COMPUTATION OF ELEMENTS OF DYNAMICS

BY

BIOKU IBRAHEEM JIMOH

PGD/mcs/176/96

A project submitted to the department of Mathematics/Computer Science, Federal University of Technology, Minna in partial fulfilment of the requirement for the award of post-graduate Diploma in Computer Science.

March 1998.

DEDICATION

To the entire **Bioku** family.

ACKNOWLEDGEMENT

It is with gratitude that I acknowledge the effort of Dr. S.A. Reju who supervised this work. He offered a great assistance. Special thanks to the academic staff of Maths and Computer Science Department especially those that took the PG students one course or the other. I also acknowledge the financial contributions of Alh. R.J. Bioku and Dr' Tunji Yussuf. Special thanks to Serifat Mojisola Lawal, S.D. Salau and Moshood Raji for their moral and usefull contributions respectively.

May the God Almighty Allah bless each an everyone of you.

CERTIFICATION

This project work has been examined and found acceptable in partial fulfillment of the requirement for the Post-graduate diploma in Computer Science of the Department of Mathematics\Computer Science, Federal University of Technology, Minna.

Dr. S.A. Iwareju Supervisor	Date
Dr. K.R. Adeboye Head of Department	Date
External Examiner	*Date*

TABLE OF CONTENT

CHAPTER FIVE:CONCLUSION

ABSTRACTS

This project work, **COMPUTATION OF ELEMENT OF DYNAMICS**, discusses the statistical analysis of system of particles whether the dependent or independent with particular reference to probability distribution laws.

It discusses the concept of dynamic. In it concepts such as scalar, vector, force, work, energy, momentum and motions shall be introduced. It will be looked at for a particle and later generalised as for system of particles.

Three basic statistical distributions will be considered and hence will classified system of particles as to which of the laws they obey.

CHAPTER ONE

PRINCIPLES OF DYNAMICS

1.1 INTRODUCTION

Everything that really exists in the world is called matter. As a branch of natural science, physics deals with the properties and interactions of matter and radiation.

The key progress in the understanding of nature is to base conclusion on the result of experimental observations (scientific methods).

Invariably, this natural science uses concepts (unexplained fundamental abstractions e.g. time, charge), theories (which connects these concepts and judge by predictive power, comprehensiveness and simplicity), models (constructions to reflect the experimentally determined facts) and laws which are deductions from the model. The laws tells us how things behave in terms of the theory.

Mechanics, a branch of physics investigates one of the natural phenomenon known as mechanical motion in relation with their causes (or Dynamics) which form the basis of this work. This work will look at bodies as if they are single particle, group of particles and as rigid body.

1.2 PHYSICAL QUANTITIES

Physical quantities which are symbols to concepts are used to express laws. There are so many of these quantities such as length, mass, time, charge, force, temperature e t.c, that some has to be selected and called them basic quantities from which others can be derived.

There are standards for each these physical quantities based on international agreement.

These standards are used to compare a given object which implies that the standard must

be accessible. Accessibility is achieved by creating more readily available secondary, tertiary e.t.c standards which are invariable. These standards are known as units of measurement (i.e. scale and units).

The following are selected basic quantities and their respective units known as system International Unit (S.I. units).

S.I. BASE UNIT

	Quantity	Symbol	Unit name	Unit symbol
1.	Length	L	meters	M
2.	Mass	M	Kilogrammes	Kg
3.	Time	T	Seconds	S
4.	Electric current	I	Ampere	A
5.	Thermodynamic Temperature	T	Kelvin	K
6.	Amount of substance		mole	mol
7.	Luminous intensity		candela	cal

Table 1.2.1: Showing S.I. unit of some basic quantities.

1.3. VECTORS

1.3.1 VECTORS & SCALARS

A change of position of a particle is called displacement. If a particle move from one position A to B, the path need not necessarily be straight between A and B but line A to B represent the net effect of the motion (not the actual motion). This net effect can be represented by a line standing for magnitude and an arrow showing the direction. Any quantity that behave like displacement is known as vectors.

A vector can then be said to be characterised by a magnitude and directions. Other

examples of vectors are force, velocity, acceleration, momentum electric and magnetic field e.t.c.

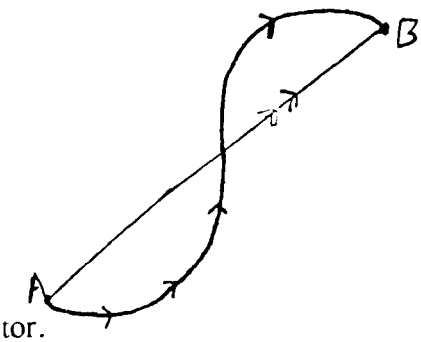


Fig. 1.3.1 Displacement Vector.

Scalar quantities are quantities that has magnitudes but no direction. That is they can be represented wholly by only magnitude.

Example of scalar quantities are time, mass speed e.t.c.

1.3.2 VECTOR ALGEBRA

(1) ADDITION OF VECTORS

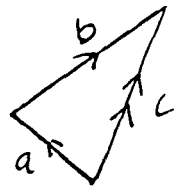


Fig 1. 3. 2 vector Addition

Two or more vector can be added together to give a resultant rector [total effect]. from fig. 1 3 1 above vector $\underline{a}$ and $\underline{b}$ added to give vector $\underline{c}$.

$$\underline{a} + \underline{b} = \underline{c} \quad \text{---------} \quad 1.3.1.$$

for instance if vectors $\underline{a}$ and $\underline{b}$ were consider in 3 - dimension as

$$\underline{a} = a_x\underline{i} + a_y\underline{i} + a_z\underline{k} \quad \text{.....} \quad 1.3.2$$

$$\underline{b} = b_x\underline{i} + b_y\underline{i} + b_z\underline{k} \quad \text{.....} \quad 1.3.3$$

$$\underline{a} + \underline{b} = \underline{c} = C_x\underline{i} + Cy\underline{i} + C_z\underline{k} \quad \text{.....} \quad 1.3.4$$

$$= (a_x + b_x)\underline{i} + (a_y + b_y)\underline{i} + (a_z + b_z)\underline{k} \quad \text{.....} \quad 1.3.4a$$

Similarly, vectors can be subtracted as

$$a - b = a + (-b) \quad \text{..... 1.3.5}$$

Addition of vectors obeyed cumulative and associative laws

$$a + b = b + a \quad \text{.....1.3.6 (commutative)}$$

$$a + (b+c) = (a+b)+c \quad \text{..... 1.3.7(distributive)}$$

ii MULTIPLICATION OF VECTORS

Vectors can be multiplied in two ways.

(a) The scalar multiple :- Also known as Dot product. If two vectors undergo this multiplication the result is always a scalar quantity.

It can generally be shown for two vectors a and b in the same plane separated by angle O as in figure 1.3.3(a) below that

$$\underline{a} \cdot \underline{b} = /a/*/b/* \cos O \quad \text{.....1.3.8}$$

(b) The vector product or cross product :- The result of cross product of two vectors a and b is always a vector quantity and always perpendicular to the plane containing a and b.

It can also be shown for two vectors a and b in the same plane separated by angle O as in figure 1.3.3 (b) below .

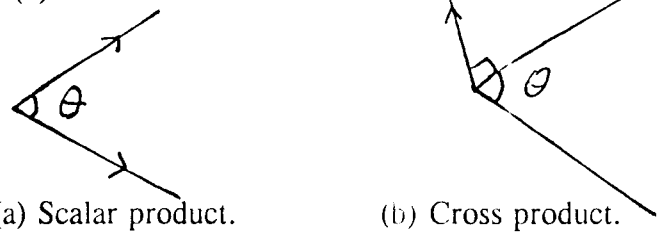


fig. 1.3.3 (a) Scalar product. (b) Cross product.

(iii) Vector Components

In some cases (e.g analytical method), we may need a given vector in a preferred

direction. We then find the projection of the vector in this preferred direction. The projected vector in this axis is known as the components of the vector. There are infinite number of components of a components of a particular vector.To find a particular component (i.e in the preferred direction) construct a unit vector U at the root of the vector along the preferred direction.

Then the component of the vector say A in this direction is

$A\hat{u} = (A\,Cos\Theta)\hat{u}$---------- 1.3.9

Where Θ is the angle between the vector A and $U^{\wedge}$ see figure 1.3.4 below;

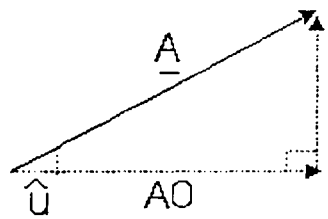


Fig. 1.3.4 component of vector A in U direction.

1. 4 KINEMATIC VARIABLES

1.4.1 POSITION VECTOR

If we consider a particle moving in space relative to an origin. Let the particles move along a curved part as show in figure 1.4.1

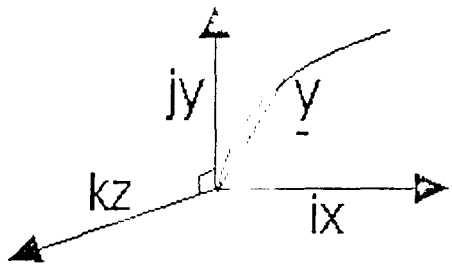


Figure 1.4.1 Particle moving in space.

Its position or displacement from the origin, is measured by the vector r known as its position vector. Explicitly r can be written as : $r = r_x\underline{i} + r_y i + r_z k$. ---------- 1.4.1.

1.4.2 AVERAGE VELOCITY.

The velocity of a particles is the rate at which its position changes with time.

Let ra represent the position vector of a particles in a given frame of reference and rb be its position vector at a later time t in the same frame of reference.

The displacement vector describing the change in position is given by

$$\underline{r} = \underline{ra} - \underline{rb} \quad \ldots\ldots\ldots\ldots\ldots 1.4.2$$

Hence Average Velocity V is given by

$$V = \frac{\Delta r}{\Delta t} = \frac{r_a - r_b}{t} \qquad 1.4.3$$

Therefore velocity is a vector quantity since it involves both direction and Magnitude (Value). It is worthy to note here that velocity is the rate of changes of displacement which can be represented as

Instantaneous velocity $$V = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \qquad 1.4.4$$

This is in one dimension.

In three dimension.

$$V = \frac{dr}{dt} = i\frac{dr_x}{dt} + i\frac{dr_y}{dt} + \frac{dr_z}{dt} \quad \ldots\ldots\ldots 1.4.5$$

$$V = iV_x + iV_y + kV_z \quad \ldots\ldots\ldots\ldots\ldots 1.4.6$$

The S.i. UNIT OF VELOCITY IS m/s.

1.4.2 ACCELERATION.

When a body moves and its velocity changes either in magnitude, direction or both we say the body accelerate. This quantity, acceleration of a particle can then be defined as the rate of change of its velocity with time.

Suppose that at the instant time t a particle is with a position vector ra moving in a plane with an instantaneous velocity va. And at a later time t2 its position vector is rb moving with a velocity vb. The average acceleration a of the particle is defined to be the change of velocity divided by the corresponding change in time.

$$a = \frac{Vb - Va}{t2 - t1} = \frac{V}{t} \quad \ldots\ldots\ldots\ldots 1.4.7$$

$$\text{Instateneous acceleration} = a = \lim_{t \to 0} \frac{V}{t} = \frac{dv}{dt} \quad \ldots\ldots\ldots 1.4.8$$

Note that if Va = Vb then the body is not accelerating i.e. the acceleration = o. Also we have a constant (or uniform) acceleration if the rate of change of velocity with time is equal throughout a motion. But if the rate of change of velocity is not same throughout them we can say the acceleration varies.

1.5 GRAPHS OF MOTION

1.5.1 DISPLACEMENT - TIME GRAPH

When the displacement r of a body under motion is plotted against time 't' as shown below, the slope gives the uniform velocity of the body at all time.

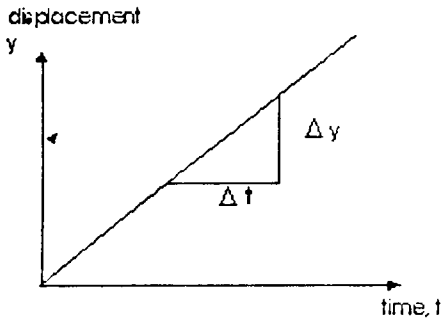


Fig. 1.5.1 Displacement -Time graph

$$\text{Slope} = \frac{r}{t} = V \quad \ldots\ldots\ldots 1.5.1$$

1.5.2 VELOCITY- TIME GRAPH

Plotting the graph of velocity of a body against time gives a straight line graph. The slope of which gives the accelerations of the body at any instant. The graph as shows in fig. 1.5.2.

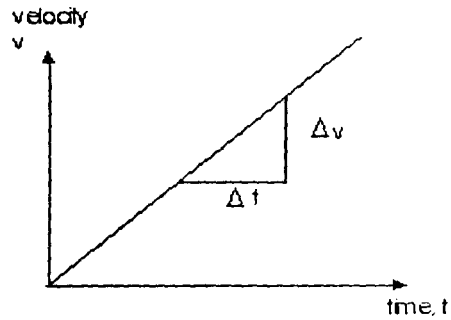


FIG. 1.5.2 velocity - time graph

Slope = = a -------- 1.5.2

Notice that the area under this graph gives the displacement of the body under considerations.

1.6 MOTIONS

In the studies of motion one is aimed at looking at the various properties of a moving particle (or body). Here I will treat all bodies as if they are single particle for convenient purpose. Motion of particle can be one -, two - or three dimensional.

1.6.1. LINEAR MOTION

The concept of positions, displacement, velocity and acceleration as concern particles in motion have been treated earlier. However, with much particular to the linea motion it is necessary to bring to fore some simple relationships between them as related by Newton knows as equations of motion.

(i) FIRST EQUATION OF MOTION

If U and V are the initial and final velocities respectively of a particle in motion within a time t. And recall equation 1.4.7 its acceleration

$$a = \frac{V-U}{t} \text{ ------- } 1.6.1$$

This can be rewritten as

$$V = U + at \text{ ---------- } 1.6.2$$

(i) FIRST EQUATION OF MOTION

If U and V are the initial and final velocities respectively of a particle in motion within a time t. And recall equation 1.4.7 its acceleration

$$a = \frac{V-U}{t} \text{ ------- } 1.6.1$$

This can be rewritten as

$$V = U + at \text{ ---------- } 1.6.2$$

(ii) 2ND EQUATION OF MOTION

Average velocity of the particle is given by

$$V = \frac{V + U}{2} \text{ --------- } 1.6.3$$

Using equ. 1.6.2.

$$V = \tfrac{1}{2}[at + U + U] \text{ -------- } 1.6.3a.$$

But displacement S is

$$S = Vt \text{ ------- } 1.6.4$$

Therefore $S = \tfrac{1}{2}[at + 2U]t$ ..... 1.6.4a.

Hence $S = Ut + \tfrac{1}{2} at_2$ 1.6.5

(iii) THIRD EQUATION OF MOTION

From 1.6.2.

$$t = (V - U)/a \text{ ..... } 1.6.6$$

using 1.6.6 in 1.6.5

$$S = U\left(\frac{V - U}{a}\right) + \tfrac{1}{2}a\left(\left(\frac{V - U}{a}\right)\right)^2 \text{ ..... } 1.6.7$$

$$S = \frac{UV - U^2}{a} + \frac{U^2 + V^2 - 2UV}{2a} \text{ ..... a.}$$

1.6.2 PROJECTILES

This described the two dimensional motion of a body th own into the air. That is the motion is along a vertical plane. The body undergoing this typ s of motion has a constant acceleration 'g' (acceleration due to gravity) which is directed ( )wnwards. To analyse this motion we consider its motion in y (vertical) and x (horizontal) directions separately.

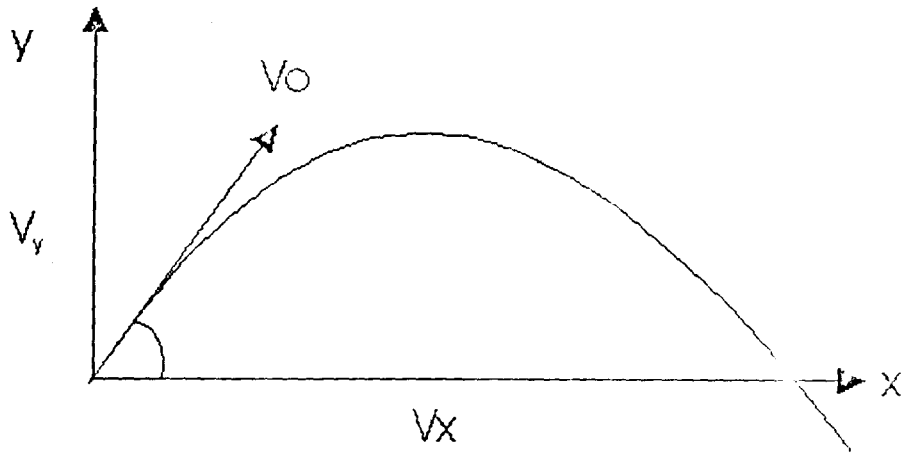


fig. 1.6.1 path of project describing projectile motion.

If the initial velocity of the body under consideration is Vo a l projected through angle Q to the horizontal:

Its vertical component = $V_y = V\sin\Theta$, ...1.6.3

Its horizontal component = $V_x = V\cos\Theta$ .....1.6. 4.

Vx is constant throu h out and hence the horizontal acceleratic n is zero.

$a_x = 0$ 1.6.5

If motion al ng positive y direction is taken as positiv then

$a_y = -g$ 1.6.6

At any instant

$$V_x = V_o \cos\theta \quad \ldots\ldots\ldots\ldots\ldots \quad 1.6.7$$

$$Vy = V_o \sin\theta - gt \quad \ldots\ldots\ldots\ldots \quad 1.6.8$$

$$Vmag = -\sqrt{V_x^2 + V_y^2} \quad \ldots\ldots\ldots \quad 1.6.9$$

$$\tan\theta = V_y/V_x \quad \ldots\ldots\ldots\ldots\ldots \quad 1.6.10$$

horizontal distance moved at any time t is

$$x = (V_o \cos\theta)t \quad \ldots\ldots\ldots\ldots\ldots \quad 1.6.11$$

vertical distance covered is

$$y = (V_o \sin\theta)\, t - \tfrac{1}{2}gt^2 \quad \ldots\ldots\ldots\ldots\ldots \quad 1.6.1$$

But from 1.6.11

$$t = x/V_o \cos\theta \quad \ldots\ldots\ldots\ldots\ldots \quad 1.6.13$$

Using 1.6.13 in 1.6.12 we have

$$y = (\tan\theta)x - \frac{(g)x^2}{(2V_o^2 \cos^2\theta)} \quad \ldots\ldots\ldots\ldots\ldots \quad 1.6.14$$

1.6.14 is a parabolic equation in x.

If $y = 0$

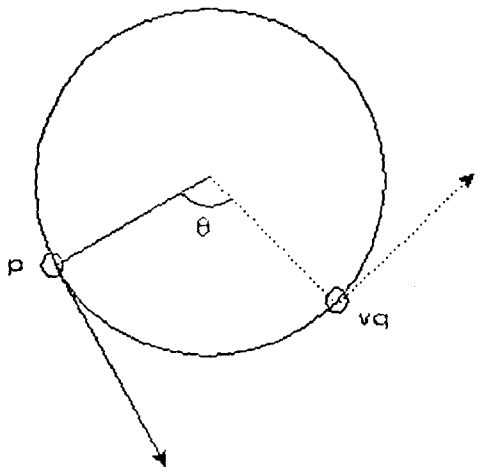
$$x = \frac{Vo^2}{g} \sin 2\theta \quad \ldots\ldots\ldots\ldots\ldots \quad 1.6.15$$

equation 1.6.15 is the maximum horizontal distance known as RANGE.

1.6.3 CIRCULAR MOTION

Circular motion could be vertical or horizontal in this section I shall treat only the horizontal circular motion of an object which travels equal distance in equal time. In this case

the magnitude of the velocity is constant but not the direction. Therefore we can say that the speed of the object is constant since speed is a scalar quantity. The direction of the velocity at any instant is along the tangent to the circle at that point.



1.6.2Showing an object describing a Circular motion

Angular velocity denoted by w (omega) is defined as angle swept out in unit time by the line joining the body and the center of the circle (radius). And it is given as

$$w = \Theta/t \quad \ldots\ldots\ldots\ldots \quad 1.6.16$$

And linear speed $= V = \frac{r\Theta}{t} = rw$ 1.6.17

The acceleration, a, of the body describing circular motion is

$a = V^2/r$ 1.6.18

or $a = w^2r$ 1.6.19

And is always directed towards the centre of the circle.

One may want to ask what keeps a body in circular motion?

Definition (centripetal force)

This is the force that is responsible for keeping a body in a circular motion.

Since $F = ma$ 1.6.20

or $F = mV^2/r$ 1.6.21

or $F = mw^2r$ 1.6.22

1.7 LINEAR MOMENTUM

Linear momentum of a body in motion can be interpreted mathematically as the product of its mass and velocity. That is if M is the mass and V is the velocity of the body under discuss we have

$P = M.V$ 1.7.1

Since P is a product of scalar (mass) and vector (velocity) then momentum is a vector quantity.

Definition (Inpulse)

Let a force F be exerted on a body during a collision. We assume that the force has a constant direction. And that the collision last for a time t we can write the change in momentum as

$dp = Fdt$ 1.7.2

$\int^{2}_{1} dp = \int^{t2}_{t1} fdt$ 1.7.3

The left side is $p_2 - p_1$ which is change in momentum of the body.

The right side, which measures the strength and duration of the collision force is called the impulse and denoted J.

$J = \int Fdt$ 1.7.4

1.8 CONSERVATION OF LINEAR MOMENTUM

Momentum possesses an interesting and important property inherent i quite a few physical quantities. This is the property of being conserved. It consists in that the geometrical sum of

momenta of bodies which interact only with each other remain unchange.

