APPLICATIONS OF DIFFERENTIAL EQUATIONS IN THE SOCIAL AND MANAGEMENT SCIENCES.

BY

MOHAMMED NDAKO DAUDA.

PGD/MCS/480/97

DEPARTMENT OF MATHS /COMPUTER SCIENCE
FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA.

DECEMBER, 1998.

Title page

APPLICATION OF DIFFERENTIAL EQUATIONS IN SOCIAL AND MANAGEMENT SCIENCES

BY

MOHAMMED NDAKO DAUDA
PGD/MCS/480/98

A PROJECT SUBMITED TO THE DEPARTMENT OF MATHEMATICS/COMPUTER SCIENCE, FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA IN PARTIAL FULFILLMENT FOR AWARD OF POST GRADUATE DIPLOMA IN COMPUTER SCIENCE

DECEMBER, 1998

CONTENT

APPROVAL PAGE

This project has been supervised and approved as meeting the requirement of the Department

Mathematics/computer sciences.

------------------------------ -----------------

:AD OF DEPARTMENT. Date

'of. K.R.Adeboye)

--------------------------- ---------------

OJECT SUPERVISOR Date

'. AIYESIMI.

---------------------------- -----------------

(TERNAL EXAMINER. Date

DEDICATION

Dedicated to my parents for their love and kindness.

ABSTRACT

This project work consists of five chapter, in chapter one, we discussed the general definition and some categorisation of differential equations

The project is introduction to the essential ideas in differential equation rather than a comprehensive account of the subject it does not, for example, Consider all the types of equations and other topics to be found in the standard books.

While in the study of most differential equations, the tools used are restricted almost completely to algebra, here in these course works, we used much of differential Calculus and sometimes integral Calculus. Therefore, the work deals essentially with equations of differentiation and integration. For example, Chapter one, we discuss the classification of differential equations, order of differential equation, degree, ordinary differential equations, linear and non-linear differential equation, in Chapter two we discuss about Review of its related literature.

In Chapter three, we also discuss mainly on types of first order, ordinary differential equation, variable seperable, First order differential equation: Population growth Model, the illiteracy-literacy Transition, First order linear ordinary differential equation and Model of price adjustment and price speculation.

In Chapter Four, we discussed about secound order ordinary differential equation. A Model of price adjustment with stocks, A Model of foreign exchange speculation under floating exchange rate.

In Chapter five, we discussed about closing of chapter and Summarised the major points.

ACKNOWLEDGEMENT

I am grateful to my project supervisor Dr. Aiyesimi in the department of Maths/Computer Science, Federal University of Technology, Minna who has made invaluable suggestions and corrections. I also like to thank other lecturers like Mr. Kola Raimi, in the Computer Centre, Dr. S. A. Reju and Mal. Audu Isah in Maths/Computer Science department.

I would also extend my gratitude to Alhaji Umar Sadiq, Registrar, Federal University of Technology, Minna for his assistance.

I would also like to thank the following people who assisted me such as Mr. Ezeako, Dr. Suleiman (Dean, Student Affairs), Mr. Y. O. Sule, Prof. K. R. Adeboye, who guided me through the project work.

I would also like to thank the following friends Vincent, Adamu Elisha, Mustapha Aliyu, Mohammed Mohammed and well wishers.

1.0 INTRODUCTION

Whenever the word mathematics is mentioned, different groups of people give it meanings based on their own perception. For instance, the man in the street will equate mathematics with Arithematic, children at schools who have been recently exposed to modern mathematics syllabuses in their schools give it meanings according to the ways they have been introduced to it. Others like businessmen may simply think of book keeping going to medical or experimental workers, they will talk of it as being just coming up with computing or statistical analysis. All of these if the diversity of mathematics is considered, they can be regarded as been correct.

This diversity of mathematics gives it a wide range of applications in the real life situations. However, despite the several applications in almost all aspects of human endeavor, mathematical application is in general were formally restricted
only to the physical science. As a result of this restriction, those with interest in social and management science related disciplines consider the knowledge of mathematics as something that is irrelevant to them and as such terminate their study of mathematics at the initial stage.

Recently over some few decade as broken out into a whole new range of application in the social and management science. Presently, mathematical causes that play important roles i planning managerial decision making and economics have been introduced. Notable among this is differential equations whose application in the social and management science is the topic of discussion of this project.

Description of many social and economic changes takes the foam of an equation relating an unknown functions of one variable and it is derivatives or unknown of several variables and their derivatives such equations are the differential equations. These differential equations help modern industrialist to apply quantified technics in analyzing problems of the decision science. This purposely to show how differential equations are used to formulate and solve the model of problems from economies, sociology and other social and management sciences related displines.

In this project the view of the differential equation is considered by first introducing the definition of differential equations and some important terminologies which are of vital important in the classification of differential equations. This is then followed by the classification of differential quations. Based on the classification mentioned above, the work is divided into chapters dealing ith the application of the differential equations under consideration in the social and management iences.

It is hoped that at the end of this project, although not all the applications of the differential quations in the social and management sciences can be brought, it will be made clear that all aspect mathematics have their respective application in the real life situation.

1.0 CLASSIFICATION OF DIFFERENTIAL EQUATIONS

DEFINITIONS

An equation which involves one or more derivatives of a function is called a differential uation.

Differential equations are broadly classified into main categories based on weather they are lking no partial or ordinary derivative of a function.

A differential equation in which only partial derivative appear is called a pertain differential uation. On the other hand, the one in which ordinary derivative of a functions are dealt with is lled ordinary differential equation.

ORDER OF DIFFERENTIAL EQUATIONS

The order of differential equation is the order of the highest power derivative

For example

This means that it has order two

$$(d^2\frac{y}{dx^2})=\frac{dy}{dx}$$

DEGREE

The degree of a differentials equation is the exponent of the highest power of the highest der derivative.

For example

$$(d^2\frac{y}{dx^2})^3+\frac{xdy}{dx}$$

e. it has degree three

ORDINARY DIFFERENTIAL EQUATIONS

If in a differential equation, the dependent variable is function of only one independent variable, then an ordinary differential will result, example

$$\frac{dy}{dx}=3x^2+7x+5$$

Ordinary differential equation is further classified as linear or non-linear, homogeneous or non-homogenous and first order or second order.

A differential is said to be linear if each term is linear (degree one or zero) in terms of all dependent variables and their derivatives, for example

$$\frac{dx}{dt}+x=t^3$$

Otherwise, it is called non-linear differential equation, example

xy = 10 This means that it's no linear equation, but equation likes y + 7x = 5 it's linear.

