

**COMPUTER APPLICATION OF LINEAR
PROGRAMMING PROBLEM IN NIGERIAN
INDUSTRIES**

A CASE STUDY OF TECHO-EQUIP NIG.LTD.LAGOS

BY

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NOVEMBER, 2003

TITLE PAGE

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**BEING A PROJECT WORK SUBMITTED TO THE DEPARTMENT
OF MATHS/COMPUTER SCIENCE IN PARTIAL FULFILMENT OF
THE REQUIREMENT FOR THE AWARD OF A POST GRADUATE
DIPLOMA IN COMPUTER SCIENCE OF THE FEDERAL
UNIVERSITY OF TECHNOLOGY, MINNA.**

NOVEMBER, 2003

APPROVAL PAGE

This is to certify that this project was carried out by DARAMOLA I.K., Reg.No. MCS/SS&SE/98/99/842 of Computer Science/Mathematics Department of Federal University of Technology Minna, under my supervision.

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DEDICATION

This project is dedicated to my beloved wife

Mrs. Charity Kayode

ACKNOWLEDGEMENT

It is inevitable to accomplish a work of this nature without being indebted to some people. However, I must thank God for his Grace and Mercy, divine wisdom, knowledge, favour, understanding and protection through his only begotten son Jesus Christ, who gave me all these virtues during the course of accomplishing this project work. Glory, Honour and Majesty be unto you Oh Lord.

Among these indebted to are my able supervisor Dr. Yomi Aiyesimi whose academic advice, a desired touch and excellent supervisor made the completion of this work a reality. God bless you. Also to all the Lecturers in the department.

My sincere thanks are due to my Pastors, they are pastor Mike Favour Onyebushi – Paul, Pastor Emmanuel Solako, Pastor Israel, Pastor Benjamin Daramola, Pastor Timothy and other pastors of Living Faith Church Minna for their encouragement and advice. I own unalloyed thanks to my parent Mr and Mrs. F.A. Daramola, my wife and all my friends especially Engr. Kingsley Ndem Jr. for their supports. Pastor S Makinwa for his encouragement and information given during the project.

Finally, glory honour, mighty and adoration to God Almighty that makes every thing possible, in Jesus Name.

ABSTRACT

This research work applies the techniques of linear programming to profit optimization on metal manufacturing.

A review of mathematical programming was made with an overall emphasis on linear programming models. Also, methods of solving these models such as graphical solutions, simplex and two-phases methods were discussed.

Chapter three runs through metal manufacturing in Techo-Equip and a model was developed for one of my studies. Computerized solution of this model as well as its implementation and documentation were carried out.

In order to provide the decision-makers of the company with valuable introduction, careful analysis of the result obtained was done. Limitations as well as recommendation were stated.

KID (SEPT.2001).

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CHAPTER ONE

1.0 INTRODUCTION

In every sphere of human endeavor, there are problems of optimization. A chief of any staff must shape his forces in such a way it will have the maximum possible striking power. The Managing Director of an Industrial Enterprise tries to increase the profit and reduce the cost of his company as much as possible. Production manager of a plant is bound to be interested in finding out how he can minimize the production cost for his plant. A typical housewife agonizing over the problem of spending her husband's income in such a manner that she can purchase as many of the goods as possible that the family would like to have. These are just a few examples of the so-called optimization problem.

In metal manufacturing industries, although it generates huge revenue, but problem still arises ranging from simple ones to severe ones which tend to defeat this objectives. Due to no systematic research plans to manufacturing, the combinations to produces that yield maximum profits are usually not known to the management and the over-production of assumed well-sold product could lead to unpredicted loss. To avoid such circumstances, well-defined policies must be followed to determine before hand the possible mix of factors require to produce the highest profit.

Statistical analysis, simulation, games theory-queueing theory, and linear programming are some of the most widely used techniques of operations research. Operation research is concerned with optimal decision making in and modeling of deterministic and probabilities system that originate from real life. The applications, which occur in government, business, engineering, economic and the natural and social science are largely characterized by the need to allocate limited resources. In these situation considerable insight can be obtained from scientific analysis such as that provided by operation research.

One of the model types of operation research is an allocation model, which our research is based on. These models are concerned with allocating scare or constrained resources in order to meet all requirements, both of supply and demand. Usually, it is aimed as maximizing profit or minimizing cost. Linear programming is an aspect of allocating models in which all valuables occurs only as linear variables. Linear programming has been used successfully in the solution of problems concerned with assignment of personnel, blending of materials, distribution and transportation and investment portfolios.

1.1 PURPOSE OF STUDY

This project aims to study the high revenue generated from metal manufacturing and develop a model for optimizing profit or minimizing cost for any production.

2.0 PROBLEM DEFINITION

Often in every metal manufacturing company like Techo-Equip Nigeria limited, the overall objectives may be seen as maximization of profit. However, the cost of materials it purchases. An optimization problem arises here as to what particular combination of supplies; quantities and materials minimized the overall cost of purchasing. Also, the production department may be faced with the problem of providing space if large quantities are purchased, say, due to reduced cost. This involves additional cost.

For this study, we shall attempt to develop computerizing solution, which will assist in deciding what units of each metal product the firm should manufacture each week to meet all contractual obligations and minimize its total cost.

1.3 THE METHODOLOGY

The following are the activities involved in carrying out the project work.

a. **Studying and formulation of problems;**

Here, the knowledge of system analysis was used, which enables us to solve there mailing other activities. The objective and inter-relationship between component in the system so also constraints among others were determined.

b. **Construction of the Model**

A model substitute representation of some objectives or systems in which changes in the component of the model, whatever they may be affect it in such a way that the result can be translated into the same kind of response in the original objective or system understudy. Here, it has been decided that value of the controllable variable should have in order to optimize the performance measure.

c. **Solution to the Model**

An iterative solution technique was used to obtain solution to the model because it has relatively big number of variables and constraints. This process involves continuous use of tentative solution until it was optional.

d. **Testing the Model and Evaluation of the Solution**

This could be achieved by using prospective test, which involves operating the actual physical system according to the solution obtained by solving the model. Otherwise, past record might be used and later compared with the derived ones.

CHAPTER TWO

2.0 FUNDAMENTAL OF LINEAR PROGRAMMING

2.1 LINEAR PROGRAMMING

Linear programming is one of the most popular mathematical techniques used in solving operation problems. It can be defined as a class of mathematical techniques or models that are concerned with the efficient use or best allocation of source and limited resources to known activities with the objectives of meeting a desired goal (for examples, maximization of profit or minimization of cost).

As a tool, it is a quantitative technique used in solving management problems in the fields or production, planning, feed mix, stock cutting, agriculture, administration, oil drilling and production, assembly industry and other areas involved with decision making.

In summary, linear programming models have been found to be the simplest and most efficient models from a computational point of view. Its distinct characteristic is that functions representing the objective and constraints are linear.

2.1.1 LITERATURE REVIEW OF LINEAR PROGRAMMING

Linear programming or optimization models are among the most commercially successful application of operation research; infact, there is

considerable evidence that they rank highest in economic impact. This because many large firms adopt its quantitative analysis technique to solve their managerial critical decision. For example, in the decades ago, problem concerning production of commodities like, which commodity or processes? Are the main question before a production managers. Alfred Marshal points out that businessman studies his production function and his input prices and substitute one input for another till his cost become minimum possible. This is done intuitively by businessman not by his trained instinct. Now, there exist a method known as linear programming, which ends maximization of profit and minimization of cost.

