**FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA**
**Department of Mathematics**
BTech. Degree *First Semester Examination 2022/2023 Session*

**Title:** ***Numerical Analysis II*** | ***Unit: 3*** | ***Time: 3 hrs.***

***Instruction: Answer any Four (4) Questions*** | ***Code: MAT 515***

1. (a) Define
a. A partial differential equation. (3 marks)
b. A boundary value problem (3 marks)
(b) Classify each of the following as hyperbolic, parabolic or elliptic at every point $(x, y)$ of the domain.
i. $xu_{xx} + u_{yy} = x^2$
ii. $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} = e^x$
iii. $e^x u_{xx} + e^y u_{yy} = u$
(9 marks)

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2. (a) Show that
i. $\nabla = -\frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$
ii. $\mu = \frac{1}{2}\, E^{\frac{1}{2}} + E^{-\frac{1}{2}}$ (9 marks)
iii. $\delta = \nabla(1 - \nabla)^{-\frac{1}{2}}$

(b) Replace the Laplace equation
$\nabla^2 u(x, y) = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ by a central 2nd order difference scheme. (6 marks)

3. Solve $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ subject to the boundary conditions;
$u(x,0) = 1$, $u(0, y) = 0$
$u(x,1) = 1$, $u(1, y) = 0$ (15 marks)
$0 \le y \le 1$, $0 \le x \le 1$

4. Consider $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ where $\dfrac{\partial}{\partial t} = L$ and $\dfrac{\partial^2}{\partial x^2} = D^2$, retaining up to the 4th order central difference show that $U_m^{n+1} = \frac{1}{2}(2 - 5r + 6r^2)U_m^n + \frac{3}{2}r(2 - 3r)(U_{m+1}^n + U_{m-1}^n) - \frac{1}{12}r(1 - 6r)(U_{m+2}^n + U_{m-2}^n)$ (15 marks)

5. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to initial conditions

I. $u = 0,\ x = 0\ \forall t$

II. $u = 2x,\ 0 \le x \le \frac{1}{2},\ t = 0$ (15 marks)

III. $u = 2(1 - x),\ \frac{1}{2} \le x \le 1,\ t = 0$

Use $r = \frac{1}{2}$

6. Using Crank Nicholson scheme solve the parabolic equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to

$u = 0,\ 0 \le x \le 1 = 0\ \forall t > 0$

$u = 2x,\ 0 \le x \le \frac{1}{2},\ t = 0$ (15 marks)

$u = 2(1 - x),\ \frac{1}{2} \le x \le 1,\ t = 0$

Use $r = \frac{1}{2}$