AUTOMATION OF SOLUTIONS OF SYSTEM OF LINEAR EQUATIONS USING GAUSSIAN ELIMINATION METHOD

BY

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PGD/MCS /1999/2000/907

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A PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF A POSTGRADUATE DIPLOMA IN MATHEMATICS/COMPUTER SCIENCE IN THE DEPARTMENT OF MATHEMATICS/COMPUTER SCIENCES, SCHOOL OF SCIENCE AND SCIENCE EDUCATION, FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA, NIGER STATE, NIGERIA.

SEPTEMBER 2001.

CERTIFICATION

This is to certify that this project work submitted to the department of Mathematics/Computer sciences has been approved as meeting the requirements of the Department of Mathematics/Computer Sciences, Federal University of Technology Minna, in partial fulfillment of the award of a postgraduate Diploma in Computer Science.

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MR L. N. EZEAKOR
(PROJECT SUPERVISOR)

SIGNATURE AND DATE

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DR S. A. REJU
(HEAD OF DEPARTMENT)

SIGNATURE AND DATE

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(EXTERNAL EXAMINER)

SIGNATURE AND DATE

DEDICATION

This project work is dedicated to ALLAH the most high and to my parents ALHAJI AND HAJIA BALA NAMAMA, my Guardian (HRM), The Maigari of Lokoja ALHAJI MOHAMMED KABIR (III), Late SULEMAN KANSILAM (Jigawa INEC Commissioner), my fiancée FATI DARDA'U and to the poor and less privilege in the society who out of financial difficulties could not further their education. All of them have been my inspiration and encouragement in the course of this project.

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I specifically wish to register my acknowledgement to all my colleagues who make my stay very conducive and friendly.

I am truly grateful.

ABSTRACT

This project is based on the various solutions of system of linear equation. A linear equation based on dependent and independent variable.

Pascal programming language is used to find the solutions to a set of linear equation containing an unknown variable that satisfies the equation $Ax = b$ which have exactly as many equations as unknown, for which the coefficient matrix A is square. For such systems, A should be inevitable in order that that the system have exactly one solution for every right hand side b.

Application of computer is used to calculate the values of the unknowns in the equation, which is achieved, by the use of GAUSSIAN ELIMINATION METHOD.

TABLE OF CONTENTS

CHAPTER ONE

1.0 INTRODUCTION

Many of the problems of numerical analysis can be reduced to the problem of solving linear systems of equations. Among the problems, which can be so treated, are the solution of ordinary or partial/differential equations by finite difference methods, the solutions of system of equations, the eigen value problems of mathematical equation, least squares fitting of data and polynomial approximation.

The use of matrix notation is not only convenient, but also extremely powerful, in bringing out fundamental relationships.

1.1 LINEAR EQUATION

An equation of the form

$$\alpha_1 X_1 + \alpha_2 X_2 + \text{-----------------------------------} \alpha_n X_n = b \text{---------------------------} (1)$$

Where $\alpha_1 \alpha_2 \text{..................} \alpha_n$ and b are in field (F) and $X_1 X_2 \text{..........................} X_n$ are the unknowns is called a linear equation, in an unknown over F.

The elements $\alpha_1, \alpha_2 \text{-----------------------------------} \alpha_n$ are called the co-efficient of the Unknown and b is called the constant term of the equation.

A Vector $C_1, C_2 \text{........................} C_n$ in f_n is called the solution of the equation (I) if and only if.

$$\alpha_1 C_1 + \alpha_2 C_2 + \text{-------------------------------------} + C_n = b \text{ is a true equation.}$$

Consider the linear equation,

$3x - 4y + 5z = 17$ --(2)

The equation is in three unknowns x, y, z, over the rational field (Q)

The vector (3, -2, 0) in Q3 is a solution of the equation (2) since

$3(3) - 4\ (-2) + 5\ (0) = 17.$

On the other hand (4, 1, 2,) is not a solution because if you fix the values in to the equation, it will not give 17, hence it is a false solution.

1.2 CONCEPT OF MATRICES AND SYSTEM OF LINEAR EQUATION

A system of M linear equation in n unknowns has the general form:

$\alpha_{11} X_1 + \alpha_{12} X_2 + \text{------------------} \alpha_{1n} X_n = b_1$

$\alpha_{21} X_1 + \alpha_{22} X_2 + \text{------------------} \alpha_{2n} X_n = b_2$

$\alpha_{m1} X_1 + \alpha m_2 X_2 + \text{------------------} \alpha_{mn} X_n = b_m$ ....(1.2)

Where the co- efficient αij (i = 1, 2,-----------,m: j = 1, 2,-------------,n) and the right sides bi, i= (1, 2, ---------------,m) are given numbers.

The problem is to find if possible, numbers Xj, j = (1, 2, ---------------, n) such that the m equations (1.2) are satisfied simultaneously. The discussion and understanding of this problem is greatly facilitated when using algebraic concepts of matrix and vector.

1.3 DEFINITION OF MATRIX AND VECTOR

A matrix is a rectangular array of (usually real) numbers arranged in rows and columns. The co-efficient of (1.2) form a matrix, which is represented by A. It is customary to display such a matrix A as follows:

$$A = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \alpha_{2n} \\ \multicolumn{3}{c}{\text{-------------------------}} \\ \multicolumn{3}{c}{\text{-------------------------}} \\ \alpha_{m1} & \alpha_{m2} & \alpha_{mn} \end{pmatrix} \text{---------------------------(1.3)}$$

At times, we can write it more briefly.

$$A = \begin{bmatrix} \alpha ij \end{bmatrix}$$

The nature A in (1.3) has m rows and n columns, or A is of order mXn for short. The (i,j) entry of A is located at the intersection of the i^{th} row and j^{th} column of A. If A is an nXn matrix, then A is said to be a square matrix of order n.

If a matrix has only one column, we call it a column VECTOR, and a matrix having only one row is called a Row VECTOR. A column vector is represented by a single lower case letter in bold type, to distinguish them from other matrices, and called VECTORS. Thus both the right-side constants bi, i = (1, 2, ...n) and the unknowns Xj,. j = (1,2 ...,n) form VECTORS. e.g.

$$B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ " \\ " \\ b_m \end{pmatrix} \qquad X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ " \\ " \\ X_n \end{pmatrix}$$

b is said to be a m-vector, and X is an n-vector.

