MATHEMATICAL MODELLING AND DYNAMIC SIMULATION OF LEAKAGES IN A STORAGE TANK SYSTEM

BY

YAHAYA IBRAHIM BAJINI

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A FINAL YEAR PROJECT WORK PRESENTED TO THE DEPARTMENT OF CHEMICAL ENGINEERING IN PATIAL FULFILMENT OF THE REQUIREMENT FOR THE AWARD OF BACHELOR OF ENGINEERING (B.ENG) DEGREE IN CHEMICAL.

NOVEMBER, 2007.

DECLARATION

I, Yahaya Ibrahim B of The Department of Chemical Engineering, School of Engineering and Engineering Technology, Federal University of Technology Minna hereby declare that this research work is my personal undertaking, executed under the guidance of my supervisor (Prof. J. O Odigure). All information utilized and their sources have been duly acknowledged.

28th November, 2007

Yahaya Ibrahim B

2001/11675EH

Date

CERTIFICATION

This is to certify that this project work titled "Mathematical Modelling and Dynamic Simulation of Leakages in a Storage Tank System" has been presented by Yahaya Ibrahim B to the Department of Chemical Engineering, School of Engineering and Engineering Technology, Federal University of Technology Minna, Niger State.

This is in partial fulfillment of the award of Bachelor Degree in Chemical Engineering.

______________________	27/11/17 ______________
Prof. J.O. Odigure	Date
Project Supervisor	
______________________	______________________
Dr M.O. Edoga	Date
H.O.D	
______________________	______________________
External Supervisor	Date

DEDICATION

This project is dedicated wholly to God Almighty, the reason why I am alive, who equally sustained me through out my stay in this institution. And also to my loving parents Rev & Mrs. Yahaya Bajini.

ACKNOWLEGDEMENT

In honour to God and with the joy of salvation in my heart, I want to express my profound gratitude to the lord God Almighty for sustaining me throughout my stay in this institution.

My special appreciation also goes to my loving parents, Rev & Mrs. Yahaya Bajini, and all my brothers, sisters, nephews, nieces and all relative for their assistance, encouragement and constant prayers to see that I made it. The lord will never forget your labour of love.

My sincere gratitude also goes to my intelligent, hardworking and dynamic supervisor Prof J.O. Odigure for his kindness, guidance, moral support and constructive criticism which led to the successful completion if this project, sir thank you for your patience and understanding.

I equally want to appreciate the entire lecturers of this great and noble department under the able leadership of Dr M.O. Edoga (H.O.D). I would not forget all my level advisers Engr. Abdulfatai Jimoh, Engr. S.O. Azeez, Engr. Kovo, Mr D. Paul, and Engr E. Eterego, thank you.

My acknowledgement goes to Engr. Hassan Funsho Akande and Mr Micheal of Maths/Computer Department, your contributions to this work is just wonderful. Also to my colleagues under the supervision of Prof J.O. Odigure and the entire graduating students of this department.

I also want to appreciate the entire membership of ECWA Student Ministry (ESM), you guys are wonderful. To all my good friends, your names are too numerous to mention, may God bless you all.

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SYMBOLS

M = Mass

m = Mass flow rate

T = Time

ρ = Density

V = Volume

V = Velocity

A = Area

L = Length

D = Diameter

R = Radius

h = Height

g = acceleration due to gravity

u = Internal energy

z = elevation

P = Pressure

Pa = Atmospheric Pressure

ABSTRACT

The work involves the modelling and simulation of leakages in a horizontal cylindrical storage tank. An experiment using a tank of 5m diameter, 7m length, and initial liquid height (level) of 4.25m. It was subjected to leak, and data was collected at interval of time (0.33 hr). A theoretical (physical) model was developed, which enables one to predict the rate at which the liquid in the tank will leak, and the amount of liquid that will leaked. From the comparism of experimental and simulated data (using mathcad software), the values are closed enough and the correlation is 0.99476 (99.5%). It is recommended that the developed model be used in managing leakages in storage tanks.

CHAPTER ONE

1.0 INTRODUCTION

In recent years, one of the most critical problems facing the global community has been technological disasters, and the consequence of leakages from energy sources. The situation is aggravated by failure of systems and criminal activities. These circumstances require an efficient solution to the problem of improving safety in the industrially developed nations. Thus there is a need of reassessment of the importance of solving problems related to increasing of safety levels and more efficient environmental friendly enterprises in the energy sector.

Among the main components of such enterprises are fluid storage tanks. Nowadays, storage tanks are becoming notably less reliable; as a result, the environment is subjected to increasingly harmful effects. Industrial objects characterized by technological (deliberate or accidental) emissions of fluid mixtures into the atmosphere, land or in water are often located near populated areas. Storage of large quantities of hazardous substances entails risk for the population, the environment and the surrounding area, following established town planning practices. Leakage of hazardous substances into the soil can lead to expensive decontamination process.

One of the main reasons for storage tank failure is corrosion. Over time, uncontrolled corrosion can weaken or destroy components, resulting in holes or possible structural failure and the release of stored products into the environment.

All the above facts demonstrate that the problem of increasing environmental safety of storage tank is urgent indeed.

In view of modern level of industrial development, solving this problem of increasing safety requires the application of numerical simulation methods. These methods allow detailed integrated analysis of complicated technological system to be made with high level of reliability and accuracy.