This method, the linear programming was first formulated by Russian Mathematician Shir L. V. Kantorovich but it was developed later in 1948 by George B Dentzig for the purpose of scheduling the complicated procurement method is being used in solving a wide range of practical business problems. The advent of electronic computers has further increased its application to solve many other problems in industry.

In the light of wide range of application of this method (Linear Programming) there are several definition of the method defined by various authors. Among these definitions, we have;

“Linear programming is a mathematical method for solving a large number class of special problem” (Anderson 1982) it applies typically to problems in which one is attempting either to maximize cost while meeting certain requirements. “ It may be defined as an analytical technique used to determined the optional solution to a decision problem” (Pappas and Brighen, 1979). They went further to define it as one of the series of analytical techniques known collectively as mathematical programming which are used to solve constrained optimization problem. Partly because linear programming has been developed much more frequently than non-linear and other programming techniques.

“Another people defined it as a “mathematical techniques for finding the best uses of a firm’s limited resources” (Donnelly, Gibson and Invancevich, 1989). The adjective linear is used to described relationship between two or more variable.

It also defined as “ Strictly an algebraic graphical technique that at least on relatively simple problems can be used without an electronic computer” (Hick and Gullet, 1981). Here the linear programming models can be used to determine the best use of scare resource to accomplish a defined objective for simple and small (in size) problem while it indirectly implies that computer applications helped to solve complex and large

problems like calculus, linear programming is a mathematical tool for solving maximization and minimization problems. It is particularly powerful solving the course of action available to the decision-maker. From all these definitions, one can conclude that linear programming is a useful analytical technique for managerial decision making.

2.1.2 NATURE OF LINEAR PROGRAMMING PROBLEM

The natures of linear programming problems are such that limited resource are to be effectively and efficiently allocated to activities with the objective of meeting a desire goal.

Hence, linear programming problems are optimization problems and it can be defined as the one where the feasible region is a subset of the non-negative portion of R^2 , defined by linear equations and inequalities and also the objective function to be minimized or maximized is linear i.e. of the form $P(x^1, x^2, \dots, X^1) = a_1x_1 + a_2x_2 + \dots + a_nx_n$.

We should note that for a problem to be solvable using linear programming models, there are certain requirement that must be met. These requirements include;

- a. There should be only one but well defined objective function. This is because the model is build in such a way as to allow for only one well defined objective function

- b. The resource to be used must be limited in supply. This means that for every linear programming problem, constraints must exist.
- c. There must be more than one constraint existing for any problem to be solvable by the linear programming model. This makes it possible to select between various combinations of manpower and automation.
- d. The variables must be inter-related this means that it should be possible to formulate a mathematical relationship between the variables describing the problem. The relationship must be in linear form and if this requirement is not met, the problem cannot be solved using linear programming model.

2.1.3 GENERAL LINEAR PROGRAMMING MODEL

Linear programming is a mathematical technique that will maximize or minimize a linear function subject to a system of linear constraints. These linear functions together with the system of linear constraints form what is called linear programming model. A general form of a linear programming model is as follows optimize: $Z = C_1 X_1 +$

$$C_2 X_2 + \dots + C_r X_r \dots \dots \dots (1)$$

$$\text{subject to: } a_{11}x_1 + a_{12}x_2 + \dots + a_{1r}x_r [\geq, \leq] b_1$$

$$= 1, 2, 3, \dots, m \dots \dots \dots (2) \quad X_j \geq 0, j = 1, 2, 3, \dots, r]$$

Where a_{ij} , b_i are assumed known constants and x_j are decision variable restriction, (2) in the model are called constraints.

These are mathematical statements, which specify such elements in the problem as the limitation of available resources, or the demand that must be met. The nature of a problem will determine which of the relation [\geq , \leq] appears in each constraints of a particular model.

2.1.4 ELEMENT OF A LINEAR PROGRAMMING MODEL

All linear programming models require the following data to be established.

- a. There must be only one well-defined objective function or goal of the decision maker.
- b. Identification of the various constraints, may be classified as follows;
 - i. Restrictions due to limited supply of resources, which may be in terms of men, machines of raw materials.
 - ii. **Policy:** The constraints reflect the various business policies of the organization.
 - iii. **Finance:** Those constraints usually take form of budget standard cost and others.
 - iv. **Market:** The most likely constraints of this nature is the sales limit determined by market demand for various products.

- c. The variables (those quantities or aspect of the process that the decision-maker is seeking control) must be related and the mathematical relationship among them must be of the linear.
- d. There must be alternative courses of action in that it must be possible to vary combination of resources to yield some desired level of output goal.

2.1.5 TYPE OF LINEAR PROGRAMMING PROBLEMS

There are three types of linear programming models, they are

- a. **Resources Allocation Problems:** This can be used to determine how to use limited assets to maximize gain or minimize loss.
- b. **Diet Problems:-** This is used to find the best food mix that would result in the maximization of profit or not gain in value and /or to minimize the cost of services.
- c. **Transport Problems:-** Finding the cheapest means of transportation of goods.

2.1.6 ASSUMPTION IN LINEAR PROGRAMMING

The following are the various assumption s in linear programming (these assumption help us to know the characteristics possessed by the linear programming techniques).

1. **Single Objective:** It is assumed that all problems that are to be solved using linear programming techniques are targeted at only one objective which can either be maximization of profit contribution or minimization of cost but not a dual objective (minimax) problem.
2. **Linearity:** This assumed that there is a linear or straight-line relationship between the variables in linear programming model. In other words, the values to the individual variables are assumed known or can be determined and serve as input to the model (that is, linear programming are deterministic models).
3. **Additivity:-** The individual cost or profit contributions of the various processes or processes that constitute the model can be added together to obtain an accurate cost or profit.
4. **Divisibility:-** This is assumed that non-integer values of variable at optimal solution must make a managing sense.
5. **Limitation of Resources:-** This resources to be employed in the achievement of the defined single objective are assumed to be limited in supply and its allocation must be in the optimal ways among the competing ends.

6. **Non-Negativity of Resources:** All decision variables are non-negative. This is based on the fact that negative resources cannot be used.
7. **Proportionality:-** This requires that the amount of each resources used must be proportional to the value of the decision variables.
8. **Mathematical Representation:** The single objectives to be achieved and the limitation militating against its achievement must be capable of being expressed as a set of equations or inequalities is called the linear programming model.
9. **Alternative Solution:-** It is assumed that there are alternative courses of action to achieve a defined objective, one, which will be the optional solution.

2.1.7 APPRAISAL OF LINEAR PROGRAMMING TECHNIQUE

linear programming techniques so the problems which are strictly mathematical or can be formulated into mathematical models linear programming model assumes that for any problem there is a single goal and this goal is known as the objective function and constraints of as the constraints. The objective function and constraints of any linear programming problem must be precisely expressed mathematically and must

be linear in form. However, there are some shortcomings of the linear programming model. These are:

1. The linearity of relationship between decision variables is not the real-life solution as required by the linear programming model.
2. Linear programming model is inapplicable to some real-life non-linear problems. Whenever a non-linear problem is encountered this has to be reached to a linear form and this can produce some inaccuracies in the results obtained.
3. Due to its assumption of only objective function, other objective functions of the problem must be dropped and this implies that the models may not solve the problem in real-life situation.
4. In cases the solution to a linear programming model is designated (that is, where the values of one or more basic variables becomes zero), the value of the objective function may never improve and the linear programming technique breaks down.
5. In most cases of unbounded solution space, the value of the objective function can increase indefinitely; hence the linear programming technique is not reliable in producing an optimal solution.