1.4 DIAGONAL AND TRIANGULAR MATRICES

If $A = (\alpha ii)$ is a square matrix of order n, then it's entries $\alpha_{11}, \alpha_{22}, \_\_\_\_\_\_\_\_ \alpha_{n\,n}$ are called the DIAGONAL entries of A and the other entries are call off-diagonal.

All entries of (αij) of A with $i < j$ are called Super Diagonal, while all entries aij with $i > j$ are called Sub Diagonal.

If all the entries of the matrix A are 1's such matrix is called a UNIT matrix and if all the entries are zero, this is called a zero matrix. If all sub-diagonal entries of the square matrix A are zero, the matrix is called an UPPER (or RIGHT) triangular matrix, while if all super diagonal entries of A are zero, then A is called LOWER (or LEFT) triangular matrix. Clearly, a matrix is diagonal if and only if all entries are zero except the main diagonal. Example,

$$A = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix} \qquad D = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}$$

$$E = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ 0 & \alpha_{22} & \alpha_{23} \\ 0 & 0 & \alpha_{33} \end{pmatrix} \qquad F = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$G = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ 0 & 0 & \alpha_{33} \end{pmatrix}$$

In the above matrix, A is a square matrix of order 3, B is a unit matrix, C is a diagonal matrix, D is a Lower triangular matrix, E is an upper triangular matrix, F is a zero matrix while G has non of those properties.

1.5 DETERMINANTS

Associated with every square matrix A of numbers is a number called DETERMINANT of the matrix and usually denoted by det (A). If A = (αij), is nXn matrix, then the determinant of A is defined by:

$$\text{Det}(A) = \Sigma\sigma_{\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}} \qquad \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix}$$

$$= \Sigma\sigma_{\alpha_{1ij}\alpha_{4ij} - \alpha_{2ij}\alpha}$$

$$= \Sigma\sigma_{\alpha_1\alpha_4 - \alpha_3\alpha_2}$$

Where the sum is taken over all n! Permutations p of degree n, and $\delta\alpha$ is l or –1 depending on whether α is even or odd. Hence if n = 1 then

$$\det(A) = \det\begin{bmatrix} \alpha_{11} \end{bmatrix} = \alpha_{11}$$

if n = 2 then

$$\det(A) = \det\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$$

$$= \alpha_{11}\ \alpha_{22} - \alpha_{12}\alpha_{21}$$

Also if n=3, six products have to be summed and for n = 10, over a million products each with 10 Factors have to be computed and summed. Hence, the definition is not very suitable for the calculation of determinants.

1.6 THE EIGEN VALUE PROBLEM

λ is an Eigen value for A iff

det $(A - \lambda_1) = 0$

The number λ is an eigen value of the matrix A iff the matrix $(A - \lambda_1)$ is not invertible.

Hence, finding all the eigen values for a given matrix A is equivalent to finding all the roots of the so called characteristic equation.

det $(A - \lambda_1) = 0$.

The matrix $(A - \lambda_1)$ differs from A only in that $-\lambda$ has been subtracted from each diagonal entries of A. Example

$$\text{If} \quad A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 4 & 1 & -4 \end{pmatrix}$$

Then the characteristic polynomial is denoted by $/A - \lambda_1/$

$$\begin{pmatrix} 1-\lambda & 1 & -1 \\ 2 & 3-\lambda & -4 \\ 4 & 1 & -4--\lambda \end{pmatrix}$$

$$= \Big[(1-\lambda)(3-\lambda)(-4-\lambda) + 4\Big] - 1\Big[-8 - 2\lambda + 16\Big] - 1\Big[2 - 12 + 4\lambda \Big]$$

$$= -8 + \lambda + \lambda^2 + 8\lambda - \lambda^2 - \lambda^3 + 2 - 2\lambda$$

$= (\lambda - 1)(\lambda - 2)(\lambda + 3) = 0$

$\therefore \lambda = 1, \lambda = 2, \lambda = -3$ are the eigen values

1.7 SOLUTION OF SYSTEMS OF LINEAR EQUATIONS

The solution of equation (1.2) is an n triple of real number X_1, X_2----------X_n

That satisfies each equation of (1.2), If (1.2) has no solution, then it is called an inconsistent system of equation. If it has a solution, it is called a consistent system of equation.

If $b_1 = b_2 =$--$b_n = 0$ then (1.2 is called an HOMOGENOUS SYSTEM OF EQUATION. The solution $X_1 = X_2 =$ ----------$X_n = 0$ is a solution of homogeneous equation called the TRIVIAL SOLUTION.

A solution of homogenous equation in which not all of the X_1, X_2----------X_n are 0 is called a NON-TRIVIAL SOLUTION.

1.8 METHODS OF SOLVING SYSTEMS OF LINEAR EQUATION

Considering the linear systems

$AX = b$

Which has one and only one solution for every right side b, we must therefore restrict attention to those solutions which have exactly as many equations as unknown, i.e. for which the co-efficient matrix A is square - A should be invertible in other that the system has exactly one solution for every right side b. Therefore assume that all linear systems under discussion have an invertible co-efficient matrix.

1.8.1 SOLVING SYSTEMS OF LINEAR EQUATION USING ELIMINATION METHOD

Example: Find all solutions of the system of equations:

$X + 2y - 2z + 3w = 2$ ....(i)

$2x + 4y - 3z + 4w = 5$ ....(ii)

$5x + 10y - 8z + 11w = 12$ ....(iii)

From (i)

$X = 2 - 2y + 2z - 3w$ ....(iv)

Substituting (iv) into (ii), we obtained.

$4 - 4y + 4z - 6w + 4y - 3z + 4w = 5$

$z - 2w = 1.$ .... (v)

Putting (iv) into (iii)

$10 - 10y + 10z - 15w + 10y - 8z + 11w = 12$

$2z - 4w = 2$

$x + 2y - 2z + 3w = 2$

$z - 2w = 1$

This shows that the system is inconsistent and thus has no solution.