The principle of conservation of linear momentum can then be stated as "when the resultant external forces acting on a system is zero, the total linear momentum of the system remain the same.

$$\therefore F = \frac{df}{df} = O \quad \ldots\ldots\ldots\ldots \quad 1.8.1$$

For instance if we consider collision between two particles, such as masses M_1 and M_2 as shown below:

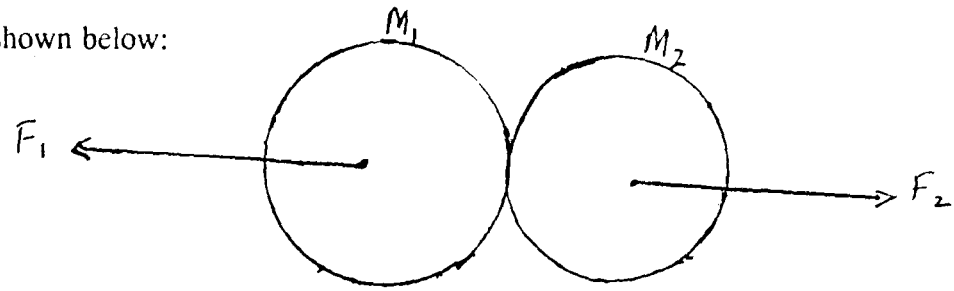


Fig. 1.8.1 collision between two bodies M_1 and M_2.

During the brief collision these particles exert large forces on each other. At any instant F_1 is the force exerted on particle 1 by particle 2 and F_2 is the force exerted on particle 2 by particle 1. By experiment it is found that F_1 and F_2 are both equal but in reverse direction.

Considering particle 1

$$P_1 = \int F_1 dt = F_1 \triangle t \quad \ldots\ldots\ldots\ldots\ldots \quad 1.8.2$$

Considering particle 2

$$\triangle P_2 = \int F_2 dt = F_2 \triangle t \quad \ldots\ldots\ldots\ldots \quad 1.8.3$$

But $F_1 = - F_2$ 1.8.4

$$\triangle P = \triangle P_1 + \triangle P_2 = O \quad \ldots\ldots\ldots\ldots\ldots \quad 1.8.5$$

1.9 LAWS OF MOTION

Here some of the laws that summarizes the general behaviour of bodies in motion shall be discussed.

Definition (FORCES)

Force could mean pull or push. In a more general term force can be said to be quantity that alter the state of object (either stops or causes motion). Alternatively we can define force in term of "acceleration a given standard mass (body) will experience when placed in a suitable environment.

Example of forces are, gases or liquid exert forces on container, tension in the rope, ruberbands and springs exerts forces on the object attached to their end, two bodies rubbed together exerts frictional forces on each other e.t.c. All these are contact forces. Gravitational, electrostatic and magnetic forces are some of the forces that does not involves contact.

In measuring forces e.g. using elastic material, it is found that the force exerted on the material (comprehension or stress) is proportional to the change in length of the elastic material.

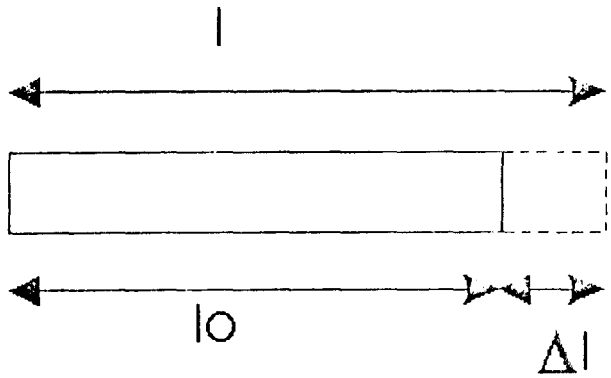


fig. 1.9.1. Elastic material being acted upon by a force.

$$F \alpha (l - l_o) = \_l \quad \text{..... 1.9.1}$$

$$F = K\_l \quad \text{..... 1.9.2}$$

Where k is the force constant for the material under consideration.

(i) NEWTON'S FIRST LAW OF MOTION

It says that 'everybody will continue in the state of rest, or uniform motion in a straight

line unless an impressed force act upon it".

Mathematically it can be iterpreted as

If F = O

$$\triangle V = O \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1.9.3$$

To best understand this law we can consider these two examples.

(a) The passangers in a stationary vehicle jack beackwards when the car suddenly moves. and

(b) The passangers in a moving vehicle jack forwards when the driver suddenly applied break.

These two examples tells us that every object don't willingly change their state and this can be generated with the Newton's first law.

(ii) NEWTON'S SECOND LAW OF MOTION

The first law did not tell us anything about the nature of the force. It is only a statement about the behaviour of objects in the absence of any force. But the second law of motion helped us to answer a fundamental question that "what effect will the same force produce on different bodies with different properties (mass)?"

The answer in the overview is thus, different acceleration will be produced.

Therefore the statement of the law says "the rate of change of momentum of a body is proportional to the force applied" and it takes place in the directions of the force.

mathematically,

$$F \alpha \frac{Mv - Mu}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1.9.4$$

$$F = \frac{KM\,(V-U)}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1.9.5$$

but $\frac{V-U}{t} = a$ 1.9.6

$$F = Kma$$

If a force of 1 Newton is applied to a mass of 1kg it produces an accelaration of 1 m/s.

Hence $k = 1$ and

$$F = ma \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1.9.7.$$

(iii) NEWTON'S THIRD LAW OF MOTION

When two bodies acted on each other, the two exert forces mutually on one another. These forces are equall but in contrary directions. Simply put Newton said "To every action (force) there is always an equal but opposite reaction (force)".

Mathematically, if F_{AB} = action of body A on body B and F_{BA} = reaction of body B on body A

$F_{AB} = - F_{BA}$. 1.9.8

(iv) KEPLER'S LAWS OF PLANETARY MOTION

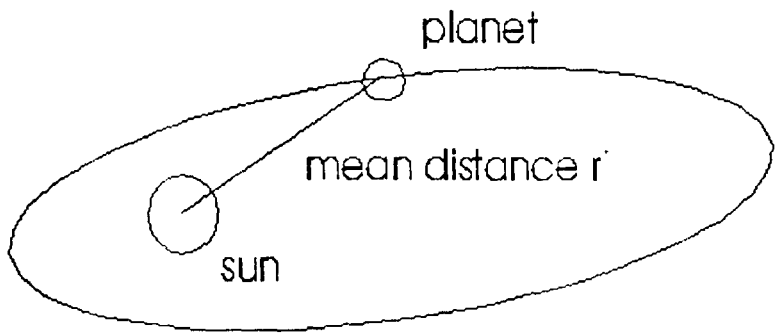
The following three statement credited to Kepler summarises the planetary motions.

(a) Each planet moves in an elliptical path with the sun at one focus.

(b) The radius (distance between centre of sun to center of planet) of the moving planet sweeps out equal area in equal time.

(c) The square of the period of revolutions of the planet (T) about the sun is

proportional to the cube of the mean distance from the sun i.e.



Path of revolution

Fig. 1.9.2. showing planetary motion named the sun.

(v) LAW OF UNIVERSAL GRAVITATION

Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of their distance apart. That is if M_1 and M_2 are two masses separated by distance r in the universe they attract each other with a force F given by

$$F \alpha \frac{M_1 M_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots 1.9.10$$

or

$$F = \frac{GM_1 M_2}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots 1.9.11$$

Where G is a constant of proportionality known as universal gravitation.

1.10 WORK AND ENERGY

DEFINITION (WORK)

Work as a concept and by its definition is more restricted. That is not all muscular efforts cab be regarded as work. Example of this is a basketball player holding a ball in his unstreched hand. Because the ball is stationary the player does no work on the ball, nevertheless he feels tired.

he feels tired.

Therefore we can say work is being done when a force acts on a body and cause it to undergo a displacement.

The unit of work is Joules (j).

(i) WORK DONE BY A CONSTANT FORCE.

From the above definition of work, if a constant force acts on a body, the infinitesimal mechanical work (DW) done by the constant force F on the body can be expressed as Dw

$= F\triangle S\text{Cos}\ \theta$ 1.10.1

where $\triangle S$ is the infinitesimal displacement and θ is the angle between the vectors displacement and force.

Equivalently, the work equation above can be express as dot product of two vectors force and displacement as

$\triangle W = F.\ DS$ 1.10.2

Since $F.DS = FDS\text{CosQ}$ 1.10.3

(ii) WORK DONE BY SET OF FORCES

Single force was been considered so far. More than one force (set of forces) can act upon a particle and the total work done on the particle is the sum of the individual workdone by each force when they act separately on the particle.

Therefore,

$\triangle W\ \text{total} = \triangle W_1 + \triangle W_2 + \triangle W_3 + \ldots\ldots + \triangle W_n$ 1.10.4

$\triangle W\ \text{total} = \triangle W_i$ 1.10.5

$\triangle W\ \text{total} = F_1.\ \triangle S + F_2.\triangle S + F_3.\triangle S + \ldots\ldots\ldots\ Fn.\triangle S$ ... 1.10.6

$$\triangle W \text{ total} = (F_1 + F_2 + F_3 + ... + Fn).\ \triangle S ..... 1.10.7$$

$$\triangle W \text{ total} = F.\ \triangle S ..... 1.10.8$$

where F is the resultant effect of all the forces.

(iii) WORKDONE BY VARIABLE FORCES

Here we examine cases in which a particular force F acting on a particle depends on the position of the particle. If we consider particle moving in a straight line say x-direction and if the force F act only in the x-direction but varies with x we have

$$\triangle W = F(x).\triangle x ..... 1.10.9$$

The total work done by F(x) during the displacement of the particle from the point A ($x=xA$) to point B($x=x_b$) when the interval between A and B is divided into M equal intervals is given by

$$W = \Sigma^{N}_{n-1}\ F(x_n).\ \triangle x_n .....1.10.10$$

$$\triangle W = F(xn).\triangle xn ..... 1.10.10$$

Alternatively we can write

$$\triangle W = \text{lin}\ \Sigma^{N}_{n-1}\ F(xn)\ \triangle xn = \int^{xb}_{xa} F(x)\ dx$$

And this can be shown graphically as below.

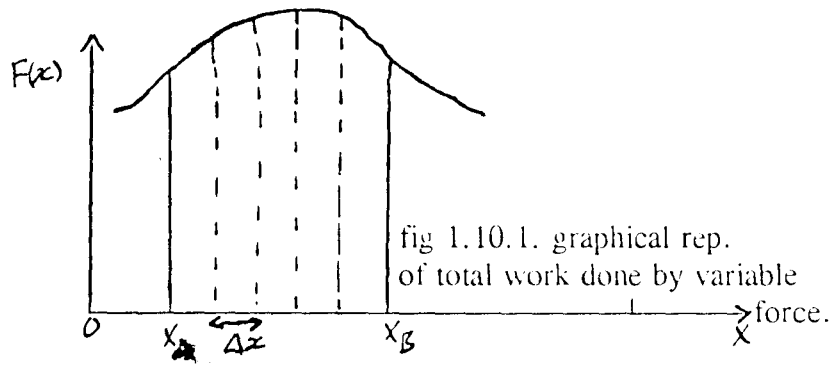


fig 1.10.1. graphical rep. of total work done by variable force.

The area under the above graph between X_A and X_B gives the total work done by the variable force on the particle.

1.11 WORK - ENERGY PRINCIPLE:

Considering a particle with mass m that is acted upon by a constant net force F. Thus leads to a constant acceleration for the particle.

$$a = F/m \quad \ldots\ldots\ldots\ldots\ldots \quad 1.11.1$$

If we consider x-direction only and that the particle is at point A with a velocity V_A. After a time t, the particle is at point B with velocity VB therefore

$$a = \frac{V_B - V_a}{t} \quad \ldots\ldots\ldots\ldots\ldots \quad 1.11.2$$

$$S = \frac{(V_B + V_A)}{2} t \quad \ldots\ldots\ldots\ldots\ldots \quad 1.11.3$$

since

$$F = ma \quad \ldots\ldots\ldots\ldots\ldots \quad 1.11.4$$

$$W = FS \quad \ldots\ldots\ldots\ldots\ldots \quad 1.11.5$$

$$W = mas \quad \ldots\ldots\ldots\ldots\ldots \quad 1.11.6$$

$$W = \tfrac{1}{2}m \frac{(V_B - V_A)}{t} (V_B + V_A) t \quad \ldots\ldots \quad 1.11.7$$

$$W = \tfrac{1}{2}m (V^2_B - V^2_A) \quad \ldots\ldots\ldots\ldots\ldots \quad 1.11.8$$

The quantities $\tfrac{1}{2}mV^2_B$ and $\tfrac{1}{2}MV^2_A$ are the particle's kinetic energies (energy due to motion) at points B and A respectively. If we denote these energies as K_B and K_A then the work equation becomes

$$W = \tfrac{1}{2}MV^2_B - \tfrac{1}{2}MV^2_A \quad \ldots\ldots\ldots\ldots\ldots \quad 1.11.9$$

Hence

$$W = \triangle K = K_B - K_A \quad \ldots\ldots\ldots\ldots\ldots \quad 1.11.10$$

This is the work -Energy principle and it holds true for both constant and variable forces.

DEFINITION (POWER)

Consider a force F(x) that is one of the forces acting on a particle. The work done by this force during a general displacement that requires a time Dt is

$$\triangle W = F_x \triangle x \quad \text{..... 1.11.11}$$

To obtain the rate at which work is done, we divide equation 1.11.11 by Dt and when taking the limit $\triangle t \rightarrow 0$ it gives

$$\frac{dw}{dt} = \lim_{\triangle t \rightarrow 0} \frac{\triangle w}{\triangle t} = \lim_{\triangle t \rightarrow 0} F_x \frac{\triangle x}{\triangle t} \quad \text{..... 1.11.12}$$

$$\frac{dw}{dt} = F_x \frac{dx}{dt} \quad \text{..... 1.11.13}$$

$$\frac{dw}{dt} = F_x V_x \text{ (in one dimension)} \quad 1.11.14$$

By generalization

$$\frac{dw}{dt} = F_x V_x + F_y V_y + F_z V_z \quad \text{..... 1.11.15}$$

Therefore $$\frac{dw}{dt} = F.V \quad \text{..... 1.11.16}$$

The rate at which work is done, $\frac{dw}{dt}$, is called the power denoted as P

Thus $P = \frac{dw}{dt} = F.V$ 1.11.16

DEFINITION (POTENTIAL ENERGY)

There is a kind of energy associated with position of particles known as potential energy.

Suppose a weight lifter raises a weight very slowly from the floor to a shelf at height h. Force applied on the weight F is just equal to that of the gravitational force

$$F_g = mg \quad \text{..... 1.11.17}$$

work done = F.h = mgh 1.11.18

If the weight is allowed to fall and its kinetic energy is measured just before it touches the ground (floor) it is always equal to the potential energy gained during raising the weight.

1.12 CONSERVATION OF ENERGY.

The work-energy principle discussed earlier only touched on the kinetic energy and work done concepts. If now we add the concept of potential energy we can now get a more revealing formulation of energy conservation.

If U_A and U_B are the potential energies of a particle at position A and B respectively then we can say

$$U_B + \frac{1}{2}MV^2_B = U_A + \frac{1}{2}MV^2_B + Fdx = E \text{ .....} 1.12.1$$

The content of the above equations can be stated as follows:

When a particle moves from a point 'A' to another point 'B', the mechanical energy at point 'B' is equal to the mechanical energy at point 'A' less the discipative work done against frictional force.

From this if we consider a close system and without friction then we can say that the total energy of a particle 'E' is constant at all point (or time).

1.13 CONSERVATIVE & NON-CONSERVATIVE FORCES

Since the potential energy depends on position of the particle in a force field, then the work done on the particle by the force field is given by

$$W(A \rightarrow B) = \int^b_A F.ds \text{ .....} 1.13.1$$

If this work done is independent of path followed by the particle then the force is a conservative force. If a particle move round from A to B and back to A following arbitrary

different path to achieve this, the potential energy of the particle at the end of the trip is the same as it has at the beginning. Gravitational force is an example of conservative force.

Any force field that does not behave in this way is said to be non-conservative.

1.14 SYSTEM OF PARTICLES

Up to this point we have been looking at body as if they are single particle. Let us now see what will happen if more than one particle are involved.

(i) CENTER OF MASS

When dealing with system of particles the interest is to enquire about the motion of the system as a whole. To do this it is helpful to use the center of mass concept. The point that corresponds to the center of mass the "effective mass center" of the system of particles. That is to say if all the mass of the object were to be concentrated at this point, the resulting point - like mass would have the same transnational kinematic behaviour as the extended mass system as a whole.

Precisely, the center of mass vector, denoted as R, of n particles is

$$R = \frac{m_1r_1 + m_2r_2 + m_3r_3 + \ldots + m_nr_n}{m_1 + m_2 + m_3 + \ldots + m_n} \quad \ldots\ldots 1.14.1$$

$$R = \frac{\Sigma^n_{i=1} M_ir_i}{} \quad \ldots\ldots 1.14.1a$$

$$R = \frac{1}{M} \Sigma^n_{i=1} M_ir_i \quad \ldots\ldots 1.14.1b$$

$$M = \Sigma^n_{i=1} Mi \quad \ldots\ldots 1.14.2$$

Considering a system of 4 particle shown in fig. 1.14.1 with reference to an origin 0.

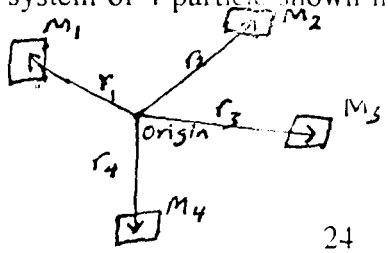


Fig. 1.14.1 system of 4 particles.

The center of mass R for this system is given by

$$R = \frac{M_1 r_1 + M_2 r_2 + M_3 r_3 + M_4 r_4}{M_1 + M_2 + M_3 + M_4}$$

Generally in three dimensions

$$ri = r_x i + r_y i + r_z k. \quad \ldots\ldots\ldots\ldots\ldots \quad 1.14.4$$

(ii) MOTION OF CENTER OF MASS

If rem is the position vector identifying the center of mass at a particular point and if M is its total mass, then

$$Mrcm = M_1 + M_2 r_2 + M_3 r_3 + \ldots. + M_n r_n \ldots\ldots\ldots \quad 1.14.5$$

By first derivative

$$\frac{Mdrm}{dt} = M_1 \frac{dr_1}{dt} + M_2 \frac{dr_2}{dt} + M_3 \frac{dr_3}{dt} + \ldots. + M_n \frac{dr_n}{dt} \ldots\ldots \quad 1.14.6$$

Where Vcm is the velocity of the anter of mass and Vn is the velocity of the nth particle.

The second derivative.

$$\frac{MdVcm}{dt} = M_1 \frac{dV_1}{dt} + M_2 \frac{dV_2}{dt} + M_3 \frac{dV_3}{dt} + \ldots.. + M_n \frac{dV_n}{dt} \ldots\ldots .. \quad 1.14.8$$

$$Macm = M_1 a_1 + M_2 a_2 + M_3 a_3 + \ldots\ldots + M_n a_n \ldots\ldots\ldots \quad 1.14.9$$

where acm is the acceleration of the center of mass.

If $$F_n = M_n a_n \ldots\ldots\ldots\ldots\ldots \quad 1.14.10$$

$$Macn = F_1 + F_2 + F_3 + \ldots\ldots + F_n \ldots\ldots \quad 1.14.11$$

From Newton's third law of motion it is clear that each pair of particles exert equal but opposite forces on one another. Hence the internal forces of a system of particle add up to zero.

Therefore

$$Macn = Fext. \quad \ldots\ldots\ldots\ldots \quad 1.14.12$$

(iii) WORK AND ENERGY

$$\text{Work done} = \int Fext\ dx_{cm} \quad \ldots\ldots\ldots\ldots\ldots \quad 1.14.13$$

$$W = \int Macm\ dx_{cm} \quad \ldots\ldots\ldots\ldots\ldots \quad 1.14.13a$$

$$= \int M \frac{dVcm}{dt} dx_{cm} \qquad 1.14.13b$$

$$= \int MVcm\ dV_{cm} \qquad 1.14.13c$$

Hence $W = \int Fext\ dxcm = \frac{1}{2} MVcm^2 \quad \ldots\ldots\ldots \quad 1.14.14$

Change in kinetic energy $\triangle K$ is given by

$$W = \triangle K = Kcm_2 = Kcm_1 = \triangle Kcm \quad \ldots\ldots\ldots\ldots \quad 1.14.15$$

This is the work - Energy therein.

(iv) LINEA MOMENTUM

Each particle has a momentum given by

$$Pn = MnVn \quad \ldots\ldots\ldots\ldots \quad 1.14.16$$

Therefore for the general system momentum is given by

$$MVcm = M_1V_1 + M_2V_1 + M_3V_3 + \ldots\ldots + MnVn \ldots\ldots \quad 1.14.12$$

$$Pcm = P_1 + P_2 + P_3 + \ldots\ldots + P_n \ldots\ldots\ldots \quad 1.14.18.$$

And that the conservation principle of linea momentum still holds for system of particles

$$\triangle Pcm = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad 1.14.19.$$

That is $P_1 + P_2 + P^{23} + \ldots\ldots\ldots\ldots + P_n = P_6 = \text{constant} \ldots\ldots 1.14.20$

1.15 ANGULAR MOMENTUM

In rotational motions, the analog of linear momentum is the angular momentum. If we consider a particle of mass 'm' and linear momentum 'P' at a position described by position vector r relative to an origin. We can define the angular momentum l of the particle with respect to the origin to be

$$l = r \times P \quad \ldots\ldots\ldots\ldots \quad 1.15.1$$

It magnitude is given by

$$l = rP\sin\Theta \quad \ldots\ldots \quad 1.15.2$$

Where Θ is the angle between r and P and the direction of the angular momentum is normal to the plane formed by r and P.

Angular momentum is often called the moment of (linear) momentum.

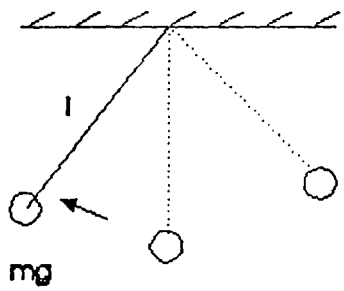
1.16 HARMONIC OSCILLATORS

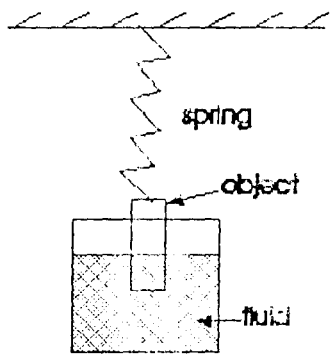
Any object that moves to and fro such that its acceleration varies with its displacement from a fixed point and that the acceleration is always directed towards the fixed point is said to be an harmonic oscillator.