Where the objective function is parallel to a binding constraint the function assumes the optimal value at more than one basic solution. In spite

of these shortcomings, linear programming models have been found to be the simplest and most efficient set of models from the computational point of view.

The advantages of linear programming model are perceived to outweigh the disadvantages. These advantages include the following.

- i. Linear programming models are the easiest in term of computation of all other techniques in its family.
- ii. Linear programming models are used in solving production bottlenecks.
- iii. Linear programming models allow for the modification of its solution for the sake of convenience through the sensitivity post-optimality analysis.
- iv. There is an improved quality of decision when using linear programming models.

The simplex method of linear programming gives the users an opportunity to compute shadow process of products (dual concepts).

2.2 SOLUTION TO LINEAR PROGRAMMING PROBLEMS

About three methods of solving linear programming problems shall be considered. To initialize the method, one must transform all inequality constraints into equalities and must know one feasible solution, non-negative

solution. However, before looking at these methods, let us define some terms to be used.

2.2.1 DEFINITION OF TERMS

In order to understand some of the problems employed in presenting facts in this project, it is necessary to present some of the linear programming terms used. These terms and their meaning are as follows.

- i. **Objectivity and Objective Functions:** Objective is the set of goals, which a linear-programming problem is set to achieve. This goal may be to minimize cost or to maximize contributions expressed in a mathematical form.
- ii. **Constraint and Constraint Function:** In linear programming the set of factors that limits the attainment of a specific objective are called constraints while constraint functions are set of equations or inequalities which rule out certain combination of variables of feasible solution.
- iii. **Decision Variables:** A variable is a character or character strings which can accept of the variable. Decision variable comprises of characters that are used in the representation of parts of a mathematical model. In other words, decision variables are used to represent those activities over which a decision-maker has control.

- iv. **Slack/Surplus Variable:** A slack variable is a variable added to the left-hand-side of less –than equal to constraint function to convert the inequality to equality. A surplus variable on the other hand is a variable subtracted from the left hand-side of a greater-than –or-equal-to constraint to convert the constraint function inequality. At the optimal solution, the value of the surplus variable is interpreted as the amount over and above the minimum requirements.
- v. **Mathematical Model:-** A model is abstract representation of a real life problem. It is the mathematical representation of the objective to be achieved as well as the set of constraints mutilating against the achievement of the objective.
- vi. **Standard/Canonical Forms:-** A linear programming problem is said to be in its standard or canonical form if all the constraint inequalities have been expressed as equalities. This can be achieved either by the addition of slack variables or subtraction of surplus variable.
- vii. **Pivot Row:-** This is the row in the simplex table all corresponding to the basic variable that will leave the solution as the algorithm iterates from one basic feasible solution to another.
- viii. **Pivot Column:-** The column corresponding to the non-basic variable that is about to be introduced into the basic feasible solution.

- ix. **Pivot Element or Pivot:-** This is the element of the simplex tableau at the intersection of both pivot row and pivot column.
- x. **Artificial Variables:-** These are variables that has no physical meaning in terms of the original linear programming problem, but serves merely to enable a basic feasible solution to be created for starting the simplex method.
- Artificial variables are assigned an objective function co-efficient of $-M$ where M is a very large number.
- xi. **Basic Feasible Solution:-** Linear programming basic feasible solution is a combination of decision and non-decision (slack, surplus and artificial) variables that satisfies all system and the non-negative conditions on the standard form of a linear programming.
- xii. **Iteration:-** An interaction of a simplex method consist of the sequence of steps performed in moving from one basic feasible solution to another.
- xiii. **Shadow Prices:-** The change in the criterion value results from a one-unit change in the amount of constraint.
- xiv. **Range of optimality:-** This is the range of values over which a c_j associated with a basic variables may vary without causing may change in the optimal solution (that is the values of all the variables

will remain the same, but the value of the objective function will change).

2.2.2 THE GRAPHICAL METHOD

This method is best achieved when we are considering only two independent variable x_1 and x_2 . If we have a system of inequalities such that its graphical representation is a polygon of finite area and if we have a related function of the form subject to the constraint occur at specific corner point of the polygon.

The diagram below illustrates this;

Simple of product Y and that each of product Z will attract 4 times as many buyers as one sample of product Y. The model is formulated as below. In the calculation $20-x-y$ will represent Z.

Hence we have

$$x + y \leq 20$$

$$x \leq 16$$

$$x \geq 8 \dots\dots\dots(1)$$

$$y \leq 10$$

$$y \geq 0$$

Also, the cost of shipping the product is represent by equation

$$C = 3x + 2y + (20 - x - y) \dots\dots\dots(2)$$

For the buyer appeal, we have

$$B = 10x + y + 4(20 - xy) \dots\dots\dots(3)$$

The president of the company is interested in knowing how many of each product should be sent to meet either of the two considerations. They are first minimization of the shipping cost, and second, maximization of buyer appeals. To discover the numbers of products yielding these extreme values, it is necessary to take the coordinates of each of the corner points in the figure (2) below and introduce these numbers into the two linear equation for cost and buyer appeal.

These are results in tabular form

				Cost of shipping	Buyer Appeal
Point	X	Y	Z	$3x+y+(20-x-y)$	
A	8	10	2	46	98
B	10	10	0	50	110
C	16	4	0	56	164
D	16	0	4	52	176
E	8	0	12	36	128

From the results in the above table, we concluded that

2.2.3 THE SIMPLEX TECHNIQUE

The simplex technique is one of the various computational techniques, which have been developed to solve linear programming problem. This technique is virtually the only one used in practical applications. Simplex method employs an attractive process that starts at a feasible corner point, normally the origin, and systematically moves from one feasible extreme point to another until the optimum point is eventually reached. The bases of the simplex method, which guarantee generating a sequence of basic solutions, are of two fundamental conditions. They are;

- a. **The Optimality Condition**:- This ensures that no interior points solutions (relative to the current solution points) are over encountered.
- b. **The Feasibility Condition**:- This ensures that starting with a basic feasible solution, only basic feasible solution is encountered during computation.

Manually, the procedure though simple, requires time and patience to be executed but it is well suited for a computer system. To us the conventional simplex technique, the problem has to be seen as the application of a hence its use of slack variables which can be seen as the application of a mathematical trick to convert statement of bounds into

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2.3.1 DESCRIPTION OF ALGORITHM FOR THE TECHNIQUES

For easy understanding, it would be better to briefly recall some of the concepts of linear programming mentioned earlier. These concepts are;

a. **Canonical Form:-** The general linear programming problem can always be put in the following form which is referred to as the canonical form.

Maximize

Subject to;

Where $i = 1, 2, \dots, m, j = 1, 2, \dots, n, x_j \geq 0$

The characteristics of the canonical form linear programming model are

- i. All decision variables are non-negative
- ii. All constraints are of the (\leq) type.
- iii. The objective function is of the maximization type.

A linear programming problem can be transferred into canonical form by performing some or all of the following operations on the linear programming problem.

- i. The minimization of a function is mathematically equivalent to the maximization of the negative expression of the same function $f(x)$

For example

Minimize $Z_0 = GX_1 + (2x_2 + \dots + (nx_n))$

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For example

Minimize $X_0 = GX_1 + (2x_2 + \dots + (nx_n)$

C. **A basic solution:-** This obtained by setting (n-m) variables equal to zero and solving for the remaining m variable provided that the determinant of the coefficient of the variable is not zero.

Where n = number of unknown variables

M = number of equations.

