

**SIMULATION OF TEMPERATURE VARIATION
IN GRAIN METAL SILOS**

BY

**OLALEKAN ISMAILA OLU ADISA
(M.ENG/SEET/003/96)**

**A PROJECT REPORT IN THE DEPARTMENT OF AGRICULTURAL
ENGINEERING SUBMITTED TO THE POST - GRADUATE SCHOOL
IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR
THE AWARD OF MASTER OF ENGINEERING DEGREE**

IN

CROPS STORAGE AND PROCESSINGS

**FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA
NIGER STATE, NIGERIA
MARCH 1999.**

ACKNOWLEDGEMENT

I thank God, the most merciful and the most beneficent, who despite all hardships along the lines, has guided me all through the period of study in Federal University of Technology, Minna, Niger state, Nigeria.

I also appreciate the untiring efforts and enormous contributions of my supervisor Dr. M. G. Yisa (Head of Department) who took his time to review this work and offer useful criticisms that have contributed to making this work a success. I thank the following staff : Dr. Akin Ajisegiri, Dr. D. Adgidzi, Messers Egharevba, Chukwu, Onuachu, Idah, Alabadan and Mrs. Osunde the mother of the department for their contribution to this work. I also appreciate the assistance of Hakeem, Zubair, Ajibola, Bola, Yinka, Wahabi, Dr. J.O.Odigure, Dr. P.A. Onwaulu and Dr. O.D. Jimoh.

My profound gratitude goes to my late Father Alhaji Gbadamosi Adisa of blessed memory and my Mother Alhaja Ike-ola Moriama Adisa for their efforts. The love shown and showered on me since I was born and their immense struggle in ensuring that I secure that essential and basic passport to a good living in life is gracefully appreciated.

To all those whose names could not be mentioned but made it possible to acquire this knowledge, thank you and God Bless.

Adisa, O. O. I.

APPROVAL AND CERTIFICATION

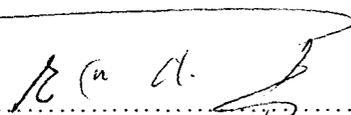
This thesis has been read and approved as meeting the requirement of the Department of Agricultural Engineering, Federal University of Technology, Minna for the award of Master of Engineering Degree(M. Eng) in Crop Storage And Processings.


.....
ENGR. DR. M.G. YISA
PROJECT SUPERVISOR

17/01/2000
DATE


.....
ENGR. DR. D. ADGIDZI
INTERNAL EXAMINER

17/01/2000
DATE


.....
ENGR. PROF. E.U. ODIGBOH
EXTERNAL EXAMINER

20/01/2000
DATE


.....
ENGR. DR. M.G. YISA
HEAD OF DEPARTMENT

24/01/2000
DATE

TABLE OF CONTENTS

Pages

APPROVAL AND CERTIFICATION-----	i
ACKNOWLEDGMENT-----	ii
Table of Content -----	iii
List Of Figures-----	iv
List Of Tables-----	v
ABSTRACT-----	vi
1.0 INTRODUCTION-----	1
1.1 Modern Methods of Storing Grain-----	1
1.2 Temperature-----	4
1.3 LITERATURE REVIEW -----	5
1.3.1 Simulation Concepts -----	5
1.3.2 Theory -----	8
1.3.3 Gauss-Jordan Method -----	11
1.3.4 Determinant -----	13
1.3.5 Solution of M Equations with N unknown -----	15
1.4 Statement of Problems-----	15
1.5 Justification-----	17
1.6 Objectives-----	19
2.0 MATERIALS AND METHODS-----	20
2.1 Description of Storage Silos -----	20
2.2 Measurement of Temperature-----	21
2.3 Generalised Mathematical Formulation-----	22
2.3.1 Model Development -----	22
2.4 Assumption and Final Governing Equation -----	26
2.5 Method of Solution -----	27
2.5.1 Finite - Difference Method -----	27

2.6	Computer Implementation -----	31
2.6.1	Development of Simulation Procedure -----	31
2.6.2	Programm Listing -----	37
2.7	Validation of Model -----	42
2.7.1	Grain Aeration Test -----	42
3.0	RESULTS AND DISCUSSION -----	44
4.0	CONCLUSION -----	48
	REFERENCES -----	49
	SYMBOLS -----	52

LIST OF FIGURES

	Pages
Fig.1 Picture of a Metal Silo	3
Fig.2 The top elevation section of an experimental with the positions of the thermocouples	26
Fig.3 A Sector of a Cylindrical Grain Silo Divided Into a Finite Number of Spatial Elements	28
Fig.4 Computational Flow Chart	32
Fig.5 Cross-section of experimental aeration Silo showing the thermocouple and Grain sampling conditions.	42
Fig.6 Measure and simulated Temperatures (dry bulb) at centres at a depth of 1.2m from the surface of the grain bulk and the dry bulb temperature of the ambient temperature. (Silo 6)	45
Fig.7 Measured and simulated temperature (dry bulb) at 8.25m fro the centre of the metal silo at a depth of 1.2m from the surface of the grain bulk and the dry bulb temperature of the ambient temperature (Silo 6).	45
Fig.8 Measured and simulated temperature (dry bulb) at 3.25m from the centre of the metal Silo at a deph of 1.2m from the surface of the Grain bulk and the dry bulb temperatures of the ambient temperature.	45
Fig.9 Measured and simulated Temperatures (dry bulb) at centres at a depth of 1.2m from the surface of the grain bulk and the dry bulb temperature of the ambient Temperature. (Silo 8)	46
Fig.10 Measured and simulated temperature (dry bulb) at 3.25m from the centre of the metal Silo at a deph of 1.2m from the surface of the Grain bulk and the dry bulb temperatures of the ambient temperature. (Silo 8)	46

Fig.11 Measured and simulated temperature (dry bulb) at 8.25m fro the centre of the metal silo at a depth of 1.2m from the surface of the grain bulk and the dry bulb temperature of the ambient temperature (Silo 8). 46