If in a differential equation, the only term consisting entirely of the independent variable is '0' then, the differential equation is called a homogeneous differential equation. An example of homogenous equation is:

$$\frac{d^2y}{dx^2}+\frac{xdy}{dx}+y=0$$

An equation which is not homogeneous is called a non-homogeneous equation. An example of non-homogeneous equation is $\frac{dy}{dx}+y=3x$

A differential equation can be linear and homogeneous at the same time. This called a linear homogeneous equation, it can also be linear non-homogeneous equation. In situation where it is linear but non-homogeneous equation. All those type of differential equation can be of first order or second order or otherwise.

These are further categorized as first order homogeneous, first order linear, first order exact and variable separable differential equations. Second order equations are also further classified as linear or non-linear, homogeneous or non-homogeneous.

CHAPTER TWO

REVIEW OF RELATED LITERATURE

Application of differential; in the social and management science has been an area of vital interest to many scholars over the few decades. Theses scholars describe many social and economic changes in form of an equation relating an unknown function of one variable and its derivatives. Such equations are the ordinary differential equations and can be either of first order or second order or more. These equations are used in modelling such phenomena as the population models and many economic changes such as fluctuations of supply and demand with changes in price.

In this project the dealt with intensively are the first order second order differential equations.

Actually, several scholars have taken much of their times treating the application of this differential equations in the social and management sciences. For instance, the application of first order differential have been treated by several scholars on different phenomenons. Notable among these work is the work of Thomas Malthus on the model of population growth using the application of first order differential equation by separation of variables in 1798. The work of Malthus is highly recognize and many authors now use his work in population study. In his work Malthus consider a given population over long period of time and formulated his theory as "both birth and death are proportional to the total population size and the time interval". This work is used by several authors such as D.N. Burghes in his book titled "the mathematical methods and models in the social management life science" This work was also used by Patrick Hayes in his book known as "the mathematical methods in the social and managerial science".

Also Patrick Hayes in the above as described another phenomenon dealing with social science application of first order differential equation. These are the transitions from illiteracy to literacy and the spread of rumor. In constricting the model of the transition from illiteracy to literacy, Hayes consider a population of large size and which he assumed that its size is fixed and the population is homogeneous. With the assumption he put forward the theory in his book that "the shift rate from the illiteracy to literacy is directly proportional to the numbers of the illiterates". Based on the above, his model was constructed

Another works are those conducted on the application of linear first order differential equation by several Authors. These works include the model of response of sales to advertisement conducted by vidale and Wolf (1957).In this model sale is considered to be a function of advertisement only and in the absence of advertisement, there are no sales. In this model they explained that sales generally increase with increasing advertising expenditures. This model was explained also in the books of D.N. Burghes and patrick Hayes mentioned above. Other important works on the application of first order linear differential equations are the model of price adjustment published in the books of patrick Hayes as above, D.N.Burghes, Robbert L.Childrens in his book titled as Calculus in business and economics rica. D. Allen of the University of California in his book (mathematical economics) and in another book (the mathematical methods and models in economic dynamics) Written by Giancarlo Gandolfo of the Universitas di-siena, Italy. All the Authors listed above considered demand as a decreasing function of price and supply as an increasing function of price and based on these assumptions the model for price adjustment was constructed.

These are many other works on the price speculation model based on the first order linear differential equation. For instance in the book of patrick Hayes he explained the model of price speculation in which variation of demand and supply were assumed not to be only dependent on the price alone, but are also stimulated or depressed by the fact that the price is rising or falling. Based on the assumption above the model of the price speculation was constructed in which the variations of market prices with time were described.

Many works have also been carried out on the applications of second order differential equations. Many of such works include the work of Glahe (1966),See Friedam (1953),Baumol (1957),Telser (1959),Cutili (1963),Kemp(1963) and Obst (1967) all of Whom were said to have based their assumption of foriegh exchange speculation model on the fact that the risk coefficient included in the definition of the Marginal revenue is an increasing function of the scale of the operation (the spot exchange rate).Other works are the works in the books of D.N. Burghes, and Hayes on the variation of supply and demand on price trends in as in the case of price speculation supply and demand or generally the market behaviours of the buyers and sellers depend ion the current price and the price trend. As they explained, price trend leads purchasers and sellers to certain expectations on the prices of commodities.

There are more several works on the model and inventory adjustment. Some of the works carried on this the book of Giancarlo Gandolfo (the Mathematical methods and models in economic dynamics).In which he assumed that producers vary their output in relation to the difference between the desired and actual level of their inventories (finished goods).

CHAPTER THREE

3.0 FIRST ORDER ORDINARY DIFFERENTIAL EQUATION

An ordinary differential is to be of first order if the highest power derivative in the equation is one. An example of such equation is

$$\frac{dy}{dx}+3x+5=0$$

First order differential equations are classified based on the fact that whether they satisfy the conditions of being homogenous, linear first order equation as defined above.

In this case the first category of differential equation of first order to be considered is "variable separable differential equation".

3.10 VARIABLE SEPARABLE FIRST ORDER DEFERENTIAL EQUATION

Many first order differential equations can be reduced into the form

$$F(y)\frac{dy}{dx}=g(x)$$

To solve this type of differential equation, the first step is putting the equation in such a way that the independent variable 'X' are separated from each other. In the case of the equation above, this aim is achieved by multiplying both sides of the equation by 'dx'

$$F(y)\frac{dy}{dx}*dx=g(x)*dx$$

This results in obtaining the equation below;

$$F(y)dy = g(x)dx$$

The process where by this equation is transformed into separate variables is called separation of variables and the equation is said to be a variable separable differential equation. The general solution of this type of equation is found by integrating the already separated equation as below

$$\int F(y)dy = \int g(x)dx$$

An example of this type of equation is shown below;

$$x(2y-3)dx + (x^2+1)dy = 0$$

In solving this equation, the first step is separating the variable by dividing both sides of the equation by $(2y-3)(x^2+1)$

Thus, yielding

This equation becomes, $x(2y-3)\frac{dx}{(2y-3)(x^2+1)}+(x^2+1)\frac{dy}{(2y-3)(x^2+1)}=0$

$$\frac{xdx}{(x^2+1)}+\frac{dy}{(2y-3)}=0$$

Upon integration the above equation becomes:-

$$\frac{xdx}{(x^2+1)}+1\frac{dy}{(2y-3)}=c1$$

$$\frac{1}{2}\ln(x^2+1)+\frac{1}{2}\ln(2y-3)=c1$$

$$\frac{1}{2}[\ln(x^2+1)(2y-3)]=c1$$

$$\Rightarrow\ln[(x^2+1)(2y-3)]=2c1$$

$$\Rightarrow(X^2+1)(2Y-3)=@^2C=C$$

$$\Rightarrow(X^2+1)(2y-3)=c$$

APPLICATIONS IN THE SOCIAL AND MANAGEMENT SCIENCE

This type of differential equation has many applications in the social and management sciences. Although it is not possible to even list all the application of this of differential equation in the social and management science, some notable ones among them are going to be discussed. These includes population growth model, spread of a minor and transition from illiteracy to literacy. These applications are going to be treated separately.