1.2 AIM OF THE PROJECT

This project work aims at developing a mathematical model to detect leakage, calculate the amount of liquid leaked the rate of leakage and hence predict the mention parameters with time in a liquid storage tank system.

1.3 SCOPE OF THE PROJECT

This project is to detect the presence of leakage using dynamics of height in a horizontal cylindrical storage tank, calculate the rate of leakage and the amount of liquid leaked.

CHAPTER TWO

2.0 LITRATURE REVIEW

2.1 Storage Tanks

Storage tanks are generally classified into Aboveground Storage Tank (AST) and Underground Storage Tank (UST).

2.1.1 Underground Storage Tank (UST)

A UST is a tank, and any underground piping system connected to the tank that has at least ten percent (10%) of its combined volume underground. Underground Storage Tanks generally fails into three different types:

- Fiberglass Reinforcement Tanks
- Steel Tanks
- Composite material tanks which are incorporated, usually a steel tank covered in fiberglass or a plastic compound for corrosion protection and to form an interstitial space

2.1.2 Aboveground Storage Tank (AST)

The AST is located generally above the ground level by means of supports. These aboveground storage tanks are used in the storage of various types of fluids. They are manufactured from tough linear or crosslink polyethylene which are durable enough to withstand extreme service environments and harsh outdoor weather. They can also be manufactured from steel and other metallic materials. AST might be rotationally moulded from a seamless piece of tank that will not lead, rust, chip or corrode.

2.1.3. Uses

In a specific sense, tanks can also be classified base on uses as;

- Galvanized corrugated and stainless corrugated tanks, which are mainly used for ground water storage in house hold, irrigation, and fire protection.

- Bolted tanks which are fabricated in accordance with standards and are utilized in rural water supply and housing subdivisions.
- Welded tanks which are epoxy coated and stainless steel welded pressure vessels, designed for portable water storage, recycled water, waste water processing, ammonia diffusion and other industrial water processing applications.
- Double wall tanks which are containment solutions for management of waste oil, transmission fluids, motor oil, and other hazardous flammable liquids.

2.1.4 STORAGE TANK CONFIGURATIONS.

Standard storage tank configurations include closed top, open top, cone bottom, cylindrical (horizontal or vertical) and a number of other specialty tanks. These tanks can be safely used for portable water storage, gray water storage, fertilizer storage, chemical storage, petroleum (and its components) storage and numerous other liquid storage applications.

2.2 LIQIUD STORED IN TANKS.

Basically, all liquids manufactured in large (and in some cases smaller) quantities need a storage device in which they can be stored for further or future use. Few examples of such liquids include water, petroleum and its refine fractions petrochemicals, etc.

2.3 LEAK CAUSES.

Leakages in a stored tank system are caused by a number of factors, which include;

- In-Tank Corrosion; Storage tanks usually fail from rust perforation due to several effect of water the tank like in the case of heating oil. Also combination of water and sulphur, bacterial action and other factors contribute to in-tank corrosion, which result in leakage.
- External rust; leaks can occur due to rust, decay or frost shift or at piping connections and leaking fittings.

- Corrosive soil; liquid storage tank (more especially steel tanks) are more likely to leak if buried in a corrosive soil, by the action and effect of the soil on the metal used in casting the tanks.
- Inadequate fill or Vent pipe diameter is also blamed for leak in AST and UST, asserting that because liquid tanks are filled under pressure from the liquid delivery pümper, a corroded, damage or poorly-plumped storage tank or one with a too small Vent opening may not withstand the pressure the filling process.
- Filling tank supports, sight gauges, valves and other equipment problems.
- Accidentally broken lines, tank tipping over, or lines severed due to falling ice, snow removal or landscaping.
- Not inspecting and maintaining the tank on regular (routine) basis.

2.4 RAPTURE HAZARD FROM LIQUID STORAGE TANK

Over the past few years, there have been several catastrophic failures of liquid fertilizer storage tanks resulting in property damage and environmental contamination. These ruptures have involved site-erected storage tanks with capacities ranging from 500,000 to 1.5 million-gallons. The tank failures, which prompted this alert, were all built by either Carolyn Equipment Company of Fairfield, Ohio, or Nationwide Tanks Inc. of Hamilton, Ohio. Both of these companies have since gone out business. (Carolyn Equipment in 1990 and Nationwide Tanks in 1995.) This alert describes some of the tank failures and identifies standards and precautions that apply to aboveground liquid storage tanks. Owners of tanks produced by these two manufacturers are advised to take extra precautions to guard against tank failure.

NOTE: Though all failed storage tanks cited in this alert have been produced by these two companies, owners of all storage tanks should be aware of the risks associated with operating a storage tank.

2.4.1 Some Selected Accident History

3/1997 in Iowa - A 1-million gallon tank containing ammonium phosphate ruptured and released its contents. The walls of the ruptured tank fell onto two other tanks and broke their valves. One tank contained 1- million gallons of a nitrogen liquid fertilizer and the other tank held ammonium Thiosulfate. Much of the release was contained by an earthen dike, but immediate construction of a secondary, temporary dike was necessary to keep the release from flowing into the nearby Missouri River. Cleanup involved pumping the liquid out of the dikes and removing all contaminated soil.