And the m variables are called the basis variables.

c. **A Basic Feasible Solution:-**This is a basic solution that satisfies the conditions that all basic variable are non-negative.

d. **The Simplex Tableau:-** Detaching the coefficient of a standard linear programming problem in canonical form and arranging them in a tableau form forms the simplex tableau . For Example.

$$Mm - 2x_1 - 3x_2 = z$$

$$S.t \quad 3x_1 + 2x_2 = < 60$$

$$-x_1 + x_2 = < 10$$

$$x_1, x_2 \geq 0$$

This can be converted to standard form. It becomes

$$\text{Max } 2x_1 + 3x_2 = Z$$

$$\text{St } 3x_1 + 2x_2 + x_3 = 60$$

$$-x_1 + x_2 + x_4 = 40$$

$$x_1, x_2, x_3, x_4, \geq 0$$

We need to make sure that there is a set of m left hand vectors, so that we can set up a starting tableau in restricted normal form. We do this by inventing new variables. There is m of these, and they are called artificial variables, we denote them by x_1 , hence we have

$$Z = 2x_1 + 3x_2$$

$$x_3 = 60 - 3x_1 - 2x_2 - x_3$$

$$x_4 = 10 + x_1 - x_2 - x_4$$

$$x_2, x_2, x_3, x_4, > 0$$

Our example is now restricted normal form. Now to translate into tableau we detach the coefficient into the table as below;

B2

Basis	RHS	x_1	x_2	x_3	x_4
Z	0	2	3	0	0
x_3	60	-3	-2	-1	0
x_4	10	1	-1	0	-1

Usually the 2nd row is the cost (c_j 's) row

e. The simplex Algorithm

The following steps could be employed on solving linear programming problems using simplex method.

- Step 1: Start problem is canonical form. Set up an initial tableau
- Step 2: If all the C_j 's (xxcept the objective) is 0, the maximum value of the objective function has been achieved stop.
- Step 3: If there exist S such that $*s \geq 0$) and a $l s \geq 0$ for all l then the objective function is not bounded below. Stop other wise, pivot to determine the pivot term.
- a. Pivot in any column with a positive C_j term. If there are several positive C_j 's
Pivoting in the column with the largest C_j may reduce the total number of steps necessary to complete the problem. Assume we pivot in column S .
- b. To determine the row of the pivot term, find that row, say row r , such that

$$\frac{b_i}{a_{is}} = \min \left\{ \frac{b_i}{a_{is}} \mid a_{is} > 0 \right\}$$

- Step 4: Replace the x - variable in the pivot row and first column by the x -variable in the first row and pivot column. The new, first column is the current of the basic variable.
- Step 5: The problem remains in canonical form at a different basic solution. Return to step 2.

The optimal solution is obtained by assigning to each variable in the first column that value in the corresponding row 2nd column. All other variables are assigned the value zero. The associated Z, the optimal value of the objective function, is the value in the second row and second column.

2.2.4 TWO PHASE SIMPLEX METHOD

If the constraints contain not only slack variable two-phase simplex method is applied. In the first phase, we create an auxiliary objective, Z' where $Z' = -Z_i$ and the simplex method is performed on the auxiliary objective function. The second phase is used to solve the real problem. As a result of this, the following modifications are incorporated into the simplex method.

Change 1: The second row of the simplex tableau is decomposed into two rows, the first of which is the cost (C_j 's) row and while the second involves the coefficient of the sum of the artificial variables.

Change 2: Step 2 of the simplex method is applied to the last row (auxiliary objective function row) created in change 1 (followed by step 3 and 4) until this row contain no positive elements. The step 2 is applied to those elements in the second row that are positioned over zeros in the last row.

Change 3: Whenever an artificial variable column ceases to be basic. It is removed from the 1st column of the tableau as a result of step 4. It is deleted from the top row of the tableau, as is the entire column under it.

Change 4: The last can be deleted from the tableau whenever it contains all zeros.

Change 5: If non-zero artificial variables are present in the final basis set, the program has no solution.

The following examples will illustrate these methods

a. Maximize $Z = 4x_1 + 3x_2$

Subject to $-x_1 + x_2 < 10$

$$x_1 + 2x_2 < 14$$

$$2x_1 + x_2 < 16$$

Solution

Let us start by introducing 3 slack variable x_3 , x_4 , and x_5 since it has 3 inequalities. This we write maximize $Z = 4x_1 + 3x_2$

$$\text{Solution to } -x_1 + x_2 + x_3 = 10$$

$$x_1 + 2x_2 + x_4 = 14$$

$$2x_1 + x_2 + x_5 = 16$$

The coefficient could be detached into the simplex tableau as follows:

B2

Basis	RHS	x_1	x_2	x_3	x_4	x_5
C_j	0	4	3	0	0	0
X_3	10	1	-1	-1	0	0
X_4	14	-1	-2	0	-1	0
X_5	16	-2	-1	0	0	-1

We select the key column which is the column containing the most positive entry in index row i.e 4.

For the key row, we divide the value of b_i column by the value of the entry in the key column and select the smallest of the absolute values.

$$x_4 \quad r = 14 / -1 = 14$$

$$x_5 \quad r = 16 / -2 = 8$$

We divide all entries in the key row by the negative of the pivot to reduce the pivot to unit.

Basis	b_i	x_1	x_2	x_3	x_4	x_5
C_j	0	4	3	0	0	0
X_3	10	-1	-1	-1	0	0
X_4	14	-1	-2	0	-1	0
X_5	8	-1	$-\frac{1}{2}$	0	-0	$-\frac{1}{2}$

Using the min row, we now operate on the remaining rows of the tableau, including the index row, to reduce the other entries in the key column to zero. New number = old - the product of the corresponding entries in the main row and the key column.

For example, in the third row (x_4)

-1 is replaced by $-1 + (-1)(-1) = 0$

-2 is replaced by $-2 + (-1/2)(-1) = -3/2$

14 is replaced by $14 + (8)(-1) = 6$. Etc.

Basis b_i	x_1	x_2	x_3	x_4	x_5
C_j	32	0	1	0	-2
X_3	18	0	-3/2	-1	-1/2
X_4	6	0	-3/2	0	1/2
X_1	8	-1	-1/2	0	-1/2

The key column consists of a single -1 and the remaining entries zero.

This column headed by x_i , which indicates that the basis variable x_5 in the main row can be replaced by x_i ,

Also, there is still a positive value in the index row, so we repeat the process for this second tableau.

Basic	Bi	X ₁	X ₂	X ₃	X ₄	X ₅
CJ	32	0	1	0	0	-2
X3	18	0	-3/2	-1	0	-1/2
X4	4	0	-1	0	-2/3	1/3
X1	8	-1	-1/2	0	0	-1/2
CJ	36	0	0	-2/3	-2/3	-5/3
X3	12	0	0	-1	1	-1
X2	4	0	-1	0	-2/3	1/3
X1	6	-1	0	0	1/3	-2/3

The basis solution is $x_1 = 6$, $x_2 = 4$

Furthermore, since there is no further positive entry in the index row, this is also the optimal solution and the optimal value C_j is given in the b_i column

i.e $C_j = 36$ hence $C_j \max = 36$

Also $x_3 = 6$, $x_4 = 0$, $x_5 = 0$

The full result, therefore is

$C_j \max = 36$, $x_1 = 6$, $x_2 = 4$, $x_3 = 6$, $x_4 = 0$, $x_5 = 0$

QN2 Maximize $Z = 5x_1 + 2x_2$

Subject to $6x_1 + x_2 > 6$

$$4x_1 + 3x_2 > 12$$

$$x_1 + 2x_2 > 4$$

With all variables non-negative i.e $x_1, x_2, > 0$

This program is put into standard form by introducing surplus variables x_3, x_4 and x_5 respectively, in the constraint in equalities, and the artificial variables