3.1.1. POPULATION GROWTH MODEL

In recent years, there have been the development of many theories of population change. For instance models have been constructed to describe the growth of population by examining the size of various age groups or the made female ratio. Many other attempt have been made but the simplest of all is that which has formulated by Thomas Malthus in 1798.

Before constructing this model several supplification assumption were made as follows;

Suppose that the size of the population grows with a constant rate 'h' decreases with a constant death rate 'd' and then the size of the population at t = 0 is $y_o > 0$. Based on the above assumptions, the Malthus theory was stated as follows:-

Births and deaths are proportional to the total population size and the time interval. In this case if y = y(t) the population size in a time interval of 't', then there are birth = 'h' and death = 'd' then, theorem above can be represented as (where k is the proportionality constant $\frac{dy}{dt} y$ i.e. $\frac{dy}{dt} = ky$

= (b-d)).'y' here is the population size at time 't'.The proportionality constant k depends on the births and death rates. If there are more deaths than births per unit time, d > h, the population size is decreasing and

Similarly if there are more births than deaths $\frac{dy}{dt} < 0 \wedge k < 0$ will be increasing and

Also when there is one $\frac{dy}{dt} > 0 \wedge k > 0$

birth for each death, h = d, the population size remains stationary.

Implying that k = 0. putting the value of k or substituting k with 'h-d' the population model is given by $\frac{dy}{dt} = 0$

by separation of variables, the above equation can be solved as follows: $\frac{dy}{dt} = (h-d)y$

Integrating both the sides we get;

$$\frac{dy}{dt} \cdot \frac{dt}{y} = (h-d) \frac{)}{1} \cdot \frac{dt}{y} \frac{dy}{y} = (h-d)dt$$

$$\text{integrate} \frac{dy}{y} = \text{integrate}(h-d)dt + c$$

===>Lny = (h-d)t + C

===> y = $Ce^{(h-d)t}$ (by inspection)

But at t = 0, y = y_0

So, this equation can be written as

Ln y_0 = (h-d)(0) + C ===> Lny_0 = C

Substituting this value of C in (3) we get;

Lny =(h-d)t + Lny_0

i.e Lny = (h-d)t + Lny_0

===>Lny -Lny_0 = (h-d)t

===> $y/y_0 = e^{(h-d)t}$

$\Longrightarrow y = y_0 e^{(h-d)t}$

From this expressing for the

size of population at any time 't', it can be seen that the population becomes infinitely large as time passes if the birth rate exceeds the death rate. That is if h-d > 0, then

$$\lim_{t \to \infty} e^{(h-d)t} = \infty,$$

and consequently $\lim_{t \to \infty} y(t) = \lim_{t \to \infty} e^{(h-d)t} = \infty$

Furthermore since $dy/dt = (h-d)y = (h-d)y_0 \, e^{(h-d)}$

$$\lim_{t \to \infty} dy/dt = \infty$$

the population increases at a greater rate as time passes.

Conversely, if h-d < 0, the population diminishes to zero as time passes.

$$\lim_{t \to \infty} dy/dt = \lim_{t \to \infty} (h-d)y_0 e^{(h-d)t} = 0$$

Conclusively, according to this model, if the initial population is zero ($y_0 = 0$), the population at any time (t) remains zero. That is if $y_0 = 0$, then $y(t) \equiv 0$ regardless of what are the sizes of h and d.

THE ILLITERACY-LITERACY TRANSITION (3.1.2)

Illiteracy-Literacy transition is one of the one way social processes which are some of the fundamental problems of mathematical sociology. These one-way social processes as characterised as having two states with a shift between the steps in any one possible direction (as from living to dead). In demonstrating the quality of these one-way social processes, we shall consider education as also a one-way social process and mathematically describe the transition from an illiterate to a literate state.

In constructing a model for illiterate to literate transition, we are to describe the rate of transition to determine the number of individuals in either date at any time 't'. In order to achieve this aim, several simplification assumptions have been made. These assumptions as follows:-

Firstly, we assume that we are dealing with a large homogeneous population of fixed size y_o all of whom are initially (at t = 0) illiterate.

Then, we consider education as the one-way social process that, operating over a period of time transform the person from the state of illiteracy to that of literacy. Also let us assume that educational facilities are improving through time, the shift rate from illiteracy to literacy depends upon the date at which people begin their education. Therefore, let us assume that the incremental rate of improvement in educational facilities is constant. Because the population is assumed to be large and homogeneous, we theorized that of change of the number illiterate with time is directly proportional to the numbers of illiterates, let 'y' denote the number of illiterate at a time t, then

(k > 0, a constant) describes this behaviour $\frac{dy}{dt} = -ky$

The constant 'K' is defined as the illiteracy to literacy transition rate and depends upon the rate of improvement in the educational facilities and the dates when the illiterates begin their education that is $k = \propto B$ Where $\propto$ is the constant rate of improvement in the educational facilities and 'B ' increases, this is a reflection of the effect of improved educational facilities. Thus, at this juncture, we can say that the illiteracy to literacy transition has been modelled for a large homogeneous population with the initial value problem that:

($k = \alpha B$) and $y(O) = y_o$

The equation $\frac{dy}{dx} = -ky$

can be solved' using separation of variable as follows:- $\frac{dy}{dt} = -ky$

$\frac{dy}{y} = -k\,dt$ ....(3.4)

Lny = -kt + C (by integrating both sides) which by inspection becomes $y = ce^{-kt}$-kt at t = 0, y = yo

$Lny_o = c$

$\Longrightarrow Lny = -kt + Lny_o$

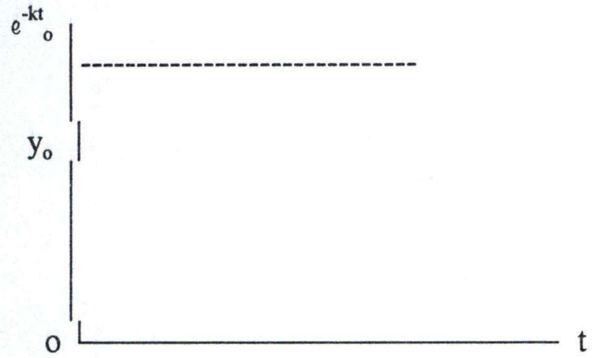
$\Longrightarrow Lny - Lny_o = -kt$

$Ln\, y/y_o = -kt$

$y/y_o = e^{-kt}$

$y = y_o e^{-kt}$ (where $k = \alpha B$) ....(3.5)