7/1999 in Michigan - A 1-million gallon tank full of ammonium polyphosphate ruptured and damaged three other tanks. Fortunately, the tanks were surrounded by earthen dikes lined with polyethylene. This minimized the environmental damage.

1/8/2000 in Ohio - A 1-million gallon tank of liquid fertilizer ruptured an damaged four adjacent tanks. The wave of liquid broke a concrete dike wall and hit five tractor-trailer rigs, pushing two of the rigs into the river. A total of 990,000 gallons of material were released. More than 800,000 gallons of the liquid spilled into the Ohio River. Sampling detected amounts of the fertilizer mixture 100 miles downstream, which is expected to increase algae growth in the river. The company has discontinued use of seven other tanks.

3/8/2000 in Ohio - At the same facility, a 1.5- million gallon tank of ammonium phosphate ruptured and damaged three nearby tanks causing them to leak. Two of the damaged tanks held phosphoric acid and the third one held 'Ice-Melt', a magnesium chloride mixture. The released liquid overflowed the dike walls into nearby creeks. The four tanks were dismantled after the incident. Over 1.8 million gallons of contaminant were recovered, with an additional 450,000 gallons of contaminated water recovered from the sewer system. The release caused evacuation of a nearby school, and the public was forced to use bottled water because of concern that the drinking water supply may be contaminated by the spilled chemicals.

2.4.2 Hazard Awareness

Defective Welds: In the incidents cited, all of the above-ground liquid storage tanks that failed appeared to have had defective welds. The tanks were all produced by either Carolyn Equipment Company or Nationwide Tanks Incorporated. Both companies have since gone out of business. The tanks were under warranty for only one year, and the welding of the tanks was done by subcontractors hired by the two companies. The companies built tanks in Michigan, Ohio, Indiana, Illinois, Missouri, and Iowa between 1980 and 1995. Because of increased frequency in tank failures, the Ohio Fire Division is creating a voluntary registry of liquid storage tanks to help track and prevent similar failures.

Chemicals Involved: The failed tanks have held liquid fertilizers, such as ammonium phosphate, which are not considered hazardous and are not regulated by the U.S. Environmental Protection Agency. However, the failure of these tanks can damage nearby tanks containing hazardous substances and cause releases. In some cases, accidents have involved tanks containing hazardous materials like anhydrous ammonia and phosphoric acid, which are used to produce the fertilizer ammonium phosphate. Hazard Identification Facilities should evaluate their storage tanks for potential catastrophic failure. Some of the factors to consider include:

- Manufacturer's record for quality workmanship.
- Evidence of weakened or defective welds.
- Signs of corrosion around the base and direct contact with ground and exposed to moisture.
- Exposure to high winds or frequent precipitation.
- Age of the tank.
- Close proximity to other storage tanks containing hazardous chemicals.

2.4.3 Hazard Reduction/Prevention

The failure of liquid storage tanks can stem from inadequate tank design, construction, inspection, and maintenance. Hazard reduction and prevention starts with good design and construction. The risk to tanks already in service can be reduced through tank maintenance and weld inspection. To minimize effects from possible tank failures, there should be a secondary containment such as a dike or a beam surrounding the tank.

2.4.4 Tank Design and Construction

A tank should be designed and constructed according to API-650, "*Welded Steel Tanks for Oil Storage*:" issued by the American Petroleum Institute (API). API-650 specifies an allowance for corrosion and for the specific gravity of the fertilizer liquid. In each of the tank failures mentioned, welding has been the main cause of failure. To ensure durability and integrity, it is imperative that the tank is welded correctly. Several standards and specifications outline the proper techniques and procedures for welding including API-653,

2.4.5 Tank Inspection, Repair, Alteration, and Reconstruction

Operational Hazards and Maintenance: Tank buyers should insist on seeing the inspection record. Although tanks should undergo a rigorous inspection by a recognized inspection authority before a manufacturer's job is complete, the tanks should still be closely inspected by the buyer prior to purchasing the unit. For liquid storage tanks, the most important item to look for is complete penetration and complete fusion of the welds joining shell plates. Once a tank has been purchased, it becomes the tank owner's duty to regularly inspect the tank. Inspection intervals may be set by using a risk-based inspection theory, as indicated by API-653. Various inspection methods can be used for those tanks already in service. Radiography is the technique applied to all tanks designed to API-650 to ensure that complete penetration and fusion of welded joints has occurred. Unfortunately, this procedure cannot detect poor mechanical properties in the welded regions. This and other standards cover what types of joints must be checked by a radiograph, as well as the number of tests that must be done. Additional inspections may be done visually or by a vacuum box for localized problems. The vacuum box, approximately 6 inches by 30 inches, is tightly sealed to the tank surface, and pressure is applied. Automated ultrasonic testing can be applied to all shell welds to examine for cracks, fusion and penetration, and porosity with greater resolution than radiography. It is also now possible to conduct floor scanning while the tank is full. Combined with chemical analysis and hardness testing, field replication can assess the toughness, or resistance to brittle failure of a weldment. If damage is found during an inspection, this needs to be assessed in accordance with API- RP579 *"Fitness for Service"* methodology. Any tanks that do not meet the acceptance requirements set by API-RP579 should be repaired or replaced.

2.4.6 Steps for Safety

Here are some additional ways to prevent rupture of liquid storage tanks:

- Realize the inherent risk of using and maintaining any storage tanks.