LIST OF TABLES

	Pages
Table 1. Thermal and physical properties of Silo wall materials	20

ABSTRACT

A mathematical model, based on the finite-difference method of calculating heat transfer has been developed to simulate the temperature in a cylindrical metal storage silo. The model was based on a 2 dimensional transient heat conduction equation with the associated boundary conditions. The model included several sub-models which predicted temperature profiles of solar radiation on silo wall at anytime of the day, and convective heat transfer coefficient for the silo wall. The main variables that can be studied using this computer model are thermal properties of the grain and type of materials of silo wall. To validate the accuracy of the model, two 19.0m diameter metal silos were used to store maize for observing grain temperature variations. In each metal Silo, 6 thermocouples were installed to measure the temperatures of grain at different depths and different radial distances from the metal silo center. The temperatures were recorded daily using a temperature recorder from the control panel. Each metal silo was filled with 660.56 and 440 metric tonnes of maize to depths of 1.64 and 1.25 metres respectively. Tests were started in May 1997 and ended in September 1997. The heat flow pattern was assumed symmetrical around the vertical and horizontal directions of the cylindrical silo, that is, the heat flow was radial and vertical and so the problem has been treated as two- dimensional. Local hourly weather data (air temperature, relative humidity, wind speed, and solar radiation on horizontal surface) and air-flow rates during aeration periods were used as model inputs to simulate the temperatures of grain during storage. Aeration in both metal silos were controlled manually with temperature limit settings. Measured and simulated grain temperatures were in close agreement for a test period of five months. Results indicated that the model and the parameter values used in the model are applicable for simulating temperatures of stored maize with and without aeration.

1.0 INTRODUCTION

The production of Agricultural commodities is seasonal, whereas the food industries require a continuous supply of raw materials. This gives rise to sophisticated storage and distribution systems. The quality is maintained in storage largely through the control of physical environment, the lowering of temperature and water activity so that the biological activity of both the stored materials and potential pests is minimised. In grains, low water activity combined with secondary control over temperature enables long-term storage. A grain bulk in storage is a human-made ecological system in which living organisms and their non-living environment interact on each other. Deterioration of stored grain results from interactions among physical, chemical and biological variables and factors. The main agents that contaminate and destroyed stored grains are insects, mites and fungi. The rate of reproduction and growth of these organism is mostly dependent on temperature and moisture content. Heat, moisture and carbon dioxide are produced by respiration of living organisms during the deterioration of grain. Therefore, grain temperature, moisture content, and increases in carbon dioxide concentrations in the intergranular air can be used as indicators of incipient grains spoilage. Metal silo have moisture condensation resulting from temperature fluctuations within the silo and hot spots due to elevated temperatures. Mathematical models can be developed to predict temperatures and moisture contents and carbon dioxide concentrations at various locations in the stored grains . The predictions can be used to decide the location of sensors for detecting incipient spoilage in the stored grains.

1.1 Modern Methods of Storing Grains.

There are three main modern methods of storing grains. They are storing in a metal or concrete silo, a crib and a warehouse. Commercial farmers and government use the metal silo which is the subject of this research mainly for shelled maize, and sorghum throughout the country . It involves extra handling like shelling/threshing, cleaning, drying and conveying into the silo; and treatment with chemicals. Silos, which are usually made of metal or concrete can be classified as horizontal (height is less than diameter) or vertical (height is greater than diameter) and can

vertical (height is greater than diameter) and can further be described as round, rectangle, cylindrical and box shaped.

The Federal Government of Nigeria started the construction of metal silos (fig 1) in the country in 1988. Ten of such metal silo complexes have been completed, across the country; while eight are in operation. These are Minna, Gombe, Akure, Ogoja, Lafiagi, Makurdi, Irrua and Jahun silo complexes. These completed metal silo complexes have been the responsibility of the Strategic Grains Reserve Storage Division (SGRSD) under the office of the Honourable Minister of the Federal Ministry of Agriculture and Natural Resources. Except Lafiagi silo complex which has a capacity to store 11,000 metric tonnes of grain, while others have capacity to store 25,000 metric tonnes. There are other silos owned by State Governments and private individuals in the country as well. Corrugated steel silos are light-weight structures, therefore their strength limitations dictate how they are filled and emptied. They must be unloaded centrally, to ensure that the perimental wall load is evenly distributed. Asymmetric emptying would create local stresses and metal failure. The speed of loading and unloading must be controlled to keep the live loads within design limits. The silos usually have a flat floor, with can be emptied by gravity flow through the central point and assisted by a sweep auger when the grain reaches its angle of repose. Each sheet of the silo is sealed to its neighbour by coating the overlap with a suitable mastic sealant. The bolts are fitted with composite washers of galvained steel and neoprene rubber, to prevent water seeping through the bolt holes. A small access door is usually provided, either near the floor level (with an internal trust plate), or just above the maximum height of fill, or in the roof.

Many storage silos installations in Nigeria and in tropics in general have failed to perform to expectation because adequate considerations were not given to the storage factors and anxillaries. The storage engineers and scientists should be involved during construction, installation and management of the silos. They understand the variables and factors of deterioration of grains during storage and hence are in the best position to advise on how to deal with them.



Figure 1 : Metal Silo

1.2 Temperature

Temperature is a key factor in the regulation of pest insect populations in stored grains. In the absence of metabolic heat release by heavy infestations of insects, weather influences are the major cause of temperature in bulk maize. The effects are transferred through the bulk by conduction and, where temperature gradients occurred, by convection (Muir, 1973). Stored product insects can survive at temperatures from 8 to 41°C (Sinha and Watters 1985). Their development and multiplication are optimum near 30°C and 50 - 70% relative humidity. Mites can develop at temperatures from 3-41°C with optimum temperature for the development and multiplication near 25°C. Fungi can develop at temperatures from -2 to 55°C with the optimum temperature near 30°C. There is considerable variation in optimum condition for different species. Conditions for optimum development of insects, mites and fungi may occur in localized regions of stored-grains ecosystem even when the average condition for the bulk prevents pest infestation. Mathematical models of heat and moisture can play a major role in designing and evaluating methods of reducing temperature and moisture gradient in stored grains, in predicting location within the silos where spoilage can occur, and in determining rates of deterioration. This project considers only the phenomenon of heat transfer in store grain bulks.

