FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA
Department of Mathematics

B.Tech Degree First *Semester Examination 2022/2023 Session*

Title: *NUMERICAL ANALYSIS I* *Code*: MAT415 *Unit*: 3

***Instruction*: Answer any Four (4) Questions** *Time*: 3hrs

1a. Enumerate 4 applications of numerical methods in (i) Biology (ii) Engineering.

1b. A ball has a high temperature at 1000K, what will be its temp after 500 secs (θ_2)? The differential equation of the temp of the ball is given by $\frac{d\theta}{dt} = -2.2067\times10^{-12}\left(\theta^4 - 81\times10^8\right)$. Employ the fourth order Runge-Kutta method to evaluate θ_2, $t_0 = 0, \theta_0 = 1000, h = 250\sec s$. **Note**: Round off k values to 6 decimal places and θ values to 4 decimal places.

2a. Explain the 3 types of error associated with numerical methods.

2b. Write the least squares equation for the Table below and find the value of y when x = 40.

x	16	33	50	28	50	25
y	2	3	6	5	9	3

3a. Derive the Jacobi method in matrix form and state the condition for its applicability.

3b. Solve the linear system given by

$$\begin{aligned} 10x_1 - x_2 + 2x_3 &= 6 \\ -x_1 + 11x_2 - x_3 + 3x_4 &= 25 \\ 2x_1 - x_2 + 10x_3 - x_4 &= -11 \\ 3x_2 - x_3 + 8x_4 &= 15 \end{aligned}$$

algebraically using the Jacobi method up to 3rd iteration, take $x^0 = (0,0,0,0)^T$ to 4 dec. places

4a. Derive the Successive Overrelaxation Relaxation method (SOR) in matrix form.

4b. Consider a linear system $Ax = b$, where $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $b = \begin{bmatrix} -1 \\ 7 \\ -7 \end{bmatrix}$, compute the first iteration by the SOR method starting at the point $x^0 = (0,0,0)^T$ for $\omega = 1.25$.

5. Find the least squares solution and the error of the system $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$

6 (a) What is the Least Squares Approximate (LSA)? Mention three (3) application in Medicine and Finance.

(b) Find the polynomial $P(x) = P_2$ which best fits the data points (0,3), (1,0), (2,1) and (3,3).

$P(x) = ax2 + bx + c$

Evaluate the least squares approximate solution to the equation $P(x)$