- Identify the manufacturers of the tanks on the property, being careful to identify any tanks built by either company mentioned in this alert. **NOTE:** If tanks were manufactured by Carolyn Equipment Company or Nationwide Tanks of Hamilton, take the following actions immediately:

- A close external inspection should be made for leaks, corrosion, or any anomalies in the surface of the tank. Vent(s) should be checked for any blockages by foreign materials, such as snow or ice. The majority of the failures have occurred during the winter months, when steel becomes more brittle and when vents can become blocked by snow and ice. If liquid is drawn out of the tank when vent are plugged or restricted, a vacuum may be pulled on the tank causing it to collapse inward.
    - If you find evidence of leakage or corrosion during the inspection, the tank should be taken out of service and if possible, drained.

- If there is no evidence of leakage or corrosion, arrange for an external evaluation by a qualified inspection agency.
    - Depending on the results of the evaluation, arrange for an internal inspection immediately or within the year.

-Ensure that employees are aware of the hazards associated with the failure of a liquid storage tank.

- Avoid overfilling tanks.

- Perform regular inspections of tanks. Be sure to look for all possible risks.

- Follow up on identified problems with repairs or replacement. Inspections are otherwise useless.

- Replace, repair, or modify any and all tanks not meeting the standards set forth in API-579, "*Fitness for Service*" methodology.

- Be on the alert for new tank regulations. (There were recently changes made to API-653 that improved the suggested calculations)

- Consider better mitigation in case of a leak to separate the content of a collapsing tank from the rest of the facility, and more importantly, prevent any leakage from going offsite.

- Develop an emergency plan that addresses a catastrophic tank failure. Information Resources

2.5 LEVEL MEASUREMENT.

A wide variety of instruments are available for measuring the level of liquid in a storage tank. Level measurement instrument includes, dipsticks, float operation, pressure measuring devices, capacitive devices, ultra sonic level gauge, radiations, vibrating level sensor and hot wire element.

2.6 LEAK DETECTION SYSTEMS.

A method for detecting and measuring leaks in an underground container such as a tank and piping containing a liquid, comprising:

- Measuring the pressure and temperature of the liquid over a predetermining time interval,

- Simultaneously measuring the liquid level in the container and the temperature of the liquid therein over the same predetermined time interval,
- Calculating from each said measuring step the leak rate of the liquid from said container, and comparing the two calculated leak rates with each other.

Accurate liquid level and contents data is vital in the process industries where inventories, batching and process efficiency are critical measurements.

Leak detection can be accomplish by variety of methods ranging from tank testing with inventory controls (requiring daily measurement), to installation monitoring equipment around the tank. Leaking storage tanks are sources of toxics in the environment.

2.6.1 SIGNS OF LEAKING TANK.

Depending on the type of fluid stored in a tank, leakage can cause indoor or air pollution and can potentially contaminants the soil, surface, ground and drinking water. Often, the UST release will not be detected unless the tank is removed from the ground. Also, the following scenarios may indicate a leaking tank,

- Death or dying vegetation around the area of the tank.
- Oily rainbow sheen on water beneath the tank, basement sump or footing drain discharge.
- Dark stained soiled compared to the surrounding native soil.
- Odour emanating from the storage tank area.
- Straining around the filled pipe.
- Increase in the rate of consumption or removal of liquid from the storage tank.

2.6.2 METHODS OF LEAK DETECTION.

There is no one leak detection system that best suits all tank sites, or a particular sensor technology that is the most appropriate for all tank installation and their unique characteristics. Identifying the best leak detection choice for any storage tank system depends on a number of factors including cost (both initial installation cost and long term operation and maintenance cost), facility requirement for piping runs and interior or exterior tank site, temperature ranges; and availability of experience installers etc.

2.6.2.1 LIQUID LEVEL INDICATION METHOD.

The basic liquid indication can be accomplished with a float switch design, usually mounted on the tank top, extending down into the tank to the lowest point where the empty indication is desired. This would be the simplest configuration for obtaining a low-level signal. This could also be accomplished by mounting a single-point float switch horizontally into the side of the tank. Such single devices are two wire electrical hook ups, and it is easy to select the position on the tank for installation.

The main consideration that needs to be made about a storage tank construction is that the walls on top of the tank must not be constructed of carbon steel, since the material will cause interface with magnet and the reed switch within the float switch. Float switches have been successfully used for both over fill and leak detection with petroleum product, gasoline, jet fuel, motor oil, kerosene, diesel fuel, and ethanol and or methanol blend.

2.6.2.2 POINT LIQUID LEVEL DETECTION

Reliable liquid level detection for high and low alarm or pump control is essential in industry. Overflows can be both dangerous and costly, while empty vessels lead to pumps or downstream processes running dry.