Grain may lose quality during drying, storage and aeration due to mould growth, protein damage and insect infestation. These factors are influenced by the moisture content and temperature of the grain, which in turn are affected by the air flow pattern in the grain bed. Within a silo, the factors which affect grain quality most are grain temperature, moisture and duration of storage.

All living organisms live and flourish within certain limits of temperature. The atmospheric temperature, temperature of the grain and the intergranular air temperature are all crucial variables for safe and prolonged storage of grains. Heat from external sources penetrates slowly into the grain bulk and it has been discovered that at about 300cm depth, the effect of variation in weather becomes minimal (Adewale, 1995). The metabolic heat production by sound dry grain is about 71J/m²·°K and that of wet grain is 54.34J/m²·°K. However, the amount of heat produced by insects, fungi and mould are considerably higher.

Therefore the bulk grain storage structure and the specific heat of the grains have to be taken into consideration. A knowledge of thermal properties and prediction of time-temperature histories of food products are required in all the food processing, storage and preservation processes. These are essential and useful for optimum design for process heat transfer equipment. Heat transfer takes place through one or more of three possible modes: conduction, convection and radiation. The temperature in various portions of the grain bulk may be different because of heat transfer through the silo wall, top and bottom surfaces of the grain bulk, as well as the variable heating of the silo wall due to solar radiation (including the heat emitted at night to the surroundings).

This study presents mathematical designs and numerical analysis to predict the time-temperature histories subjected to both heat and mass transfer effects. Two-dimensional conduction of heat transfer equations in radial and vertical directions for cylindrical storage bins were formulated and solved numerically to predict temperature changes during storage of maize in this project.

1.3.0 Literature Review

1.3.1 Simulation concepts

Simulation analysis is a natural and logical extension to the analytical and mathematical models inherent in operations research. It is the only tool that might be used in many situations which can not be represented mathematically due to the stochastic nature of the problem; the complexity of problem formulation, or the interactions needed to adequately describe the problem under study. The word simulation has been used rather uncare. Naylor et.al(1966) defined simulation as “A numerical technique for conducting experiments on a digital computer, which involves certain types of mathematical and logical relationship necessary to describe the behaviour and structure of a complex real-world system over extended periods of time”.

Simulation has also been described as the process of creating the essence of reality without over actually attaining that reality itself. Recent advances in simulation methodologies, soft ware availability and technical development have made simulation one of the most widely used and accepted tools in system analysis. Simulation modelling is often more of an “art” than a

science. This art is best cultivated rather than taught, although the basic tools and modelling logic can be gained through diligent study of simulation methodologies. The design of a simulation model itself is a critical portion of any study but not the only one with which the user must be concerned.

Simulation process involves:

Pre- Simulation Activities

(Problem definition, system analysis, Objectives and Aims)



Development Activities

(Design and Implementation)



Operational Activities

(Tactical design of Implementation)

It is worthy of note that in spite of the availability of simulation as one of the easiest tools in management science, it is also probably one of the hardest to apply properly and perhaps the most difficult from which to draw accurate conclusions.

To maintain grain quality during storage, grain must be protected from weather, insects, and growth of microorganisms. Many investigators have developed mathematical models of heat transfer for prediction of temperature in the bulk grain (Muir, 1980; Converse et al., 1973; Lo et al., 1975; Yaciuk et al., 1975; Metzger and Muir, 1983; Alagusandaram, 1994; Basunia et al. 1992). The heat transfer model has been solved for one dimensional, two dimensional and three dimensional configuration by the above researchers. The heat transfer model has been solved using analytical (Converse et al., 1975), finite difference (Muir et al. 1980) and finite element (Alagusandaram, 1994 and Basunia et al. 1992) methods. Investigator (Converse et al., 1995) also reported experimental data of wheat, barley and rape seeds use for validation of their models. Bruce (1985) and Sokhansanj (1986) showed that for accurate prediction of moisture and temperature of grain during storage, and thus grain quality, internal gradients must also be considered in simulation. In addition to conduction, (Muir et al., 1980) included a sub-model for

air movement by natural convection within the grain bulk. This did not significantly improve temperature predictions and computing time increased by about 25 times over that for two-dimensional conduction alone.

Two dimensional conduction heat transfer equations in radial direction for cylindrical storage silos were formulated and solved numerically to simulate the temperature variation during the storage of maize in the study. The temperatures of a sector of cylindrical bin are simulated using the finite - difference method of calculating heat transfer under transient conditions. Analytical methods have been developed to study temperature movement in cylindrical bins of wheat (Converse et. al;1969)but the solutions were limited by initial and boundary conditions. This led to the use of numerical methods with fewer limitations and more degree of accuracy.

The history of the measurement of heat transfer coefficient in grain beds is one of increasing estimates; as techniques for reducing moisture transfer effects have been developed. It is useful at this stage to consider the range of Reynolds numbers encountered in the deep bed drying and cooling of agricultural crops. Wakao and Kaguei(1982) have brought together much of the data in this area for both heat and mass transfer. Selectively examining the literature and correcting the transfer coefficient for the axial dispersion effect at low Reynolds number, the following correlation was proposed.

$$Nu = 2 + 1.1Re^{0.6} Pr^{1/3}$$

Where Nu, Re and Pr are the Nusselt, Reynolds and Prandtl numbers respectively.

In reviewing the experimental data on grain-to-air heat transfer coefficient in a packed bed, two sources of data are considered. Measurement made specifically on grain are presented first in the form of the volumetric heat transfer coefficient. Also, the upper-Reynolds number limits on the correlations examined to assess their suitability for use in the description of heat and moisture convection in the cooling of high moisture crops, where larger diameters and velocities occur. All the experiments reviewed employ some form of transient method and attempt to minimise the effect of moisture transfer. This is achieved by bringing the grain to equilibrium with hot air stream and then cooling the sample in a sealed container prior to the heat transfer experiment.