From the above expression, we see that at time passes, the number of illiterates decreases. This happens because at 't' becomes large ($t \to \infty$) $e^{-kt} \to 0$ and hence, the number of literates tends to zero . i. e $y(t) = y_o e^{-kt} \to 0$



The above graph, explained the number of illiteracy against time and the value of graph shown us that time tend to infinity Similarly, at any time t_o the number of illiterates $y_o e^{-kt}_o$ also decreases as the rate of improvement in the educational facilities '∝' increases. That is, since $k = \alpha B$ increases as α increases for a constant B, e^{-kt}_o decreases.

Finally, Substituting (3.5) into (3.4) yields an expression for the rate of change in the number of illiterates at any time t.

$$\frac{dy}{dt} = \frac{dy}{dt} = -ky$$

$-ky_o e^{-kt}$ ....(3.6)

How do variations in 'α' affect this rate of change at a particular time t_o ?

A more systematic techniques for obtaining solutions of (i) i.e

$y` + ky = 0$ is as follows:

Note that for any constant k, multiplication of (iii) by e^{kt} yields

$e^{kt} Y` + k e^{kt}y = 0$ which is the derivatives of the product $e^{kt}y$. That is, $(e^{kt}y)` = 0$

therefore, there exists a constant ('c') such that $e^{kt}y = c$ or $y = Ce^{-kt}$

'c' can be obtained using the initial value problem as done above.

3.2.0 FIRST ORDER LINEAR ORDINARY DIFFERENTIAL EQUATION

A differential equation of 1st order is called linear if it can be put into the form

$$\frac{dy}{dx} + Py = Q \text{.............(1)}$$

Where 'p' and 'Q' are functions of x.

To solve this differential equation a function P = f(x) is found such that, if equation (i) is multiply by ' P ' the left hand side becomes the derivatives of the product Py as follows:

Multiplying by $\mathcal{P}$ the equation becomes $\frac{dy}{dx}+Py=Q$

thsidesby 'y' weget $\frac{dp}{} = \wp p$ ....*(iii)separatingthevariablesbymultiplyingbothsidesby* $\frac{dx}{\wp}$ *theeq*

Upon integration of both sides we get:

$\ln \mathcal{P} = \int Pdx + C \Longrightarrow \mathcal{P} = e^{\int pdx}$

$\Longrightarrow \ln \mathcal{P} = \int pdx + \ln C$

$\Longrightarrow \ln \mathcal{P} - \ln C = \int pdx$

$\Longrightarrow \ln \mathcal{P} - \ln C = \int p\, dx$

$\equiv \Rightarrow \ln \frac{\wp}{\ln} c = \int pdx$

$\mathcal{P} / C = e^{\int Pdx}$

$\mathcal{P} = Ce^{\int Pdx}$

The '$\mathcal{P}$' found above is called the integrating factor of equation (i)

From Equation (ii)

$d(\wp y) = pQ$

$\Longrightarrow \mathcal{P}y = \int (PQ)dx + C$ (by integration). An **example** of order linear ordinary differential equation is as follows:-

$\frac{dy}{dx}+2y=e^{-x}$

Here $P = 2$, $Q = e^{-x}$

Then, the integrating factor '$\mathcal{P}$' $= e^{\int P\, dx}$

$\Longrightarrow \mathcal{P} = e^{\int 2dx} = e^{2x}$

Since we know that from above

$\mathcal{P}y = \int (PQ)\, dx + C$ then,

$e^{2x} y = \int (e^{2x} . e^{-x})\, dx + C$

$e^{2x} y = \int e^{x} + C$

$\Longrightarrow y = e^{x} / e^{2x} + C/e^{2x}$

$\Longrightarrow y = e^{-x} + Ce^{-2x}$

ALGORITHMS AND FLOWCHARTS

An algorithm is a step by step method or rules for solving a problem in a finite sequence of steps.

Flowcharting is one of the widely used techniques for specifying a algorithm in computer science. And this is simply defined as a diagrammatic representation of algorithms. For **example of** solving the equation like y = x^3 + x^2 by using algorithm system program flowcharts.

i.e. Algorithms method

let y = x^3 + x^2

==> dy/dx = 3*x^2 + 2*x

APPLICATIONS OF FIRST ORDER LINEAR ORDINARY DIFFERENTIAL EQUATION IN THE SOCIAL AND MANAGEMENT SCIENCES

In this case we are going to consider how first other linear ordinary differential equations are used to construct the models of many social and management science processes. Such processes include model of price adjustment, response of sales to advertisement, price speculation etc.

3.2.1 MODEL OF PRICE ADJUSTMENT

In describing the model of price adjustment, a simple model of a market for one community will be developed. In constructing this model, the interested things are determining the equilibrium price (the price at which supply and demand are equal) and an expression for the price at any time 't'.

In this model, we assume that demand and supply at a time 't' are functions solely of the price of the commodity at the time 't'. Demand 'u' is assumed to be a decreasing function of price 'p' (du/dp < 0) and supply 's' (ds/dp > 0). This behaviour is expressed with the following simple functional relationships:

$U(t) = a_o + a_1 P(t)$------(3.16)

$S(t) = b_o + b_1 p(t)$------(3.17)

Where $a_1 < 0$, $b_1 > 0$ and U(t),S(t) and P(t) are the demand, supply and price of commodity respectively at time 't'. Considering the above relationship for the demand and supply,when the supply is set to be equal to the demand i.e S(t) = U(t) we get the equilibrum price as follows.