It is important to select a level switch to suit the application, which is why Mobrey offers a choice of technologies and products, including the original Mobrey float operated liquid level switches, vibrating fork liquid level switches and ultrasonic gap sensors and switches. ,

2.6.2.3 PASSIVE VAPOUR MONITORING METHOD

Passive vapor monitoring of underground storage tanks (USTs) containing volatile hydrocarbons at locations external to the tank (an external system) is touted as a fast and effective method of leak detection. However, major gaps remain in our knowledge of the physical processes that relate a measured vapor concentration to the leak rate, thus making network design according to a quantitative design criterion nearly impossible, and differentiation between surface spills and a leaking UST requires certain levels of sophistication in the leak detection system and in the analysis that are not usually available. Heavier-than-air vapors from the constituents of stored hydrocarbons could result in a density-driven convective propagation component that complicates the design of leak detection systems, and finally, detection times are highly sensitive to concentration detection threshold levels set by the system. The use of inadequate systems and analyses can lead to either wasted efforts or excessive subsurface contamination. This paper discusses the physical processes involved, explores the above aspects of external passive vapor leak detection design, and suggests some alternatives as they pertain to gasoline service stations.

2.6.2.4 CONTINOUS LEVEL INDICATION.

Continuous level sensors with a self contained compact alarm display, in which case the user can program the point. This method is at a higher cost and can also be provided with reed switch devices. If this type of level indication is required, a 5mm resolution may be specified this specification should take into consideration tank capacity, tank configuration (linear or non-linear) and the cost benefit of material been monitored. With this type of device, logging of the tank's inventory may help in detection of leakage if the operator begins to record inventory losses between the fills. This will not be considered a highly reliable method of early tank leak detection however.

A float switch can also be use to monitor the interface level between two different liquids, such as water and gasoline. This type of application requires the use of another float configuration on the switch.

2.7 RATE OF LEAKAGE.

At any time interval, the rate at which liquid content of a storage tank leaks can be calculated. This is done by reading the level of content at interval of time and dividing by the time interval. This can be done using a plot of change in height against time interval, and the slope of the graph give that rate at which the liquid content leaks.

2.8 STORAGE TANK INSPECTION.

Tank-flow failures are insidious because liquid can leak in the sub-surface and into the ground water for years before leak is detected. When failure occurs, leak occurs and litigation often follows to determine blame and responsibility of clean-up. The cost of clean-up of such ground water is very high. Some of the large leaks can cause billions of naira to remediate.

Senior engineering consultants are required for conducting investigation on tank to determine the cause and also the associated ground-water remediation. The following steps are taken,

- Routine inspection and maintenance of storage tank, scrutinizing the fill lines valves, gauge, and tank support (for AST) for any visible sign of rust, decay or dark stains on them.
- Specifying coatings, linings and galvanic system for storage tank corrosion prevention.
- Performing hazardous waste integrity assessment.
- Conducting root-cause analysis of failed storage tank.

2.9 POTENTIAL HEALTH EFFECTS.

The Environmental Protection Agency (EPA) is issuing alert as part of its ongoing effort to protect human health and the environment by preventing chemical accidents. EPA is striving to learn the causes and contributing factors associated with chemical accidents and to prevent their recurrence. Major chemical accidents cannot be prevented solely through regulatory requirements. Rather, understanding the fundamental root causes, widely disseminating the lessons learned, and integrating these lessons learned into safe operations are also required. EPA publishes Alerts to increase awareness of possible hazards. It is important that all those concerned be inform and take appropriate steps to minimize risk.

The risk to human health and environment is high in some cases where leaked substances exposure to humans is possible short term exposure to such substances can cause headache, nausea, dizziness, increased blood pressure, difficulty in respiration and concentration. Long term health effect has been associated with an increased risk of developing cancer.

CHAPTER THREE

3.0 MATHEMATICAL MODELLING AND DYNAMIC SIMULATION

The mathematical model of liquid leaking from a horizontal storage tank is given below:

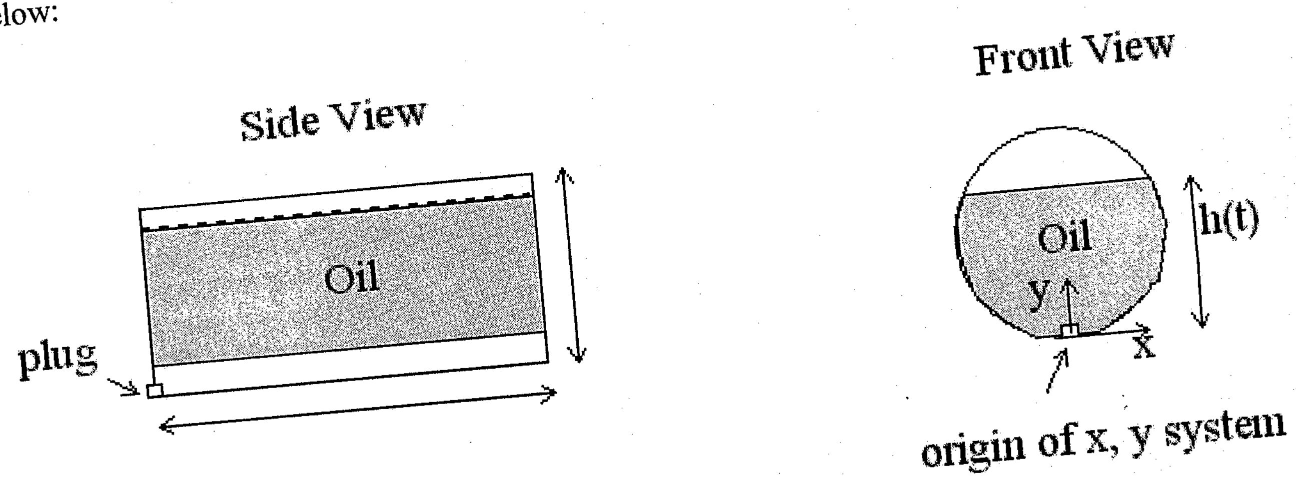

Fig. 3.1: A sketch of the side and front views of the damaged tank.