1.8.2 SOLVING SYSTEMS OF LINEAR EQUATION BY FORMING MATRIX

Example: - To solve the set of equation

$X_1 + 2X_2 + X_3 = 4$

$3X_1 - 4X_2 - 2X_3 = 2$

$5X_1 + 3X_2 + 5X_3 = +1$

First write the set of equations in matrix form which gives

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$$

i.e. $A. X = b$ therefore

$X = A^{-1} . B$

Next step is to find the inverse of A i.e. A – 1 where A is the matrix of the coefficient of x.

$$/A/ = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{pmatrix}$$

$= -14 - 15 + 29$

$= 29 - 64$

$\therefore /A/ = -35$

Co-factor

$A_{11} = (-20 + 6) = -14$

$A_{12} = -(15 + 10) = -25$

$A_{13} = (9 + 20) = 29$

$A_{21} = -(10-3) = -7$

$A_{22} = (5 - 5) = 0$

$A_{23} = -(3-10) = 7$

$A_{31} = (-4 + 4) = 0$

$A_{32} = -(-2-3) = 5$

$A_{33} = (-4 - 6) = -10$

$$\therefore C = \begin{pmatrix} -14 & -25 & -29 \\ -7 & 0 & 7 \\ 0 & 5 & -10 \end{pmatrix}$$

Therefore

$$\text{Adj } A = C^T = \begin{pmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{pmatrix}$$

Now /A/ = -35 therefore $A^{-1} = \frac{AdjA}{/A/}$

$$= \frac{-1}{35} \begin{pmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{pmatrix}$$

$X = A^{-1} . b$

$$= \frac{-1}{35} \begin{pmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$$

By multiplying,

$$X = \begin{pmatrix} -70 \\ -105 \\ 140 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$$

$\therefore$ Finally $X = X_1 = 2$

$X_2 = 3$

$X_3 = -4$

1.9 APPLICATIONS OF LINEAR SYSTEM OF EQUATION

1. Many of the problems of numerical analysis can be reduced to the problem of solving linear systems of equation. Example is the solution of an ordinary or partial differential equations by finite – difference methods.
2. It is used in statistics to find the least-squares fitting of data by calculating a line, which minimizes the total of the square deviations of the actual approximations from the calculated line.
3. Eigen values play a major role in the study of convergence of iterative method for solving linear system of equations. The stability of an aircraft, for example, is determined by the location of the Eigen values of a certain matrix in the complex plane. The natural frequencies of vibrations of beams are actually Eigen values of a matrix.

CHAPTER TWO

2.0 THE NUMERICAL SOLUTION OF LINEAR SYSTEMS

Considering only the linear system.

$$Ax = b$$

which have one and only one solution for every right side b. Attention should be restricted only to those systems which have as many equations as unknown i.e. for which the co-efficient matrix A is square, for such systems, A should be invertible in order that the system have exactly one solution for every right side b. Assume that all linear system under discussion have an invertible co-efficient matrix.

A frequently quoted text for invertibility of a matrix is based on the concept of the determinant. The relevant theorem states that the matrix A is invertible if and only if

$\text{Det}(A) \neq 0$.

If $\det(A) \neq 0$, then it is possible to express the solution of $Ax = b$ in terms of determinants, by crammer's rule. Nevertheless, determinants are of the practical interest for the solution of linear systems since the calculation of one determinant is in general of the same order of difficulty as solving the linear systems.

2.1 NUMERICAL METHODS OF SOLVING LINEAR SYSTEMS

The method of solving linear systems may be divided into two types:

1. Direct Method
2. Iterative Method

2.11 DIRECT METHODS

Direct methods are those which, in the absence of round off or other errors, will yield the Exact solution in a finite number of elementary arithmetic operations. In practice, because a computer works with finite word length, direct methods do not lead to exact solutions. Indeed, errors arising from round-off, instability and loss of significance may lead to extremely poor or useless results. A large part of numerical analysis is concerned with only and how these errors arise, and with the search for methods which minimize the totality of such errors. The fundamental method used for direct solutions is GAUSS ELIMINATION, but even within this class there are a variety of choices of methods, which vary, in computational efficiency and accuracy.

2.12 ITERATIVE METHODS

Iterative methods are those which starts with an initial approximation and which by applying a suitable chosen algorithms, lead to successfully better approximations. Even if the process converges, we can only hope to obtain an approximation solution by iterative methods. Iterative methods vary with the algorithm chosen and in their rates of convergence – some iterative methods may actually diverge; others may converge so slowly that they are computationally useless. The important advantage of iterative methods are the simplicity and uniformity of the operations to be performed, which make them well suited for use on computers and their relative insensitivity to the growth of round-off errors.

2.2 GAUSSIAN ELIMINATION METHOD

For solving a set of linear equation, i.e.

$$Ax = b$$

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \cdots\cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \cdots\cdots & \alpha_{2n} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \cdots\cdots & \alpha_{3n} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{n1} & \alpha_{n2} & \alpha_{n3} \cdots\cdots & \alpha_{nn} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \cdots \\ \alpha_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ \cdots \\ b_n \end{pmatrix}$$

All the information for solving the set of equations is provided by the matrix of coefficient A and the column matrix b. If the element b is written within the matrix A, we obtain the augmented matrix B of the given set of equations i.e.

$$B = \left(\begin{array}{cccc|c} \alpha_{11} & \alpha_{12} & \alpha_{13} \cdots\cdots & \alpha_{1n} & b_1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \cdots\cdots & \alpha_{2n} & b_2 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \cdots\cdots & \alpha_{3n} & b_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha_{n1} & \alpha_{n2} & \alpha_{n3} & \alpha_{nn} \cdots\cdots & b_n \end{array}\right)$$

(a) by eliminating the elements other than all from the first column by subtracting α_{21}/α_{11} times the first row from the second row and α_{31}/α_{11} times the first row from the third row, etc.

