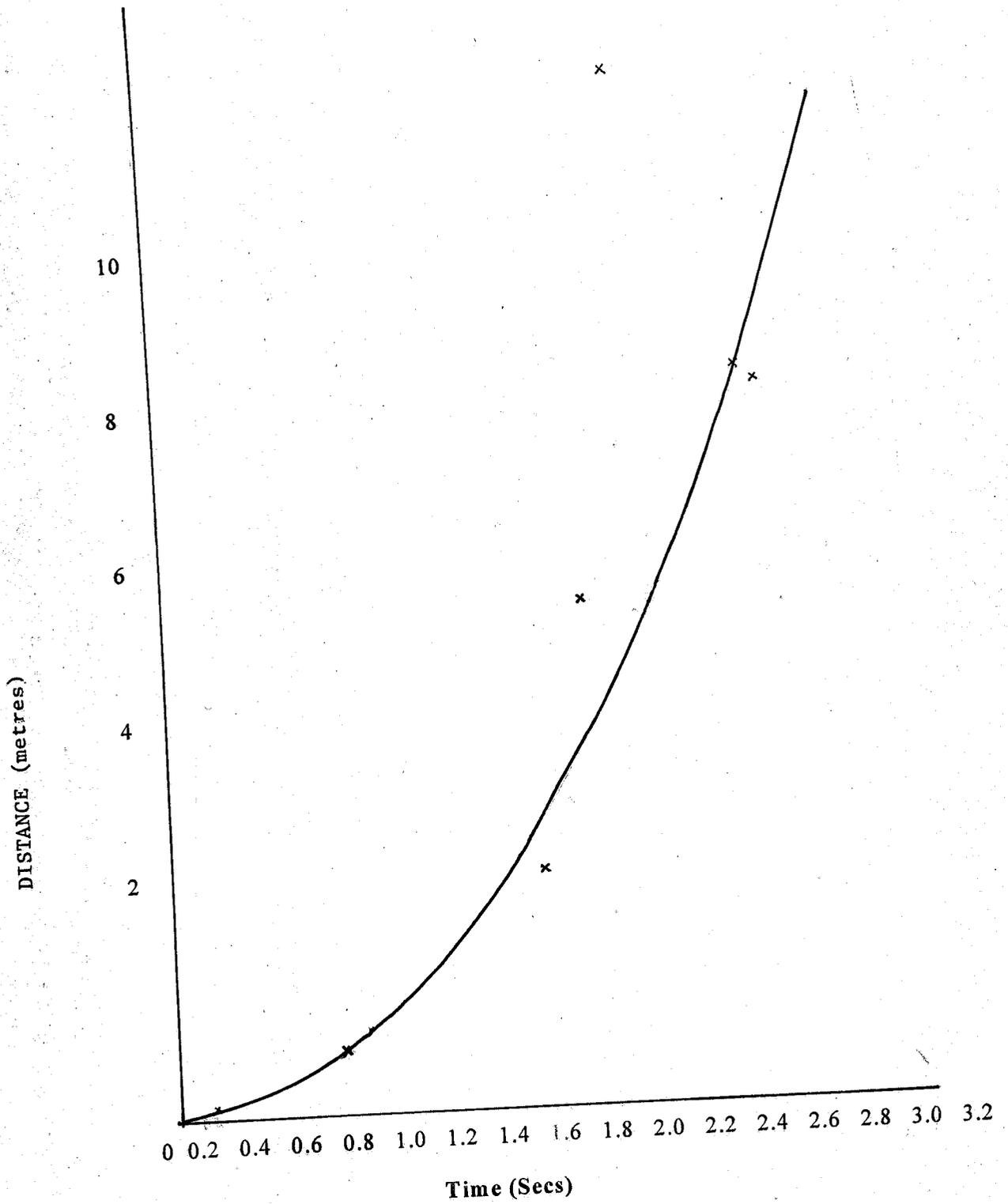