If there is no frictional and external forces acting the motion is known as simple harmonic oscillation.

If a frictional force is present then the motion is Damped harmonic oscillation while it is a forced harmonic oscillation when there is an external force driving the object.

Example of S.H.M is a simple pendulum while object attached to a spring balance inserted in fluid experiences damped harmonic oscillation.





(i) SIMPLE HARMONIC MOTION

The general equation for objects describing S.H.M is

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = O \ldots\ldots 1.16.1$$

eq. 1.16.1 has a general solution given by

$$x = A \text{ Sin } wt + B\text{Cos}wt \ldots\ldots 1.16.2$$

$$\frac{dx}{dt} = WA\text{Cos}wt - WB\text{Sin}wt \ldots\ldots 1.16.3$$

$$\frac{d^2x}{} = -W^2(A \text{ Sin}wt + B\text{Cos}wt) \ldots\ldots 1.16.4$$

If we use equations 1.16.2 and 1.16.4 in 1.16.1 we have

$$W^2 = k/m \ldots\ldots 1.16.5$$

$$\Longrightarrow \quad W = \sqrt{k/m} \ldots\ldots 1.16.5a$$

This is the angular frequency of the particle.

At every $2\pi/w$ interval of time the motion repeat itself. Therefore, $2\pi/m$ is the period of the motion T.

$$T = 2\pi/w \ldots\ldots 1.16.6$$

$$T = 2\pi\sqrt{m/k} \ldots\ldots 1.16.7$$

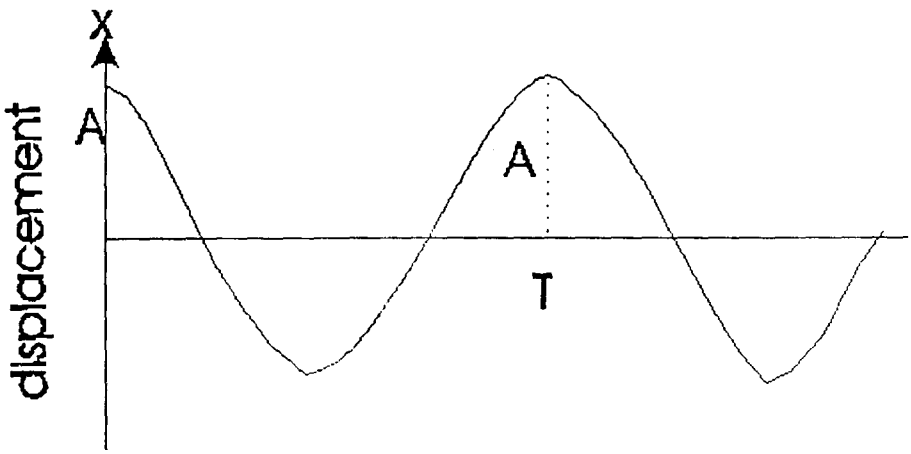
This is the period of all motions govern by 1.16.1.

The frequency of S.H.M is given by

$F = 1/T$ 1.16.8

$F = 1/2\pi(\sqrt{k/m})$ 1.16.9

If we plot the displacement x against the time t for S.H.M described by equation 1.16.1 we have



Recall $x = A\sin wt + B\cos wt$ 1.16

The velocity of an object describing S.H.M is

$$V = \frac{dx}{dt} = WA\cos wt - WB\sin wt \text{ ..... } 1.16.11$$

And the acceleration is

$$a = \frac{d^2x}{dt^2} = -W^2(A\sin wt + B\cos wt) \text{ ..... } 1.16.12$$

Hence $a = -W^2x$ 1.16.13

All these can be represented graphically as

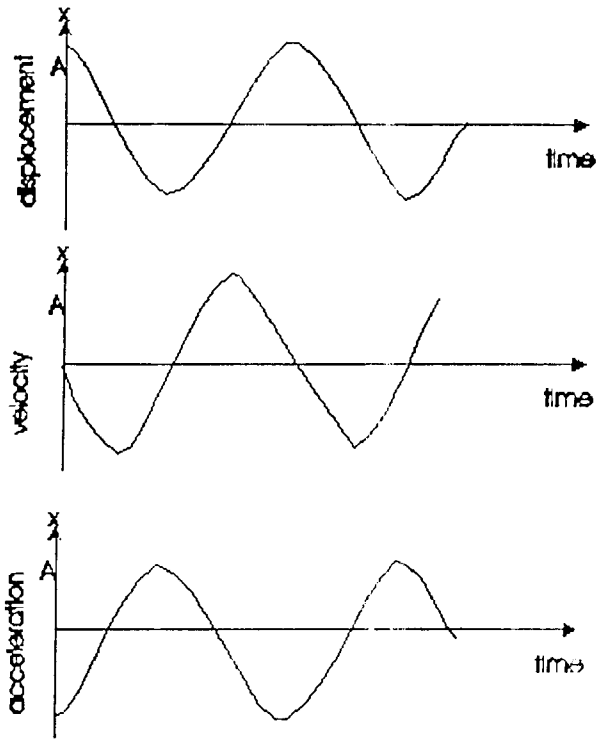


Fig. 1.16.4 Relationship between x, v, and a in simple H.M.

(2) ENERGY CONSIDERATION OF S.H.M

For all harmonic motions including S.H.M in which no descriptive force acts, the total mechanical energy E is given by

$$E = K + U \quad \ldots\ldots\ldots 1.16.14$$

And that it is conserved.

Let displacement $x = A\text{Sin}wt$ 1.16.15

The potential energy $U = \frac{1}{2} kx^2$ 1.16.16

$$= \frac{1}{2} KA^2 \text{Sin}^2 wt \ldots. 1.16.17$$

This has a maximum value $\frac{1}{2} KA^2$ and a minimum value O.

The velocity is given by

$$V = \frac{dx}{dt} = AW\text{Cos}wt \ldots\ldots\ldots 1.16.18$$

Hence The kinetic Energy is given by

$$Ke = \frac{1}{2}MV^2 \ldots\ldots\ldots 1.16.18$$

$$Ke = \frac{1}{2}MW^2 A^2 \text{Cos}^2 wt \ldots\ldots 1.16.20$$

$$Ke = \tfrac{1}{2}KA^2 \cos^2 wt \ldots\ldots\ldots 1.16.21$$

It also has a min. value 0 and a maximum value $\tfrac{1}{2}KA^2$ during the motion.

Recall $E = U + K$

$$\therefore \quad E = \tfrac{1}{2}KA^2\cos^2 wt + \tfrac{1}{2}KA^2 \sin^2 wt \ldots\ldots\ldots 1.16.22$$

$$= \tfrac{1}{2}KA^2(\cos^2 wt + \sin^2 wt)$$

But $\cos^2\Theta + \sin^2\Theta = 1$

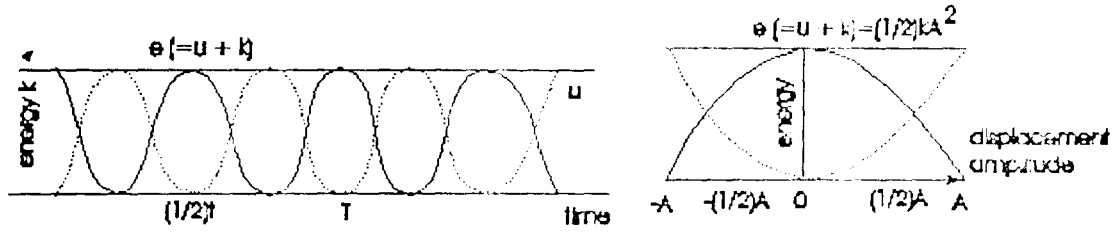
$$\therefore \quad E = \tfrac{1}{2}KA^2 \ldots\ldots\ldots\ldots 1.16.23$$

$$E = \tfrac{1}{2}MV^2 + \tfrac{1}{2}Kx^2 = \tfrac{1}{2}KA^2 \ldots\ldots\ldots 1.16.24$$

From this relationship

$$V = \frac{dx}{dt} = \pm \sqrt{(k/m(A^2 - x^2))}\ldots\ldots\ldots\ldots 1.16.25$$

$E(=U + K)$ $\qquad E = U+K = \tfrac{1}{2}KA^2$



(3) DAMPED HARMONIC OSCILLATION

Let us now treat oscillations experiencing frictional force f which is proportion to the magnitude of the velocity of the oscillator.

The general equation governing D.H.O is given by

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = O \ldots\ldots 1.16.26$$

And the general solution satisfying equation 1.16.26 is

Subject to restriction to restriction of conservation of energy in the process

$E_i + E_j = E_k + E_i$ 2.3.9

(1) PARTITION FUNCTION

In the above discussion if we treat M_i, M_k, M_j, N_i as function of energies E_i, 2. 3. 8 and 2.3.9 requires that

$N_i = \alpha Exp\{\beta E_i\}$ 2.3.10

for indices i, j, k,l where α and ß are constants independent of which of the four stata we look at

Therefore

$n_i = \alpha Exp\{\beta E_i\}$ 2.3.11

For isolated system the additional restrictions are

(1) total molecule $N = {}_i\Sigma\ \alpha\ Exp\{\beta E_i\}$ 2.3.12

(2) and total energy $E = \Sigma_i\ \alpha\ E_i Exp\{\beta E_i\}$ 2.3.13

from equation 2.3.12

$$\alpha = \frac{N}{{}_i\Sigma Exp\{\beta E_i\}} \quad \text{..... 2.3.14}$$

Therefore equation ..... 2.3.11 becomes

$$ni = \frac{N\ Exp\{\beta E_i\}}{\Sigma Exp\{\beta E_i\}} = \frac{N Exp\{\beta E_i\}}{Z} \quad \text{..... 2.3.15}$$

Where $Z = \Sigma Exp\{\beta E_i\}$ 2.3.16

is an enormously important quantity the single components partition function

(2) QUANTUM STATISTICS

We are still considering a gas of non-interacting (or weakly interacting) identical particles

```
        SBE ← SBE * SBE(i)
        IF MSBE > SBE(i)
        MSBE ← MSBE
        ELSE
        MSBE ← SBG(i)
        ENDIF
        ENDFOR
        AVEN ← A/TBE
        FOR i ← 1 to n
OUTPUT i, SBE(i), Ni(i), gi(i)
        ENDFOR.
OUTPUT   "Average particle in states, AVEN
OUTPUT "Total number of microstates" TBE
OUTPUT  "Most probable state", MSBE
                CASE  F

          SFD = 1
OUTPUT  Particles are identical, inistinguishable and maximum of one particle
        in a level.
        FOR  i ← 1 to n
OUTPUT Supply the value of degeneracy
INPUT  gi (i)
INPUT  "No. of particles" Ni(i)
```

ENDFOR

FOR i ← 1 to n

LET V ← gi(i)

TRANSFER CONTROL TO SUBROUTINE FOR FACTORIAL

Restate the answer into ANSI

LET V ← gi(i) - Ni(i)

TRANSFER CONTROL TO SUBROUTONE FOR FACTORIAL

Restate the answer into Ans2

LET V ← Ni

TRANSFER CONTROL TO SUBROUTINE FOR FACTORIAL

Restate the answer into Ans3

$$SFD(i) \leftarrow \frac{ANSI}{ANS2 * ANS}$$

SFD ← SFD * SFD(i)

OUTPUT i, gi(i), Ni(i), SFD(i)

ENDFOR

OUTPUT "Total microstate is", SFD.

CASE Z

SMB ← 1

OUTPUT No restriction, identical but distinguishable particles.

INPUT "total particle in the system" N

FOR i ← 1 to n

INPUT "particle in the level" Ni(i)

INPUT "degeneracy" gi(i)

LET V ← N

TRANSFER CONTROL TO SUBROUTINE FOR FACTORIAL

Restate the answer into ANS4

LET V ← Ni(i)

TRANSFER CONTROL TO SUBROUTINE FOR FACTORIAL

Restate the answer into ANS5

ANS6 ← gi^{Ni}

SMB(i) ← $\frac{ANS4 * ANS6}{ANS5}$

SMB = SMB * SMB(i)

OUTPUT i, Ni(i), gi(i), SMB(i)

ENFOR

OUTPUT "Total state is" SMB

ENDSELECT

OUTPUT MORE COMPUTATIONS ? Y/N

INPUT CH

ENDO

SUBROUTINE (FACTORIAL)

FACT ← 1

FOR i ← 1 to V

FACT ← FACT * i

ENDFOR

RETURN

CHAPTER FOUR

EXPERIMENTAL EXAMPLES

4.1 INTRODUCTION

In the previous chapter, systems were designed to solve some specific physical problems. Here the systems are implemented with real data and the output presented. The following were the inputed data.

4.2 HARMONIC OSCILLATOR

In this section, let us consider a harmonic experiment of a mass 10kg attached to a spring of spring constant of 1000 and an amplitude of 1. Using these data, values in the harmonic oscillator system designed, the following properties of the harmonic oscillator (body) were conspicuous as seen in the output.

The output ranges from the table of values to graphs of various parameters as they vary in relation to one another.

MENU KEYS

(1) Calculation Properties

(2) Table of values

(3) Graphs

(4) displacement _ time graph

(5) velocity _ time graph

(6) acceleration _ time graph

(7) kenergy _ time graph

(8) penergy _time graph

(9) kenergy _distance graph

(10) penergy _distance graph

(11) Exit

options ? 1

```
The value of Mass .............? 10

The value of Spring Constant ...? 1000

The value of Amplitude.........? 1
```

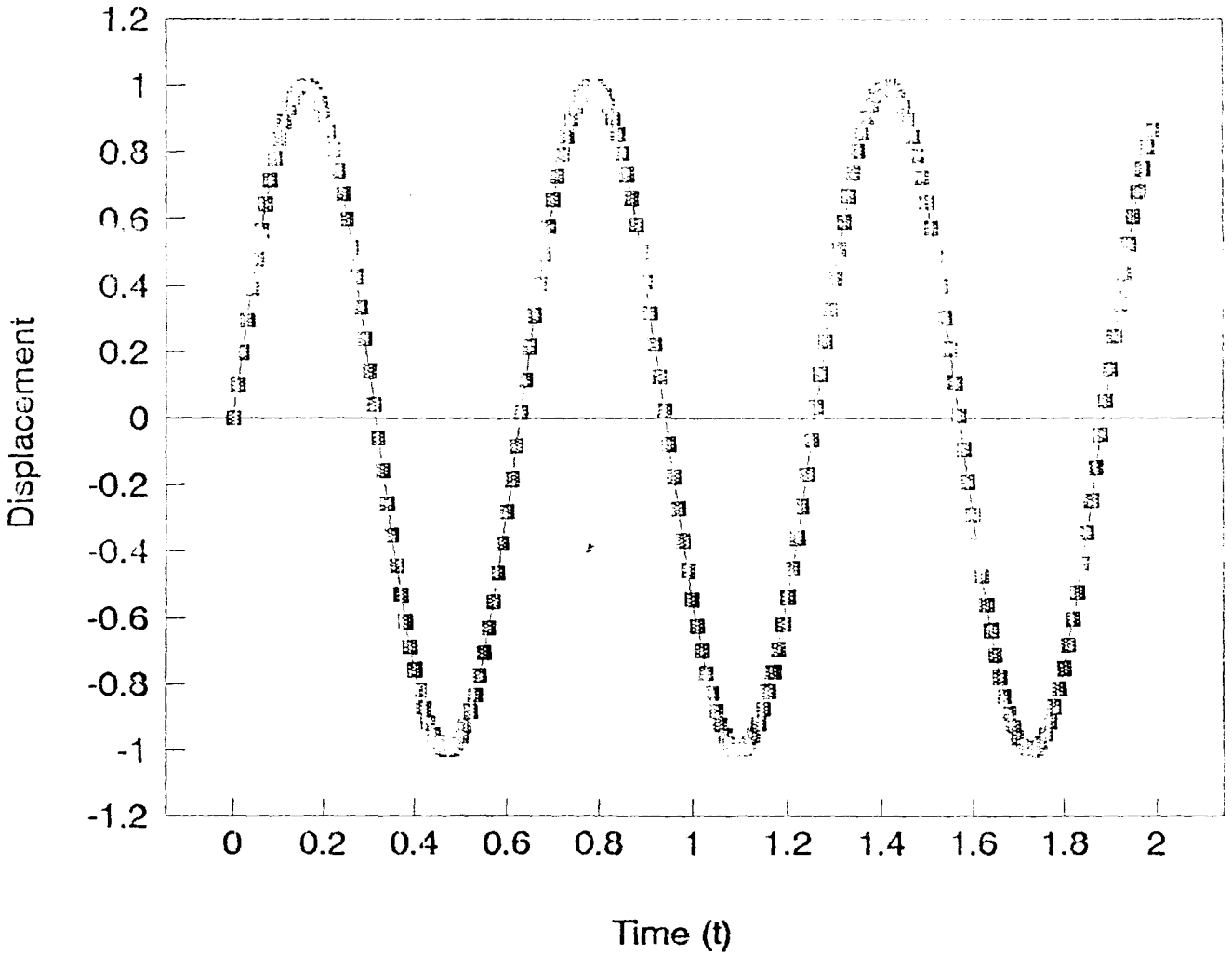
diplacement	velocity	acceln	penergy	kenergy	tenergy	time
0.000	10.000	0.000	0.000	500.000	500.000	0.000
0.100	9.950	-9.983	4.983	495.017	500.000	0.010
0.199	9.801	-19.867	19.735	480.264	500.000	0.020
0.296	9.553	-29.553	43.666	456.334	500.000	0.030
0.389	9.211	-38.942	75.823	424.177	500.000	0.040
0.479	8.776	-47.941	114.924	385.076	500.000	0.050
0.565	8.253	-56.461	159.411	340.589	500.000	0.060
0.644	7.648	-64.422	207.508	292.492	500.000	0.070
0.717	6.967	-71.776	257.300	242.700	500.000	0.080
0.783	6.216	-78.333	306.801	193.199	500.000	0.090
0.841	5.403	-84.147	354.037	145.963	500.000	0.100
0.891	4.536	-89.121	397.125	102.875	500.000	0.110
0.932	3.624	-93.204	434.348	65.652	500.000	0.120
0.964	2.675	-96.356	464.222	35.778	500.000	0.130
0.985	1.700	-98.545	485.556	14.444	500.000	0.140
0.997	0.707	-99.749	497.498	2.502	500.000	0.150
1.000	-0.292	-99.957	499.574	0.426	500.000	0.160
0.992	-1.288	-99.166	491.700	8.300	500.000	0.170
0.974	-2.272	-97.385	474.190	25.810	500.000	0.180
0.946	-3.233	94.630	447.742	52.258	500.000	0.190
0.909	-4.161	-90.930	413.411	86.589	500.000	0.200
0.863	-5.048	-86.321	372.565	127.435	500.000	0.210
0.808	-5.885	-80.850	326.833	173.167	500.000	0.220
0.746	-6.667	-74.570	278.038	221.962	500.000	0.230
0.675	-7.374	-67.546	228.125	271.875	500.000	0.240
0.598	-8.011	-59.847	179.084	320.916	500.000	0.250
0.516	-8.569	-51.550	132.871	367.129	500.000	0.260
0.427	-9.041	-42.738	91.327	408.673	500.000	0.270
0.335	-9.422	-33.499	56.108	443.892	500.000	0.280
0.239	-9.710	-23.925	28.620	471.380	500.000	0.290
0.141	-9.900	-14.112	9.957	490.043	500.000	0.300
0.042	-9.991	-4.158	0.864	499.136	500.000	0.310
-0.058	-9.983	5.837	1.704	498.296	500.000	0.320
-0.158	-9.875	15.775	12.442	487.558	500.000	0.330
-0.256	-9.668	25.554	32.651	467.349	500.000	0.340
-0.351	-9.365	35.078	61.524	438.476	500.000	0.350
-0.443	-8.968	44.252	97.912	402.088	500.000	0.360
-0.530	-8.481	52.984	140.363	359.637	500.000	0.370
-0.612	-7.910	61.186	187.185	312.815	500.000	0.380
-0.688	-7.259	68.777	236.511	263.489	500.000	0.390
-0.757	-6.536	75.680	286.375	213.625	500.000	0.400
-0.818	-5.748	81.828	334.788	165.212	500.000	0.410
-0.872	-4.903	87.158	379.822	120.178	500.000	0.420
-0.916	-4.008	91.617	419.680	80.320	500.000	0.430
-0.952	-3.073	95.160	452.773	47.227	500.000	0.440
-0.978	-2.108	97.753	477.782	22.218	500.000	0.450
-0.994	-1.122	99.369	493.711	6.289	500.000	0.460
-1.000	-0.124	99.992	499.923	0.077	500.000	0.470
-0.996	0.875	99.616	496.172	3.828	500.000	0.480
-0.982	1.865	98.245	482.607	17.393	500.000	0.490
-0.959	2.837	95.892	459.768	40.232	500.000	0.500
-0.926	3.780	92.582	428.567	71.433	500.000	0.510
-0.883	4.685	88.316	390.247	109.753	500.000	0.520
-0.832	5.544	83.227	346.336	153.664	500.000	0.530
-0.773	6.347	77.277	298.584	201.416	500.000	0.540
-0.706	7.087	70.554	248.895	251.105	500.000	0.550
-0.631	7.756	63.117	199.250	300.750	500.000	0.560
-0.551	8.347	55.089	151.628	348.372	500.000	0.570
-0.465	8.855	46.460	107.929	392.071	500.000	0.580
-0.374	9.275	37.388	69.893	430.107	500.000	0.590
-0.279	9.602	27.992	39.037	460.963	500.000	0.600
-0.182	9.833	18.217	16.592	483.408	500.000	0.610
-0.083	9.965	8.307	3.452	496.548	500.000	0.620