$x_1, x_2,$ and x_3 respectively, in the resulting equation,

Hence we have

$$\text{Maximize } Z \ 5x_1 + 2x_2$$

$$\text{Subject to } 6x_1 + 2x_2 - x_3 + \dots + x_6 = 6$$

$$4x_1 + 3x_2 - x_4 + \dots + x_7 = 12$$

$$x_1 + 2x_2 - x_5 \dots + x_8 = 4$$

Problem becomes

$$\text{Maximize } W \quad = -1 (x_6, +x_7, x_8)$$

Subject to

$$6x_1 + x_2 - x_3 + \dots + x_6 = 6$$

$$4x_1 + 3x_2 - x_4 \dots + x_7 = 12$$

$$x_1 + 2x_2 - x_5 \dots + x_8 = 4$$

$$5x_1 + 2x_2 \dots = Z$$

$$x_6 = -6 + 6x_1 + x_2 - x_3$$

$$x_7 = -12 + 4x_1 + 3x_2 - x_4$$

$$x_8 = -4 + x_1 + 2x_2 - x_5$$

$$x_6 - x_7 - x_8 = -6 + 6x_1 + x_2 - x_3 - 12 + 4x_1 + 3x_2 - x_4 - 4 + x_1 + 2x_2 - x_5$$

$$w = -22 + 11x_1 + 6x_2 - x_3 - x_4 - x_5$$

Problem becomes

$$W + 22 = 11x_1 + 6x_2 + x_3 - x_4 - x_5$$

$$6x_1 + x_2 - x_3 + x_6 = 6$$

$$4x_1 + 3x_2 - x_4 + x_7 = 12$$

$$x_1 + 2x_2 - x_5 + x_8 = 4$$

$$5x_1 + 2x_2 = Z$$

Then, applying the two-phase method and rounding all calculation to two significant figures we generate sequentially the following tableau in each of which the pivot element is marked by an asterisk.

Tableau 1

Basis	RHS	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈
Real	0	5	2	0	0	0	0	0	0
X ₆	6	-6	-1	1	0	0	-1	0	0
X ₇	12	-4	-3	0	1	0	0	-1	0
X ₈	4	-1	-2	0	0	1	0	0	-1
C _j 's	22	11	6	-1	-1	-1	1	1	1

Table 2

Basis	RHS	X ₁	X ₂	X ₃	X ₄	X ₅	X ₇	X ₈
Rear	5	0	1.107	0.834	0	0	0	0
X1	16	-1	0.167	0.167	0	0	0	0
X7	8	0	-2.333	-0.668	1	0	-1	0
X8	3	0	-1.833	-0.167	0	1	0	-1
C _j s	11	0	4.1766	0.8335	-1	-1	1	1

Basis	RHS	x1	x2	x3	x4	x5	x7
Real	6.9	0	0	0.727	0	-0.638	0
X1	0.73	-1	0	0.455	0.183	-0.091	0
X7	4.2	0	-1	-0.091	0	0.546	0
X2	1.6	0	-1	-0.091	0	0.546	0
C _j s	4.2	0	0	-0.455	-1	1.273	0

Tableau 4

Basis	RHS	x1	x2	x3	x4	x5
Real	9.0	0	0	0.500	0.500	0
X1	43	-1	0	0.214	-0.071	0
X5	3.3	0	0	-0.357	0.786	-1
X2	3.4	0	-1	-0.286	0.429	0

C'js 0.0 0 0 0.000 0.000 0

Tableau 5

Basis	RHS	x1	x2	x3	x4	x5
Real	12.0	-7.00	0	2.00	0	0
X4	6.0	-14.00	0	3.00	-1	0
X5	7.9	-11.00	0	2.000	0	-1
X2	6.0	-6.00	-1	1.000	0	0

Tableau 4 is the first tableau containing no artificial variables in its first column. Hence, with change 3 section 2

Implemented, the last row of the tableau should be zero. To within round error. It is zero, so we delete it from the tableau. Tableau 5, however, present, a problem that cannot be ignored, the work column is the x3 – column and all the elements in that column are positive. It follows from step 2 section 2 that the original program has no solution.

CHAPTER THREE

3.0 DEVELOPMENT AND FORMULATION OF MODELING FOR TECHO-EQUIP NIG. LTD.

3.1 HISTORY OF THE COMPANY

Tech-Equip, was founded and licensed to manufacture ornamental wrought iron to produce household products in various ways. The company has a good reputation in the manufacture of household metal products and fabricated iron and steel for Agro-allied product machines. The company started in the mid 80's and later diversifies into other markets, products and activities through self-help plants and equipment update as well as product development. The company now has well equipped workshops, offices, and facilities along Idimu-Egbe road at Ikotun in Lagos – State.

The company fully appreciates the challenges of the future in the economic development of Nigeria. Being wholly indigenous company its vision is to become a multidisciplinary engineering offering total engineering services to the construction, agro-allied and packaging industries in Nigeria.

It intends to achieve these objectives through the manufacture of products and offer of services that conform to specifies standards with added value for the benefit of all her customers.

3.2 PRODUCTION PROCESS OF METAL PACKAGING MANUFACTURING AT TECHO-EQUIP

Due to increase in demand for large quantity of metal products mainly needed for packing finished product by companies that need them prompted her to go into metal packaging manufacturing. Today, however, various metals such as can boxes, this contains metals products are produced. The following are the processes involved in metal packaging manufacturing.

- a. Machine:-** This is the first process and it includes activities like cutting, bending into various shape, joining or welding, pressing, and drilling holes among others, into metals sheet.
- b. Polishing:-** In order to prevent rusting and the machined metals, to give a better look, different types of chemicals includes painting, greasing and labeling are used in this process.
- c. Assembling:** The metal sheets products parts are fitted or put together by any type of joining process with bolt or rivet and then packed and arranged in order for sale or exporting.

3.3 DEVELOPING A MODEL FOR TECHO-EQUIP NIGERIA

In one of our studies, four different types of metal products are producing at this packaging section, each of which must be machined, polished and assembled. The specific time requirement (in minutes) for each product are as follows:

	Machine	Polishing	Assembling
	Min	Min	Min
Product I	3	2	1
Product II	2	1	1
Product III	2	2	2
Product IV	4	1	3

The firm has available to it on a weekly basis, 48000 min of machine time, 40000 mins of polishing time, and 40000 mins of assembling time. The unit profit on the products are ~~N60~~, ~~N40~~, ~~N60~~, and N80 respectively, together with those needed by the company. The firm has a contract with some distributor to provide 5000 units of products I and 10,000 unit of any combination of products II and III each week. Through other customers, the firm can sell each week as many units of product II, III and IV as it can produce; but only a maximum of 2500 units of product IV.

3.3.1 THE OBJECTIVE FUNCTION

The objective is to find out how many unit of each product should the firm manufactured each week to meet all contractual obligations and maximized its total profit.

3.3.2 THE CONSTRAINTS

The constraints in this problem include number of units to be provided for the distributors and numbers of units that can be sold in each week. The objective is to maximize profit (in Naira), which we denote as Z setting.

X_1 = number of product I to be manufactured in a week

X_2 = number of product II to be manufactured in a week

X_3 = number of product III to be manufactured in a week

X_4 = number of product IV to be manufactured in a week.