$$a_o + a_1P(t) = b_o + b_1P(t)$$

$a_o - b_o = b_1P(t) - a_1P(t)$

$a_o - b_o = (b_1 - a_1)P(t)$

$$\Longrightarrow P(t) = \frac{a_o - b_o}{b_1 - a_1}$$

That is the equilibrum price at time 't' is :

$$P_e = \frac{a_o - b_o}{b_1 - a_1}$$

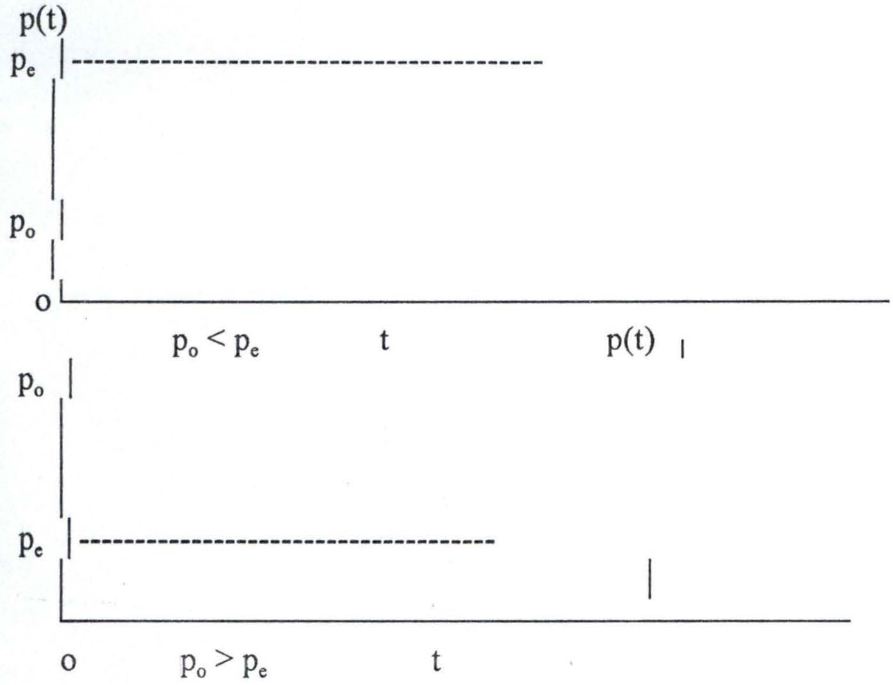
Note:- Since $c > 0$, $b_1 > 0$, $a_1 < 0$, we have $C(a_1 - b_1) < 0$, and hence

$$\lim_{t \to \infty} e^{c(a_1 - b_1)t} = 0$$

Therefore, since $\lim_{t \to \infty} P(t) \lim_{t \to \infty}[p_e + (p_o - p_e)e^{c(a1-b1)t}]$

$P(t) \to p_e$ as $t \to \infty$. That is the price tends to an equilibrium state in the limit, and therefore, the market is stable.

Note that this equilibrium condition holds regardless of whether $p_o > p_e$ or $p_o < p_e$ as shown in the graph below:-



3.2.2 PRICE SPECULATION

In this model the response of consumers demand to alternative prices of a product is going to be considered. Actually, the response of consumer demand to alternative prices is of great interest in mathematical economics. Mathematical models of the responsiveness has regarded price as independent of time, but market prices for a commodity usually vary with time. Consequently, the demand for (and supply of) the commodity also change with time for whether the price is going up or down.

In actual sense, demand and supply are often not merely functions of price alone but are also stimulated or depressed by the fact that the price is rising or falling. For instance, business is usually good when prices are rising and not so good when the prices are falling. This is because when prices are rising buyers will try to make purchases when prices are relatively low before reaching higher prices, while in the case where the price are falling, buyers will wait until the prices fall further. This market behavior is known as the effect of price speculation.

Here, the effect of speculation in mathematical model describing the variation of market price as time passes will be incorporated under the initial assumption that the demand and supply are in equilibrium. This implies that the market behavior under consideration is showing that demand and supply at time 't' depend upon the price and rate of change of the price. This mathematically implies that the demand at time 't' u(t) and the supply at time 't' s(t) satisfy the relationship of the form

$$s(t) = s(y(t), dy/dt)$$

$$u(t) = u(y(t), dy/dt)$$

Where y(t) is the price of the commodity at a time 't', the rate of change in price or the measure of the responsiveness of demand to variations in the price is the partial derivative of $U = u(t)$ with respect to $Y = y(t)$, or $W = dy/dt$. If for instance $W = dy/dt$ is held constant while the price level is increased by a small amount Dy, the demand will vary by an amount Du. The average variation in demand due to this price increase is Du/Dy. The limiting value is du/dy, ie the marginal demand with respect to price. If demand increase with price increase, demand is an increasing function of price or $du/dy > 0$

If demand decreases with increase i price, he demand is said to be a decreasing function of price. Thus $du/dy < 0$.

In the same way, the marginal demand with respect to the rate of change in price is defined as du/dw in this discussion, it is assumed that higher prices cause a decreased demand. That is du/dy < 0. Also it is assumed that demand increases when prices are rising which mathematically means du/dw > 0, where w = dy/dt. That means the faster prices rise as time passes, the greater the demand. This market demand behaviour is approximated by the following linear function. $U(t) = a_0 + a_1(t) + a_2 dy/dt$ ..(3.21) Where $a_1 < 0$ and $a_2 > 0$.

Note here that du/dy > 0 and du/dw > 0 (w = dy/dt)

Similarly, if supply is assumed to be an increasing function of price and an increasing function of the rate of price increase, s (t) must satisfy ds/dy > 0 and ds/dw > 0, where w = dy/dt. This behaviour is exhibited by the linear function $s(t) = b_0 + b_1 y(t) + b_2 dy/dt$ ....(3.22) where $b_1 > 0$ and $b_2 > 0$

Under the assumption that the supply balances the demand, we have

$$S(t) = U(t)$$

$$b_0 + b_1 y(t) + b_2 dy/dt = a_0 + a_1 y(t) + a_2 dy/dt \quad .....(3.23)$$

Also at t = 0, $y(t) = y_0$

Equation (iii) is a first order linear equation when transformed as follows:-

$$b_2 dy/dt - a_2 dy/dt + b_1 y(t) - a_1 y(t) = -(b-a_0)$$

$$\Longrightarrow (b_2 - a_2) dy/dt + (b_1 - a_1) y(t) = -(b_0 - a_0)$$

dividing both sides of the equation with $(b_2 - a_2)$ we get

$$dy/dt + b_1 - a_1/b_2 - a_2 (y) = -b_o - a_o/b_2 - a_2 \quad ....(3.24)$$

This equation is a linear first order differential equation with the integrating factor and from

$$p = b_1 - a_1/b_2 - a_2 \text{ and } Q = -b_o - a_o/b_2 - a_2$$

so, the integrating factor

$$\wp = e^{\int p\,dt}$$

That is $\wp = e^{\int (b1-a1/b2-a1)\,dt}$

$$\wp = e^{(b1-a1/b2-a2)\,t}$$

But from the general method of the linear first order differential equation

$$\rho y = \int (PQ)dt + C$$

$$\Longrightarrow y\, e^{(b1-a1/b2-a2)t} = \int (e^{(b1-a1/b2-a2)t} .(-bo-ao/b2-a2))dt + C$$

This solution is found as

$$y(t) = y_1 + (y_o - y_1)e^{-(b1-a1/b2-a2)} \text{ ....(3.25)}$$

Where $y_1 = -(b_o-a_o)/(b_1-a_1)$

This gives a simple relationship between the price of a commodity and time under the assumptions of our model.