0.117	9.932	-11.65	6.791	493.208	500.000	0.640
0.215	9.766	-21.51	23.138	476.862	500.000	0.650
0.312	9.502	-31.154	48.528	451.472	500.000	0.660
0.405	9.144	-40.485	81.950	418.050	500.000	0.670
0.494	8.694	-49.411	122.072	377.927	500.000	0.680
0.578	8.157	-57.844	167.295	332.705	500.000	0.690
0.657	7.539	-65.69	215.814	284.186	500.000	0.700
0.729	6.845	-72.89	265.696	234.304	500.000	0.710
0.794	6.084	-79.36	314.952	185.048	500.000	0.720
0.850	5.261	-85.04	361.619	138.381	500.000	0.730
0.899	4.386	-89.87	403.837	96.163	500.000	0.740
0.938	3.466	-93.80	439.921	60.079	500.000	0.750
0.968	2.513	-96.79	468.433	31.567	500.000	0.760
0.988	1.534	-98.817	488.238	11.762	500.000	0.770
0.999	0.540	-99.85	498.544	1.456	500.000	0.780
0.999	-0.460	-99.89	498.942	1.058	500.000	0.790
0.989	-1.455	-98.93	489.416	10.584	500.000	0.800
0.970	-2.435	-96.98	470.344	29.656	500.000	0.810
0.941	-3.392	-94.07	442.489	57.511	500.000	0.820
0.902	-4.314	-90.21	406.959	93.041	500.000	0.830
0.855	-5.193	-85.46	365.172	134.828	500.000	0.840
0.798	-6.020	-79.84	318.793	181.207	500.000	0.850
0.734	-6.787	-73.44	269.672	230.328	500.000	0.860
0.663	-7.486	-66.29	219.767	280.233	500.000	0.870
0.585	-8.111	-58.49	171.067	328.933	500.000	0.880
0.501	-8.654	-50.10	125.513	374.487	500.000	0.890
0.412	-9.111	-41.21	84.923	415.077	500.000	0.900
0.319	-9.477	-31.91	50.914	449.086	500.000	0.910
0.223	-9.748	-22.29	24.841	475.159	500.000	0.920
0.124	-9.922	-12.44	7.745	492.255	500.000	0.930
0.025	-9.997	-2.47	0.307	499.693	500.000	0.940
-0.075	-9.972	7.51	2.823	497.177	500.000	0.950
-0.174	-9.847	17.43	15.194	484.806	500.000	0.960
-0.272	-9.624	27.17	36.925	463.075	500.000	0.970
-0.366	-9.304	36.64	67.151	432.849	500.000	0.980
-0.458	-8.892	45.75	104.667	395.333	500.000	0.990
-0.544	-8.391	54.40	147.976	352.023	500.000	1.000
-0.625	-7.806	62.50	195.354	304.646	500.000	1.010
-0.700	-7.143	69.93	244.909	255.091	500.000	1.020
-0.768	-6.408	76.76	294.668	205.332	500.000	1.030
-0.828	-5.610	82.78	342.645	157.355	500.000	1.040
-0.880	-4.755	87.95	386.930	113.070	500.000	1.050
-0.923	-3.853	92.27	425.755	74.245	500.000	1.060
-0.957	-2.913	95.66	457.573	42.427	500.000	1.070
-0.981	-1.943	98.09	481.117	18.883	500.000	1.080
-0.995	-0.954	99.54	495.446	4.554	500.000	1.090
-1.000	0.044	99.99	499.990	0.010	500.000	1.100
-0.995	1.042	99.45	494.568	5.432	500.000	1.110
-0.979	2.030	97.91	479.396	20.604	500.000	1.120
-0.954	2.997	95.40	455.078	44.922	500.000	1.130
-0.919	3.935	91.93	422.585	77.415	500.000	1.140
-0.875	4.833	87.54	383.211	116.789	500.000	1.150
-0.823	5.683	82.28	338.527	161.473	500.000	1.160
-0.762	6.476	76.19	290.313	209.687	500.000	1.170
-0.694	7.204	69.35	240.492	259.508	500.000	1.180
-0.618	7.861	61.81	191.051	308.949	500.000	1.190
0.537	8.438	53.65	143.959	356.041	500.000	1.200
450	8.932	44.96	101.095	398.905	500.000	1.210
358	9.336	35.82	64.167	435.833	500.000	1.220
263	9.647	26.32	34.648	465.352	500.000	1.230
166	9.862	16.56	13.714	486.286	500.000	1.240
066	9.978	6.63	2.200	497.800	500.000	1.250
034	9.994	-3.36	0.565	499.435	500.000	1.260
	9.911	-13.33	8.874	491.126	500.000	1.270

0.232	9.728	-23.150	26.796	473.204	500.000	1.280
0.327	9.449	-32.747	53.617	446.383	500.000	1.290
0.420	9.075	-42.016	88.267	411.733	500.000	1.300
0.509	8.610	-50.865	129.364	370.636	500.000	1.310
0.592	8.059	-59.207	175.271	324.729	500.000	1.320
0.670	7.428	-66.956	224.157	275.843	500.000	1.330
0.740	6.722	-74.037	274.074	225.926	500.000	1.340
0.804	5.949	-80.378	323.030	176.970	500.000	1.350
0.859	5.117	-85.916	369.075	130.925	500.000	1.360
0.906	4.234	-90.595	410.373	89.627	500.000	1.370
0.944	3.308	-94.369	445.278	54.722	500.000	1.380
0.972	2.350	-97.201	472.397	27.603	500.000	1.390
0.991	1.367	-99.061	490.650	9.350	500.000	1.400
0.999	0.372	-99.931	499.309	0.691	500.000	1.410
0.998	-0.628	-99.803	498.029	1.971	500.000	1.420
0.987	-1.621	-98.677	486.861	13.139	500.000	1.430
0.966	-2.598	-96.566	466.250	33.750	500.000	1.440
0.935	-3.549	-93.490	437.018	62.982	500.000	1.450
0.895	-4.465	-89.480	400.330	99.670	500.000	1.460
0.846	-5.336	-84.575	357.649	142.351	500.000	1.470
0.788	-6.153	-78.826	310.676	189.324	500.000	1.480
0.723	-6.910	-72.289	261.284	238.716	500.000	1.490
0.650	-7.597	-65.030	211.442	288.558	500.000	1.500
0.571	-8.208	-57.121	163.138	336.862	500.000	1.510
0.486	-8.737	-48.641	118.296	381.704	500.000	1.520
0.397	-9.179	-39.675	78.706	421.294	500.000	1.530
0.303	-9.529	-30.313	45.944	454.056	500.000	1.540
0.206	-9.785	-20.648	21.317	478.683	500.000	1.550
0.108	-9.942	-10.776	5.807	494.193	500.000	1.560
0.008	-10.000	-0.797	0.032	499.968	500.000	1.570
-0.092	-9.958	9.190	4.222	495.778	500.000	1.580
-0.191	-9.816	19.085	18.211	481.789	500.000	1.590
-0.288	-9.577	28.789	41.441	458.559	500.000	1.600
-0.382	-9.241	38.206	72.985	427.015	500.000	1.610
-0.472	-8.814	47.241	111.586	388.414	500.000	1.620
-0.558	-8.298	55.804	155.706	344.294	500.000	1.630
-0.638	-7.700	63.810	203.584	296.416	500.000	1.640
-0.712	-7.024	71.178	253.313	246.687	500.000	1.650
-0.778	-6.278	77.834	302.910	197.090	500.000	1.660
-0.837	-5.470	83.714	350.398	149.602	500.000	1.670
-0.888	-4.607	88.756	393.883	106.117	500.000	1.680
-0.929	-3.698	92.912	431.631	68.369	500.000	1.690
-0.961	-2.752	96.139	462.139	37.861	500.000	1.700
-0.984	-1.778	98.406	484.190	15.810	500.000	1.710
-0.997	-0.787	99.690	496.904	3.096	500.000	1.720
-1.000	0.212	99.977	499.775	0.225	500.000	1.730
-0.993	1.209	99.266	492.688	7.312	500.000	1.740
-0.976	2.194	97.563	475.926	24.074	500.000	1.750
-0.949	3.157	94.885	450.157	49.843	500.000	1.760
-0.913	4.089	91.259	416.408	83.592	500.000	1.770
-0.867	4.979	86.721	376.026	123.974	500.000	1.780
-0.813	5.820	81.316	330.619	169.381	500.000	1.790
-0.751	6.603	75.100	281.998	218.002	500.000	1.800
-0.681	7.320	68.132	232.101	267.899	500.000	1.810
-0.605	7.963	60.484	182.918	317.082	500.000	1.820
-0.522	8.527	52.232	136.409	363.591	500.000	1.830
-0.435	9.006	43.458	94.429	405.571	500.000	1.840
-0.342	9.395	34.249	58.651	441.349	500.000	1.850
-0.247	9.690	24.699	30.501	469.499	500.000	1.860
-0.149	9.888	14.901	11.102	488.898	500.000	1.870
-0.050	9.988	4.955	1.228	498.772	500.000	1.880
0.050	9.987	-5.041	1.270	498.729	500.000	1.890
0.150	9.887	-14.986	11.229	488.771	500.000	1.900
0.248	9.688	-24.782	30.707	469.293	500.000	1.910

0.343	9.392	-34.330	58.928	441.072	500.000	1.920
0.435	9.003	-43.535	94.766	405.234	500.000	1.930
0.523	8.523	-52.305	136.792	363.208	500.000	1.940
0.606	7.958	-60.553	183.332	316.668	500.000	1.950
0.682	7.314	-68.195	232.530	267.470	500.000	1.960
0.752	6.597	-75.156	282.424	217.576	500.000	1.970
0.814	5.813	-81.366	331.025	168.975	500.000	1.980
0.868	4.972	-86.764	376.396	123.604	500.000	1.990

Fig. **Graph of Displacement against Time.**



Graph of Velocity against Time.

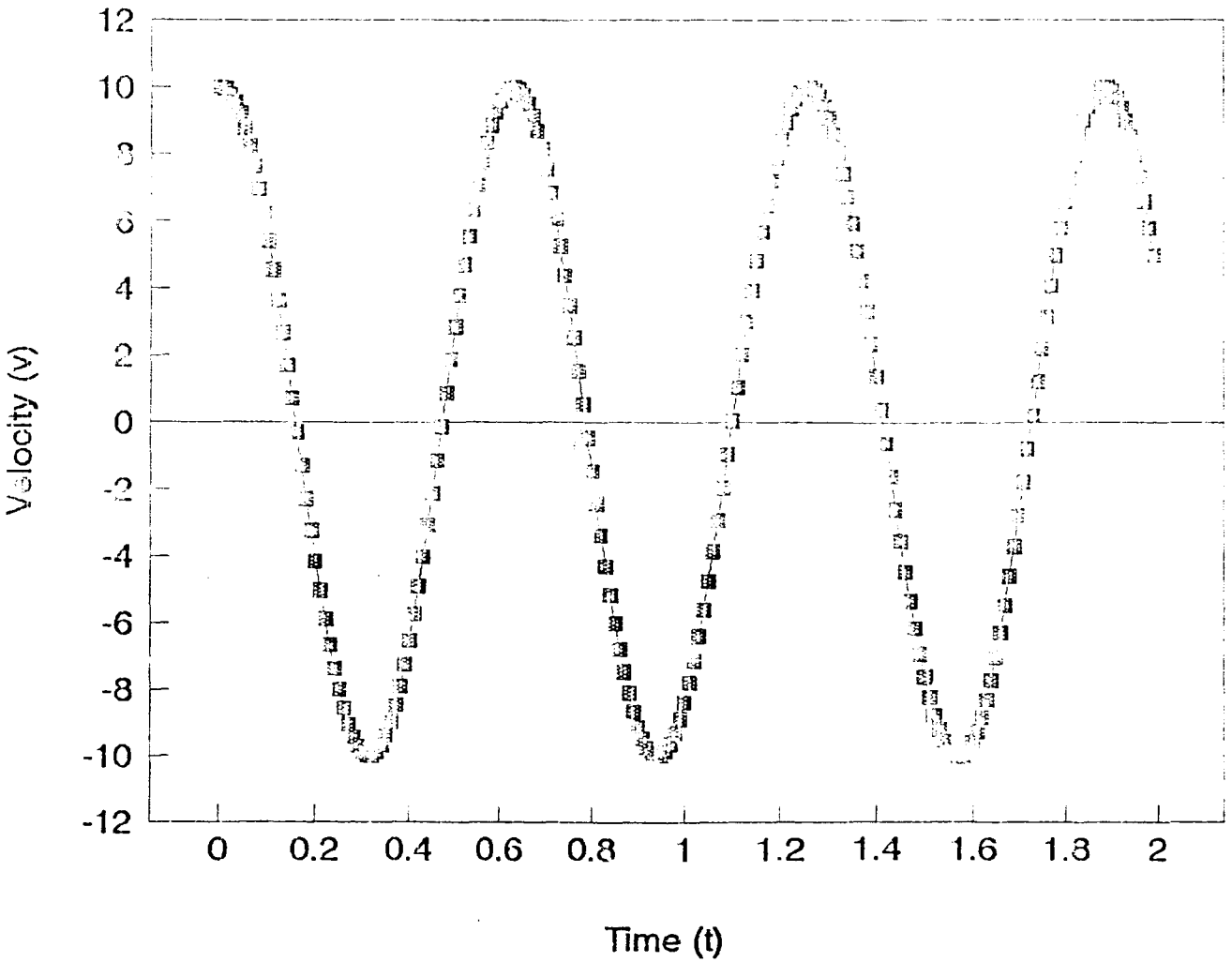


Fig. **Graph of Acceleration against Time.**

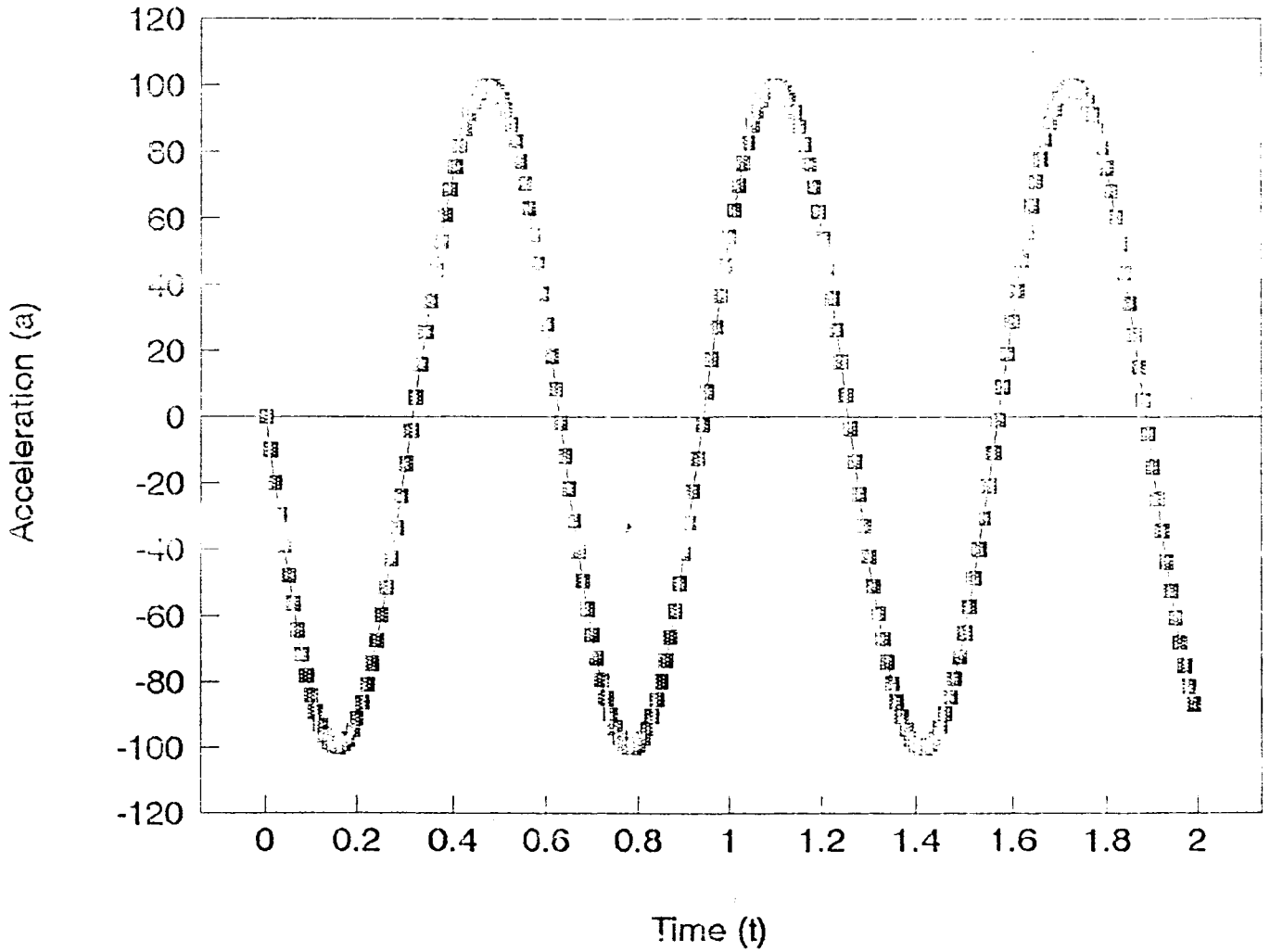


Fig. Graph of Kinetic Energy against Time.

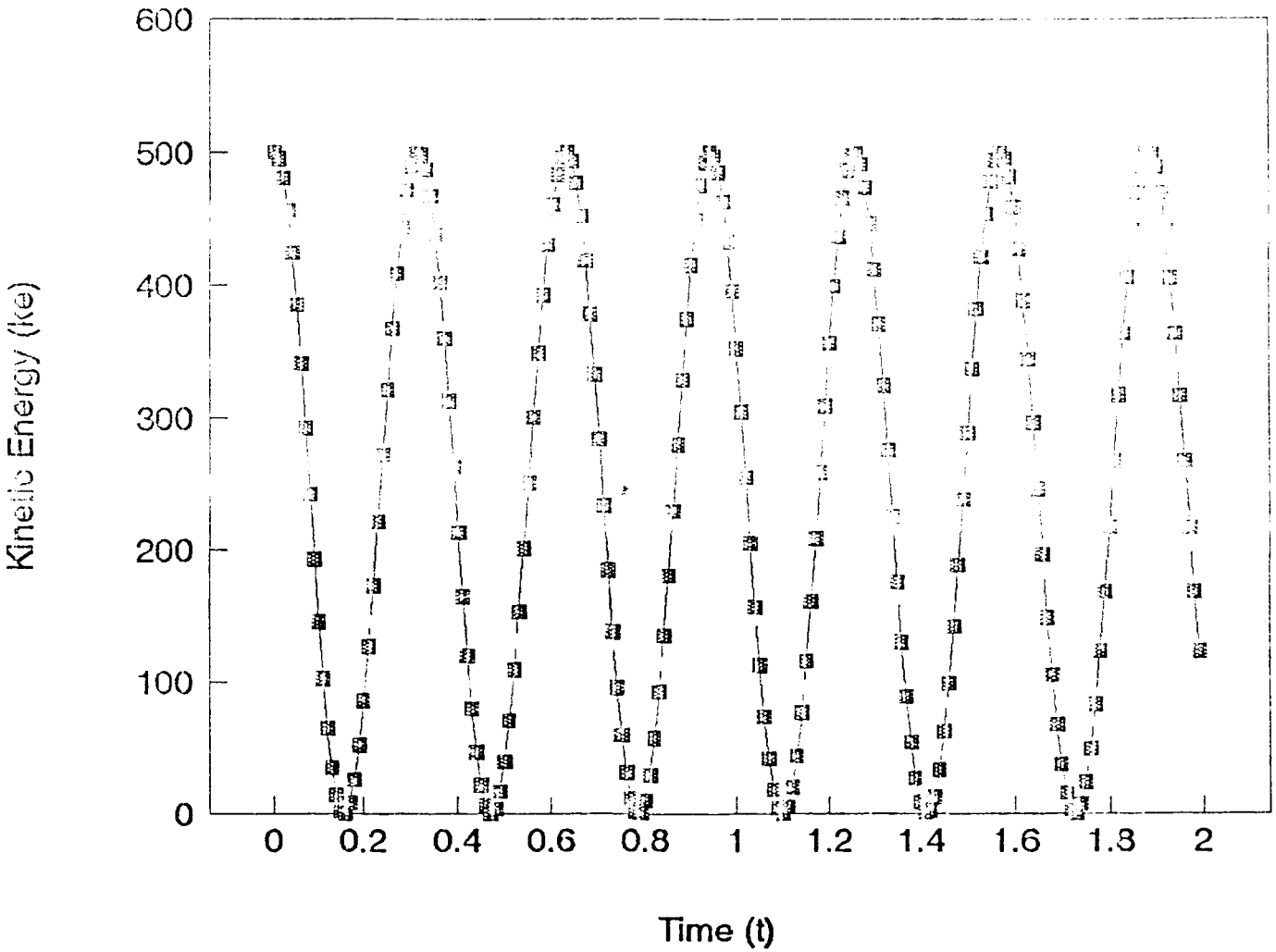
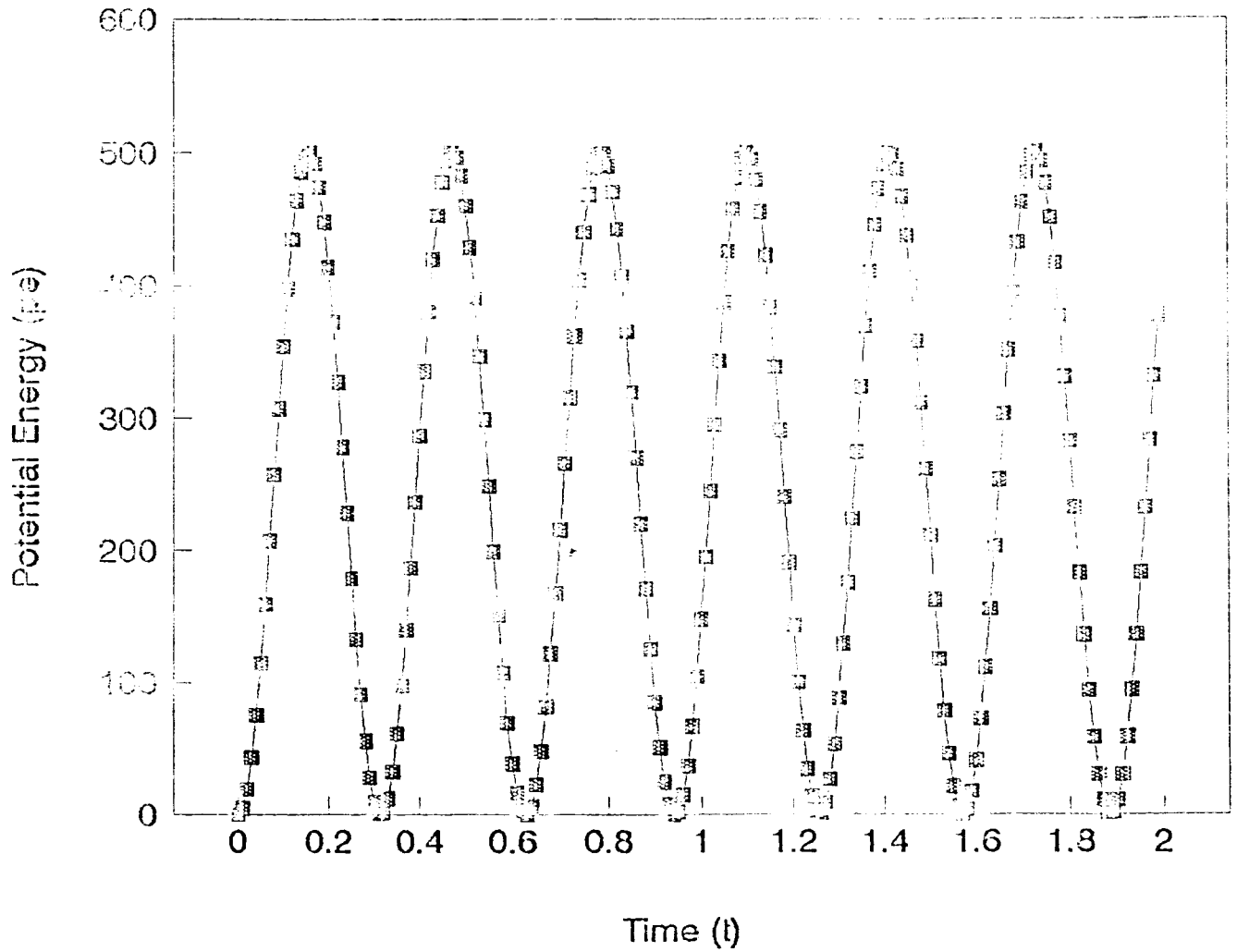


Fig. Graph of Potential Energy against Time.