(b) This gives a new matrix of the form

$$\left(\begin{array}{ccccc|c} \alpha_{11} & \alpha_{12} & \alpha_{13} & \cdots & \alpha_{1n} & b_1 \\ 0 & C_{22} & C_{23} & \cdots & C_{2n} & d_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & C_{n2} & C_{n3} & \cdots & C_{nn} & d_n \end{array}\right)$$

The process is then repeated to eliminate C_{22} from the third and subsequent rows.

Solution of vector X can now be determined when A is upper triangular with all diagonal elements non-zero. Then the right hand column is back to its original position.

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} \cdots & C_{1,\ n-1} & C_{in} \\ 0 & C_{22} & C_{23} \cdots & C_{2,\ n-1} & C_{2n} \\ 0 & 0 & C_{33} \cdots & C_{3,\ n-1} & C_{3n} \\ | & & & & \\ | & & & & \\ | & & & & \\ | & & & & \\ 0 & 0 \cdots & \cdots & & C_{nn} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ | \\ | \\ | \\ | \\ X_n \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ | \\ | \\ | \\ | \\ d_n \end{pmatrix}$$

In particular, the last equation involves only x_n, ……… hence since $C_{nn} \neq 0$, we must have

$$X_n = \frac{d_n}{C_{nn}}$$

Since we know now x_n, the second last equation involves only one unknown, namely

x_n^{-1} as $C_n^{-1}{}_{n-1} \neq 0$, it follows that

$C_{n-1}{}^{'}{}_{n-1} X_{n-1} + C_{n-1}{}^{'}{}_{n} X_n = d_{n-1}$

Therefore $X_{n-1} = \dfrac{d_{n-1} - C_{n-1},{}_{n} X_n}{C_{n-1}{}^{'}{}_{n-1}}$

With X_n and X_{n-1} now, determine the third last equation $C_{n-2}{}^{'}{}_{n-2} + C_{n-2}{}^{'}{}_{n-1} X_{n-1} + C_{n-2}{}^{'}{}_{n} X_n$

$= d_{n-2}$

Contains only one true unknown, namely, X_{n-2}. Once again,

Since $\alpha_{n-2,\, n-2} \neq 0$, we can solve for X_{n-2}

$$X_{n-2} = \frac{d_{n-2} - C_{n-2,\, n-1} X_{n-1} - C_{n-2}{}^{'}{}_{n} X_n}{C_{n-2,\, n-1}}$$

In general, with $X_{K+1}, X_{K+2}, \ldots\ldots\ldots\ldots, X_n$ already computed, the k^{th} equation can be uniquely solved by X_k, since $C_{kk} \neq 0$ to give

$$X_K = \frac{d_K - \sum_{j=K+I}^{n} C_{KJ} X_J}{C_{KK}}$$

Example: Solve

$X_1 + 2X_2 - 3X_3 = 3$

$2X_1 - X_2 - X_3 = 11$

$3X_1 + 2X_2 + X_3 = -5$

The augmented matrix becomes

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 3 \\ 2 & -1 & -1 & 11 \\ 3 & 2 & 1 & -5 \end{array}\right)$$

Now , subtracting 2 times the first row from the second row and 3 times the first row from the third row, this gives.

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -5 & 5 \\ 0 & -4 & 10 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 11 \\ -14 \end{pmatrix}$$

Now, subtraction -4/-5, i.e. 4/5, times the second row from the third row.

The matrix become

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -5 & 5 \\ 0 & 0 & 6 \end{pmatrix} \quad \begin{pmatrix} 3 \\ 5 \\ -18 \end{pmatrix}$$

Note, as a result of these steps, the matrix of co-efficient of X has been reduced to a triangular matrix and finally, by detecting the right-hand column back to its original position.

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & -5 & 5 \\ 0 & 0 & 6 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -18 \end{pmatrix}$$

then, by 'back-substitution', starting from the bottom row we obtain.

$6X_3 = -18$ therefore $X_3 = -3$

Similarly

$-5X_2 + 5X_3 = 5$ therefore

$-5\,X_2 = 5 + 15 = 20$

$X_2 = -4$

Also

$X_1 + 2X_2 - 3X_3 = 8$ therefore

$X_1 - 8 + 9 = 3$

$X_1 = 2$

In General $X_1 = 2$, $X_2 = -4$, $X_3 = -3$.

The following qualities are associated with augmented matrix.

(a) Two rows can be interchanged

(b) Multiply any row by a non-zero factor

(c) Add (or subtract) a constant multiply of any one row to (or from) another.

These operations are permissible since we are really dealing with the co-efficient of both sides of the equations.

CHAPTER THREE

ANALYSIS AND DESIGN OF THE PROPOSED SYSTEM

3.0 COMPUTER SYSTEM

A system is defined as a set of programs on input and files to provide desired outputs. Every system must be analysed, designed, programmed, debug where necessary, tested and periodically evaluated.

3.1 AIMS AND OBJECTIVES OF THE SYSTEM

The aim of this project is the application of the computer in solving problems of system of linear equation using GAUSSIAN ELIMINATION

With advancement in technological development in the whole world as well as the introduction of computer system, Calculation has been made easier and one can accomplish his goals in a very short period of time, the result are also accurate and reliable.

The short coming of human nature is over come by the use of computers in that there is bound to be a difference no matter how little, analysis when correctly assigned to the computers take very short time to analyze and produce the some results when tested repeatedly.

The programming language of choice to the application of this project is PASCAL LANGUAGE, This is chosen because it is well structured and its syntax makes it easy to write programs that are modular hence easy to understand.

3.2 SYSTEM ANALYSIS

Analysis of needs and resources, to select and plan effective data applications and the equipment and human resources needed to carry them out. The primary aim of this activity is to produce the best answer to identified needs. The most economical solution consistent with management and operating objectives, equipment capabilities and personal resources. This activity includes such task as.

(a) Selecting of systems to be developed, incorporating in a general development plan, and declaration of projects within the plan. This can include:- review with user departments and copy management of information needs and their translations into agreed objectives and requirements considered for the life span of the applications.

To determine whether cost are justified by the purpose to be served. Analysis of proposes application to determine whether they fit into the overall plan of data processing development.