3.1 Model Development:

A mass balance for the liquid in the tank can be written as

$$\frac{dM}{dt} = -\dot{m}_{out} \qquad 3.1$$

The mass, M, is the oil density, ρ, times the volume, V, which changes with time.

$$M = \frac{\rho dV}{dt} \qquad 3.2$$

Since the density is constant, we have

$$\frac{dM}{dt} = \rho \frac{dV}{dt} = \rho L \frac{dA}{dt} \qquad 3.3$$

The volume is simply the length times the cross sectional area perpendicular to the tank's main axis that is occupied by the liquid. When the tank is completely full, the area is given by

$$A_{full} = \pi \frac{D^2}{4} \qquad 3.4$$

However, when only partially filled to a height, h, the cross-sectional area is given by the following development (as shown in Fig 3.1):

In Cartesian coordinates, the relationship between surface radius, R and diameter D is given by

$$\mathbf{R = D/2} \qquad \mathbf{3.5}$$

Therefore, the x, y coordinates along the surface is given by

$$x^2 + (y - R)^2 = R^2 \qquad 3.6$$

Now, solving this for x(y) gives

$$x^2 = R^2 - (y^2 - 2Ry + R^2)$$

or

$$x(y) = \pm\sqrt{2Ry - y^2} \qquad 3.7$$

Now, focusing on only the positive x values, the shaded area in the front view of the tank for any height, h, is given by

$$A(h) = 2\int_0^h x(y)dy \qquad 3.8$$

In finding an expression for **dA/dt**, Leibnitz Rule is used.

3.2 Leibnitz Rule

Leibnitz rule is applied in differentiating integral expressions that have variable limits of integration, the Rule is stated as follows:

$$F(t) = \int_{a(t)}^{b(t)} \phi(x,t)dx \qquad 3.9$$

Then with suitable conditions on a(t), b(t), and $\phi(x,t)$ [i.e. that they are well behaved with no discontinuities], we have

$$\frac{d}{dt}F(t) = \int_{a(t)}^{b(t)} \frac{\partial}{\partial t}\phi(x,t)dx + \phi[b(t),t]\frac{d}{dt}b(t) - \phi[a(t),t]\frac{d}{dt}a(t) \qquad 3.10$$

Using Leibnitz Rule for differentiation of an integral with variable limits of integration, we have

$$\frac{dA}{dt} = 2\left[\int_0^h \frac{\partial x(y)}{\partial t}dy + x(h)\frac{dh}{dt} - x(0)\frac{d}{dt}(0)\right] \qquad 3.11$$

or, since the first and last terms of equation 3.11 are zero, the equation now reduces to

$$\frac{dA}{dt} = 2x(h)\frac{dh}{dt} \qquad 3.12$$

Now, substituting the above expressions into the left hand side of the mass continuity equation gives

$$\frac{dM}{dt} = 2\rho L\sqrt{2Rh - h^2}\,\frac{dh}{dt} \qquad 3.13$$

Now, the mass flow rate out of the tank is given by

$$\overset{\circ}{m}_{out} = \rho A_{out} V_{out} \qquad 3.14$$

Performing an energy balance on a small control volume at the exit gives

$$0 = \dot{m}\Delta u + \frac{\dot{m}}{2}\left(v_1^2 - v_2^2\right) + \dot{m}g\Delta z + \frac{\dot{m}}{\rho}\left(P_1 - P_2\right) \qquad 3.15$$

3.3 ASSUMPTIONS

1. The energy and mass storage in this small control volume are zero.

2. Also assume negligible change in internal energy and elevation (i.e. Δu and Δz are zero)

3. The liquid velocity in the tank just prior to the exit (denoted as v_1) is small compared to the exit velocity (denoted as v_2).

Then the conservation of energy equation indicates that the change in flow work across the orifice is related to the increase in fluid velocity across the open plug, or

$$v_2 = v_{out} = \sqrt{\frac{2}{\rho}\left(P_1 - P_2\right)} \qquad 3.16$$

However, since the pressure at the bottom of the tank is simply the atmospheric pressure at the top plus the pressure due to a column of oil of height h, we have

$$P_1 = P_a + \rho g h \qquad 3.17$$

Since the opening is also at atmospheric pressure, (i.e. $P_2 = P_a$), the exit velocity is given as

$$v_{out} = \sqrt{2gh} \qquad 3.18$$

Finally, the overall mass balance gives

$$2\rho L\sqrt{2Rh - h^2}\frac{dh}{dt} = -\rho A_{out}\sqrt{2gh}$$

or

$$\frac{dh}{dt} = -\frac{A_{out}}{2L}\left[\frac{\sqrt{2gh}}{\sqrt{Dh - h^2}}\right] = -\frac{A_{out}}{2L}\sqrt{\frac{2g}{D-h}} \qquad \textbf{3.19}$$

$$\frac{dh}{dt} = \frac{-A_{out}}{2L}\sqrt{\frac{2g}{D-h}} \qquad \textbf{3.20}$$

With the initial condition, $h(0) = h_0$, being determined by the tank geometry and the amount of oil initially stored in the tank.