The simple form of this speculative model i.e equation (2.4) gives rise to further analysis. For instance, the equation may be used to determine the market equilibrium price. The equilibrium price occurs whenever the price is stationary with respect to time. That is dy/dt = 0. If dy/dt = 0, then equation (2.4) implies that the equilibrium price is $y_1 = -(b_o-a_o/b_1-a_1)$.

CHAPTER FOUR

4.0 SECOND ORDER ORDINARY DIFFERENTIAL EQUATION

The general second order differential equation is of the form

$F(x, y, dy/dx, d^2y/dx^2) = 0$

The above equation usually appears in two different forms as follows:

$F(y, dy/dx, d^2y/dx^2) = 0$....................(i)

and

$F(y, dy/dx, d^2y/dx^2) = 0$.....................(ii)

As can be seen from the equations above, in the first equation that is equation (i), the independent variable 'y' is missing.

This equation can be solved by a suitable method of substitution as follows:

let $P = dy/dx$

The equation now becomes $\frac{d^2y}{dx^2} = \frac{dp}{dx}$

$F(x, dy/dx, dp/dx) = 0$ which is of first order in p. This can be solved for p as a function of x.

Suppose p is a function of x, then

$p = Q(x, c1)$

$y = \int dy/dx = \int Pdx$

$\therefore\ y = \int Q(x,c1) + c_2$

Type(2) is of the form:

$F(y, dy/dx, d^2y/dx^2) = 0$ as already shown. This does not contain 'x' explicitly to solve this equation:-

let $p = dy/dx$

Then, $d^2y/dx^2 = dp/dx = dp/dy.\ dy/dx$

$= Pdp/dy$

The equation is then transformed into the following way:

F(y,p,pdp/dy) =0 which is of first order in terms of p.

Example of type (1)

solve the equation:

$$d^2\frac{y}{dx^2}+\frac{dy}{dx}=0$$

solution:-

let p = dy/dx

===> dp/dx = d^2y/dx^2

by substituting the values of `p' and dp/dx into the given equation we get:

dp/dx + p = 0

by separation of variable we get

dp/dx * dx/p+ p* dx/p = 0

===> dp/p + dx = 0

4.1.2 A MODEL OF PRICE ADJUSTMENT WITH STOCKS

In treating the application of first order linear equation, a model of price adjustment was considered in that case the model describe the responsiveness of the supply and demand of a commodity with variation in it's price as time passed . In that case the price is set so that the demand clears the supply . In the model of price adjustment with stocks, the model of price adjustment is extended so that it account for changing stocks. Although the buyers demand for the commodity is realised in sales to merchants who are assured to hold stocks. The variation of price depends on how merchants set prices relative to stocks. So, in this case in addition to considering the buyers and suppliers as in the case of price adjustment model already treated, this third group that is the merchants who hold stock and make sales is going to be considered. In this case a simplifying assumption is made as follows:-

That merchant are assumed to buy and sale the commodity at the same price and they set prices according to the levels of stocks.

Let us also assumed that the stocks of very continuous overtime as do the variation in demand "u", supply"s" and price 'p' if there is no time lag, stocks will increase with time if there an excess behaviour is mathematically described as,

dq/dt = K(S-U)-----------------------(4.5)

Where K is a positive constant, that is to say stock is increasing overtime ($dq/dt > 0$) when there is an excess supply $s > (u)$ and is decreasing overtime when there is excess demand $s < (u)$.

In addition to the above considerations, we also assumes that merchant set the price 'P(t) at time(t) so that the rate of increase is proportional to the amount by which stocks fall short of a given level q_o

$dp/dt = -M(q-q_o)$-----------------(4.6)

Where 'M' > 0 is a constant.

From equation (ii) we see that if $q > q_o$ prices are falling ($dp/dt < 0$) and if $q < q_o$, prices are increasing and ($dp/dt > 0$). Differentiating equation (ii) yields the following:-

$d^2/dt^2 = -Mdp/dt$..........(4.7)

That is the acceleration of the price increase is proportional to the rate of increase of stocks. Combining equation (4.6) and (4.7) we see that $d^2p/dt^2 = -MK(S-U) =$

$MK(U-S)$..........(4.8)

Since demand and supply are assumed to satisfy the relationships.

$U(t) = a_o + a_1p(t)$

$S(t) = b_o + b_1p(t)$

as in the model of price adjustment already treated, where $a_1 < 0$ and $b_1 < 0$, these functions are substituted into equation as shown bellow,

$d^2p/dt^2 - MK(a_1- b_1)p = MK(a_o-b_o)$

Assuming that initially at $t = 0$, $p(0) = p_o$ and $p^1(o) = p_1$, this price- adjustment model accounting for the variation in stock is given by the initial value problem

$p^{11}(t) -K^1(a_1- b_1)p(t) = K^1(a_o-b_o)$ ...(4.9)

$p(o) = p_o, p^1(o) = p_1$.....(4.10) where $K^1 = MK$. For convenience it is now assumed, that

$p_o > (a_o-b_o/a_1-b_1)$

The above equation is a second order linear non homogenous equation with constant coefficients. A particular solution clearly is $-(a_o-b_o/a_1-b_1)$

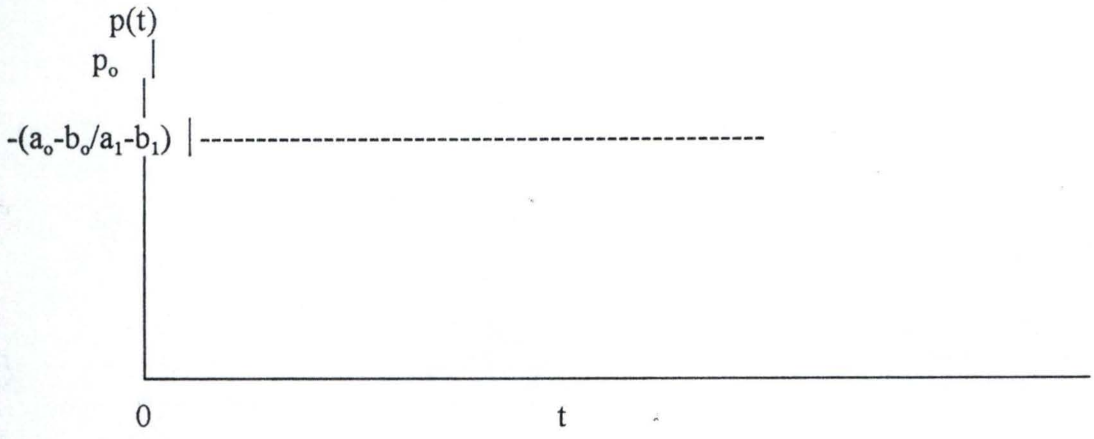
The general solution to the corresponding homogenous equation is