4.3 THERMODYNAMIC PROBABILITIES

The thermodynamic probabilities system so designed is implemented by using the following set of data values.

H = No of micro states = 3

J = Degeneracy of each micro state = 2

MAC1 = $g_1 = 1$, $N_1 = 3$, $g_2 = 3$, $N_2 = 3$

MAC2 = $g_1 = 1$, $N_1 = 4$, $g_2 = 2$, $N_2 = 2$

MAC3 = $g_1 = 1$, $N_1 = 1$, $g_2 = 3$, $N_2 = 3$

Using Bose Einstein statistic We find

(1) Most probable state

(2) Total number of microstates

(3) Average occupation number.

CHAPTER FIVE

5.1 CONCLUSION

I have been able to examined in this work an introduction into some basic concepts that serve as foundation for the entire work in its early parts. The principles of dynamics as it relates to matters. Some mathematical and statistical concepts were defined in terms of what they are all about. Precisely in chapter one, the concept of scalars, vectors, force, work, energy momentum and motions e.t.c, were well established to the minimum requirement of this job. The concepts were initially considered for bodies as if they are single particle and later extended (or generalized) as it applied to system of particles.

In chapter two, systems of independent particles were fully treated in terms of their specific characteristic that distinguish are system from the other. A simple analogical example in the motion of cross-section in classical physics was drawn to depict how statistics comes into play in mechanical physics. Three main particle were considered in relation to the probabilities distribution laws. These particles are Bosons, fermions, and Boltzons. For each group of particles, equations for distribution of a number of particle among energy levels (state) were derived. Also, the mean distribution and mean square deviation from a center position X were derived from the probability, Pi of finding a particular particle i, out of N particles in an energy state i. A bit of interacting system was also looked into. The

interaction system were also grouped into three; microcanonical approach, cannonical approach and Grandcannonical approach.

The equation governing their distributions among energy states was established. And hence the probability Pi of finding a system in the ith energy state were derived for each approach. The thermodynamic properties such as entropy, S, pressure P were then expressed in terms of probabilities Pi. And the energy state Ei. To conclude the chapter, partition function for each approach were derived and properties stated. Also the fluctuations were established.

The basic principles of chapters one and two were applied to physical problems such as harmonic oscillator (simple one) and thermodynamic probabilities. The problem specification was drawn in full details. Algorithm written in pseudocode and flowchart drawn. Computer program to solve such problem (and related ones) was written in BASIC language.

In chapter four, programs written was implemented on a particular question or set data and the output presented.

5.2 REMARKS

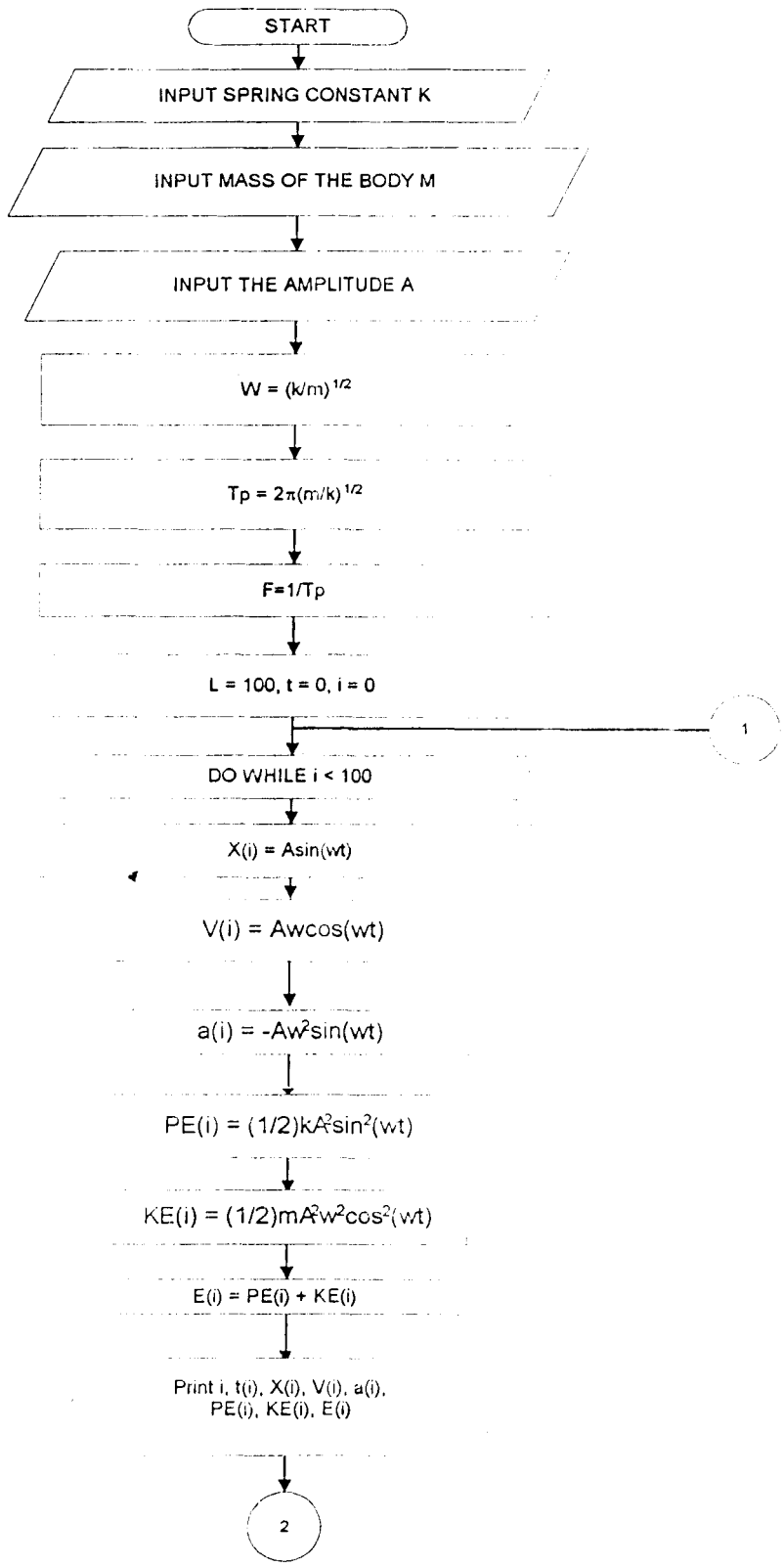
With the establishment of the basic principles of dynamics, statistics, mechanics matters, system of independent and interacting particles based on established (or laid down) rules one could say the work is complete. But the reservoir of knowledge can not dry up. There are still a lot to be explore in this area

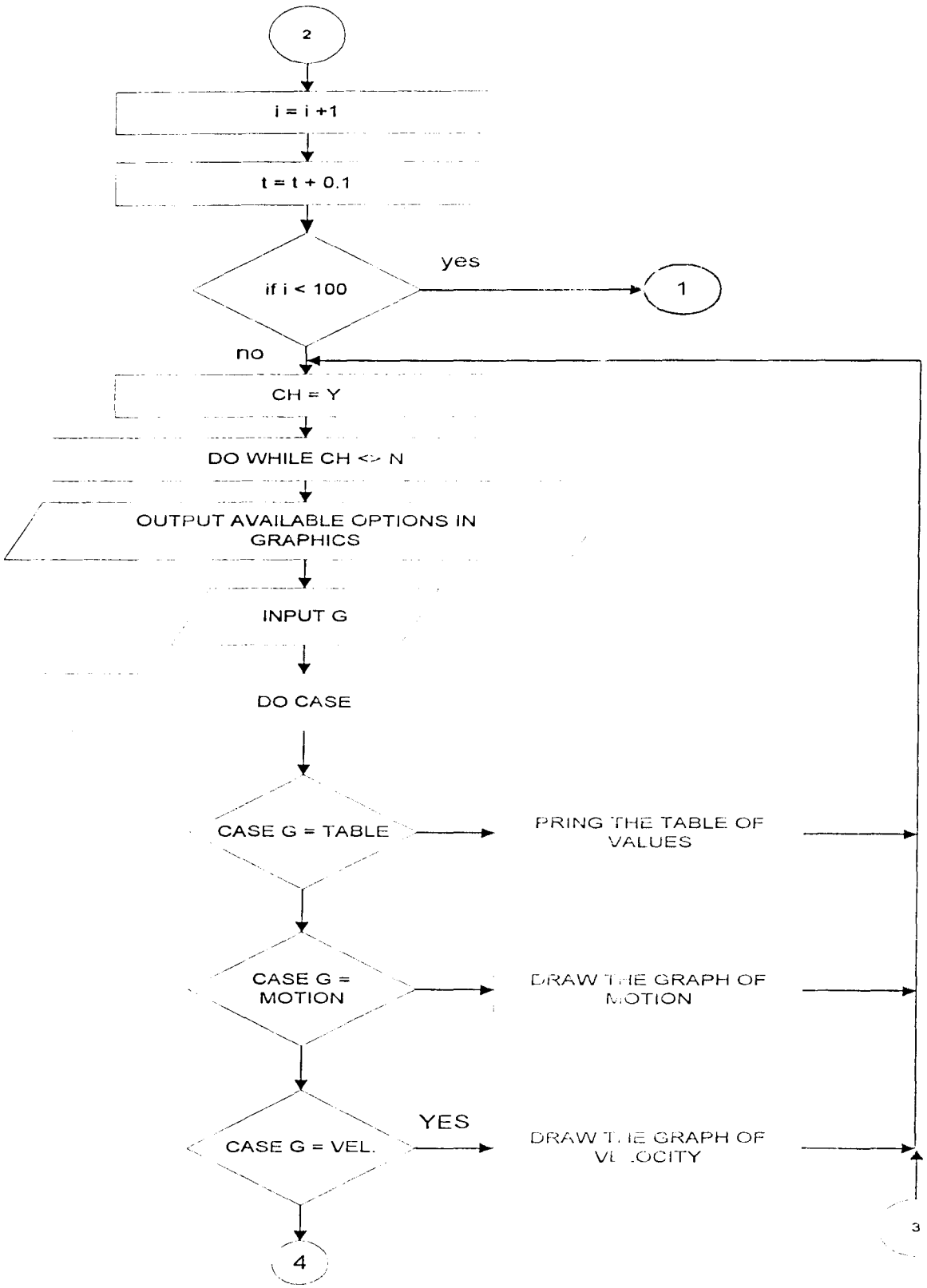
of study. Especially in the area of interacting system. I therefore recommend that a special project should be dedicated to the area which will be an improvement on this job.

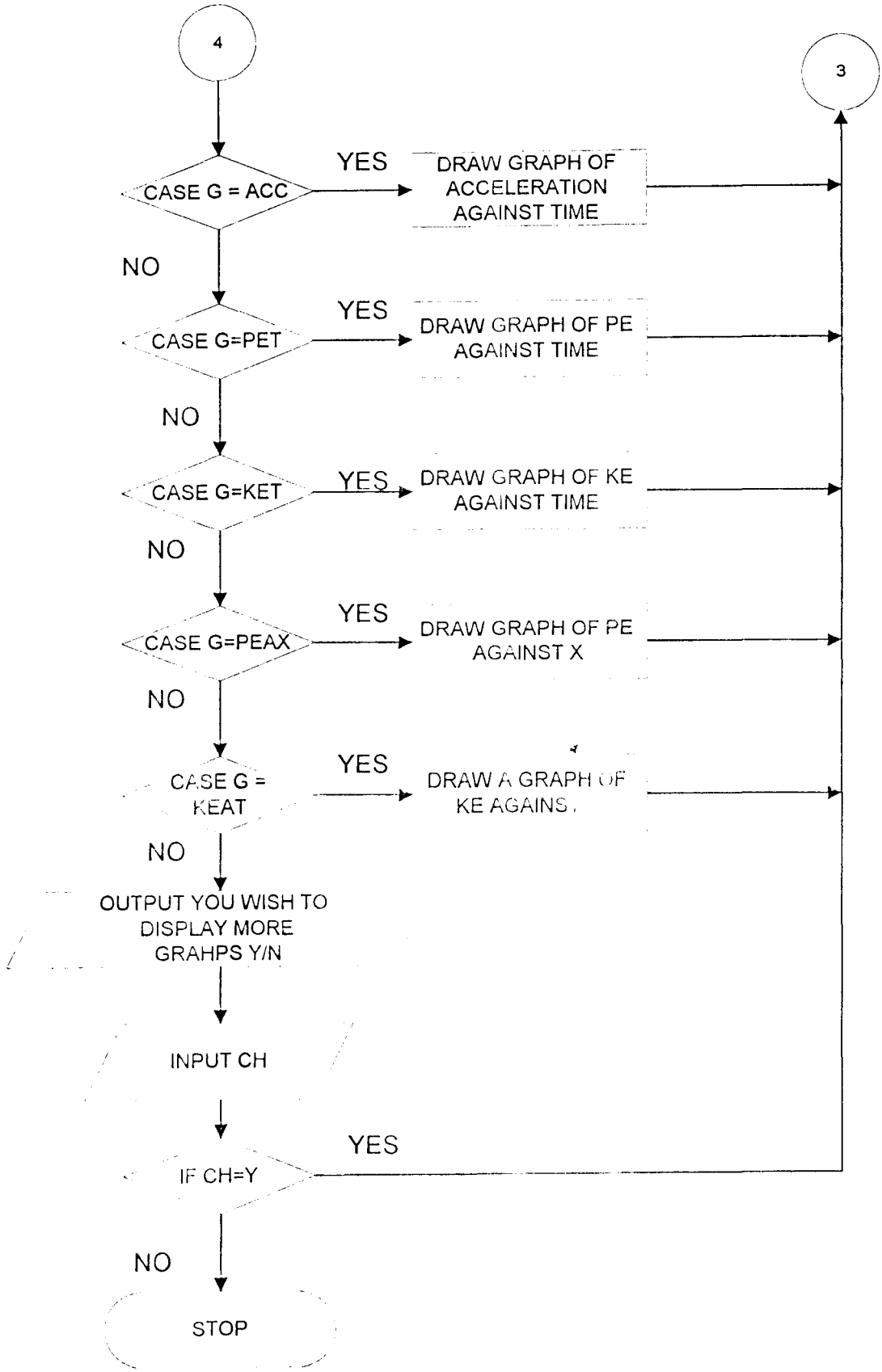
REFERENCES

1. Akhiezer A.I, Peletminski S.V	Methods of Statistical Physics	Pergomon Press	1981
2. Bowler M.G	Lectures on Statistical Mechanics	Pergomon Press	1982
3. Gupia M.C	Statistical thermodynamics	John Willey & sons	1990
4. Heither W.	Elementary wave mechanics 2nd edition with application to quantum chemistry.	Oxford	1956
5. John L, Byron A.	Principles of mechanics	Mc Graw-Hill	1970
6. Mandl F.	Statistical Physics 3rd edition	Wiley	1971
7. Mark W.Z, Dittman R H	Heat and thermodynamics International Student edition	Mc Graw-Hill	1981
8. Ter Haar D.	Lectures on selected topics in Statistical mechanics.	pergamon Press	1977

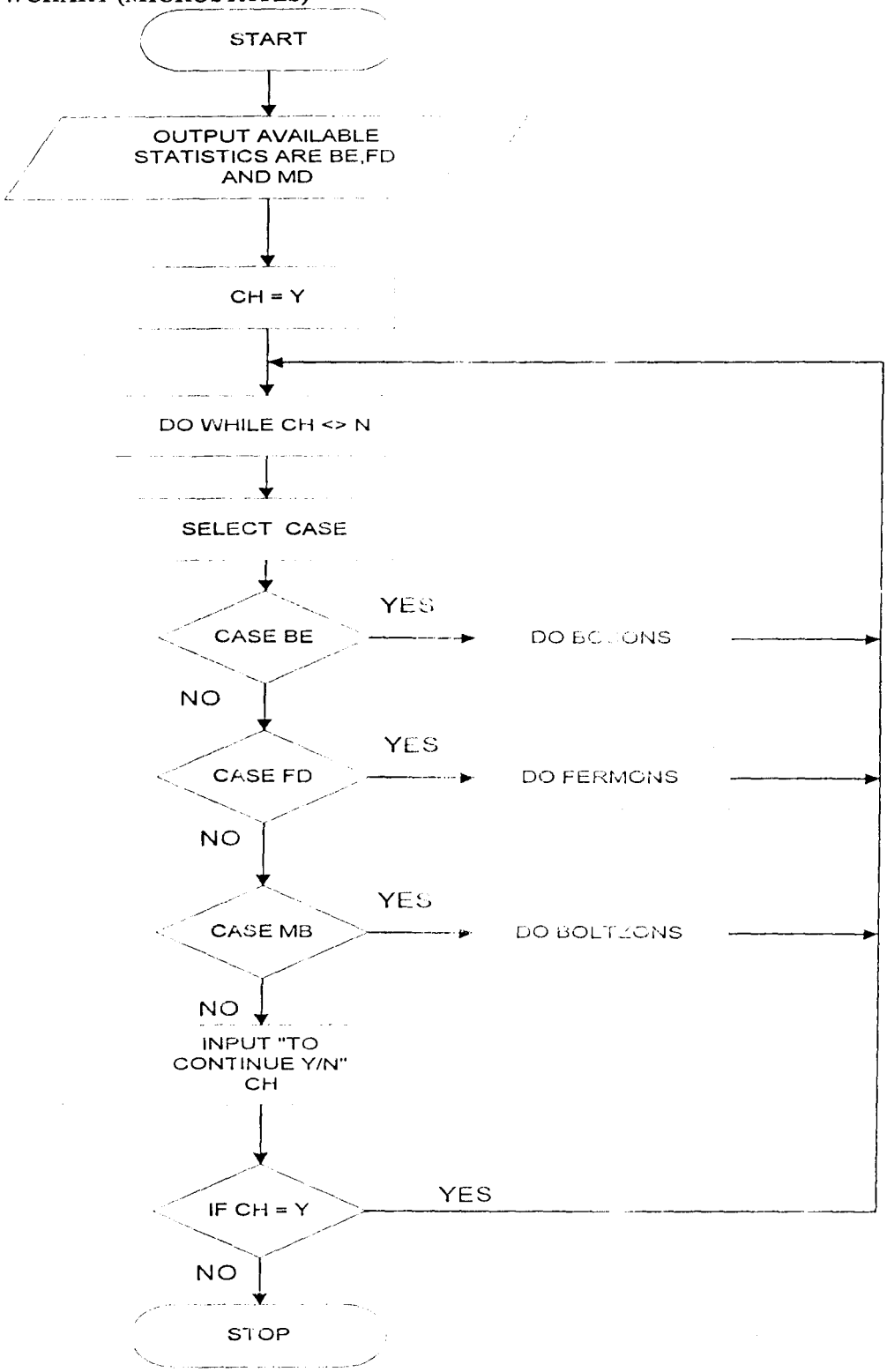
FLOWCHART (SHM)