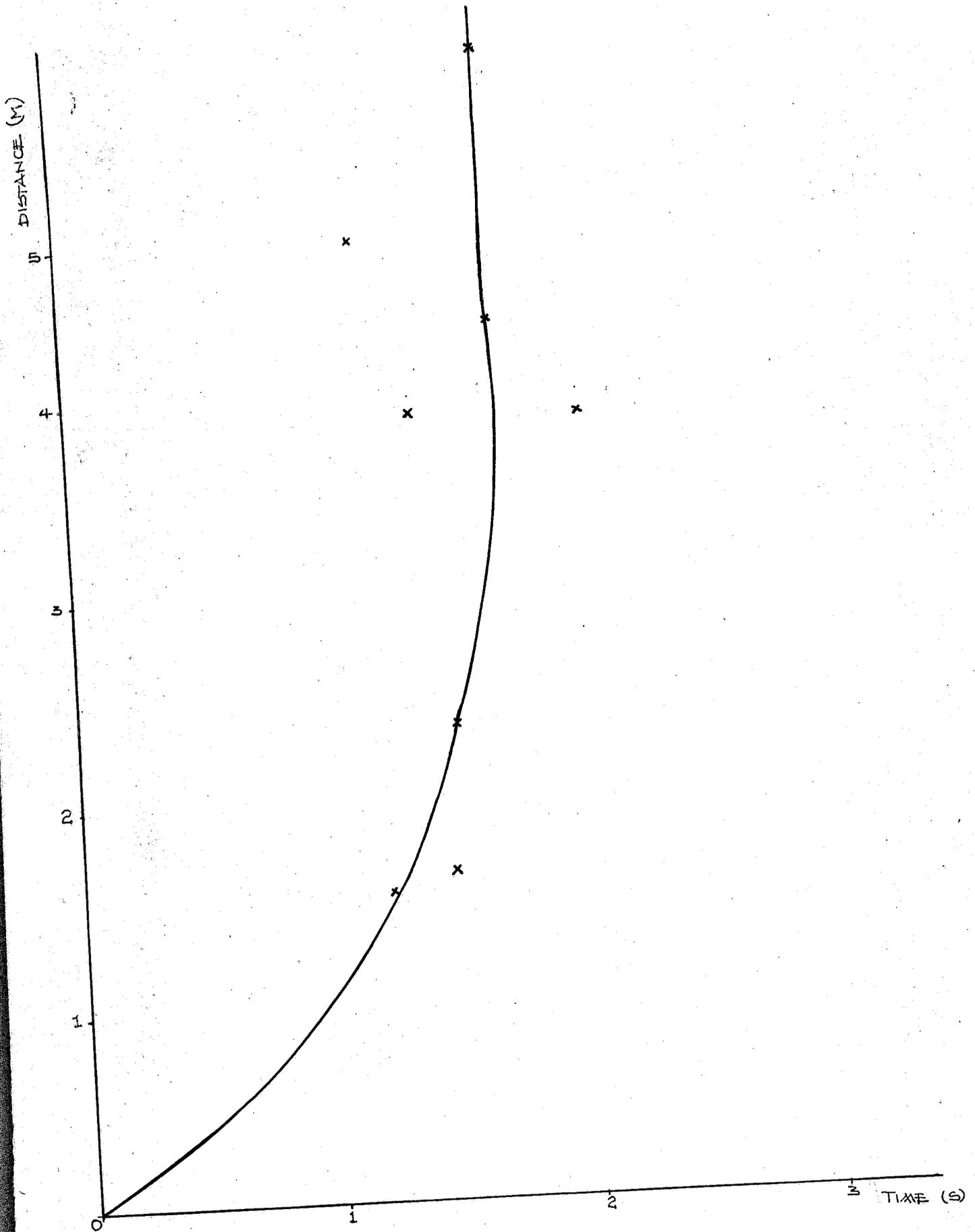