Continuing liaison with the operating department and company to review the need for new applications are satisfying objectives.

(b) Cost/benefit analysis of alternative approaches to system design and comparism to present and projected cost

(c) Selection with user management of the most cost effective approach to its documentation in a system requirement statement.

(b) Analysis of data base requirements and development of database structures to serve a set of application.

(c) Analysis of current and projected operations to determine possible improvement and to serve as the row material for sub-system design. This include gathering data about current systems and methods, application objectives, cost, volumes, reports and recording such data in a manner suitable for analysis e.g. flow chart, table, graphs etc.

3.3 SYSTEM DESIGN

This is the transformation of the problem statement into a detailed design specification. After last chapter, the next step is to design the; new or proposed system.

Before designing a system, the problem to be solved or objectives to be satisfied by a system must be specified.

At this stage, the new system that will meet a set of objectives will be created. These objectives are the driving force behind the design process.

Designers, also must select the equipment needed to implement the new system, specify new programs or changes to the existing programs and specify the data structure used.

There are basically two approaches to design:

(i) initial design.

(ii) detailed design

3.3.1 INITIAL DESIGN

The initial design for the problem is the basic steps to be followed while transforming the original problem statement into required designed specification. Therefore, the initial design for the Gaussian elimination will be:

Enter the dimension of the matrix

Read data into matrix

Read data into vector (result)

Perform forward substitution

Perform backward substitution

Compute the result

3.3.2 DETAILED DESIGN

The detailed design does not begin until the initial design is complete. The processes used in initial design are used to specify users procedures. Detailed design proposes the user interface of the new system thereby helping to make sure that the system does what is required of it.

3.4 INPUT FILE SPECIFICATION

The input designed for the program is such a way that it will accept the values of the dimension of the matrix A, the coefficient of the unknowns of the matrix A and the values of the vector B. These values are used to calculate the values of the unknowns in the matrix.

3.5 OUTPUT FILE SPECIFICATION

The output file specification (the required output of the system) depends on the input file specification.

The output form for the system is shown below:-

Matrix coefficients

X_1 X_2..X_n = b_1

.... = ...

.... = ...

.... = ...

.... = b_n

Final computed result

value of X(1) is =

value of X(2) is =

"   "   "

value of $X(_n)$ is =

3.6 HARDWARE/SOFTWARE REQUIREMENTS

For this project one should have access to Pentium 233 MMX, Dos 6.22, a compatible machine either a monochrome or colour monitor, the input device i.e. Keyboard. A dual floppy drive or a fixed (hard) drive with a single floppy drive and finally, it is essential to have a printer to generate program listing and record program output and Pascal programming language.

CHAPTER FOUR

SYSTEM DESIGN AND IMPLEMENTATION.

4.1 INTRODUCTION

System design and implementation is the process of converting the required specification into the system requirement, coding, testing and documentating programs in the system, it also involve development of quality assurance procedures, including data security, backup and recovery and system controls, this involves testing programs, with both artificial and live data and training users and operating personnel.

4.2 PROGRAM DOCUMENTATION

This is the description of the program in the proper from for users and to enhance maintainability it describe the workings of the program and how expected problems could be solved. Documentation may be internal (in form of comments which exist within the program) or external (in the form of written description and structured diagrams).

The programmer documentation provided for these project are:

1 program operation flow chart shown in appendix A

2 source code shown in appendix B

4.3 CHOICE OF PROGRAM LANGUAGE

The language used in designing this project work is PASCAL PROGRAMMING LANGUAGE. It is chosen because it is well structure and its syntax makes it easy to write programs that are modular and therefore easy to understand and maintain. Another characteristics of Pascal are that it is strongly typed. Also it is friendly user since it appears in English like.

CHAPTER FIVE

EVALUATION AND SUGGESTION FOR FURTHER STUDIES.

5.1 EVALUATION OF THE SYSTEM

This is referred to the review of the system for the following reasons:

(a) to ensure that the outline goals and objective for the computation is obtained.

(b) an adequate optimization, utilization and maintenance of the system is achieved.

(c) to foresee any problem that may arise while the program is running.

For this project, the evaluation is done in two ways.

(i) Workability of the system

(ii) Limitation of the system.

5.1.1 WORKABILITY OF THE SYSTEM

This is the change over from the manual system to the computerized system. It is often a complete and separate system task in itself, involving fact finding, analysis, data capture, design of electrical methods and computer processes, form design and production of special training course for easy usage. Setting up new master file for large system can involve the transfer of hundreds of thousands of records, which may be beyond manual handling capability.

The change over may be achieved in a number of ways, the most common methods are:

(i) **Direct method:-** this is the direct replacement from the old to the new system in one move

(ii) **Parallel running:-** This involve processing both the new and old method to cross check the result.

(iii) **Pilot running:-** This is more like the parallel running method. Here, data from one or more previous periods for the while or part of the system is run on the new system after results have been obtained from the old system the new results are compare with the old one.

(iv) **Stage change over:-** This is when the new system is been introduced piece-by-piece.

5.1.2 LIMITATION OF THE SYSTEM

In the process of determining the nature and scope of this project, the following was observe and correct measures taken.

(a) If the problem is incorrectly or incompletely defined, the entire study will address the wrong issues.

(b) The equation used Ax=b should have exactly as many equations as unknown and must be a square matrix, also A should be invertible in order that the system have exactly one solution for every right side b.

5.2 SUMMARY AND CONCLUSION

The introduction of computer system to solve system of linear equation was made to improve on the manual system of solving linear systems of equation, which is tedious and time wasting.

Since the inception of the computer it has always been easier and more convenient to execute such problems, which is fast, accurate and reliable.

5.3 RECOMMENDATION

This program is recommended for practical problems to determined one and only one solution for every right side. The computation of Gaussian elimination replaces the old manual system. It makes calculation simple and produces accurate result of the statistical data.

Since this increases efficiency and saves time, it would be recommended that this program should be put into proper usage so that, we will all enjoy and utilize the advantages of the system.