The basic computational methods for solving unhomogeneous sets of linear equations are the Gauss and Gauss-Jordan eliminations. However, a minimal amount of matrix/vector notations and basic rules are introduced. Then, two related subjects, namely, matrix inversion and the determinants are discussed. Finally, solution of m equations with n unknowns is described. Where m is the number of equations and n is the number of the unknowns. The air temperature at any geographical location has a direct bearing on the grain temperature within a silo. The grain-temperature with at any point within the silo is governed by the amplitude and the mean of the air – temperature wave. For locations with a climate that is similar to savannah forest (Middle Belt) such as Minna, the fluctuations in the grain – temperature wave are small.

1.3.2 Theory

To determine and predict teperature changes in a stored product,it is desirable to know the thermal conductivity,(K),and the specific heat transfer,(C), of the product. Therefore heat transfer is an essential operation in providing the energy for evaporation, distillation and drying. Heat energy is transferred by three mechanisms: conduction,convection and radiation. In many systems, all three operate simultaneously.

Conduction is primarily a molecular phenomenon requiring a temperature gradient as a driving force. A quantitative expression relating temperature gradient, the nature of the conducting medium, and rate of heat transfer is attributed to Fourier who in 1882 presented the relation

$$\frac{qx}{A} = -k \frac{dt}{dx} \quad \text{.(Met. and Muir, 1983)} \quad \text{—————} \quad (1)$$

Where qx = directional heat flow rate in watts,

A = area normal to the direction of heat flow in [m²]

x dt/dx = temperature gradient in the x direction in [m/ °K];and

k = thermal conductivity [W/mK]

The ratio qx/A, having the unit W/m^2 ; is referred to as the x - directional heat flux.

Therefore, the complete expression for the heat flux is

$$q/A = -k\Delta T \text{—————} \quad (2)$$

σ = Stefan-Boltzmann constant, 5.67×10^{-8} (W/m² K⁴)

T = Temperature of the air above the grain surface. (° C)

In the steady-state heat conduction in two dimensions (2D), there will be more than one significant space variable involved and the solution to Laplace's or Poisson's equation becomes much more involved. Techniques of solving the 2-dimensional Laplace equation are emphasised in this report with only numerical method to be discussed.

Numerical method is very versatile, it can handle bodies with complicated geometries and boundary conditions. Two dimension with heat flux will give this equation.

$$\frac{d^2 T}{dx^2} + \frac{d^2 T}{dy^2} + \frac{q}{k} = \frac{1}{\alpha} \frac{dT}{dt} \quad (5)$$

Where q - heat generated per unit volume

This could be converted to basic differential equations which can be solved either by iteration method or Gauss seidel, Gauss inversion and relaxation method.

Heating and/or cooling of bodies of different geometries under various boundary conditions have been applied to determine thermal properties by several investigators. (Myncke et al 1964) and Myncke and Ihebeak(1966) made use of the initial temperature rise within a certain region of finite body, due to a constant heat flux source which is the same as that in infinite solid. In another study, Abbounda(1984) employed the equivalent coefficient of thermal conductivity, which was defined as the sum of the heat transfer coefficients for the conduction and natural convection, in the conduction equations to predict temperatures of grain sorghum in small cylindrical steel bins (0.76m and 1.37m diameter). His results showed an improved accuracy for temperature predictions with the inclusion of natural convection.

Directing our attention to equation (5), several differencing schemes could be used to replace the time and space derivatives.

For initial approach, we shall write in central difference form so that, in two dimensions, we have,

$$\frac{T_{i-1,j} - 2T_{i,j} + T_{i,j+1}}{\Delta x^2} + \frac{T_{i-1,j} - 2T_{i,j} + T_{i,j+1}}{\Delta y^2} + \frac{q}{k} = \frac{1}{\alpha} \frac{dT_{i,j}}{dt} \quad (6)$$

The temperature of node i,j designated $T_{i,j}$ relates to the cylindrical section in Fig. 6. The cylindrical areas shown comprise a rectangular grid with which a total conducting medium is subdivided. The center points of such rectangles or sub-volumes also form a rectangular grid with points of intersection designated as nodes. Property value at a node point are assumed representative of the subvolume having that node as centre point. Properties are thus “lumped” as single values for each node in an array. Figure 6 shows an array where spacing in the x-direction are equal, with value Δx , and Δy is the constant spacing in the y direction. However, $\Delta x \neq \Delta y$.

1.3.3 Gauss Jordan Method

The Gauss-Jordan method is a variation of Gauss elimination. The major difference is that when an unknown is eliminated in the Gauss-Jordan method, it is eliminated from all other equations rather than just the subsequent ones. In addition, all rows are normalised by dividing them by their pivot elements. Thus, the elimination steps result in an identity matrix rather than a triangular matrix; as stated below. Consequently, it is not necessary to employ backward-substitution to obtain the solution.

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ a_{21} & a_{22} & a_{23} & c_2 \\ a_{31} & a_{32} & a_{33} & c_3 \end{array} \right]$$

↓

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & c_1^{(n)} \\ 0 & 1 & 0 & c_2^{(n)} \\ 0 & 0 & 1 & c_3^{(n)} \end{array} \right]$$

↓

$$\begin{array}{l} X1 \quad \quad \quad = c_1^{(n)} \\ \quad \quad X2 \quad \quad = c_2^{(n)} \\ \quad \quad \quad X3 = c_3^{(n)} \end{array}$$

Graphical depiction of the Gauss-Jordan method

To elucidate the difference between this technique and Gauss elimination.

The superscript (n)'s mean that the elements of the right-hand-side vector have been modified n times (for this case, n = 3).

One application of the inverse occurs when it is necessary to solve several systems of equations of the form:

$$[A] \{X\} = \{C\} \quad (7)$$

differencing only by the right-hand side vector {C}. Rather than solve each system individually, an alternative approach would be to determine the inverse of the matrix of coefficients. However, Gauss-Jordan elimination used backward elimination rather than backward substitution.