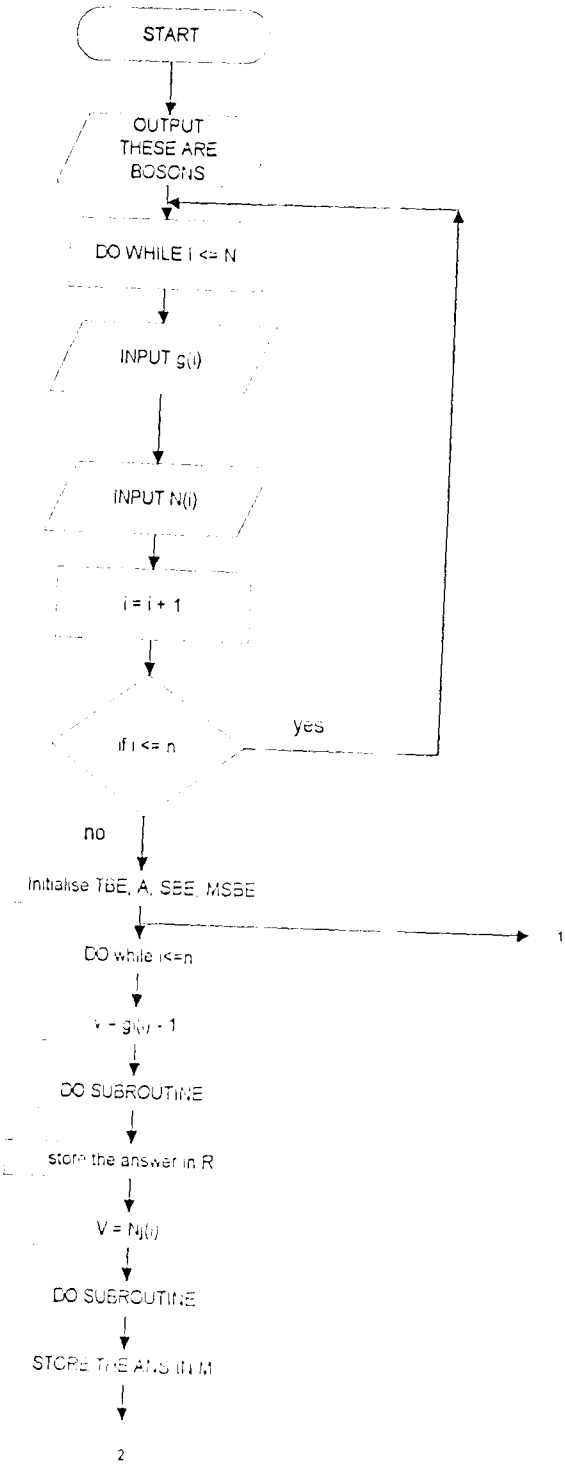

2
i = i +1
t = t + 0.1
if i < 100
yes
1
no
CH = Y
DO WHILE CH <> N
OUTPUT AVAILABLE OPTIONS IN GRAPHICS
INPUT G
DO CASE
CASE G = TABLE
PRING THE TABLE OF VALUES
CASE G = MOTION
DRAW THE GRAPH OF MOTION
CASE G = VEL.
YES
DRAW THE GRAPH OF VELOCITY
3
4

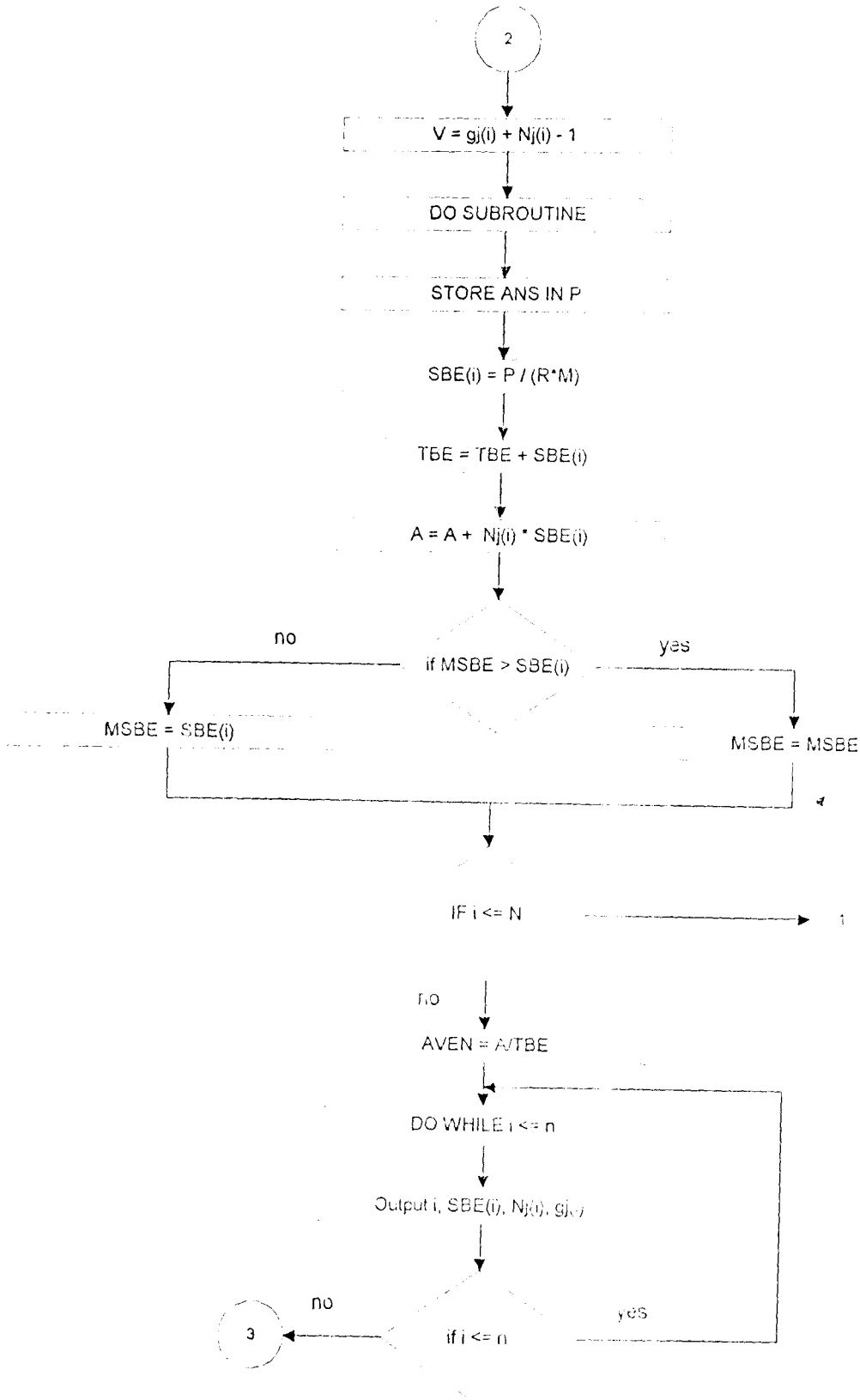

4
3
CASE G = ACC
YES
DRAW GRAPH OF ACCELERATION AGAINST TIME
NO
CASE G=PET
YES
DRAW GRAPH OF PE AGAINST TIME
NO
CASE G=KET
YES
DRAW GRAPH OF KE AGAINST TIME
NO
CASE G=PEAX
YES
DRAW GRAPH OF PE AGAINST X
NO
CASE G = KEAT
YES
DRAW A GRAPH OF KE AGAINS
NO
OUTPUT YOU WISH TO DISPLAY MORE GRAHPS Y/N
INPUT CH
IF CH=Y
YES
NO
STOP

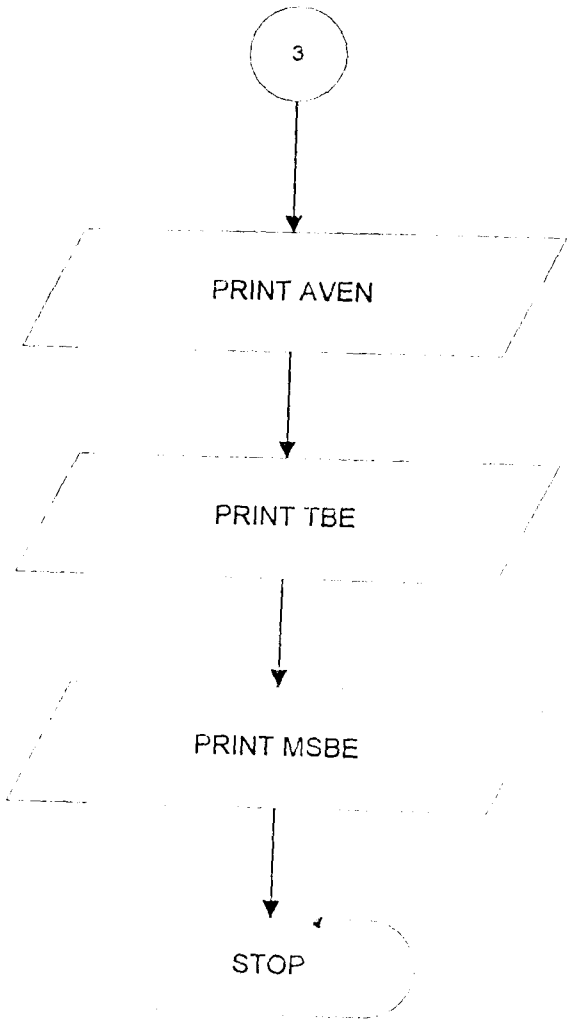
FLOWCHART (MICROSTATES)



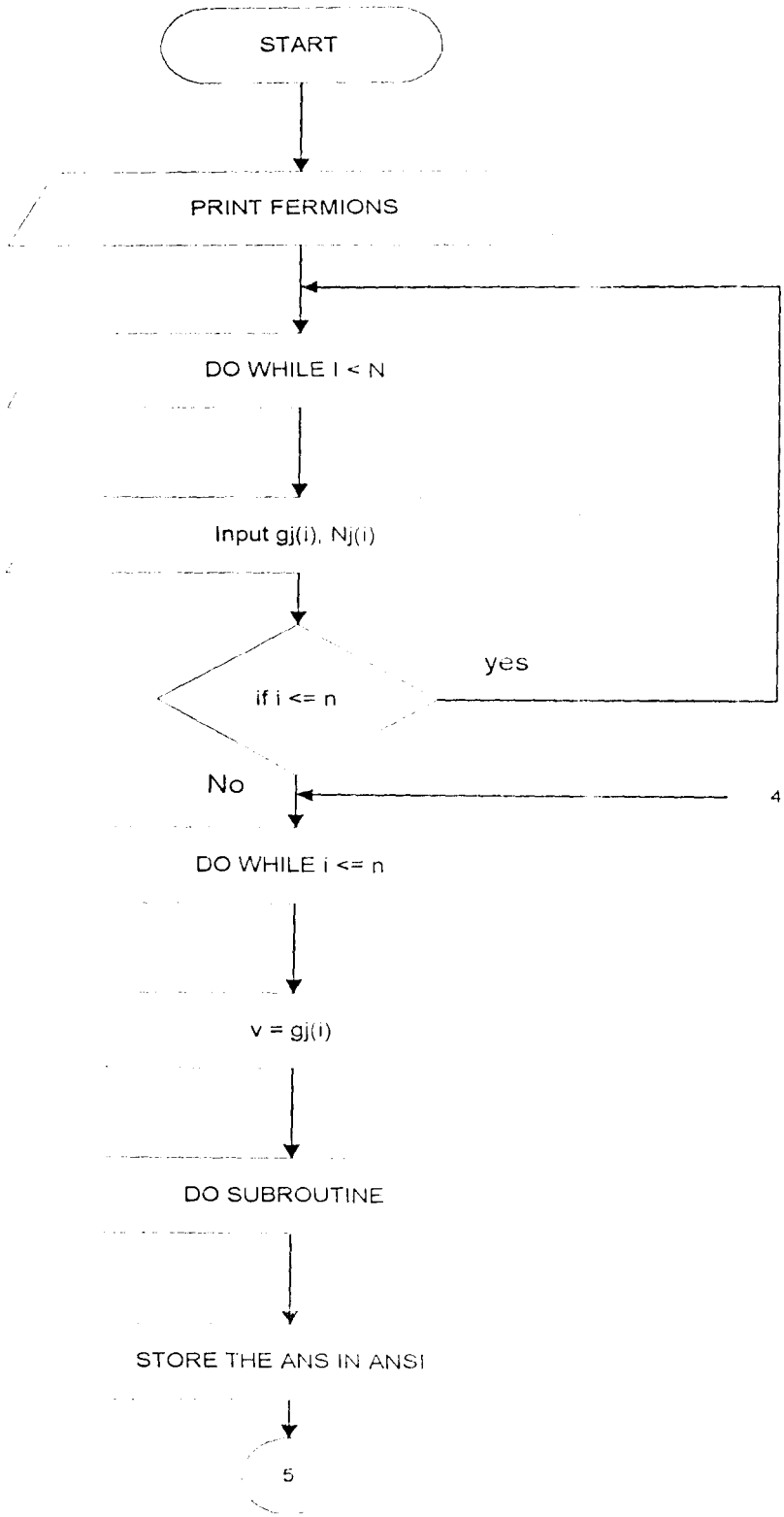
CASE BE

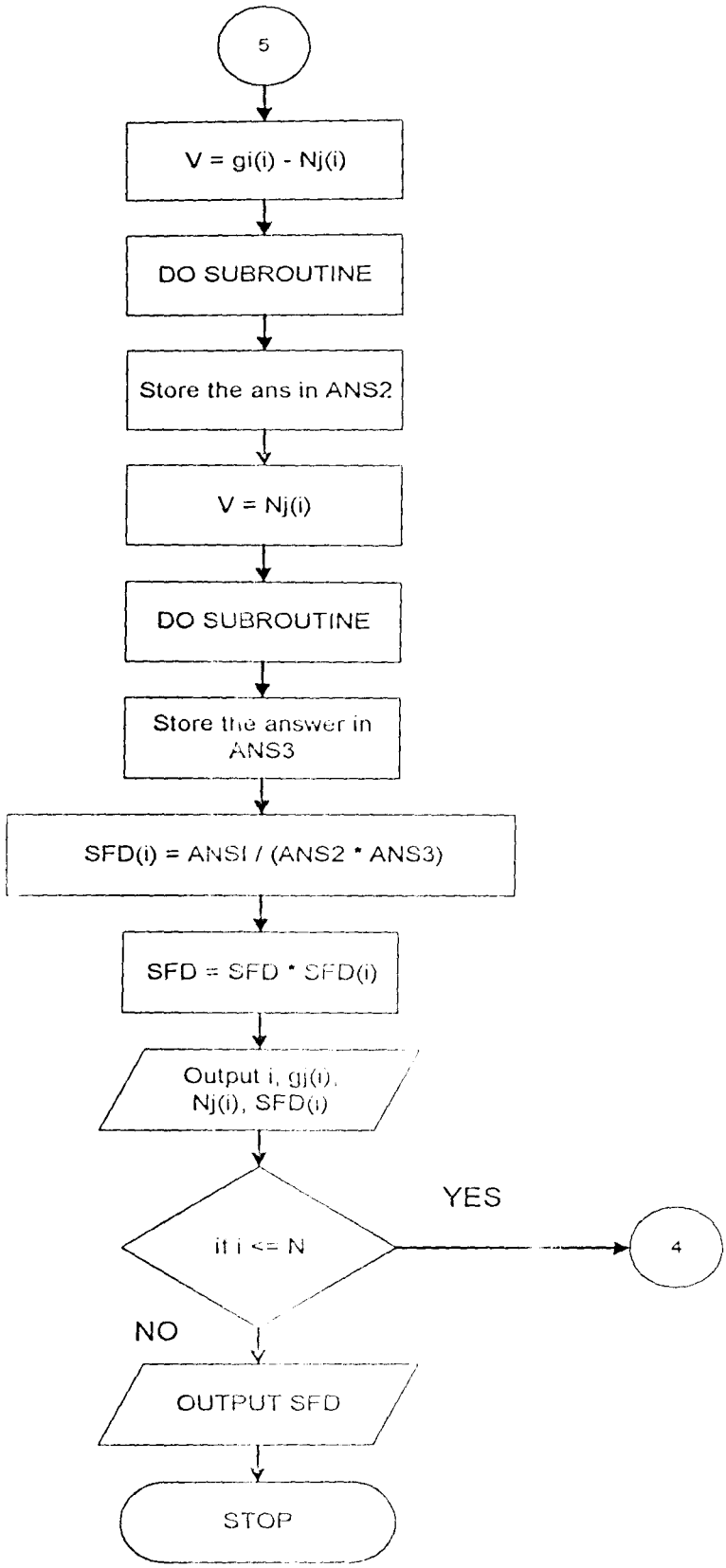

START
OUTPUT
THESE ARE
BOSONS
DO WHILE I <= N
INPUT g(i)
INPUT N(i)
i = i + 1
if i <= n
yes
no
Initialise TBE, A, SEE, MSBE
1
DO while i<=n
v = g(i) - 1
DO SUBROUTINE
store the answer in R
V = N(i)
DO SUBROUTINE
STORE THE ANS IN M
2


2
V = gj(i) + Nj(i) - 1
DO SUBROUTINE
STORE ANS IN P
SBE(i) = P / (R*M)
TBE = TBE + SBE(i)
A = A + Nj(i) * SBE(i)
no
if MSBE > SBE(i)
yes
MSBE = SBE(i)
MSBE = MSBE
IF i <= N
1
no
AVEN = A/TBE
DO WHILE i <= n
Output i, SBE(i), Nj(i), gj(i)
no
yes
3
if i <= n

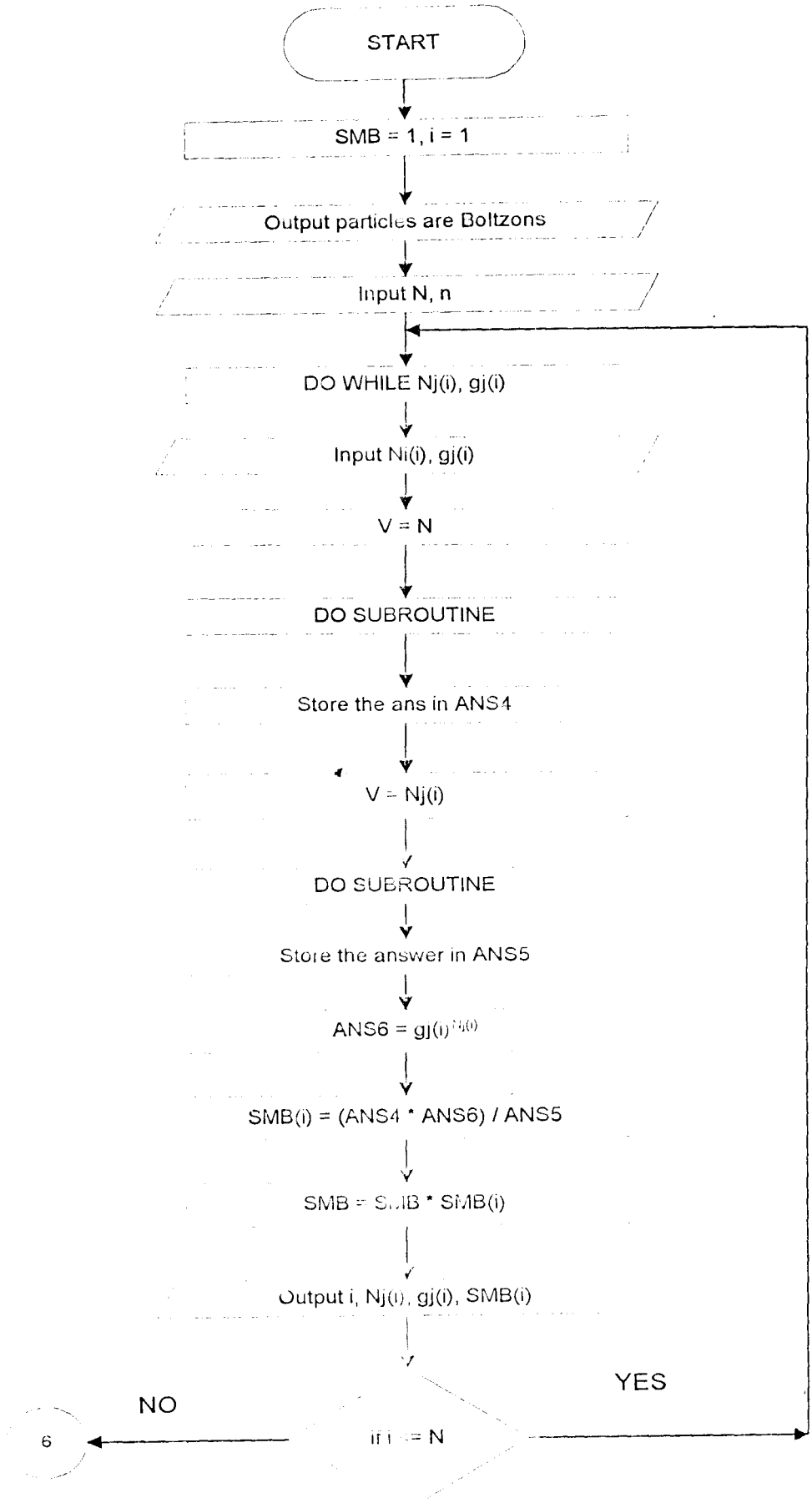

3
PRINT AVEN
PRINT TBE
PRINT MSBE
STOP

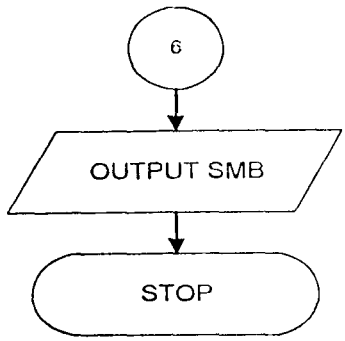
CASE FD


START
PRINT FERMIONS
DO WHILE I < N
Input gj(i), Nj(i)
if i <= n
yes
No
4
DO WHILE i <= n
v = gj(i)
DO SUBROUTINE
STORE THE ANS IN ANSI
5

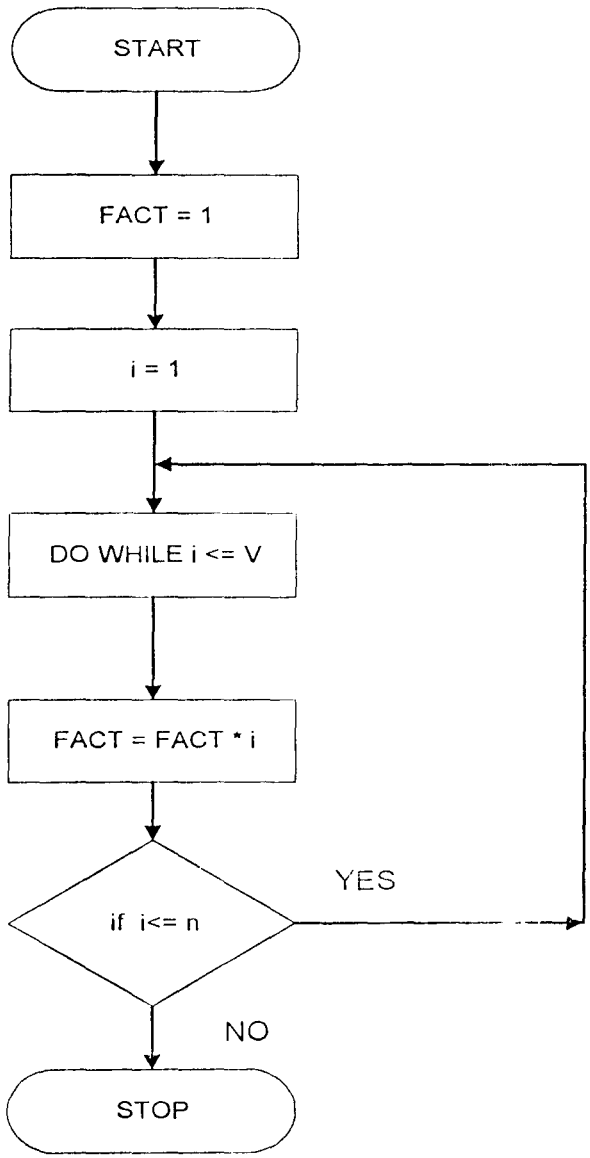

5
V = gi(i) - Nj(i)
DO SUBROUTINE
Store the ans in ANS2
V = Nj(i)
DO SUBROUTINE
Store the answer in ANS3
SFD(i) = ANSI / (ANS2 * ANS3)
SFD = SFD * SFD(i)
Output i, gj(i), Nj(i), SFD(i)
if i <= N
YES
4
NO
OUTPUT SFD
STOP