$C_1\sin\propto t + C_2\cos\propto t$, where $\propto = [K^1(b_1-a_1)]^{1/2}$

Hence, the general solution of second order linear equation given above is

$-(a_o-b_o/a_1-b_1) + C_1\sin\propto t + C_2\cos\propto t$ and the unique solution satisfying the initial condition (4.10) is $p(t) = -(a_o-b_o/a_1-b_1) + p_1/\propto \sin\propto t + p_0 + (a_0-b_0/a_1-b_1)\cos\propto t$ $p(t)$ represents the time path of the price of the commodity in our model.

This price time path is obviously Oscillatory in behaviour as shown in the diagram below.



From the above diagram $p(t)$ fluctuates about the value $-(a_o-b_o/a_1-b_1)$ with period 2π

The maximum and minimum values of the price-time path occur at the critical points of $p(t)$; that is Maximizing and minimizing points $p(t)$ satisfy the equation

$p^1(t^*) = p_1\cos\alpha t^* - \alpha(p_o + a_o - b_o/a_1 - b_1)\sin\alpha t^* = 0$

A MODEL OF FOREIGN EXCHANGE SPECULATION UNDER FLOATING EXCHANGE RATE

From the elementary theory of maximizing behaviour it follows that, in general, a speculator will be in equilibrium when the marginal revenue from his operations which is represented by the difference between the expected and the current value of the variable (in this case, the spot exchange rate) equals the marginal cost, the marginal cost whose main 'objective' elements are the costs of the operation and interest forgone at home net of interest earned abroad (or vice versa) for the period of operation is defined as including a risk coefficient. This coefficient can be assumed to be an increasing function of the scale of the operations, so that the overall marginal cost is an increasing function even if it's 'objective' components are constant. Thus, if marginal revenue increases, the scale of operations will be increased (up to the point where marginal cost has increased to match marginal revenue), and so, the amount of founds employed by speculators is an increasing function of the difference between the expected and the actual exchange rate. For simplicity a relation of proportionality is assumed i.e.

$E_s(t) = M[E_R(t)-S_R(t)]$, $M > 0$ ......(4.11) where $E_s(t)$ is the speculative excess demand of foreign exchange at time 't', $E_R(t)$ the expected rate of exchange at time 't' and $S_R(t)$ the current spot rate of exchange at time 't' (we consider only speculation in the spot market).

The rate of exchange is conventionally defined as the price of one unit of foreign currency in terms of local currency. This definition applies to both expected and the current rate of exchange. Equation (4.11) says that if the expected rate of exchange is greater than the current exchange rate (i.e. if the exchange rate is expected to depreciate speculators excess demand is positive, if on the other hand the expected rate is less than the current rate (i.e. if the exchange rate is expected to appreciate) the speculaters excess demand is negative and the greater in absolute value the difference between E_R and S_R.

Now speculators are also operating in the foreign exchange market, e.g. traders engaged in commercial operations with foreign counting. Their excess demand $E_n(t)$ depends only on the current rate of exchange.

$E_n(t) = a_o + a_1 S_R(t) + \beta coswt$, $a > 0$, $a_1 > 0$, $\beta > 0$

............(4.12)

Where $\beta coswt$ represents external factors, such as seasonal influences, acting on the demand and on the supply side. When speculators are absent, the equilibrium rate of exchange is determined putting $E_n(t) = 0$ so that,

$S_R(t) = \beta_1 coswt + \beta_2$, $\beta_1 \equiv B/-a_1$, $B_2 \equiv a_o/-a_1$ ....(4.13)

The values of the parameters must be such that non-positive values of the exchange rate cannot occur. Since the interval of variation of coswt is ±1, non-positive values of $S_R(t)$ are exchanged if $\beta_1 < B_2$, i.e if $B<0$.

When speculators are operating, the equilibrium rate of exchange is determined by the notation

$E_n(t) + E_s(t) = 0$............(4.14). i.e

$ME_R(t) + (a_1-M)S_R(t) + a_oBcoswt = 0$ .....(4.15)

CHAPTER FIVE

SUMMARY AND CONCLUSION

In closing this chapter, we collect and summarize the major points as follows:

An equation which involves one or more derivatives of a function is called a differential equation.

A differential equation in which only partial derivatives appear is called a partial differential equation. While the ordinary derivatives of a functions is called ordinary differential equation.

The order of a differential equation is the order of the highest power derivative.

The degree of a differential equation is the exponent of the highest power of the highest order derivative.

A differential equation is said to be linear if terms of all dependent variables and their derivatives.

A differential equation can be linear and homogenous at the sometime. This is called a linear homogeneous equation.

The equation of application in social and management sciences described equations are used in Modelling such phenomena as the population models and many economic changes such as fluctuations of supply and demand with changes in price.

The Thomas Malthus on the model of population growth using the application of first order differential equation by separation of variable in 1798.

Birth and death are proportional to the total population size and the time interval.

Patrick Hayes explained the model of price speculation in which variation of demand and supply were assumed not be only dependant on the price alone, but are also stimulated or depressed by the facts that the price is rising or falling.

The Glahe (1966), see Friedam (1953), Bausmol (1957), Telser (1959), Cutili (1963), Kemp (1963) and Obst (1967) all of whom were said to have based their assumption of foreign exchange speculation model on the fact that the risk coefficient included in the definition of the marginal revenue is an increasing function of the scale of the operation (the spot exchange rate). TheD.N Burghes, and Hayes on the variation of supply and demand on price trends in as in the case of price speculation, supply and demand generally the market behavious of the buyers and sellers depend on the current price trend to expectations on the prices of commodities.

Births and deaths are proportional to the total population size and the time interval .

e.g. $dy/dt \propto y$

Implies that $dy/dt = Ky$(where K is the proportionality constant).

The proportionality constant K depend on the births and death rates.

The rate of exchange of the number of illiterates with time is directly proportional to the number of literate.

$dy/dt \propto y$ Implies that $dy/dt = -Ky(K > 0)$ is a constant as the illiteracy to literacy transition rate.