REFERENCES

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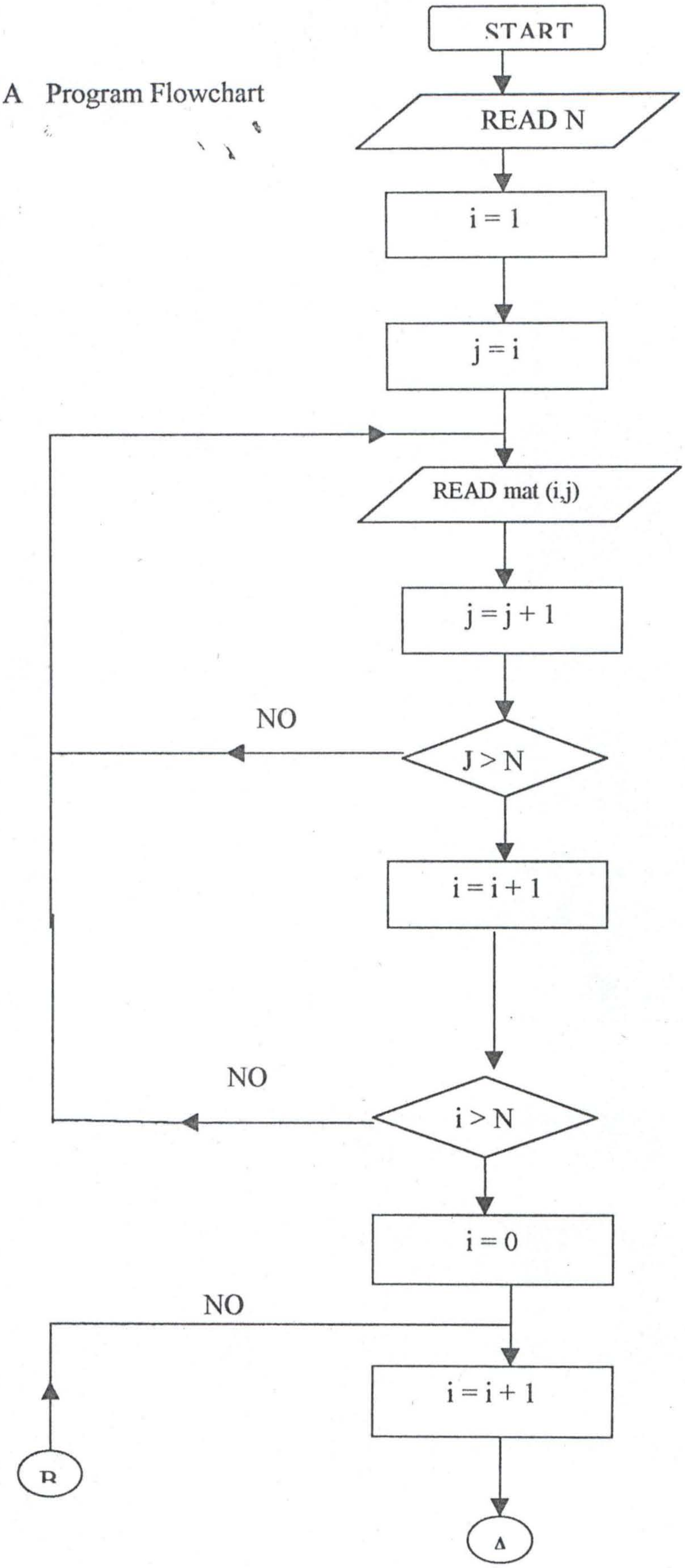
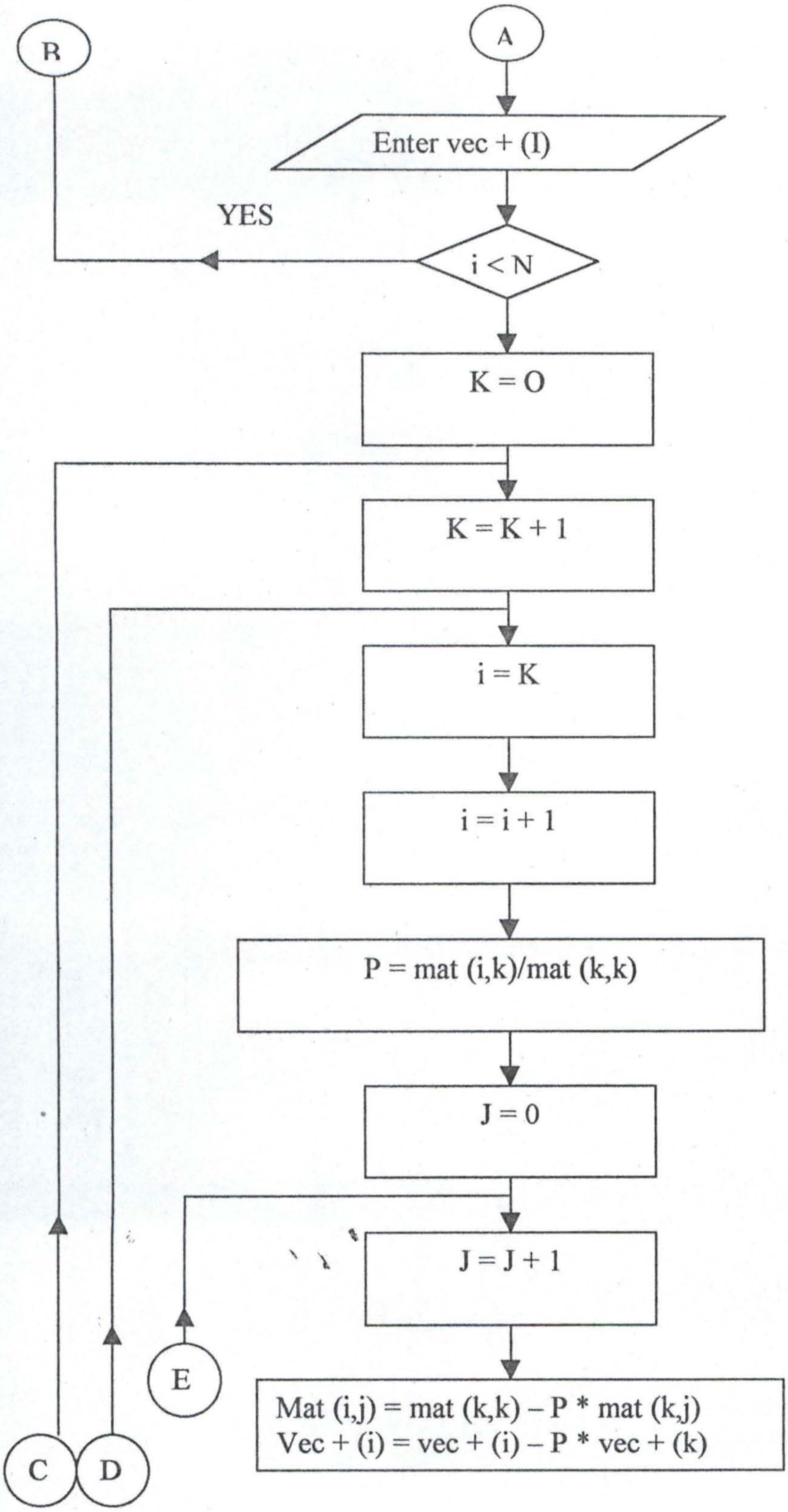
3. Faddeev, D.K., and V.H. Faddeeva (1962). Computational Methods of Linear Algebra" W.H. Freeman and Company, Francisco.