The inverse of a matrix may be calculated by applying Gauss-Jordan elimination. Consider a linear equation in matrix notation

$$Ax = y \quad (8)$$

where A is a square matrix. Assuming that no pivoting is necessary, a premultiplication of eq. 8 by a square matrix G yields

$$G = Ax = Gy \quad (9)$$

If G is chosen to be the inverse of A, namely A^{-1} , equation 8 reduce to:

$$x = A^{-1}y \quad (10)$$

which is the solution. In other words, Gauss-Jordan elimination is equivalent to premultiplication by

$$G = A^{-1} \quad (11)$$

Therefore, if the same operations performed in Gauss-Jordan elimination to the identity matrix (that is, multiplying rows by the same numbers as used in Gauss-Jordan elimination and subtracting rows in the same manner), then the identity matrix must be transformed to A^{-1} . This may be written symbolically as

$$GI = A^{-1} \quad (12)$$

To compute A^{-1} , we write A and I in an augmented array form

$$\left[\begin{array}{ccc|ccc} a_{1,1} & a_{1,2} & a_{1,3} & 1 & 0 & 0 \\ a_{2,1} & a_{2,2} & a_{2,3} & 0 & 1 & 0 \\ a_{3,1} & a_{3,2} & a_{3,3} & 0 & 0 & 1 \end{array} \right]$$

↓

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & a_{11}^{-1} & a_{12}^{-1} & a_{13}^{-1} \\ 0 & 1 & 0 & a_{21}^{-1} & a_{22}^{-1} & a_{23}^{-1} \\ 0 & 0 & 1 & a_{31}^{-1} & a_{32}^{-1} & a_{33}^{-1} \end{array} \right]$$

[I] [A]

Graphical depiction of the Gauss-Jordan method with matrix inversion

Note that the superscript -1's denote that the original value have been converted to the matrix inverse. They do not represent the value 1/aij.

Then we follow Gauss - Jordan elimination in exactly the same way as in solving a linear set of equations. When the left of the augmented matrix is reduced to a unit matrix, the right half becomes A^{-1} .

Pivoting is necessary for matrix inversion because the inversion scheme is essentially a Gauss elimination. However, the inverse matrix is not affected by a change in the sequential order of equations. One straight forward way to compute the inverse is using the Gauss- Jordan method. To do this, the coefficient matrix is augmented with an identity matrix . Then the Gauss - Jordan method is applied in order to reduce the coefficient matrix to an identity matrix. When this is accomplished, the right-hand side of the augmented matrix will contain the inverse.

1.3.4 Determinant

The determinant is an important quantity associated with each square matrix. Indeed, an inhomogeneous set of linear equations can not be uniquely solved if the determinant of the coefficient matrix is zero. On the other hand, a homogeneous set of linear equations has solutions

only when the determinant is zero.

The determinant of a matrix A of order N is denoted by $\det(A)$ and defined by

$$\det(A) = \sum (\pm) a_{i_1} a_{j_2} a_{k_3} \dots a_{r_N} \quad (13)$$

where the summation is extended over all permutations of the first subscripts of a, and (\pm) takes plus if the permutation is even and minus if it is odd.

For a 3 x 3 matrix, the determinant is

$$\det(A) = \det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11} a_{22} a_{33} + a_{21} a_{32} a_{13} + a_{31} a_{12} a_{23} - a_{11} a_{32} a_{23} - a_{21} a_{12} a_{33} - a_{31} a_{22} a_{13} \quad (14)$$

The product along the solid lines have positives signs in equation (14). The product of three numbers along dotted lines have negative signs in equation(14). This rule can not extended to matrix of 4x4 or greater. As the order of a matrix exceeds 3, direct calculation of the determinant by equation(13) becomes impractical because the amount of computations increase very rapidly. Indeed, a matrix of order N has N-factorial permutations, so the determinant of a 5x5 matrix, for example has 120 terms each of which needs four multiplications. The determinant of 10x10 matrix has more than 2×10^8 terms, each requiring nine multiplications. A practical of calculating the determinant is to use the forward elimination process of Gauss elimination.

The two important rules of determinants of determinants are:

Rule 1: $\det(BC) = \det(B) \det(C)$.

Which means that the dterminant of a product of matrices is the product of the determinants of all the matrices.

Rule2: $\det(M) =$ the product of all diagonal elements of M if M is an upper or lower triangular matrix. For example ,if all the diagonal elements of a triangular matrix are unity ,the determinant is unity . Therefore when the forward elimination is completed ,the original matix has been transformed to the U matrix of the LU decomposition. Then, the determinant can be

calculated by taking the product of all the terms along the diagonal line, then multiplying by 1 or -1 depending on whether the number of pivoting operations performed is even or odd respectively. This is the algorithm implemented in the program computing the determinant.

1.3.5 Solution of m Equations with n unknowns

In general, the number of equations m , can be less than the number of unknowns, n . The equation for such a problem can be written in the form of $Ax = y$ where the matrix A is not square but rectangular. For $m < n$, the number of solutions is infinite, but numerical value of a solution can not be determined uniquely. In this project, nonunique solutions of m equations with n unknowns, where $m < n$ were found. For consistency, a linear equation of $m < n$, can be considered as:

$$X + y = 1 \quad \text{-----} \quad (15)$$

where $m = 1$ and $n = 2$. The solution may be written as $x = 1 - y$ or $y = 1 - x$.

If we have m linearly independent equations for n unknowns and $m < n$, we can find m basic variable and $n - m$ free variables. By plotting the basic variable on the left side of equations and all the free variables on the right side, the set of m equations may be solved for the basic variables in terms of free variables.

The only requirement in choosing basic variables is that the m equations for the basic variables are not singular. Then, the selection of basic variables is not always unique. Nevertheless, they can be found systematically by using Gauss (or Gauss - Jordan) elimination devised for $m \times n$ matrices as explained in the program presented in pages (37-41).