Fig 17: Motion of the tractor on untarred paved road

GRAPH B

SCALE = 25 cm to 0.5 UNITS
BOTH AXIS

TARMAC ROAD.



The initial velocity of the experimental data is higher than that of the simulated data, though the shape and slope of the curves has close correlation. Some of the factors that could possibly caused these deviations and imperfections are:

- (i) Parallax error in the measurement of the braking distance.
- (ii) Slight delay between the actuation of the timing clock and the build - up of the braking force.
- (iii) Human error in maintaining a constant force on the brake pedal.

CHAPTER 6

CONCLUSION AND RECOMMENDATION

Computer simulation of the tractor has been conducted for the purpose of establishing its stability region and the design parameters that affect the safe regions. From the simulation, it is seen that the dynamic stability of the tractor is a relative term, depending on the value of the stability parameters at different operating conditions. Generally, it has been seen that the tractor is more stable at lower speed than higher speed.

Also, from the simulation and experiment conducted, it can be concluded that most assembling plants in Nigeria, consider to a large extent, our operating conditions before designing the tractors. This can be seen in the behaviour of the tractor especially during braking which shows that the problem of "Jackknifing" (caused by locking of the tractor wheels during braking) has been taken care of.

Also, since the safety regions of the tractor reduces with increasing speed, the design of the tractor to operate at low speed is like a "stability check".

Finally, it can be concluded that since the simulated data has close correlation with experimental data, the simulation methodology is thus satisfactory.

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THE PROGRAM

```
/* A PROGRAM THAT CALCULATES THE VELOCITY AND DISTANCE MOVED BY  
A TRACTOR IN IT'S LONGITUDINAL,LATERAL AND YAW MOTIONS **/
```

```
# Include <stdio.h>  
# Include <stdlib.h>  
# Include <conio.h>  
# Include <math.h>
```

```
type def Struct {  
double R[4];  
double T;  
double Q[4];  
double F;  
} Force;  
type def struct {  
double A;  
double B;  
double C;  
} Constant;
```

```
void paramtr(void);  
void readinput(void);  
void readout(void);  
void Runge(void);  
Constant funct2(void);  
force funct1(void);  
f(k, V, t, h);  
double fN(char a);  
double fQ(int m);
```

```
double ax,ci,b1,d1,h,g,rc,r,W,l,  $\delta_1$ ,  $\delta_2$ , l1, l2, l3, l4, F, Izz, m,  
Wc, V[30], A, B, C, b2, d2
```

```
Main ()  
{  
Char ch, Pc;  
clrscr ();  
Printf ("Enter (R) to retrieve existing parameters values \n");  
Printf (" OR (N) to create new parameters ");  
Pc = (to upper (scanf ("%C", &ch)));  
If (Pc == 'R') {  
write output ();  
}  
else if  
(Pc == 'N');  
{  
read parameter ();  
read input ();  
}  
Runge ();  
Clrscr ();  
return;
```

Void parameter (void)

```
{
printf("\n enter ax →"); scanf ("%f", & ax);
/* ax is the longitudinal acceleration of the tractor */
printf("\n enter CI →"); scanf ("%f", & CI);
/* CI is the cone index of the soil */
printf("\n enter the front tyre section width, b1 →"); scanf ("%f", & b1);
printf("\n enter the overall front tyre diameter, d1 →"); scanf ("%f", & d1);
printf("\n enter the height of centre of gravity, h →"); scanf ("%f", & h);
printf("\n acceleration due to gravity, g →"); scanf ("%f", & g);
printf("\n effective rolling radius, re →"); scanf ("%f", & re);
printf("\n free rolling radius of tyre, r →"); scanf ("%f", & r);
printf("\n Weight of tractor, W →"); scanf ("%f", & W);
printf("\n Tractor wheel base, l →"); scanf ("%f", & l);
printf("\n steer angle of one of the front tyre, δ1 →"); scanf ("%f", & δ1);
printf("\n steer angle of the other front tyre, δ2 →"); scanf ("%f", & δ2);
printf("\n enter length, l1 →"); scanf ("%f", & l1);
printf("\n enter length, l2 →"); scanf ("%f", & l2);
printf("\n enter length, l3 →"); scanf ("%f", & l3);
printf("\n enter length, l4 →"); scanf ("%f", & l4);
printf("\n enter value for braking force, Fr →"); scanf ("%f", & Fr);
printf("\n enter value for moment of inertia about Z -axis, Izz →");
scanf ("%f", & Izz);
printf("\n mass of the tractor, m = "); scanf ("%f", & m);
printf("\n values for the lateral coefficient constants=A,B,C "); scanf ("%f %f %f", & A,
&B, &C);
printf("\n enter rear tyre section width, b2 →"); scanf ("%f", & b2);
printf("\n enter the overall rear tyre diameter, d2 →"); scanf ("%f", & d2);

return
}
```

Void read input (void)

```
{
FILE * input file;
if((input file = fopen("||paramtr.doc", "a+"))==NULL)
printf("file could not be opened |n");
else
{

fprintf(inputfile, "%f %f %f %f %f %f", dx, CI, b, d, h);
fprintf(inputfile, "%f %f %f %f %f", g, re, r, W, l);
fprintf(inputfile, "%f %f %f %f %f", δ1, δ2, l1, l2, l3);
fprintf(inputfile, "%f %f %f %f %f", l4, Fr, Izz, m, Wc);
fprintf(inputfile, "%f %f %f", X, Y, Z);

}
fclose (input file);
return;
}
```

void write output(void) {

```

if((outputfile=fopen("\\paramtr.doc", "r")==NULL)
printf("file could not be opened\n");