We can formulate the objective as;

$$\text{Maximize } Z = 60x_1 + 40x_2 + 60x_3 + 80x_4$$

There are constraints in the total time available for machining, the total time available for polishing and the total time available for assembling. These are respectively, modeled, together with product and selling constraints by,

$$3x_1 + 2x_2 + 2x_3 + 4x_4 = < 48000$$

$$2x_1 + x_2 + 2x_3 + x_4 = < 40000$$

$$x_1 + x_2 + 2x_3 + 3x_4 = < 40000$$

$$x_4 = < 2500$$

$$x_2 + x_3 \geq 5000$$

$$x_1 \geq 10000$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Hence, the problem could be stated as:

$$Z = 60x_1 + 40x_2 + 60x_3 + 80x_4 - a_1 x_9 - a_2 x_{10}$$

$$3x_1 + 2x_2 + 2x_3 + 4x_4 + x_5 = 48000$$

$$2x_1 + x_2 + 2x_3 + x_4 + x_6 = 40000$$

$$x_1 + x_2 + 2x_3 + 3x_4 + x_7 = 40000$$

$$x_4 + x_8 = 2500$$

$$x_2 + x_3 - x_9 = 10000$$

$$x_1 - x_{10} = 5000$$

$$x_1, x_2, x_3, x_4 > 0$$

CHAPTER FOUR

4.0 COMPUTERIZED SOLUTION MODEL

Two-phase simplex method shall be used to solve problem. The information needed for both the first and second phases of the overall simplex method is got from the simplex table below. The underlying problem to solve is as posed in equation (1) and (2) of section 3.2.

		X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉	X ₁₀
Z	0	60	40	60	80	0	0	0	0	0	0
X ₅	48000	-3	-2	-2	-4	-1	0	0	0	0	0
X ₆	40000	-2	-1	-2	-1	0	-1	0	0	0	0
X ₇	40000	-1	-1	-2	-3	0	0	-1	0	0	0
X ₈	2500	0	-1	-1	-1	0	0	0	-1	0	0
X ₉	10000	0	-1	-1	0	0	0	0	0	1	0
X ₁₀	5000	-1	0	0	0	0	0	0	0	0	1
Z ₁	-15000	-1	1	1	0	0	0	0	0	-1	-1

4.1 IMPLEMENTATION

4.1.1 THE MACHINE CONFIGURATION

The project was carried out on the follow platform.

Machine:- Pentium 3

64MB Ram

20GB HDD

4.1.2 INPUT SPECIFICATION

Input to the program is from file, SIMPLEX. The table entries inside the box of the double lines above are no more than coefficients of the original problem. These entries, along with the value of N (number of independent variables), M (number of constraints), M_1, M_2, M_3 (number of $<$, $>=$, $=$ respectively) are the only input that is needed by the simplex method routine used. The coefficients of the auxiliary objective function (bottom row) are just the negative of the column sums of the rows above, which are easily calculated automatically.

4.1.3 OUTPUT SPECIFICATION

The output is written to an output file simplex.out. The output from these routines are:

- i. A flag, I case, telling whether a finite solution, no solution, or an unbounded solution was found.
- ii. An updated tableau A . This tableau is indexed by two returned arrays of integers. $IPOSV(J)$ contains for $J = 1..M$, the number I whose original variable x_I is now represented by row $J+1$ of A . This are thus

the right-hand variables in the solution. Likewise, IZROV(J) contains, for $J = 1 \dots N$, the number 1 whose original variable x_1 is now a right-hand variable, represented by column $J+1$.

4.2 ANALYSIS OF RESULT

The simplex method requires 7 iterations to solve the problems. The following are the results obtained:

- i. At the end of the process (after 7 iterations), the maximum profit obtained by producing the different product is ₦1,250,000.00. The variable x_2 (product II) is not present in the solution. This implies its production does not affect profitability.
- ii. The initial basic solution does not increase the objective function from zero since none of the basic variable is present.
- iii. After the 1st iteration, x_1 entered the set of basic variables with a significant effect; the profit raised up from 0 to ₦300,000.
- iv. The profit increased by ₦400,000 after the 2nd iteration. This was a result of x_2 with quantity ₦10,000 entering the solution.
- v. After the 3rd iteration, both quantities of x_2 and x_1 remained unchanged while the profit increased to ₦900,000 with x_4 equals ₦2500 entering the solution.

- vi. The 4th iteration indicates that x_1 increased by 1000 while x_4 and x_2 remained unchanged. The profit rose to ₦960,000.
- vii. After the 5th iteration, the variable x_3 replaced x_2 in the solution. This implies x_3 increased the profit than x_2 . The profit rose by ₦200,000.
- viii. By the 6th iteration, x_3 increased to 11500 and the profit rose to ₦1,190,000 while quantities x_1 and x_4 remained unchanged
- ix. By the 7th iteration x_1 remained unchanged, x_3 increased to 14500 and x_4 decreased to 1000. The profit, however rose to ₦1,250,000 which is optimal. The reduction in the quantity of x_4 coinciding with the increase in profit explained that the production of x_4 is expensive although it gives high profit.
- x. Lastly, the quantities x_6 , x_7 and x_{10} being positive in the solution set indicate the quantities of residential or wastes which could be treated in order to increase the company's profit.

4.3 LIMITATIONS

This program is limited to maximum of 100 independent variables and 100 constraints to be solved. However, this can be increased, if need be, by changing the dimension of the input arrays.

Also, the programs can not adequately solve integer programming problems i.e. when integer independent variables are required.

Lastly, the solution to understand variables as discussed in section 2.4.1 (H) can not be implemented with this routine.

CHAPTER FIVE

CONCLUSION

For this study 1000 units of product IV, 14500 units of product III and 5000 units of product I should be manufactured, while product II should not be produced at all in order to yield the optimal profit of ₦1,250,000.00.

In order to avoid unnecessary waste in production, we recommends that operations research workers should be part of the production crews. Apart from been able to build linear programming models, they should also be very goo in scientific programming languages(s) such as PASCAL,C,FORTRAN etc. Alternatively, Techo-Equip should employ the service of professional's consultants who can provide such operations research advice.

In conclusion, it is obvious from the implementation of this project that 'real' manufacturing decision cannot be based on the intuition of the manager(s) but is better rested on strong mathematical formulations has exhibited in this project.

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```

SUBROUTINE SIMPLEX (A,M,N,MP,NP,M1,M2,M3, ICASE, IZROV,
IPOSV)
PARAMETER (MMAX=100, EPS=1.E-6)
DIMENSION A (MP,NP), IZROSV(M), L1 (MMAX), L2 (MMAX), L3 (MMAX)
WRITE(*, 7)
7 FORMAT(5X, 'enter input data')
  Read(*,*) A, M, N, MP, NP, M1, M2, M3
IF (M.NE.M1+M2+M3) PAUSE
NL1=N
DO (11) K=1,N
  L1 (K)=K
  IZROV (K)=K
  (11) CONTINUE
NL2=M
DO (12) I=1,M
  IF (A (I+1, I) .LT. 0) PAUSE
  L2 (I)=I
  IPOSV (I)=N+I
  (12) CONTINUE.
DO (13) I=1, M2
  L3 (I)=1
  (13) CONTINUE
IR=0
IF (M2+M3.EQ.0) GO TO 30
IR=1
DO (15) K=1, N+1
  Q1=0
  DO (14) I=M1+1, M
    Q1=Q1+A (I+1, K)
    (14) CONTINUE
  A (M+2, K)=-Q1

```