3.4 SIMULATION

The simulation of the mathematical model for detection of leakage in a horizontal cylindrical storage tank is carried out using mathcad 2001i professional. The simulation in mathcad is given below.

The following data is needed. Diameter of tank, D, Diameter of the orifice (of leakage), D_{out}, Length of tank, L, Area of orifice, A_{out}, Acceleration due to gravity, g, Initial height of liquid in tank (level), h_0, Time step, Δt, Number of solution steps, n.

Getting started (on mathcad) clear all

Specify problem data

$D := 5.($ m $\quad h_0 := 4.25$ m $\quad D_{out} = 1.0$ m $\quad L := 7.0$ m

$g = 9.807 \frac{m}{s}$

$A_{out} := \{[\pi . D_{out} /4]$

$A_{out} := 0.786$

Time Step $\quad \Delta t := \frac{1}{30}$ hr

Total number of solution steps $\quad n := 0..11$

$$h_{n+1} := h_n + \left[\left(\frac{-A_{out}}{2 \cdot L}\right) \cdot \sqrt{\frac{2 \cdot g}{D - h_0}}\right] \cdot \Delta t$$

h =

	0
0	4.25
1	4.011
2	3.772
3	3.533
4	3.294
5	3.055
6	2.816
7	2.577
8	2.337
9	2.098
10	1.859
11	1.62
12	1.381

$h_{exp_n} :=$

4.25
4.050
3.8
3.55
3.4
3.25
3.100
2.950
2.800
2.600
2.450
2.350

CHAPTER FOUR

4.0 RESLTS AND DISCUSSION OF RESULTS.

4.1 EXPERIMENTAL RESULT.

Table 4.1 Experimental Data.

Time (hr)	Height (m)
0	4.24
0.33	4.05
0.67	3.81
1.00	3.55
1.33	3.40
167	3.25
2.00	3.10
2.33	2.95
2.67	2.80
3.00	2.60
3.33	2.45
3.67	2.35

4.2 SIMULATED RESULTS.

Table 4.2 Simulated Result ($h_0 = 4.25$)

Time (hr)	Height (m)
0	4.22
0.33	4.02
0.67	3.78
1.00	3.53
1.33	3.29
167	3.05
2.00	2.82
2.33	2.58
2.67	2.34
3.00	2.10
3.33	1.86

Table 4.3 Correlation

Experimental	Simulated
4.25	4.25
4.05	4.011
3.8	3.772
3.55	3.533
3.4	3.294
3.25	3.055
3.1	2.816
2.95	2.577
2.8	2.337
2.6	2.098
2.45	1.859
2.35	1.62
2.25	1.381

Correlation 0.994764

Table 4.4 Simulated Result (h_0 = 1/3 D)

Time (hr)	Height (m)
0	1.67
0.33	1.55
0.67	1.44
1.00	1.32
1.33	1.21
167	1.10
2.00	0.99
2.33	0.87
2.67	0.76
3.00	0.65
3.33	0.67

Table 4.5 Simulated Data (h_0 = 1/4 D)

Time (hr)	Height (m)
0	1.25
0.33	1.14
0.67	1.04
1.00	0.93
1.33	0.82
167	0.72
2.00	0.61
2.33	0.50
2.67	0.39
3.00	0.29
3.33	0.18

Table 4.6 Simulated Data (h_0 = 1/5 D)

Time (hr)	Height (m)
0	1.00
0.33	0.90
0.67	0.80
1.00	0.69
1.33	0.59
167	0.48
2.00	0.39
2.33	0.28
2.67	0.17
3.00	0.07

Table 4.7 Simulated Data (h_0 = 1/6 D)

Time (hr)	Height (m)
0	0.83
0.33	0.73
0.67	0.63
1.00	0.53
1.33	0.43
167	0.33
2.00	0.23
2.33	0.12
2.67	0..02
3.00	2.10

4.3 DISCUSSIONS OF RESULTS

The aim of this research work is to develop a model to detect leakage in a liquid storage tank system. An experiment was carried out using a horizontal cylindrical tank of 5.0m diameter, 1.0m diameter of orifice, 7.0m length and an initial liquid level (height) of 4.25m. Mathcad 2000i Professional software was used in the simulation of model developed. The result obtained shows that liquid level decreases with time if there is leakage.

Table 4.1 shows the results of the experiment, while Table 4.12 shows the simulated result using the model. Statistical analysis (correlation) was carried (using Excel) out to compare the two results in order to ascertain the validity of the model. The correlation is 0.9947 (99.5%) which is high, thus showing the validity of the model developed.

Various values of initial height (h_0) with respect to the diameter of the tank (5.0m) were also calculated for h_0 = D/3, D/4, D/5, D/6, as shown in Tables 4.4, 4.5, 4.6 and 4.7 respectively. The result obtained shows a general trend of liquid level (height) decreasing with time ones there is leakage in the tank.

CHAPTER FIVE

5.0. CONCLUSION AND RECOMEMDATION.

5.1 CONCLUSION

A mathematical model for detection of leakages, and also to predict the rate and amount of liquid that will leak at specified time in a liquid storage tank has been developed. The comparism of results of the model using mathcad 2001i to simulate and that of the experimental shows a negligible difference as shown on the plot of height of liquid against time.