```

```

else

```

```

{

```

```

fscanf(outputfile, "%lf %lf %lf %lf %lf", &ax, &CI, &b, &d, &h);
fscanf(outputfile, "%lf %lf %lf %lf %lf", &g, &re, &r, &W, &l);
fscanf(outputfile, "%lf %lf %lf %lf %lf", &δ1, &δ2, &l1, &l2, &l3);
fscanf(outputfile, "%lf %lf %lf %lf %lf", &l4, &F, &Izz, &M, &Wc);
fscanf(outputfile, "%lf %lf %lf", &X, &Y, &Z);

```

```

return;

```

```

}

```

```

Void Runge (void){

```

```

int I, no_of_eqns, j, k, n, ns;

```

```

double k1[11], k2[11], k3[11], k4[11], h, hh, Pi, t_old, t_max, t_mid, t_new, Va[30];

```

```

Void f();

```

```

Clrscr();

```

```

FILE * resultfile;

```

```

if((resultfile = fopen ("\\result.doc "a+")==NULL)

```

```

printf ("file could not be opened\n");

```

```

else

```

```

{

```

```

printf("\n fourth-order kunge kutta scheme \n");

```

```

printf("\n for a set of differential equation\n");

```

```

while (1) {

```

```

no_of_eqns = 6; /*number of equations*/

```

```

for (I=1; I<=no_of_eqns; I++)

```

```

V[i] = 0.0; /*initial conditions of the velocities and distances at t=0*/

```

```

printf("\n interval of t for printing ?");

```

```

scanf ("%lf", &Pi);

```

```

printf ("number of steps in one printing interval ?");

```

```

scanf ("%d", &ns);

```

```

printf ("maximum t to stop calculation ?");

```

```

scanf ("%lf", &t_max);

```

```

h = Pi/ns

```

```

printf ("h=%g \n", h);

```

```

t_new=0; /*initializing time*/

```

```

/* t_new: t for the new point */

```

```

hh=h/2;

```

```

printf(" t      V1      V2      V3      V4      V5      V6 \n");
fprintf(resultfile, "%s", V1      V2      V3      V4      V5      V6 \n");

```

```

printf ("%5.2lf", t_new);

```

```

fprintf(resultfile, "%5.2lf", t_new);

```

```

for (I=1; i<=no_of_eqns; i++)

```

```

printf ("%7.5e", V[i])

```

```

fprintf(resultfile, "%7.5e", V[i]);

```

```

do {
    for (n=1; n<=ns; n++)
    {
        t_old = t_new;      /* t_old; t for previous point*/
        t_new = t_new + h;  /* new time */
        t_mid = t_old + hh; /*midpoint time*/

```

```

for (i=1; i<=no_of_eqns; i++)
    Va[i]=V[i];
f(k1, Va, st_old, &h);

```

```

for (i=1; i<=no_of_eqns; i++)
    Va[i] = V[i] + k1(i)/2;
f(k2, Va, st_mid, &h);

```

```

for (i=1; i<=o_of_eqns; i++)
    Va[i] = V[i] + k2(i)/2;
f(k3, Va, st_mid, &h);

```

```

for (i=1; i<=o_of_eqns; i++)
    Va[i] = V[i] + k3(i);
f(k4, Va, &t_mid, &h);
for (i=1; i<=o_of_eqns; i++)
    V[i] = V[i] + (k1(i) + k2(i) + k3(i) + k4(i)) /6;
};

```

```

printf("%5.2lf", t_new);
fprintf(resultfile, "%lf", t_new);
for ( i=1; i<=no_of_eqns; I++) {
    printf("%7.5e", V[i]);
    fprintf(resultfile, "%7.5e", V(i);
    print ("\n")
}
}

```

```

while (t_new<t_max);
print(type 1 to continue, or 0 to stop.\n");
scanf("%d",&k);
if(k!=1) exit (0);
}
}

```

```

void f(k, V, t, h)
double k[ ], V[ ], t, h;
constant Cstant;
double A, B, C;
cstant = funct 2 ( );
    A = cstant. A;
    B = cstant. B;
    C = cstant. C;