CASE MB


START
SMB = 1, i = 1
Output particles are Boltzons
Input N, n
DO WHILE Nj(i), gj(i)
Input Ni(i), gj(i)
V = N
DO SUBROUTINE
Store the ans in ANS4
V = Nj(i)
DO SUBROUTINE
Store the answer in ANS5
ANS6 = gj(i)^Nj(i)
SMB(i) = (ANS4 * ANS6) / ANS5
SMB = SMB * SMB(i)
Output i, Nj(i), gj(i), SMB(i)
YES
NO
6



SUBROUTINE FACTORIAL



APPENDIX III

```
5 cls
screen 2
locate 1,17:print"MENU  KEYS"
LINE(2,17) - (2,26)
locate 3,8:print" (1) Calculation Properties"
locate 5,8:print" (2) Table of values"
locate 7,8:print" (3) Graphs"
locate 9,8:print" (4) displacement _ time graph"
locate 11,8:print" (5) velocity _ time graph"
locate 13,8:print" (6) acceleration _ time graph"
locate 15,8:print" (7) kenergy _ time graph"
locate 17,8:print" (8) penergy _time graph"
locate 19,8:print" (9) kenergy _distance graph"
locate 21,8:print" (10) penergy _distance graph"
locate 23,8:print" (11) Exit"
9 locate 25,8:input" options "; q
if q= 1 then goto 10
if q = 2 then goto 100
if q = 3 then goto 200
if q = 4 then goto 300
if q = 5 then goto 400
if q = 6 then goto 500
if q = 7 then goto 600
if q = 8 then goto 700
if q = 9 then goto 800
if q = 10  then goto 900
if q = 11 then goto 1000
cls:locate  10,10:print  "You enter wrong number..retype.."
delay 5:goto 5
10:cls
dim x(200),t(200),v(200),acc(200),pe(200),ke(200),e(200),xx(200)
cls
locate 5,10:input "The value of Mass ..... .....";m
locate 7,10:input "The value of Spring Constant ...";k
locate 9,10:input "The value of Amplitud .....";a
locate 15,10:print "Calculating Aspect.. ....."
w =( k / m)^0.5
f = w/(2 * 3.142)
pr = 1 / f
t(1)=0
for i = 1 to 200
x(i) = a*sin(w * t(i))
V(i) =a*w*cos(w*t(i))
acc(i) = -1*a*(w^2)*sin(w*t(i))
pe(i)= k*((a*sin(w*t(i)))^2/2)
ke(i)= m*(a*w*cos(w*t(i)))^2/2
e(i)= pe(i)+ke(i)
t(i+1)=t(i) + 0.01
print using"####.###";x(i);v(i);acc(i);pe(i);ke(i);e(i);T(i)
next i
delay 5
cls
goto 5
100 cls
```

```
print " diplacement velocity  acceln  penergy  kenergy tenergy
time"
for k = 1 to 200
print using "####.###";x(k);v(k);acc(k);pe(k);ke(k);e(k);t(k)
next k
delay 5
cls
goto 5
200 cls
300 cls
locate 2,10:print "THIS IS THE GRAPH OF DISPLACEMENT AGAINST
TIME"
SCREEN 2
LOCATE 0,0: PRINT "+1"
LOCATE 25, 1: PRINT"-1"
locate 13 ,60:  print"TIME"
LOCATE  1,2 : PRINT"DISP"
LINE (0,100) -( 400,100)
LINE( 1,0) - ( 1,300 )
FOR I= 1 TO 200
pset (t(i)*200 ,100*(1 -x(i))):next i
delay 15: cls:goto 5
400:cls
locate 2,10 :print "this is the graph of velocity against time"
SCREEN 2
LINE (0 ,100) -(400 ,100)
LINE (1,0)- (1,300)
for k = 1 to 200
pset (t(k)*200,10*(10 + v(k)))
next k
delay 15:cls:
goto 5
500 cls
locate 2,10 : print"this is the graph of acceleration against
time"
SCREEN  2
LINE (0 ,100)-(400 ,100)
LINE (1,0 )-(1,300)
for k = 1 to 200
pset (t(k)*200,(100 +acc(k))
next k
delay 15:cls
goto 5
600 cls
locate 2,10 :print"this is the graph of kenergy against time"
SCREEN 2
LINE (0,168) -(400 ,168)
LINE (1,0 )- (1,300)
for k =1 to 30
pset (t(k)*2000,ke(k)/3)
next k
delay 15 : cls
goto 5
700 cls
```

```
locate 2,10 : print"this is the graph of penergy against time"
SCREEN  2
LINE (1, 0)-(1,300)
LINE (0,168) -(400,168)
for i = 1 to 50
pset (t(i)*2000,pe(i)/3)
next i
delay 15 : cls
goto 5
800 cls
locate 2,10 :print"this is the graph of kenergy against
displacement"
SCREEN 2
LINE (1 ,0)-(1 ,300)
LINE (0 ,168) -(400,168)
for i = 1 to 50
pset  ((1+ x(i))*200,ke(i)/3)
next i
delay 15: cls
goto 5
900 cls
locate 2,10: print"this is the graph of penergy against
displacement"
SCREEN 2
LINE (1,0) -(1 ,300)
LINE (1 ,168)-(400,168)
for i = 1 to 50
pset ((1+ x(i))*200,pe(i)/3)
next i
delay 15 :cls
goto 5
1000 cls
ch = y
print"you wish to quit ( y/n)"
input ch$
IF ch$ = "n" THEN GOTO 5
PRINT"YOU TRY , BYE"
END
```

APPENDIX IV

```
1  CLS
REM DECLARE FUNCTION IFACT! ()
REM Compute the number of microstate that make up a microstate
LOCATE 1, 20: PRINT "M A I N   M E N U"
REM LINE (2, 20)-(12, 28)
LOCATE 4, 12: PRINT "(1) Bozons"
LOCATE 6, 12: PRINT "(2) fermions     "
LOCATE 8, 12: PRINT "(3) Boltzon"
LOCATE 10, 12: PRINT "(4) Exit   "
LOCATE 14, 12: INPUT "Option "; Q
      IF Q = 1 THEN GOTO 100
      IF Q = 2 THEN GOTO 200
      IF Q = 3 THEN GOTO 300
      IF Q = 4 THEN GOTO 400
      CLS : LOCATE 16, 12: PRINT "You enter wrong number==>Re-type":
GOTO 1
REM------------------------------------------------------
100  CLS
a = 0: tbe = 0: sbe = 1
LOCATE 10, 10: INPUT "Enter number of n value "; n
LOCATE 12, 10: INPUT "Enter number of x  value"; x
FOR h = 1 TO n
FOR j = 1 TO x
CLS
LOCATE 5, 5: INPUT "Enter value of gj   "; gj(h, j)
LOCATE 7, 5: INPUT "Enter value of nj  "; nj(h, j)
v = nj(h, j)
GOSUB 2000
nfac = ifact
z = gj(h, j) + nj(h, j) - 1
v = z
GOSUB 2000
gnfac =ifact
y = gj(h, j) - 1
v = y
GOSUB 2000
gfac = ifact
sbe(h, j) = gnfac / (gfac * nfac)
tbe = tbe + sbe(h, j)
a = a + nj(h, j) * sbe(h, j)
sbe = sbe * sbe(h, j)
IF msbe > sbe(h, j) THEN
```

```
 msbe = msbe
 ELSE
 msbe = sbe(h, j)
 END IF
aven = a / tbe
NEXT j
NEXT h
rem   PRINT "NUMBER OF MICRO STATE IN A MACRO STATE"
PRINT
PRINT
PRINT
   FOR h = 1 TO n
    FOR j = 1 TO x
 PRINT h, j, nj(h, j), gj(h, j), sbe(h, j): NEXT j: NEXT h

'-----------------------------------------------
rem  FOR I = 1 TO H
rem   FOR K = 1 TO J
rem   PRINT Gj(I, K), MJ(I, K)
rem   NEXT K: NEXT I
  rem  FOR I = 1 TO H
rem    for k = 1 to j
  rem  PRINT "FOR MAC"; I
rem   print sbe(i,k)
 rem   NEXT K: NEXT I
   PRINT
   PRINT "AVERAGE PARTICLE IN THE STATE = "; AVEN
   PRINT
   PRINT "TOTAL NUMBER OF MICRO STATE "; TBE
   PRINT
   PRINT "MOST PROBABLE STATE IS ARE WITH"; MSBE
   delay 15: GOTO 1
200    CLS
      SFD = 1
      INPUT "HOW MANY MICROSTATE PRESENT"; H
      INPUT "THE DEGENERACY OF EACH MACROSTATE"; J
      FOR I = 1 TO H
      FOR K = 1 TO J
      PRINT "GI"; J: INPUT GI(I, K)
      PRINT "NJ"; J: INPUT NI(I, K)
      NEXT K: NEXT I
      FOR I = 1 TO H
      FOR K = 1 TO J
      V = G(I, K)
```

```
      GOSUB 2000
      Q = IFACT
      V = Gj(I, K) - Nj(I, K)
      GOSUB 2000
      S = IFACT
      V = Nj(I, K): GOSUB 2000: L = IFACT
      SFDj(I, K) = Q / (S * L)
      SFD = SFD * SFDj(I, K): NEXT K: NEXT I
      FOR I = 1 TO H
      PRINT "FOR MAC"; I
      FOR K = 1 TO N
      PRINT "Gj"; K; "="; Gj(I, K)
      PRINT "Nj"; K; "="; Nj(I, K)
      PRINT SFDj(I, K)
      NEXT K: NEXT I
      PRINT SFD
     delay 10: GOTO 1
300 CLS
      SMB = 1
      INPUT "TOTAL NUMBER OF PARTICLES N", N
      INPUT "HOW MANY MACROSTATE PRESENT", H
      INPUT "THE DEGENERACY OF EACH MACROSTATE"; J
      FOR I = 1 TO H
      FOR K = 1 TO J
      PRINT "Gj"; K: INPUT Gj(I, K)
      PRINT "Nj"; K: INPUT Nj(I, K)
      NEXT K: NEXT I
      FOR I = 1 TO H: FOR K = 1 TO J
      V = Nj(I, K)
      GOSUB 2000
      X = IFACT
      Y = Gj(I, K) ^ Nj(I, K)
      SMBj(I, K) = X / Y: SMBi = SMBi * SMBj(I, K): NEXT K
      V = N: GOSUB 2000
      Z = IFACT
      SMB = Z * SMBi: PRINT SMB: NEXT I: delay 10: GOTO 1
400 END

2000  CLS
      IFACT = 1
      FOR I = 1 TO V
      IFACT = IFACT * I
      NEXT I
     RETURN
```

with discrete single - particle states 1, 2, ..., r.... with energies Ei $\leq E_2 \leq$.... for a gas of N particles.

Let the state of the gas be specified as a whole by the occupation numbers $n_1, n_2, \ldots, n_r \ldots$

There are two approaches to what number can the occupation number n_i assume.

(i) BOSE EINSTEIN STATISTICS

Here there is no restriction on the occupation numbers n_i, they can and do assume all integer values

Factors $n_i = 0, 1, 2, \ldots$ for all i.

This type of statistics is known as Bose - E - Einstein Statistics. It was used to derive plank radiation Law.

All particles that obey this statistics are known as BOSONS and examples are Photons and mesons.

(ii) FERMI - DIRAC STATISTICS

In this type of Statistics the occupation numbers n_i are restricted: at most are particle can be in any state. That is the occupation number can only be

$n_i = 0, 1$ for all i.

All particles obeying this statistics are known as FERMIONS. Examples are elections, protons, Neutrons, positrons etc.

This is the Paul exclusive principle for non-interactive particles: No two identical fermons can be in the same state.

2. 4 EQUIPARTITION OF ENERGY

Though in a system with evenly spaced component energy levels, the mean component

energy is

Sumesntion fact $E_i = KT$ ..... 2.4.1 and in an ideal gas the mean component energy is

$E = 3/2KT$ 2.4.1

We may want to calculate the mean energy for an ideal gas by a different route:

$$E = \frac{\int^{\infty} EP^2 dp Exp^{-}\{E/KT\}}{\int^{\infty} P^2 dp Exp-\{E/KT\}} \qquad .....2.4.3.$$

where

P = momentum of the particle

$E = P^{2/2m}$ 2.4.4

In three dimensions

$E = \frac{1}{2}m\{P^2_x + P^2_y + P^2_z\}$ 2.4.5

and P^2dp is proportional to dp_x, dp_y, dp_z

we can then write mean energy as

$$E = \frac{\frac{1}{2}m \int^{\infty}\{P^2_x + P^2_y + P^2_z\} \exp - \{(P^2_x + P^2_y + P^2_z)/2 mKT\}\, dp_x dP_y dp_z}{\int_{o\infty} \exp - \{(P^2_x + P_y^2 + P^2_z)/2mKT\} dP_x dP_x dP_y dP_z} \quad ....2.4.6$$

And since the exponential factors into a product of terms depending only on P_x, P_y or P_z then 2.3.6 is equvalent to

$$\acute{E} = \frac{\frac{1}{2}m[\int_{o\infty} P^2_x e^{-P2x/2mKT} dP_x + \int P^2_y e^{-Py2/2mKT} dP_y + \int P^2_z e^{-P2z/2mKT} dP_z]}{[\int_o^{\infty} e^{Px2/2mKT} dP_x \quad \int e^{-Py2/2mKT} dP_y \quad \int e^{-Pz/2mKT} dP_z\ ]} \quad ...2.4.7$$

From here it is clear that the mean energy associated with motion in each of the three arthogional directions is equal to

$E_x = E_y = E_z = \frac{1}{2}KT$ 2.4.8

Therefore one can say that

$$E = \tfrac{1}{2}KT \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2.4.9$$

is the mean energy associated with each of the three degree of freedom of each particle (free to move in the x, y and z directions indepedently.

2. 5 SOME THERMODYNAMIC FUNCTIONS

Let us try to investigate (evaluate) the pressure of a gas from the idea of virtual work. The change of energy in the system for an infinitesimal expansion is the work done. So we can identify

$$P = -\frac{(\delta E)}{(\delta v)ni} \quad \ldots\ldots\ldots\ldots\ldots \quad 2.5.1$$

If the volume increase is done slowly, in fact very very slowly, so that the level sag but the population remain the same (i.e. ni = constant).

Now

$$E = \Sigma Eini \quad \ldots\ldots\ldots\ldots \quad 2.5.2$$

So $\triangle E = \Sigma ni \triangle Ei + \Sigma Ei \triangle ni \quad \ldots\ldots\ldots \quad 2.5.3$

The first term represent work done on the system and the second term represent the heat change.

$$dE = đw + đQ \quad \ldots\ldots\ldots\ldots\ldots \quad 2.5.4$$

Equation 2.4.4 is the first law of thermodynamics which says that any change in energy is accounted for by work done and quality of heat.

Note that đw and đQ are infinitesimal change not differential

using 2.5.2

$$P = -\Sigma ni \frac{\delta Ei}{\delta v} \quad \ldots\ldots\ldots \quad 2.5.5.$$

There are many ways of working this expression. We have

$$ni = \frac{NExp - \{Ei/kT\}}{\Sigma Exp-\{Ei/kT\}} = \frac{NExp-\{Ei/kT\}}{Z} \quad \text{--------2.5.6}$$

Note that

$$\frac{\delta}{\delta V}\{InZ\}_T = \frac{1}{Z}\frac{(\delta Z)_T}{\delta V} \quad \text{..... 2.5.7}$$

$$= -\Sigma\frac{\delta Ei}{\delta V}\frac{1}{KT}Exp\{Ei/kT\} \quad \text{... 2.5.8}$$

So that

$$P = NKT\frac{\delta}{\delta V}\{InZ\}_T \quad \text{.....2.5.9.}$$

For our gas

Let $Z = \int Exp - \{E/kt\}\frac{4\pi P^2 dp_V}{(2\pi h)_3}$ 2.5.10

and if we compare 2. 4. 9 with equation of state

$$PV \quad NKT \quad \text{.....2.5.11}$$

we have

$$\frac{\delta}{\delta V}\{InZ\}_T = \frac{1}{V} \quad \text{.....2.5.12}$$

$$\text{If } P = -\frac{\{\delta F\}_T}{\delta V} \quad \text{.....2.5.13}$$

It is convenient to define an energy function

$$F = NKTLn\,Z \quad \text{.....2.5.14}$$

Taking exponential of this we have

$$Z^N = Exp - [F/kT] \quad \text{.....2.5.15}$$

The internal energy can also be expressed in terms of the partition function, $\mathcal{Z}$ as

$$E = \frac{N\Sigma Ei\ Exp\ -\{Ei/KT\}}{\mathcal{Z}} = NkT^2\ \frac{\delta}{\delta T}\ \{Ln\mathcal{Z}\} \quad \ldots\ldots\ 2.5.16$$

Now if we consider the environment of the system and see what we can control, we have

$$-\left(\frac{\delta F}{\delta V}\right)_T = NkT\left\{\frac{\delta In\mathcal{Z}}{\delta V}\right\}_T = P \quad \ldots\ldots\ 2.4.17$$

and work done on the system is

$$dw = -Pdv = \left(\frac{\delta F}{\delta v}\right)dv \quad \ldots\ 2.5.18$$

The equation of state links P, V, and T so we may write

$$dF = \left(\frac{\delta F}{\delta V}\right)_T dv \quad \left(\frac{\delta F}{\delta T}\right)_V dT \quad \ldots\ldots\ 2.5.19$$

So that

$$-Pdv = dF - \left(\frac{\delta F}{\delta T}\right)_V dT \quad \ldots\ldots\ 2.5.20$$

and

$$\left(\frac{\delta F}{\delta T}\right)_V = -NKln\mathcal{Z} - \frac{NKT}{\mathcal{Z}}\ \frac{\delta \mathcal{Z}}{\delta T} \quad \ldots\ldots\ 2.5.21$$

The quantity F is the Helmholtz free energy and is such that the differential with respect to volume, at constant temperature gives the pressure.

Note that the differential of internal energy with respect to V at constant temperature does not give pressure.

2.6 CANOMICAL AND GRAND CANOMICAL APPROACHES

The statistical mechanics which has been developed so far is applicable only to isolated systems of independent and non-interacting particles. Then we are able to express energy of the

system as the sum of the energies of the constituent particles, i.e.

$$E = n_1E_1 + n_2E_2 + \ldots\ldots n_iE_i + \ldots\ldots \quad 2.6.1$$

The weaknesses of such a development of statistical mechanics are:

(1) The energy of the system is specified

(2) The system contain speciefied number of particles

(3) There is no interaction among the constituent particles.

We may want to consider a situation where none (or some) of these restrictions holds.

DEFINITION (MICROCANOMICAL APPROACH)

Any system of particles that is treated with weak interaction among the constituent particles is known as microcanomical Ensembles. That is systems with specified number of particles and weak interaction among the components, such treatment enables us to calculate thermodynamic properties of materials in ideal gas state from molecular data.

DEFINITION (CANOMICAL APPROACH)

In an assemblage of systems in which each system is assumed to be closed and with constant volume but separated from its neighbours by diathermic walls, so that all systems are in a thermal equilibrium. That is the system is characterized by constant temperature, volume and number of particle N. This is a closed Isothermal system and is called the Canomical ensembles. The particle can exchange energy with each other. It can be shown schematically as in fig. 2.6.1.

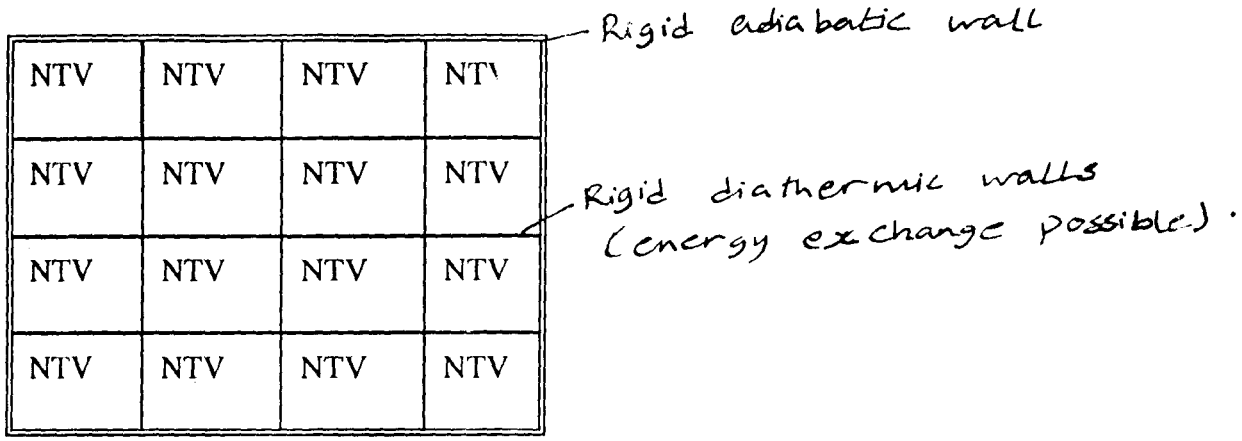


fig. 2.6.1 canonical ensemble of n systems with const. N, V, T,

DEFINITION (GRAND CANONICAL APPROACH)

In this case, each system is consider to be of volume but open and separated from its neighbours by diathermic permeable membrane. So that energy and materials can be exchanged.