From the results, the correlation of experimental and simulated data using mathcad 2001i is 0.994764 (99.5%). It can be concluded that the model can be used in the dynamics of height in liquid storage hank.

5.2. RECOMMENDATION.

Other mathematical models should be developed using another rule (instead of Leibnitz rule) and different mathematical simulation softwares be used in the simulation.

A model should also be developed that will give a detailed explanation on how to detect and locate the exact point of leak in a liquid pipeline transportation system.

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APPENDIX

Volume of liquid leaked

Cross sectional area of the horizontal cylindrical tank is given by

$$A(h) = 2\int_0^h x(y)dy$$

where $x(y) = \pm\sqrt{2Ry - y^2}$

therefore, initial volume of oil stored $[V(h_0)]$

$$V(h_0) = L*A(h_0)$$

Volume of liquid at any time t

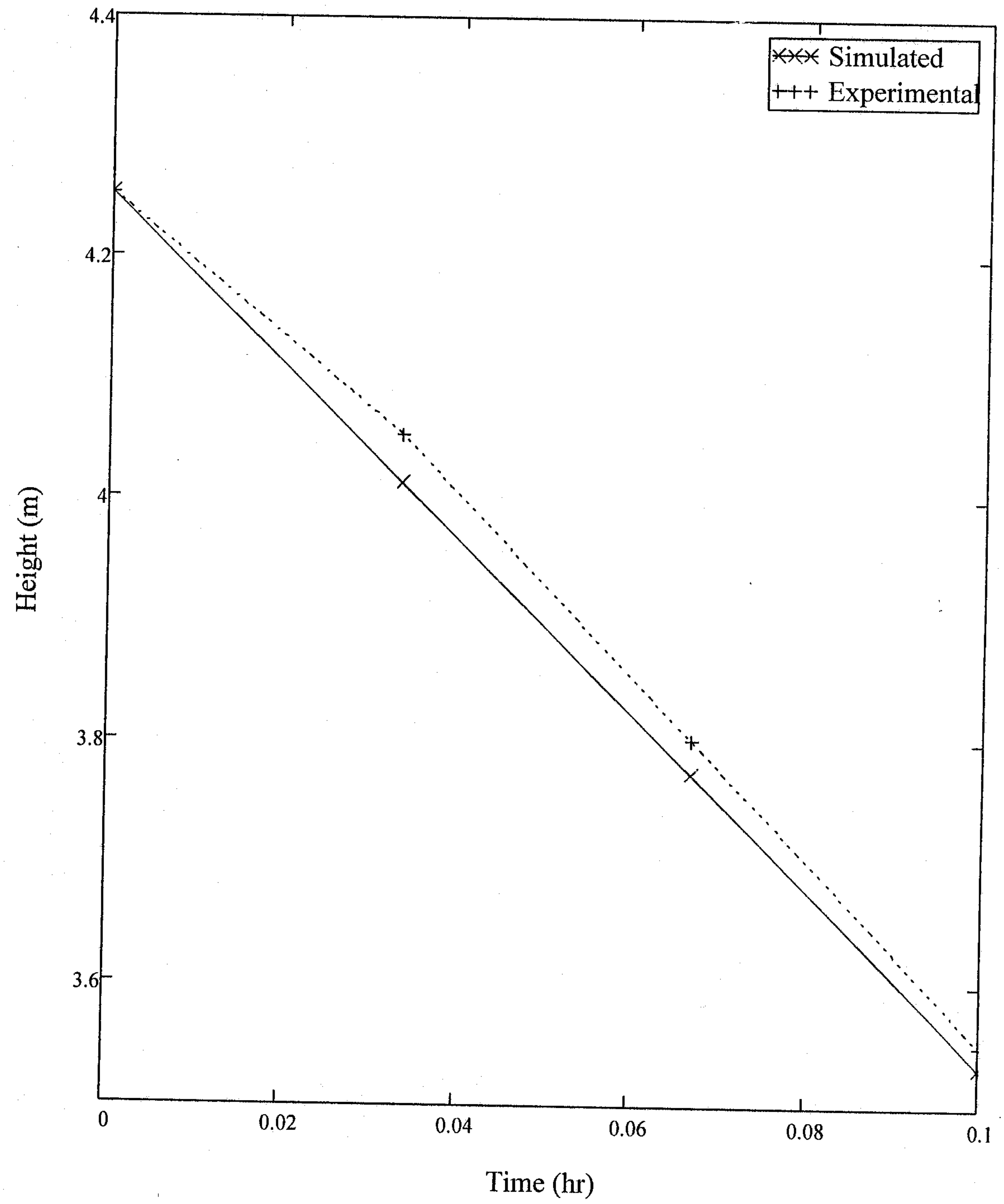
$$V(h_t) = L*A(h_t)$$

Volume of liquid leaked

$$V_L = V(h_0) - V(h_t)$$

Rate of leakage

$$r_t = \frac{\Delta h}{\Delta t} = \frac{h_{t=0} - h_{t=t}}{t_{t=t} - t_{t=0}}$$



Simulated
Experimental
Height (m)
Time (hr)
4.4
4.2
4
3.8
3.6
0
0.02
0.04
0.06
0.08
0.1