```

```

K[1] = (V(3)*V(2) + A/m)**h;
K[2] = (-V(3)*V(1) + B/m)**h;
K[3] = Wc**h;

```

```
K[5] = (V(1)*sin(V(6)) + V(2)* cos((V(6))**h;
k[6] = V(3)**h;
```

```
/* V(1) is longitudinal velocity (Vx)*
/* V(2) is lateral velocity (Vy)*
/* V(3) is angular velocity (ω)*
```

```
return
}
```

```
constant funct 2 (void) {
constant cont_var;
force load;
double Rf, Rr, Tr, Qr, Qfl, Qrl, Fr;
```

```
load = funct 1 ();
Rf = load . R[1]; /*motion resistance force*/
Rr = load . R[2]; /* on front and rear wheels respectively*/
Tr = load . T; /* Traction force*/
Qfl = load . Q[1]; /*lateral force*/
Qrl = load . Q[2]; /* on different*/
Qr = load . Q[3]; /*wheels*/
fr = load . f; /*braking force*/
```

```
cont_var.A = Qrl * sin(δ2) + Rf * cos(δ2) + Qfl * sin(δ1) - Rf * cos(δ1) - (Tr - (Rr + Fr));
cont_var.B = Qr + Qr + Qrl * cos(δ2) + Qfl * cos(δ1) - Rf * sin(δ2) - Rf * sin(δ1);
cont_var.C = l4 * (Tr - Rr - Fr - Tr + Rr + Fr) - l3 * (Qrl * sin(δ2) + Rf * cos(δ2)
- Rf * cos(δ1) - Qfl * sin(δ1)] + l2 * (Qrl * cos(δ2) + Qrl * cos(δ1)
- Rf * sin(δ2) - Rf * sin(δ1));
```

```
Wc = (Cont_var.C)/Izz
```

```
/* Izz is the moment of inertia about z- axis */
```

```
Return (cont_var);
}
```

```
Force = funct 1 (void)
{
force force-var;
double Cn, μr, μT, μn2, Nf, Nr, S, μc1, μc2, μc3;
```

```
/* Cn1 = wheel numeric for front wheels*/
/* Cn2 = wheel numeric for rear wheels*/
/* μr = coefficient of rolling resistance force*/
/* μT = coefficient of traction*/
/* Nf = Normal load on front wheel*/
/* Nr = Normal load on rear wheel*/
/* S = tyre slip*/
/* μc = coefficient of lateral force*/
```

```
Nf = fN('f');
```

$$\mu_r = 1.2 / C_{n1} + 0.04;$$

$$\text{force_var.R}[1] = \mu_r * N_f;$$

$$\text{force_var.R}[2] = \mu_r * N_r;$$

$$C_{n2} = (CI * b_2 * d_2) / N_r;$$

$$S = 1 - re/r;$$

$$\mu_T = 0.75(1 - \exp(-0.3 * C_{n2} * s));$$

$$\text{force_var.T} = \mu_T * N_r;$$

$$\text{force_var.F} = F_i;$$

$$\mu_{c1} = A + B * fQ(1) + C * \text{Pow}(fQ(1), 2);$$

$$\mu_{c2} = A + B * fQ(2) + C * \text{Pow}(fQ(2), 2);$$

$$\mu_{c3} = A + B * fQ(3) + C * \text{Pow}(fQ(3), 2);$$

$$\text{force_var.Q}[1] = \mu_{c1} * N_f;$$

$$\text{force_var.Q}[2] = \mu_{c2} * N_f;$$

$$\text{force_var.Q}[3] = \mu_{c3} * N_r;$$

return (force_var);

}

double fQ(int m) {

double α , V;

/* α is slip angle of wheel */

$\alpha = 0.0;$

V = V(2)/V(1);

if (m == 1)

$\alpha = \text{atan}(V) - \delta_1;$

else if (m == 2)

$\alpha = \text{atan}(V) - \delta_2;$

else if (m == 3)

$\alpha = \text{atan}(V);$

else

$\alpha = \alpha;$

return (α)

}

double fN(char a)

{

double N;

if (a == 'f' !! a == 'f')

$N = W * [l_2/l - (a_x * h)/(g * l)];$

else

$N = W * [l_3/l + (a_x * h)/(g * l)];$

return (N);

}