This is an open Isothermal system characterized by constant volume V, temperature T and chemical potential U of the components.

This is called Grand Canonical Ensembles.

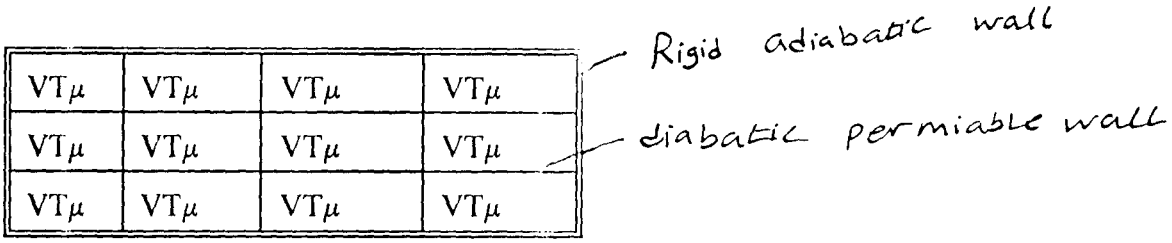


fig. 2.6.2 Grand Canonical Ensembles.

Let us consider a system with energy state Ei in contact with a heat reservoir which is made up of a large number of our system and with a total energy E.

Ei is the energy of the system and not of the component. Each of these systems is identical and localized and suppose we know their system energy levels.

If there are Mi system with energy Ei, then the number of microstates making up this is

$$W = N!.\pi \frac{g_i^{Ni}}{Ni!} \quad \ldots\ldots\ldots\ldots\ldots 2.6.2$$

If we consider these constraints

(i) $\Sigma Ni = N$ 2.6.3

(ii) $\Sigma EiNi = E$ 2.6.4

Maximizing lnW we have

$$Ni = \frac{Ngi\ Exp - \{Ei/KT\}}{\Sigma Exp - \{Ei/KT\}} \quad \ldots\ldots 2.6.5$$

The probability that our system is to be found in a single state of energy Ei is given by

$$Pi = \frac{Exp - \{Ei/KT\}}{\Sigma Exp - \{Ei/KT\}} \quad \ldots\ldots\ldots\ldots 2.6.6$$

Then the function

$$\mathcal{Z} = \Sigma Exp - \{Ei/KT\} \quad \ldots\ldots\ldots\ldots 2.6.7$$

Summed over all system energy levels and is the system partition function.

The mean energy of the system is

$$E = \Sigma PiEi \ldots\ldots\ldots\ldots 2.6.8$$

and therefore

$$dE = \Sigma PidEi + \Sigma EidPi \ldots\ldots\ldots 2.6.9$$

gives the change in energy. The first term represent work done on the system by the environment while the second term is heat.

Now the system energy levels Ei changes in response to some external constraints X (such as volume, magnetic field e.t.c).

Differentiating the system partition function $\mathcal{Z}$ with respect to constraint parameter, X at constant temperature.

$$\left(\frac{\delta \ln \mathcal{Z}}{\delta V}\right)_T = 1/\mathcal{Z}\left(\frac{\delta \mathcal{Z}}{\delta x}\right)_T \quad \ldots\ldots\ldots 2.6.10$$

$$= \frac{-1}{KT} \sum \frac{\delta Ei}{\delta x} \frac{Exp-\{Ei/KT\}}{\mathcal{Z}} \quad \ldots\ldots\ldots 2.6.11$$

$$= \frac{-1}{KT} \sum P_i \frac{\delta E_i}{\delta x} \quad \ldots\ldots\ldots 2.6.12$$

The work term can be written as

$$dw = \left(\frac{\delta F}{\delta x}\right)_T dx \quad \ldots\ldots\ldots 2.6.13$$

Where F is the Helmholt free energy for our system defined as

$$F = - KT \ln \mathcal{Z}. \quad \ldots\ldots\ldots 2.6.14.$$

We may want to guess at what will happen if the number of components in our system is variable (i.e. viewing from grand canonical approach). It is assumed that the energy (and energy levels) of the system will change if the number of component change. So we expect that the probability of finding the system in a state with n components and energy Ei(n) will be given by

$$P \alpha Exp \{(\mu n - Ei)/KT\} \quad \ldots\ldots\ldots 2.6.15$$

and the system's partition function is defined as

$$\mathcal{Z} = \sum Exp \{^{\mu n - Ei(n)})/KT\} \quad \ldots\ldots\ldots 2.6.16$$

where μ is some kind of work function known as chemical potential.

$\mathcal{Z}$ is the grand partition function.

2.7 MEAN VALUES AND FLUCTUATIONS

We are now in a position to tackle the problem of fluctuations in an approximate way. let our system have energy E and consist of N components. The probability of the system having energy E, keeping N constant, is given by the Boltzmann factor

$$P \alpha e^{-E/KT} \qquad \ldots\ldots\ldots 2.7.1$$

The number of states between E and (E + δE) is roughly proportional to

$$\frac{(E)^{N-1}}{N\triangle} \frac{\delta E}{\triangle} \qquad \ldots\ldots\ldots 2.7.2$$

The probability of finding the system between E and (E + δE) is then given by the number of states in △, each multiplied by the probability of occupation, so it is

$$P(E)\,\delta E \approx \frac{(E)^{N-1}}{N\triangle} \frac{\delta E}{\triangle} \text{ Exp } - \{E/KT\} \ldots\ldots\ldots\ldots 2.7.3$$

The term $(E/D)^N$ grows hugely with E if N is a large number, while the exponential term falls, P(E) has a maximum which can be defined.

$$\text{If } P(E) \approx \text{Exp } \{N\ln E - E/KT\} \qquad \ldots\ldots\ldots 2.7.4$$

the maximum occur (if we take lnP(E) and different late) at

$$\frac{\delta}{\delta E} \{N\ln E - E/KT\} = O \qquad \ldots\ldots\ldots 2.7.5$$

i.e. $E = NKT$ 2.7.6

Let us now study the variation of P(E) with small excursions of the energy

$$N\ln E - E/KT = N\ln (Emax + \triangle E) - (Eman + \triangle E)/KT \ldots\ldots 2.2.7$$

If we expand to second order

$$= N\ln Emax - Eman/KT + N\ln \left(1 + \frac{\triangle E}{Emax}\right) - \triangle E/KT \qquad \ldots\ldots\ldots 2.7.8$$

Since Emax = NKT the first order term vanishes and

$$P(Eman + \triangle E) = P(Eman)\ Exp - \tfrac{1}{2}N\ (\frac{\triangle E}{Emax})^2 \quad \ldots\ldots\ldots\ldots\ 2.7.9$$

So the width of this approximately gaussian distribution is ~ Emax $\sqrt{N}$ and this is the size of the fluctuations in energy which occur.

Fluctuations in the number of components may be studied through the same approximation

$$\text{If } P(N) \approx Exp\ \{\mu N/KT + (N-1)\ln E/NA\} \quad \ldots\ldots\ldots\ 2.7.10$$

The maximum occurs at

$$\frac{\delta}{\delta N}\ \{\frac{\mu N}{KT} + (N-1)\ [\ln E - \ln N - \ln\triangle]\} = O \quad \ldots\ldots\ldots\ 2.7.11$$

If Nmax satisfy this equation

If we set

$$N = Nmax + \triangle N \quad \ldots\ldots\ldots\ldots\ 2.7.12$$

And if we expand up to second order in DN, terms in DN vanishes because of the maximum and we find that

$$P(N) \approx P(Nmax)\ Exp\text{-}\tfrac{1}{2}\ (\frac{\triangle N}{Nmax})^2 \quad \ldots\ldots\ldots\ 2.7.13$$

So that the fractional variation of N is expected to be negligible for a macroscopic object.

This is an elementary consideration which can not take care of circumstances in which macroscopic system exhibit large fluctuations.

Therefore, we can develop a method for estimating fluctuations for real macroscopic systems by proceeding more formally.

Suppose some quantity q (eg particle number, energy e.t.c) has an approximately

gaussian distribution

$$P(q) \approx \text{Exp} - \frac{\{q - q_o\}^2}{2\Delta^2} \quad \ldots\ldots\ldots\ldots 2.7.14$$

The average value of q is

$$\bar{q} = \frac{\int qP(q)dq}{\int P(q)dq} \quad \ldots\ldots\ldots\ldots 2.7.15$$

$$= \frac{\int q\text{Exp} - \{(q-q_o)^2\}dq}{\int \text{Exp} - \{(2\Delta)^2\}dq} = q_o \quad \ldots\ldots\ldots\ldots 2.7.16$$

Setting $\frac{1}{2}\Delta^2 = \alpha$

the average value of q^2 is

$$\frac{\int q^2 P(q)dq}{\int P(q)dq} = \frac{\int q^2\text{Exp} - \{\alpha(q-q_o)^2\}dq}{\int \text{Exp} - \{\alpha(q-q_o)^2\}dq} \quad \ldots\ldots\ldots\ldots 2.7.17$$

let $q - q_o = x$

$$q^2 = \frac{\int (x-q_o)^2 \text{Exp} -(\alpha x^2)dx}{\int \text{Exp}-(\alpha x^2)dx} \quad \ldots\ldots\ldots\ldots 2.7.18$$

$$= \frac{\int x^2 \text{Exp} -(\alpha x^2)dx}{\int \text{Exp} -(\alpha x^2)dx} + q_o^2 \quad \ldots\ldots\ldots\ldots 2.7.19$$

But

$$\int x^2 e^{-\alpha x^2} dx = \tfrac{1}{2}\alpha\sqrt{\pi/\alpha} \quad \ldots\ldots\ldots\ldots 2.7.20$$

and

$$\int \text{Exp} -(\alpha x^2)dx = \sqrt{\pi/\alpha} \quad \ldots\ldots\ldots\ldots 2.7.21$$

The ratio equals

$$\frac{1}{2}\alpha \dashrightarrow \Delta^2$$

so that

$$\Delta^2 = q^2 - q_o^2 = q^2 - q^2 \quad \ldots\ldots\ldots\ldots 2.7.22$$

This is the mean square fluctuation of any quantity q by calculating the quantity.

$$\Delta^2 = q^2 - q^2 \quad \ldots\ldots\ldots\ldots\ldots \quad 2.7.23.$$

If we want the fluctuation on particle number.

$$N = \frac{\Sigma\Sigma N \text{Exp}\{\mu N/KT - Ei/KT\}}{Z} = \frac{KT}{Z}\frac{\delta Z}{\delta \mu} \quad \ldots\ldots\ldots\ldots \quad 2.7.24$$

$$N^2 = \frac{\Sigma\Sigma N^2 \text{Exp}\{\mu N/KT - Ei/KT\}}{Z} = \frac{K^2T^2}{Z}\frac{\delta^2 Z}{\delta \mu^2} \quad \ldots\ldots\ldots \quad 2.7.25$$

Hence

$$N^2 - N^2 = KT\frac{\delta N}{\delta \mu} \quad \ldots\ldots\ldots\ldots \quad 2.7.26$$

Similarly for energy and for convenient let us set $\beta = 1/KT$

$$E = \frac{\Sigma\Sigma Ei \text{Exp}\{(\mu N - Ei)\beta\}}{Z} = 1/Z\frac{\delta Z}{\delta \beta} \quad \ldots\ldots\ldots\ldots \quad 2.7.27$$

$$E^2 = \frac{\Sigma\Sigma Ei^2 \text{Exp}\{(\mu N - Ei)\beta\}}{Z} = 1/Z\frac{\delta^2 Z}{\delta \beta^2} \quad \ldots\ldots\ldots\ldots \quad 2.7.28$$

Since ($T = -1/KB$)

$$\text{Hence } E^2 - E^2 = KT^2\frac{\delta E}{\delta T} \quad \ldots\ldots\ldots\ldots\ldots \quad 2.7.29$$

But $\frac{\delta E}{\delta T} = Cv$ (in thermodynamics)

Then $E^2 - E^2 = KT^2Cv$

The fluctuation in number only becomes large if $\frac{\delta N}{\delta \mu}$ is large, i.e. N is sensitive to small change in μ. And fluctuation in energy only gets larage if E is sensitive to T or if Cv is large

CHAPTER THREE

STATISTICAL MECHANIC PROBLEMS ALGORITHMS

3.1 INTRODUCTION

What we have established so far can be applied to physical problems such as the harmonic oscillators, thermodynamic probabilities, steller, intersteller gas, relaxation time problems e.t.c. to mention but few.

In this chapter, we shall consider some of these problems namely; harmonic oscillator and the thermodynamic probabilities problems, and then write algorithm for them.

Prior to each problem algorithm we shall establish (state) some of the procedure and relationships that are relevant to the problem. If possible see through same derivations.

3.2 THE PROBLEMS SPECIFICATIONS

In the harmonic oscillation, we shall be restricted to the simple ones where there is no frictions and no external force influence. With this we shall study the displacements, velocities, accelerations and various energies as varies with time and positions of the body as appropriate. Also, we shall try to present some graphical relationships between these quantities.

In the case of thermodynamic probabilities problem, we want to know how probable a microstate is and the number of microstates that make up a microstate. These will be done viewing particles from three statistics, namely: Bose-Einstein, Fermi-Dirac and Maxwell-Boltzman statistics

3.3 ALGORITHMS

An algorithm is a finite set of instructions for carrying out a specific procedural task. Examples of algorithms are program (which are specifically expressed in third generation high

level languages capable of execution by a computer), pseudocodes flow charts, NSSF-diagram formulae decision tree and English language.

In this work, pseudocodes is used to express (write) the algorithms of the problems considered.

Pseudocode is a logical representation of an algorithm using such third generation languages style like DO, WHILE, IF... THEN, ELSE, FOR, NEXT, ENDIF e.t.c.

3.4 SIMPLE HARMONIC MOTION

Any harmonic oscillator which moves without frictions and external force influence is known as simple harmonic motion and its motion is known as simple harmonic motion.

The equation describing the motion is:

$$m\frac{d^2x}{dt^2} + kx = O \quad \ldots.3.4.1$$

If the solution to the above problem is

$$x = A\sin wt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.4.2$$

Taking its second derivation and substitute in (3.4.1) above it gives

$$w = \sqrt{k/m} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.4.3$$

which is the angular frequency.

Where k and m are spring constant and mass respectively.

The velocity of the oscillator is given by

$$V = \frac{dx}{dt} = Aw\cos wt \quad \ldots\ldots\ldots 3.4.4$$

and its acceleration is

$$a = \frac{d^2x}{dt^2} = -Aw^2\sin wt \quad \ldots\ldots\ldots\ldots 3.4.5$$

The period of the motion is

$$T = 2\pi\sqrt{(m/k)} \quad .. \quad ..... \quad 3.4.6$$

and its frequency is

$$F = \frac{1}{2\pi}\sqrt{(k/m)} \quad ..... \quad 3.4.7$$

The potential energy of the body is given by

$$P.E. = \tfrac{1}{2}kx^2 = \tfrac{1}{2}KA^2Sin^2wt \quad ..... \quad 3.4.8$$

and its kinetic energy is

$$KE = \tfrac{1}{2}MV^2 = \tfrac{1}{2}MA^2W^2Cos^2wt \quad ..... \quad 3.4.9$$

ALGORITHM (S.H.M)

{computation of the properties of a body undergoing simple harmonic oscillation}.

DECLARE

K, V, W, M, A, T, t, F, PE, KE, a, x; real.

CONST; π, K

CHAR; G, CH

EXECUTE

ARRAY ; V, PE, KE, a, x, E.

OUTPUT You are about to compute simple harmonic oscillation problem.

OUTPUT What is the spring constant of your spring in use.

INPUT K

INPUT "Amplitude", A

OUTPUT what is the mass of the body attached to the spring.

INPUT m

OUTPUT Compute the angular frequency, period and frequency of the motion.

$W \leftarrow (k/m)^{1/2}$

$TP \leftarrow 2\ \pi(m/k)^{1/2}$

$F \leftarrow (k/m)^{1/2}/2\pi$

OUTPUT Initialise start time

$t \leftarrow 0$

For $i \leftarrow 1$ to L

$X(i) \leftarrow ASINWt$

$V(i) \leftarrow AWCOSWt$

$a(i) \leftarrow AW^2$ Sin wt

$PE(i) \leftarrow \frac{1}{2}\ KA^2SIN^2wt$

$KE(i) \leftarrow \frac{1}{2}\ MA^2w^2Cos^2wt$

$E(i) \leftarrow PE(i) + KE(i)$

OUTPUT i, t, x(i), V(i), a(i), PE(i), KE(i) E(i) $t = t + 0.1$

ENDFOR

{In plotting various graphs of this motion we see a lot of beautiful and interesting figures.}

OUTPUT The following options are available in graphics. Choose your appropriate desire by pressing the corresponding code.

OUTPUT 1. TABLE: for table of values

OUTPUT 2. MOTION: for graph of X against t

OUTPUT 3. VEl: for graph of V against t

OUTPUT 4. ACC: for graph of a against t

OUTPUT 5. PET: for graph of PE against t

OUTPUT 6. KET: for graph of KE against t

OUTPUT 7. PEAX: for graph of PE against x

OUTPUT 8. KEAX: for graph of KE against x

INPUT "what is your desire", G

CH ← Y

WHILE CH < > N

SELECT CASE G

CASE "TABLE"

FOR i ← 1 to L

OUTPUT i, t(i), x(i), v(i), a(i), PE(i), E(i)

ENDFOR

OUTPUT Table of values

CASE "MOTION"

CLEAR

TRANSFER CONTROL TO SUBROUTINE FOR SETTING SCREEN AND COLOUR AND DRAW AXIS.

LABEL the vertical axis as displacement.

Label the horizontal axis as time

MARK The scale on vertical (x) axis

Mark the scale on time axis

Draw the graph.

OUTPUT Graph of displacement against time.

CASE "VEL"

CLEAR

TRANSFER CONTROL TO SUBROUTINE FOR SETTING.

Label vertical axis as velocity

Label horizontal axis as time.

Mark the scale on velocity axis.

Mark the scale on time axis.

Plot the graph

OUTPUT Graph of velocity against time.

CASE ACC"

CLEAR

TRANSFER CONTROL TO SUBROUTINE FOR SETTING

Label vertical axis as acceleration

Label the horizontal axis as time

Mark the scale on acceleration axis

Mark the scale on time axis

Plot the points

OUTPUT Graph of acceleration against time.

CASE "PET"

CLEAR

TRANSFER CONTROL TO SUBROUTINE FOR SETTING

Label vertical axis as potential energy

Label the horizontal axis as time

Mark the scale of PE axis

Mark the scale on t axis

Plot the points

OUTPUT Graph of potential energy against time

CASE "KET"

CLEAR

TRANSFER CONTROL TO SUBROUTINE FOR SETTING

Label the vertical axis as kinetic energy

Label the horizontal axis as time

Mark the scale on KE axis

Mark the scale on t axis

Plot the points

OUTPUT Graph of kinteic energy against time.

CASE "PEAX"

CLEAR

TRANSFER CONTROL TO SUBROUTINE FOR SETTING

Label the vertical axis as potential energy

Label the horizontal axis as Displacement

Mark the scale on PE axis

Mark the scale on x axis

Plot the points

OUTPUT Graph of potential energy against displacement.

CASE "KEAX"

CLEAR

TRANSFER CONTROL TO SUBROUTINE FOR SETTING

Label the vertical axis as kinetic energy

Label the horizontal axis as Displacement

Mark the scale of KE axis

Mark the scale of x axis

Plot the points

OUTPUT Graph of kinetic energy against displacement.

ENDSELECT

OUTPUT Do you want to display more graphs (Yes/No).

INPUT "decision" CH

ENDWHILE

END S H M

3.5 THERMODYNAMIC PROBABILITY

To estimate the number of microstates that make up a microstate in a given system, one

will need to consider three statistics available for particles to obey, namely: Bose-Einstein, Fermi- Dirac and Maxwell-Boltzmann statistics.

(A) Bose-Einstein statistic (B-E statistic)

When considering B-E statistics we speak of identical indistinguishable particles and there is no restriction to the number of particle that can occupy a microstate.

If gi the degeneracy of a particular energy level and Ni is the number of particles in i microstate, the microstate contains

$$S_{BE} = \pi \frac{(gi + Ni - 1)!}{(gi-1)!\, Ni!} \quad 3.5.1$$

And the total number of microstates in the system is given by

$$T = \Sigma_k S_k \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.5.2$$

The most probable microstate is given by Max SBE.

And the average occupation number Ni for a given degeneracy is given by

$$Nig = \frac{1}{T} \Sigma Ni_k S_k \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.5.3.$$

(B) FERM - DIRAC (F-D) statistics.

The particles here are also identical and indistinguishable but maximum of only one (not more than one) particle can occupy an energy state.

Using the same particle parameters above the microstate contain

$$S_{F-D} = \pi \frac{gi!}{i(gi - Ni)!Ni!} \quad \text{------}3.5.4$$

(C) Maxwell-Boltzman (M-B) statistics

Like the B-E statistic, the number of particles that occupy a state are not restricted but the particles are distinguishable and identical

Hence

$$S_{MB} = N! \ \pi \frac{g_i^{N_i}}{N_i!} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3.5.5$$

ALGORITHM (MICROSTATE)

{Compute the number of microstate that make up a microstate in a system}.

DECLARE

S, T, AVEN : real

gi, Ni,V, n : Integer

CONST:

CHAR: F,B, Z, CH.

ARRAY: gi, Ni

EXECUTE

OUTPUT what type of statistics is to be consider

{F, for F-D, B for B-E and Z for M-B}

CH ← Y

DO WHILE CH < > N

SELECT CASE

CASE B

OUTPUT particles are identical, indistinguishable and no restriction to number of

particles.

OUTPUT what is the value of degeneracy

For i ← 1 to n

INPUT gi(i)

INPUT "No of particle in the level", Ni(i)

ENDFOR

INITIALISE

TBE = O, A = O, SBG = 1 MSBG=O

OUTPUT set temporary locations

FOR i ← 1 to n

Let V ← gi(i) - 1

Transfer control to subroutine for factorial

Restate the answer, into R

Let V ← (gi(i) +Ni(i)-1

Transfer control to surontine for factorial

Restate the answer into P

Let V ← N,(i)

Transfer control to subroutine for factorial

Restate the answer, into M

$$SBG(i) \leftarrow \frac{P}{RM}$$

TBE ← TBE + SBE(i)

A ← A + Ni(i)* SBE(i)