1.4 Statement of Problem

Warm air can hold more moisture than cold air, hence when it cools down, it gets rid of some water in the form of dews. So, warm air in the grain near the silo bin walls cools and moves down along the inside wall surface of the silo, where the air and grain are warm. The grain, whether it be cereal, oilseed or legume, is alive and as such, is continually respiring. This respiration produces heat and moisture which, if present in excessive quantities, produce conditions suitable for growth of other injurious organisms, insects and moulds. This in turn,

will cause a loss in both quantity and quality of the grain whilst in storage. The rate of respiration and consequent heat production depend on the moisture content of the grain, the type of grain, its degree of maturity and ambient storage temperatures. Food grains should be properly dried after harvest to such a level of moisture that bacterial and fungi will not grow, while the development of mites and insects is considerably retarded. It is essential that food grains are dried quickly and yet efficiently. The drying procedure has to be adopted to suit the particular food grain because they differ in their biological make-up.

The maximum moisture content for safe storage in a metal silo is normally described as that at which the rate of respiration is low enough to prevent an accumulation of heat and consequent grain deterioration. The moisture content will be dependent of the ambient temperature, the higher this is, the lower must be the storage moisture content. The safe maximum moisture content which is in equilibrium with air at 70 percent relative humidity, at which level fungi development is controlled at 13%.

Seasonal fluctuations in atmospheric change the temperature pattern throughout the grain silo. Although grain may be stored at acceptable moisture, the moisture content may change in both time and space as a result of weather influences. Under non-isothermal conditions, the convection current caused by temperature gradients dominates the moisture movement in grain. Temperature and moisture change in stored grain result from internal and external sources of heat. The production of heat and moisture by respiration of grain, insects, mites, fungi and bacteria; and thermal (specific heat and thermal conductivity) and physical (bulk density and porosity) properties of the bulk are the main internal factors that affect the movement of heat and moisture in stored grain. External heat comes largely from atmospheric environment around the storage silo. The experiment for the project was done during the wet season; the temperature outside the storage silo is lower than the grain temperature inside the storage silo. Conversely during dry season months, grain temperature in the grain silo is lower than the outside air temperature. This difference between the centre temperature of the grain bulk and the outside ambient air temperature causes convection current in the grain accompanied by movement of moisture from high temperature to low temperature areas, which further enhances the outbreak of mould

growth. This causes accumulation of moisture from condensation either at the bottom or top of the silo depending upon the direction of the natural convection of the air within the stored grains. Also during the wet season, the convection air movement is reversed from that when the grains are cooling. The coolest grain is in the centre and the warmest near the silos walls and the top surface. As the warm air rises near the silo walls it is replaced with cooler air from the grain in the silo centre. As the warm air from the silo over space is drawn downward and contact the centre grain, it is cooled, its relative humidity increases; and moisture is transferred to the cooled grain. The rate of moisture transferred is usually slower during warm – up than when the grain is cooling during the dry season.

A knowledge of temperature distribution and moisture content distribution in the stored grain not only helps in identifying active deterioration, but also gives indication of the potential for deterioration. The metal silo used for this project was originally made for temperate region where the temperatures are not as high as in tropical region. The humid tropical is associated with high temperature fluctuation. Metal silo which therefore has high thermal conductivity in this region would conduct more heat to the grain and thereby increasing the bulk temperature. This results in moisture condensation, hot spots, mould, caking and eventual deterioration of the stored product.

The configuration of the problem is such that the silo is packed with the grains from the bottom of silo to a height less than the height of the silo. In the packed mode, the air spaces within the grain are trapped such that there is no free flow of air as to aid convection. The packed grains could therefore be approximated as a solid to which conduction is the significant mode of heat transfer. The problem is therefore reduced to a heat conduction one; in which the heat transfer from a section of the grain to another is essentially from particle to particle (conduction).

1.5 Justification

Crops grown for food fall into two categories, perishable and non-perishable crops. This refers to the rate at which a crop deteriorates after harvest and thus the length of time it can be stored. The need for grain storage in metal silo has been established. The use of metal for

storage structure have been proved successful in the climate region for long time periods. Therefore, due to seasonal changes, investigations into the performance of metal silos in the tropical region for long term storage period is needed to ascertain to a high degree the suitability of such structure year-round.

The simulation of temperature of stored grain in metal silo is important because it helps to prevent deterioration of grains. The parameters that are important are the outside temperature, the initial temperature of the grain, the moisture content of the grain, the relative humidity of the atmosphere, the humidity of the grain, the specific heat of the air and the grain; that of water, the density, the mass flow rate of air and both the conduction and convective heat and mass transfer coefficient.

It is therefore necessary to monitor the conditions within the silo, especially the temperature and moisture. This could be realised using temperature probes within the silo but this would be expensive, labour intensive and slow.

Collection of the temperature data and moisture content of various point in grain storage silos over a period of time is one way of finding the temperature content distribution. But this is an inefficient method, requiring a lot of time, cost and labour. This made mathematical model useful, based on physical principle that can potentially predict with accuracy the temperature and moisture in a grain storage silo. Further, using mathematical models, the effect of silo size, silo wall material and location of the temperature distribution among moisture content can be studied.

Although many researchers has done works on 2-dimensional transient heat transfer of temperature distribution in grain silo in the temperate region but not much has been done in Nigeria (tropical region). This made me to base my study on the temperature variation in grain silo in Nigeria, where there are high relative humidity and temperature.

1.6 Objectives

The objectives of this project are:

1. To develop a computer simulation technique for the prediction of temperature in grains stored in metal silos, under tropical conditions; using two dimensional mathematical model of heat transfer based on the finite difference method.
2. To measure the temperature variations at different locations of the storage silo in order to Validate the model developed in (1)

2.0 MATERIALS AND METHODS

2.1 Description of Storage Silos.

The silos under study is corrugated steel silos as shown in Fig.1. Steel silos are from corrugated galvanised steel sheets which have been rolled to a curve. Sheets are bolted together to form a vertical cylinder. This is anchored to a floor-level ring which is fixed to a concrete pad. A conical roof of steel sheets is fastened on top of the cylinder. Corrugation profiles are relatively shallow. They are specially designed to shed grain kernels without imposing excessive vertical loads on the structure. The cylinder of sheets support itself and its contents, with stiffening rings only at the base at eaves level, and around any access hatch. Vertical stiffeners, which bear the vertical load exerted by the grain, are fitted to the outside of the silo.