The rate of change of price is proportional to the difference between demand and supply. If $u(t) > s(t)$, then prices are rising and $dp/dt > 0$.

if $u(t) < s(t)$, then prices are falling and $dp/dt < 0$

The market behaviour under consideration is showing that demand and supply at time 't' depend upon the price and the rate change of the price.

The rate of change in price or the measure of the responsiveness of demand to variations in the price is the partial derivative of $U = u(t)$ with respect to $Y = y(t)$ or $W = dy/dt$. If for instance, $W = dy/dt$ is held constant while the price level is increased by a small amount Δy, the demand will vary by an amount Δu.

The average variation in demand due to this price increase is $\Delta u/\Delta y$. The limiting value is du/dy i.e. the marginal demand with respect to price, if demand increases with price increase, demand is an increasing function of price or $du/dy > 0$. If demand decreases with increase in price, then demand is said to be a decreasing function of price, thus $du/dy < 0$.

The variations of prices depend on how merchants set prices relative to stocks.

The merchants are assumed to buy and sell the commodity at the same price and they set prices according to the level of the stock.

Also assumed that merchants set the price $p(t)$ at time 't' so that the rate of increase is proportional to the amount by which stocks fall short of a given level q_o .

$dp/dt = -M(q-q_o)$ where $M > 0$ is a constant

The rate of exchange is conventionally defined as the price of one unit of foriegh currency in terms of local currency. This definition applies to both expected and the current rate of exchange. If the expected rate of exchange is greater than the current exchange rate the depreciate speculators excess demand is positive and if the exchange rate is expected to appreciate the speculators demand is negative.

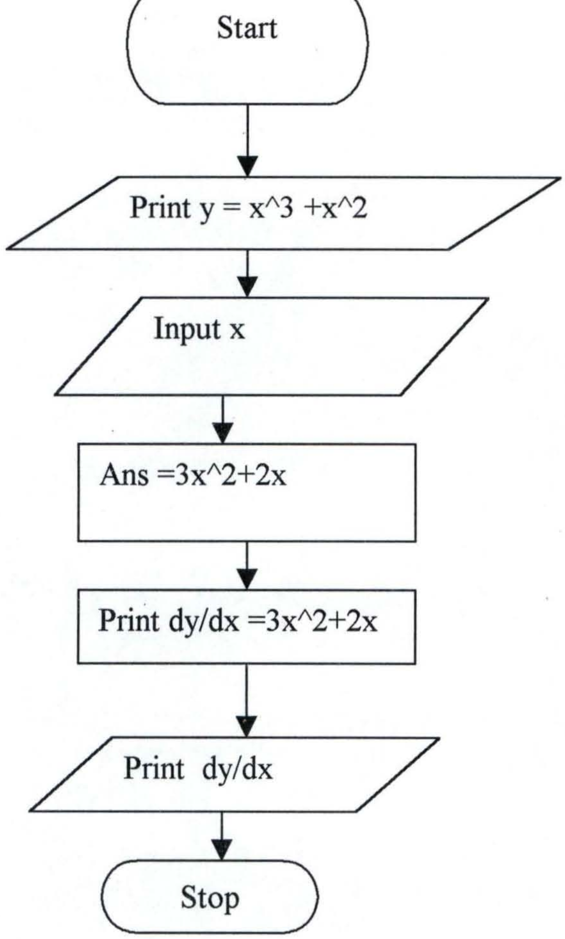
CONCLUSION

This project has shown the basic definitions of differential equations and applications of differential equation in social and management sciences, it widely explained and fields work in sciences and management sciences. Such that the rate of change of price is proportional to the different between demand and supply i.e. if $u(t) > s(t)$, then prices are rising and $dp/dt > 0$, if $u(t) < s(t)$, then prices are falling and $dp/dt < 0$. The market behaviour under consideration is showing that demand and supply at time 't' depend upon the price and the rate of change of the price. Also, if demand decreases with increase in price then demand is said to be decreasing function of price, thus $du/dy < 0$ and if demand increase with price increase demand, an increasing function of price, thus $du/dy > 0$.

REFERENCES

1. Differential equation with application and Historical Notes by George F. Simmon
2. Differential equation by David A. Sanchez
3. Schaums - Outline of theory and problems of fourier analysis by Mc. Graw-Hill
4. Mathematical techniques in chemistry by Joseph B. Dence
5. A differential equation for social sciences by P. W. Atkins

SYSTEM PROGRAM FLOWCHART



```
10 CLS:  Screen 9
20 LOCATE 2, 3: Print "APPLICATION OF DIFFERENTIAL EQUATIONS"
30 LOCATE 4, 3: Print "IN THE SOCIAL AND MANAGEMENT SCEINCES."
40 LOCATE 10, 40: Print "BY"
50 LOCATE 12, 10: Print "MOHAMMED NDAKO DAUDA."
60 LOCATE 14, 10: Print "PGD/MCS/480/97"
70 LOCATE 16, 10: Print "DEPARTMENT OF MATHS/COMPUTER SCEINCE"
80 LOCATE 18, 8: Print "FEDERAL UNIVERSITY OF TECHNOLOGY,MINNA"
90 LOCATE 19, 30: Print "DECEMBER,1998."
100 LOCATE 20, 20: PRINT "STRIKE ANY KEY TO CONTINUE": A$ = INPUT$(I)
CLS
110 CLS
120 LOCATE 3, 10: Print "MENU"
130 Line (10, 6)-(40, 260), 5, B
140 LOCATE 12, 10: Print "[1] DIFFERENTIAL EQUATION"
150 LOCATE 14, 10: Print "[2] ADD MORE VALUE OF X (Y/N)?"
160 LOCATE 16, 10: Print "[3] EXIT"
170 LOCATE 20, 10: INPUT " SELECT YOUR CHOICE"; CH
180 If CH >= 1 Or CH = 3 Then GoTo 200 Else GoTo 190
190 CLS:  Beep: LOCATE 10, 10: Print "INVALID CHOICE"; CH
200 On CH GoSub 140, 150, 160
210 Rem  ON CH
220 CLS
230 Print "Y = X^3+X^2"
240 INPUT "INPUT THE VALUE OF X"; X
250 ANS = (3 * X ^ 2) + 2 * X
260 Print "dy/dx = 3X^2+2X"
270 Print "dy/dx = 3("; X; ")^2+2("; X; ")"
280 Print "dy/dx ="; ANS
```

```
290 LOCATE 25, 10: INPUT "PRESS Y or y TO GOTO THE NEXT PAGE"; R$
300 If R$ = "Y" Or R$ = "y" Then GoTo 140 Else GoTo 160
310 End
```