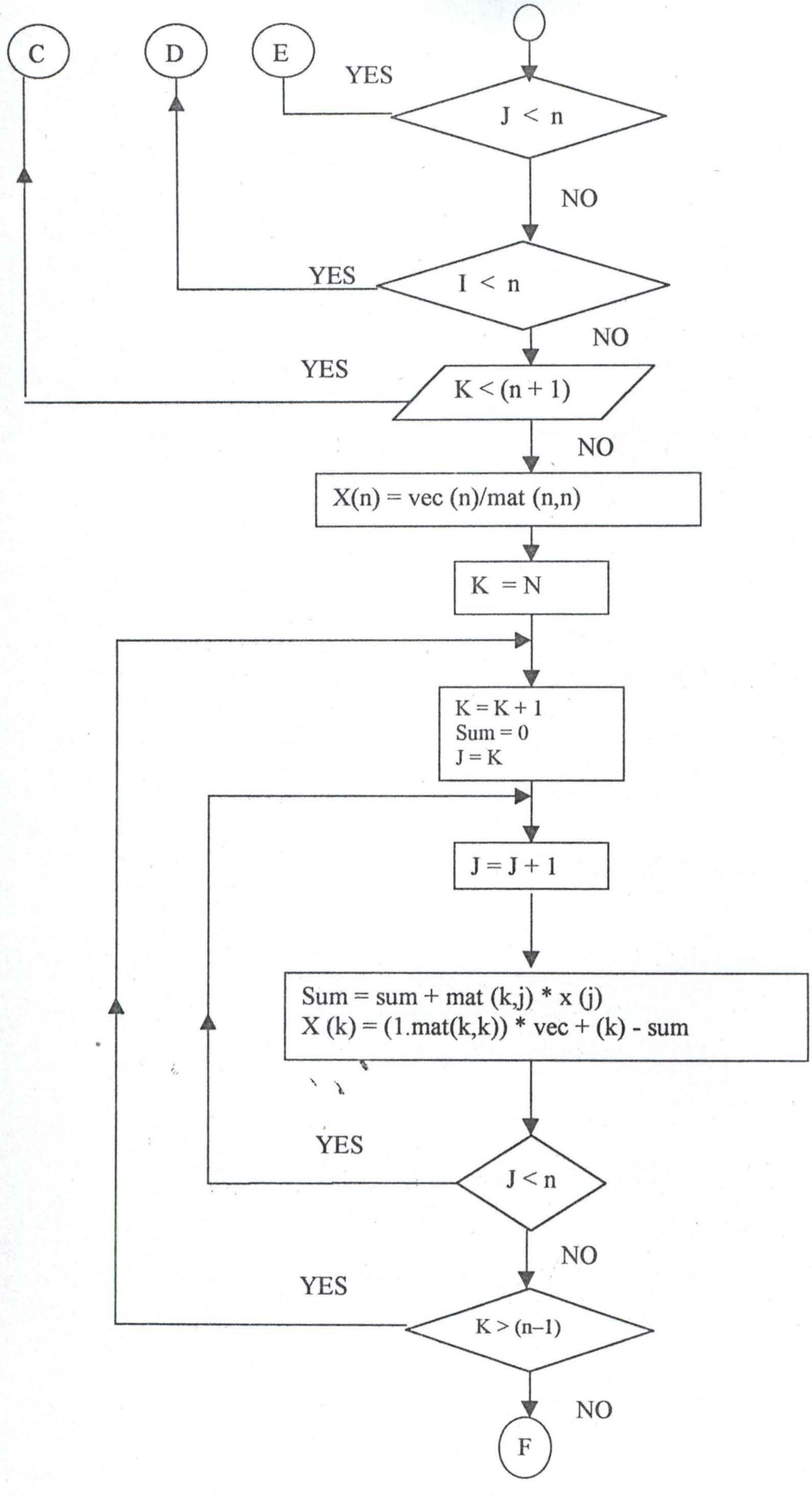
4. Muller, D. E. (1956) A Method of Solving Algebraic Equations Using an Automatic Computer Mathematical Tables and Other Aids to Computation (MTAC)