For the simulation model, two 19.0m of metal cylindrical storage silos; the height of each is 11.0m. Fig. 1. The thickness of the metal sheet was 1.25mm (Table.1).

Table 1. Thermal and Physical properties of silo wall material.

Properties	Steel
Thickness (mm)	1.25
Density (kg/m ³)	780
Specific heat (kJ/kg.k)	0.2265
Thermal conductivity (W/m.k)	64
Thermal diffusivity (m ² /h)	0.1393

Source : Sachdeve.(1993)

The floor was perforated for the aeration and discharge auger was installed through the centre of the floor to the other silos.

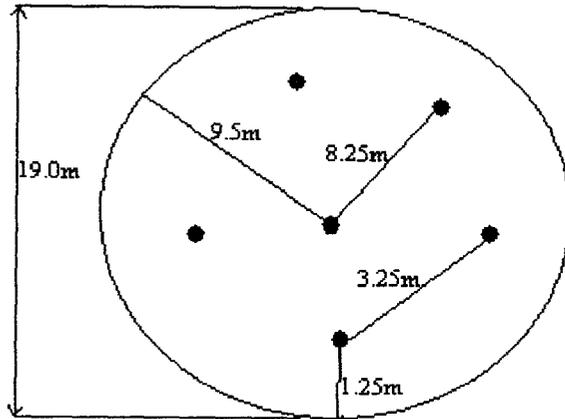


Fig. 2. The top elevation section of an experimental cylindrical storage silo with the positions of thermocouples.

Metal doors were made in the cylindrical side of the silos for the collection of moisture content samples from the desired locations of the silos. The doors were normally kept closed, so that both silos were almost air tight. Internal dimensions of both the storage silos, and positions of the thermocouples are shown in fig. 2.

Both silos were completely filled with cleaned maize at capacity of 660.56 and 440 metric tonnes to a depth of 1.74m and 1.25m respectively; at an initial moisture content of 10.5%. The silos were constructed on 1m raised concrete outside to the atmosphere. So all the surfaces of the silos were under almost similar ambient conditions.

2.2 Measurement of Temperature

To monitor the temperatures of the grain with the variation of ambient temperature, copper constantan thermocouples probes were attached and inserted at six positions in each storage silos including one at the centres as shown in Fig. 3. Thermocouples were set at depths of 0.35m from the floor of the silos and were placed at 3.25m interval from 1.25m from the wall of the silos respectively. Each of the five thermocouples were at a radius of 6.25m from the centre of the silos. Thermometer was used to measure the dry and wet- bulb temperature. Thermocouple probes were connected through an interface of AD converter then to the control panel for data

collection. The temperature readings from the thermocouple probes were recorded every eight hours. Data collected from May 1st- September 30th , 1997 (153 days) was used to validate the models.

2.3 Generalised Mathematical Formulation

2.3.1 Model Development

In developing the model, a sector of the cylindrical grain bin was divided into a finite number of spatial elements in the radial direction (Fig. 3).

The differential equations of greatest interest in conduction heat transfer are the heat equations, Poisson's equation and Laplace's equation.

The guiding heat transfer equation is of the Poisson type.

$$i.e \nabla^2 + \frac{q}{k} = 0 \Rightarrow -k\nabla T \quad \text{for conduction} \quad (16)$$

$$\nabla T = \frac{q}{hA} \Rightarrow q = \nabla ThA \quad \text{for convection} \quad (17)$$

$$Q = 1bv + Id \quad \text{for radiation} \quad (18)$$

Total heat transfer is therefore

$$Q_T = k\nabla^2 T + \nabla T + h_c 2\pi l (T_s - T_{air}) + I_{bv} + I_d \quad (19)$$

The solution to the above equation forms the basis for the temperature distribution with in the given silo.

From equation 16: The cylinder is considered as 2 - dimensional.

$$\text{Therefore } \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + q/k = 0 \quad (20)$$

Converting to numerical method of solution, replacement of time and space derivatives by finite

difference involves expressing the derivatives in terms of a truncated Taylor series of expansion. For x-direction, one-dimensional space T expanded is given by:

$$T(x + \Delta x) = T_{(x)} + \Delta x \frac{dT}{dx} + \frac{(\Delta x)^2}{2!} \frac{\partial^2 T}{\partial x^2} + \frac{(\Delta x)^3}{3!} \frac{\partial^3 T}{\partial x^3} + \dots + \frac{(\Delta x)^n}{n!} \frac{\partial^n T}{\partial x^n} \quad (21)$$

$$\text{and } T(x - \Delta x) = T_{(x)} - \Delta x \frac{dT}{dx} + \frac{(\Delta x)^2}{2!} \frac{\partial^2 T}{\partial x^2} - \frac{(\Delta x)^3}{3!} \frac{\partial^3 T}{\partial x^3} + \dots \pm \frac{(\Delta x)^n}{n!} \frac{\partial^n T}{\partial x^n}$$

* negative when n = odd and positive when n = even.

Similarly the equation for y direction are expandable in the same form.