```

(15) CONTINUE.
10 CALL SIMP1 (A,MP,NP,M+1,L1,NL1,0,KP,BMAX) .
  IF (BMAX.LE.EPS.AND.(M+2,1).LT._EPS) THEN
    ICASE=- 1
    RETURN
  ELSE IF (BMAX.LE.EPS.AND.A(M+2,1).LE.EPS) THEN
    M12=M1+M2+1
    IF (M12.LE.M) THEN
      DO (16) IP=M12,M
    IF (IPOSV(IP).EQ.IP+N) THEN
      CALL SIMP1 (A,MP,NP,IP,L1,NL1,1,KP,BMAX)
      IF (BMAX.GT.O) GO TO 1

    END IF
    (16) CONTINUE.
  END IF.
  IR=0
  M12=M12-1
  IF (M1+1.G.T.M12) GO TO 30
  DO (18) I=M1+1,M12
    IF (L3(I-M1).EQ.1) THEN
      DO (17) K=1, N+1
        A(I+1, K)=-A(I+1,K)
        (17) CONTINUE.
      END IF
    (18) CONTINUE
  GO TO 30
  END IF
  CALL SIMP2 (A,M,N,MP,NP,L2,NL2,IP,KP,Q1)
  IF (IP.EQ.O) THEN
    I CASE=-1

```

```

        RETURN
    END IF
1 CALL SIMP3 (A,MP,NP,M+1,N,IP,KP)
    IF (IPOSV(IP).GE.N +M1+M2+1) THEN
        DO(19) K=1,NL1
            IF(L1(K).EQ.KP)GO TO2
        19 CONTINUE
2 NL1=NL1-1
    DO(21) IS=K,NLI
        L1(IS)=L1(IS+1)
        (21)CONTINUE.
    ELSE
        IF(IPOSV(IP).LT.N+M+1)GO TO 20
KH=IPOSV(IP)-M1-N
    IF (L3(KH).EQ.0 ) GO TO 20
    L3(KH)=0
END IF
A(M+2, KP+ 1)=A(M+2,KP+1)+1
DO(22) I=1,M+2
    A(I,KP+1)=_A(I,KP+1)
    (22)CONTINUE
20 IS=IZROV(KP)

    IZROV(KP) =IPOSV(IP)
    IPOSV(IP)=IS
    IF (IR.NE.O) GO TO 10

30 CALL SIMP1 (A,MP,NP,O,L1,NL1,O,KP,BMAX) .
    IF(BMAX.LE.O.) THEN
        ICASE =0
        RETURN
    END IF

```

```

CALL SIMP2 (A,M,N,MP,NP,L2,NL2,IP,KP,Q1)
IF(IP.EQ.O) THEN
    ICASE=1
    RETURN
END IF
CALL SIMP3 (A,MP,NP,M,N,IP,KP)
GO TO 20
END

```

```

SUBROUTINE SIMP1 (A,MP,NP,MM,LL,NLL,IABF,KP,BMAX)
DIMENSION A (MP,NP,) LL, (NP) .
KP =LL (1)
BMAX = A(MM+1,KP+1)B
DO(11) K=2,NLL,
    IF(IABF.EQ.O) THEN
        TEST =A(MM+1, LL(K)+1) -BMAX
    ELSE
        TEST=ABS(A(MM+1) ) -ABS(BMAX)
    END IF
    IF(TEST.GT.O) THEN
        BMAX=A(MM+1,LL(K)+1)
        KP= LL(K) .
    END IF
    (11)CONTINUE
RETURN
END.

```

```
SUBROUTINE SIMP2 (A,M,N,MP,NP,L2,NL2 IP,KP,Q1) .
```

```
PARAMETER (EPS=1.E-6)
```

```
DIMENSION A(MP,NP),L2(MP)
```

```
IP =0
```

```
DO (11) I = 1,NL2
```

```
IF(A(L2(I)+1,KP+1).LT-EPS) GO TO 1
```

```
(11)CONTINUE
```

```
RETURN
```

```
1 Q1=- A(L2(I)+1,1)/A(L2(I)+1,KP+1
```

```
IP=L2(I)
```

```
DO(13) I=I+1,NL2
```

```
II=L2(I)
```

```
IF(A(II+1,KP+1).LT.-EPS) THEN
```

```
Q=-A(II+1,1)/A(II+1,KP+1)
```

```
IF (Q.LT.Q1) THEN
```

```
IP=II
```

```
Q1=Q
```

```
ELSE IF(Q.EQ.Q1) THEN
```

```
DO(12)K=1,N
```

```
QP=-A(IP+1,K+1)/A(IP+1,KP+1)
```

```
QO=- A(II+1,K+1)/A(II+1,KP+1)
```

```
IF(QO.NE.QP) GO TO 2.
```

```
(12)CONTINUE
```

```
2 IF(QO.LT.QP) IP=II
```

```
END IF
```

```
ENDIF
```

```
(13)CONTINUE
```

```
RETURN
```

```
END.
```

```

SUBROUTINE SIMP3 (A,MP,NP,I1,K1,IP,KP)
DIMENSION A (MP,NP)
PIV=1/A(IP+1,KP+1)
DO(12) II=1,I1+1
    IF(II-1.NE.IP) THEN
        A(II,KP+1)=A(II,KP+1)*PIV
        DO(11) KK=1,K1+1
            IF(KK-1.NE.KP) THEN
                A(II,KK)=A(II,KK)-A(IP+1,KK)*
A(II,KP+1)
            END IF
        (11) CONTINUE
    END IF
(12) CONTINUE
DO(13) KK=I,K1+1
    IF(KK-1.NE.KP) A(IP+1,KK)=-A(IP+1,KK)*PIV
(13) CONTINUE
A(IP+1,KP+1)=PIV
RETURN
END.

```

PROFIT OPTIMIZATION ON METAL MANUFACTURING AT

TECHO-EQUIP

THE FOLLOWING VALUES WERE READ:- M, N, M1, M2, M3

- M - NUMBER OF CONSTRAINTS
- N - NUMBER OF VARIABLES
- M1 - NUMBER OF <= CONSTRAINTS
- M2 - NUMBER OF >= CONSTRAINTS
- M3 - NUMBER OF = CONSTRAINTS

M = 6, N = 4, M1 = 4, M2 = 2, M3 = 0

THE INPUT MATRIX IS:

	b _i	X(1)	X(2)	X(3)	X(4)
X(0) *	0.00	60.00	40.00	60.00	80.00
X(5) *	48000.00	-3.00	-2.00	-2.00	-4.00
X(6) *	40000.00	-1.00	-1.00	-2.00	-4.00
X(7) *	40000.00	-2.00	-1.00	-2.00	-1.00
X(8) *	2500.00	0.00	0.00	0.00	0.00
X(9) *	5000.00	-1.00	0.00	0.00	-1.00
X(10) *	10000.00	0.00	-1.00	-1.00	0.00
W(0) *	-10000.00	1.00	1.00	1.00	0.00

BASIC SOLUTION 1
 =====
 X(5) = 48000.00
 X(6) = 40000.00
 X(7) = 40000.00
 X(8) = 2500.00
 X(9) = 5000.00
 X(10) = 10000.00

THE OBJECTIVE HAS TO BE SOLVED IN 2 PHASES.

FIRST PHASE:

ITERATION 1