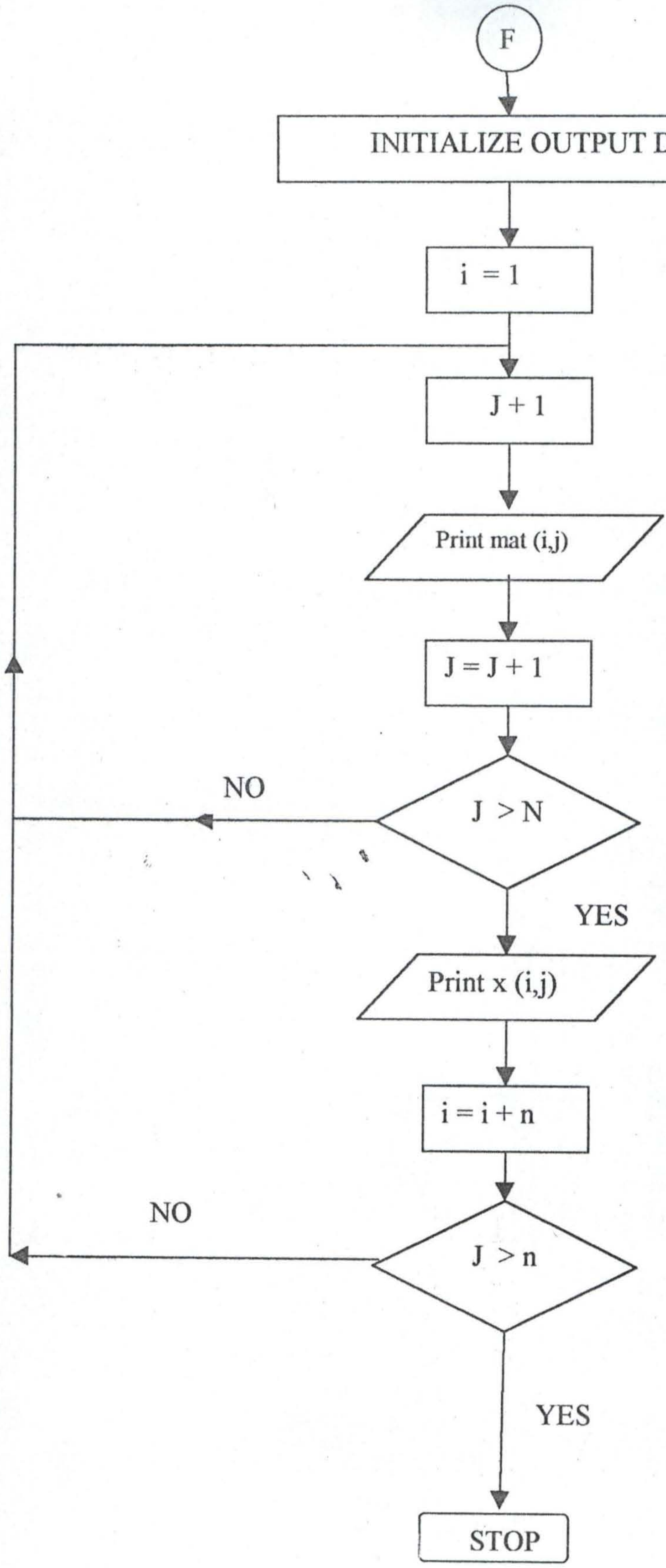
5. Stroud, K.A. and D. Secrest (1966). Gaussian Quadrature Formula, Prentice-Hall, Inc., Englewood, Englewood, Cliff, N. J.

APPENDIX A Program Flowchart




B
A
Enter vec + (I)
YES
i < N
K = O
K = K + 1
i = K
i = i + 1
P = mat (i,k)/mat (k,k)
J = 0
J = J + 1
E
Mat (i,j) = mat (k,k) – P * mat (k,j)
Vec + (i) = vec + (i) – P * vec + (k)
C
D


C
D
E
YES
J < n
NO
YES
I < n
NO
YES
K < (n + 1)
NO
X(n) = vec (n)/mat (n,n)
K = N
K = K + 1
Sum = 0
J = K
J = J + 1
Sum = sum + mat (k,j) * x (j)
X (k) = (1.mat(k,k)) * vec + (k) - sum
YES
J < n
NO
YES
K > (n–1)
NO
F


F
INITIALIZE OUTPUT DEVICE
i = 1
J + 1
Print mat (i,j)
J = J + 1
NO
J > N
YES
Print x (i,j)
i = i + n
NO
J > n
YES
STOP

```
{Program to solve equations using Gaussian Elimination
method
{
{program written by:
        Name:        Abdulrahman Bala Suleiman
               Matric Number:  PGD/MCS/99/2000/907
}

Program Gaussian (Input,Output);
      uses crt;
      Type
           index = 1..10;
      Var
           k,n,i,j:integer;
           MATRIX,jac: array [index,index] of real;
           x: array [index] of real;
           VECTOR,fn:array [index] of real;
           sum,p:real;
Begin{beggining of data entry}
     clrscr;
     writeln ('enter matric dim n:');
     read(n);
     writeln ('enter',n,'by',n,'matrix coefs by row');
     writeln ('=====================================');
     for i:= 1 to n do
         for j:= 1 to n do
         begin
           read (jac[i,j]);
           matrix[i,j]:=jac[i,j];
         end;
     writeln ('enter the input vector:');
     for i:= 1 to n do
     begin
         read(fn[i]);
      vector [i]:=fn[i];
     end;
     {end of data entry}
     {begining of forward substitution ...}
     for k:= 1 to n-1 do
       for i:= k+1 to n do
          begin
        p:=jac[i,k]/jac[k,k];
```

```
    for j:= 1 to n do
      jac[i,j]:=jac[i,j]-p*jac[k,j];
      fn[i]:=fn[i]-p*fn[k];
  end;
  {end of forward substitution....};
  {begining of backward substitution...};
        x[n]:=fn[n]/jac[n,n];
  for k:= n-1 downto 1 do
  begin
       sum:=0;
       for j:= k+1 to n do
           sum:=sum+jac[k,j]*x[j];
           x[k]:=((1/jac[k,k]*(fn[k]-sum)));
  end;
  {end of backward substitution....}
  {output of result....}
  clrscr;
  writeln ('Matrix coefficients:');
       for i:= 1 to n do
          begin
             for j:= 1 to n do
             begin
                write (MATRIX[i,j]:8:2)
     end;
             write('=',VECTOR[i]:5:2);writeln;
          end;
  writeln ('final computed result:');
     for j:=1 to n do
     begin
         writeln ('value of x(',j,') is
:',x[j]:10:2);
     end;
end.
```