```

=====

```

	bi	X(9)	X(2)	X(3)	X(4)

0) *	300000.00	60.00	40.00	60.00	80.00
5) *	33000.00	-3.00	-2.00	-2.00	-4.00
6) *	35000.00	-1.00	-1.00	-2.00	-3.00
7) *	30000.00	-2.00	-1.00	-2.00	-1.00
8) *	2500.00	0.00	0.00	0.00	-1.00
9) *	5000.00	-1.00	0.00	0.00	0.00
0) *	10000.00	0.00	-1.00	-1.00	0.00
0) *	-10000.00	0.00	1.00	1.00	0.00

BASIC SOLUTION 2

- ```

=====

```
- 5) = 33000.00
  - ( 6) = 35000.00
  - ( 7) = 30000.00
  - ( 8) = 2500.00
  - ( 1) = 5000.00
  - ( 2) = 10000.00

ITERATION 2

```

=====

```

|         | bi        | X(9)  | X(10) | X(3)  | X(4)  |
|---------|-----------|-------|-------|-------|-------|
| *****   |           |       |       |       |       |
| X( 0) * | 700000.00 | 60.00 | 40.00 | 20.00 | 80.00 |
| X( 5) * | 13000.00  | -3.00 | -2.00 | 0.00  | -4.00 |
| X( 6) * | 25000.00  | -1.00 | -1.00 | -1.00 | -3.00 |
| X( 7) * | 20000.00  | -2.00 | -1.00 | -1.00 | -1.00 |
| X( 8) * | 2500.00   | 0.00  | 0.00  | 0.00  | -1.00 |
| X( 9) * | 5000.00   | 1.00  | 0.00  | 0.00  | 0.00  |
| X(10) * | 10000.00  | 0.00  | 1.00  | -1.00 | 0.00  |
| W( 0) * | 0.00      | 0.00  | 0.00  | 0.00  | 0.00  |

BASIC SOLUTION 3

---

- 5) = 13000.00
- 6) = 25000.00
- 7) = 20000.00
- 8) = 2500.00
- 1) = 5000.00
- 2) = 10000.00

THE SECOND PHASE:

\*\*\*\*\*

ITERATION 3

---

|         | bi        | X(9)  | X(10) | X(3)  | X(8)   |
|---------|-----------|-------|-------|-------|--------|
| X( 0) * | 900000.00 | 60.00 | 40.00 | 20.00 | -80.00 |
| X( 5) * | 3000.00   | -3.00 | -2.00 | 0.00  | 4.00   |
| X( 6) * | 175000.00 | -1.00 | -1.00 | -1.00 | 3.00   |
| X( 7) * | 175000.00 | -2.00 | -1.00 | -1.00 | 1.00   |
| X( 4) * | 2500.00   | 0.00  | 0.00  | 0.00  | -1.00  |
| X( 1) * | 5000.00   | 1.00  | 0.00  | 0.00  | 0.00   |
| X( 2) * | 10000.00  | 0.00  | 1.00  | -1.00 | 0.00   |

BASIC SOLUTION 4

---

- X( 5) = 3000.00
- X( 6) = 175000.00
- X( 7) = 175000.00
- X( 4) = 2500.00
- X( 1) = 5000.00
- X( 2) = 10000.00

ITERATION 4

|         | bi        | X(5)   | X(10) | X(3)  | X(8)  |
|---------|-----------|--------|-------|-------|-------|
| X( 0) * | 960000.00 | -20.00 | 0.00  | 20.00 | -0.00 |
| X( 9) * | 1000.00   | -0.33  | -0.67 | 0.00  | 1.00  |
| X( 6) * | 165000.00 | 0.33   | -0.33 | -1.00 | 1.00  |
| X( 7) * | 155000.00 | 0.67   | 0.33  | -1.00 | -1.00 |
| X( 4) * | 2500.00   | 0.00   | 0.00  | 0.00  | -1.00 |
| X( 1) * | 6000.00   | -0.33  | -0.67 | 0.00  | 1.33  |
| X( 2) * | 10000.00  | 0.00   | 1.00  | -1.00 | 0.00  |

BASIC SOLUTION 5

- X( 9) = 1000.00
- X( 6) = 165000.00
- X( 7) = 155000.00
- X( 4) = 2500.00
- X( 1) = 6000.00
- X( 2) = 10000.00

ITERATION 5

|         | bi         | X(5)   | X(10) | X(2)  | X(8)  |
|---------|------------|--------|-------|-------|-------|
| X( 0) * | 1160000.00 | -20.00 | 0.00  | 20.00 | -0.00 |
| X( 9) * | 1000.00    | -0.33  | -0.67 | 0.00  | 1.00  |
| X( 6) * | 6500.00    | 0.33   | -0.33 | -1.00 | 1.00  |
| X( 7) * | 5500.00    | 0.67   | 0.33  | -1.00 | -1.00 |
| X( 4) * | 2500.00    | 0.00   | 0.00  | 0.00  | -1.00 |
| X( 1) * | 6000.00    | -0.33  | -0.67 | 0.00  | 1.33  |
| X( 2) * | 10000.00   | 0.00   | 1.00  | -1.00 | 0.00  |

BASIC SOLUTION 6

- X( 9) = 1000.00
- X( 6) = 6500.00
- X( 7) = 5500.00
- X( 4) = 2500.00
- X( 1) = 6000.00
- X( 3) = 10000.00

ITERATION 6

|         | bi         | X(5)   | X(9)   | X(2)   | X( |
|---------|------------|--------|--------|--------|----|
| *****   |            |        |        |        |    |
| C( 0) * | 1190000.00 | -30.00 | -30.00 | -20.00 | 40 |
| X(10) * | 1500.00    | -0.50  | -1.50  | 0.00   | 2  |
| X( 6) * | 4500.00    | 1.00   | 2.00   | 1.00   | -1 |
| X( 7) * | 4500.00    | 1.00   | 1.00   | 1.00   | -2 |
| X( 4) * | 2500.00    | 0.00   | 0.00   | 0.00   | -1 |
| X( 1) * | 5000.00    | -0.00  | 1.00   | 0.00   | 0  |
| X( 3) * | 11500.00   | -0.50  | -1.50  | -1.00  | 2  |

BASIC SOLUTION 7

|         |          |
|---------|----------|
| X(10) = | 1500.00  |
| X( 6) = | 4500.00  |
| X( 7) = | 4500.00  |
| X( 4) = | 2500.00  |
| X( 1) = | 5000.00  |
| X( 3) = | 11500.00 |

ITERATION 7

|         | bi         | X(5)   | X(9)   | X(2)  | X  |
|---------|------------|--------|--------|-------|----|
| *****   |            |        |        |       |    |
| C( 0) * | 1250000.00 | -16.67 | -16.67 | -6.67 | -1 |
| X(10) * | 4500.00    | 0.17   | -0.83  | 0.67  |    |
| X( 6) * | 3000.00    | 0.67   | 1.67   | 0.67  |    |
| X( 8) * | 1500.00    | 0.33   | 0.33   | 0.33  |    |
| X( 4) * | 1000.00    | -0.33  | -0.33  | -0.33 |    |
| X( 1) * | 5000.00    | 0.00   | 1.00   | 0.00  |    |
| X( 3) * | 14500.00   | 0.17   | -0.83  | -0.33 |    |

BASIC SOLUTION 8

|         |          |
|---------|----------|
| X(10) = | 4500.00  |
| X( 6) = | 3000.00  |
| X( 8) = | 1500.00  |
| X( 4) = | 1000.00  |
| X( 1) = | 5000.00  |
| X( 3) = | 14500.00 |

THE BASIC FEASIBLE SOLUTION IS:

---

4) = 1000.00

(1) = 5000.00

(3) = 14500.00

THE OPTIMISED OBJECTIVE FUNCTION IS #1250000.00