The heat balance is in the form of

$$\text{Input} - \text{Output} = \text{Storage} \quad (22)$$

This implies that:

$$\begin{aligned} q(x)\Delta y\Delta z\Delta t - q(x + \Delta x)\Delta y\Delta t &= \Delta x\Delta y \rho_c \Delta T && \text{for x direction} \\ q(y)\Delta x\Delta z\Delta t - q(y + \Delta y)\Delta x\Delta t &= \Delta y\Delta x \rho_c \Delta T && \text{for y direction} \end{aligned} \quad (23)$$

$$\text{Therefore } \frac{q(x, y) - q(x, y + \Delta[x, y])}{\Delta(x, y)} = \frac{\rho_c \Delta T}{\Delta t} \quad (24)$$

Taking the limit yields

$$-\frac{\partial q}{\partial x} = \rho c \frac{\partial T}{\partial t} \quad \text{in the x direction} \quad (25)$$

Substituting Fourier's law of heat conduction produces

$$K \frac{d^2 T}{dx^2} = \frac{\partial T}{\partial t} \quad \text{and} \quad \frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1}^a - 2T_i^a + T_{i-1}^a}{\Delta x^2} \quad (26)$$

$$\text{and } \frac{dT}{dt} = \frac{T_i^{a+1} - T_i^a}{\Delta t}$$

This implies that

$$K \left[\frac{T_{i+1}^a - 2T_i^a + T_{i-1}^a}{\Delta x^2} \right] = \frac{T_i^{a+1} - T_i^a}{\Delta t} \quad (27)$$

$$\text{if } \frac{K\Delta t}{(\Delta x)^2} = \lambda$$

T_{i+1}^a = temp increment

T_{i-1}^a = Previous temp.

then $T_i^{a+1} = T_i^a + \lambda(T_{i+1}^a - 2T_i^a + T_{i-1}^a)$ for 1 - dimensional system _____ (28)

For 2 - dimensional system

$$T(x + \Delta x) + T(x - \Delta x) = 2T(x) + 2 \left[\frac{\Delta x^2}{2!} \right] \frac{d^2 T}{dx^2}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T(x + \Delta x) + T(x - \Delta x) - 2T(x)}{(\Delta x)^2}$$

and similarly $\frac{\partial^2 T}{\partial y^2} = \frac{T(y + \Delta y) + T(y - \Delta y) - 2T(y)}{(\Delta y)^2}$

$$\therefore \frac{d^2 T}{dx^2} + \frac{d^2 T}{dy^2} = \frac{T(x + \Delta x) + T(x - \Delta x) - 2T(x)}{(\Delta x)^2} + \frac{T(y + \Delta y) + T(y - \Delta y) - 2T(y)}{(\Delta y)^2} \quad (29)$$

Incorporating the bin and stored product parameters BS:

$$T_{inc} - \frac{(\Delta x)^2 q}{K} = \lambda K Temp$$

$$T_{inc} = \lambda K Temp + \frac{(\Delta x)^2 q}{K} \quad (30)$$

Incorporating the (B) and (S) results:

$$BST_{inc,r} = \lambda K Temp + \frac{(\Delta x)^2 q}{K} \quad (31)$$

where $B = 2\pi l \Delta_3 = 2\pi l = y \frac{\Gamma_n}{R} \sin \theta$

i.e. $\left(y \frac{\Gamma_n}{R} \cos \theta - \frac{\Gamma_n}{R} \sin \theta \right) (k / e_c) T_{inc} = (\Delta x)^2 (q / k) + (K \lambda) Temp$

$$B = y \frac{\Gamma_n}{R} \cos \theta - \frac{\Gamma_n}{R} \sin \theta$$

where y = silo level (i.e. length)

Γ_n = present radius under consideration

θ = included angle of the chord under consideration

R = maximum radius

$S = \alpha$ = thermal diffusivity = K/e_c

That is

$$\left(y \frac{\Gamma_n}{R} \cos \theta - \frac{\Gamma_n}{R} \sin \theta \right) (k / e_c) T_{incr} = (\Delta x)^2 (q / k) + (K\lambda) \text{Temp}$$

$$= \frac{(\Delta x)^2}{S} \frac{(q / k)}{Q} + \frac{(K\lambda)}{L} \text{Temp}$$

$$B\alpha T_{incr} = L \text{Temp} + SQ \quad \text{-----} \quad (32)$$

$$T_{incr} = (L \text{Temp} + SQ) / B\alpha$$

The guiding equation for convection is of the form

$$q = h_c A (T_{surf} - T_{air}) = h_c 2\pi r l (T_{surf} - T_{ir})$$

The only unresolved parameter is h_c which is the convective heat transfer equation.

This parameter has been solved by Kreth (1965) as

$$h_c = 0.0239 \frac{K}{D} \left(\frac{VD\rho_A}{\mu} \right)^{0.805} \quad \text{-----} \quad (33)$$

where k = thermal conductivity of air

D = Bin parameter

v = wind speed

ρ_a = density of air

μ = viscosity of air

During low air flow rate of less or equal to 0.005 m/s ρ_a has been found to be

$$\rho_a = 1.293 / (1 + 0.00367 T_a) \quad \text{-----} \quad (34)$$

T_a = air temperature

The given temperature model was based on a two-dimensional transient heat conduction equation with the associated boundary conditions and was solved using the finite difference method. For a cylindrical geometry with Gauss - Jordan elimination procedure for solving a set of m equations with n unknowns ($m < n$, or $m = n$).

It was assumed that temperature distributions in the grain bin are symmetric about the vertical central axis, that is, heat flow in the circumferential direction is negligible. In developing

the model, a sector of the cylindrical grain bin was divided into a finite number of spatial elements in the vertical and radial directions (Fig.3). There was a total of 143 spatial elements in the grain section by setting $\Delta r = 1.085$ m and $\Delta z = 1.1$ m in this study. The equation for predicting the temperature of any interior spatial grain element (m, n) at the end of the time increment, $t+\Delta t$, can be developed by a heat balance method and written in the finite difference form.

2.4 Assumption and Final Governing Equations

The following simplifying assumptions are made for deriving the governing equation of heat flow through bulk maize.

- (1) The physical (bulk density) and thermal (specific heat capacity and thermal conductivity) properties of grain are constant and uniform throughout the silos;
- (2) heat transfer by conduction in the circumferential direction is negligible;
- (3) heat is transferred at the centre of the silo by mass transfer of moisture in the radial direction;
- (4) heat flow patterns are symmetrical around the vertical centre line axis of the cylindrical silos;
- (5) internal heat generation within the grain silo is negligible;
- (6) air relative humidity within the grain bulk is dependent upon dry and wet- bulb temperature of the air within the grain bulk and is affected by changes in outside air temperature; and
- (7) temperature differences between the air within the grain bulk and grain is negligible. With the above assumptions, the general differential equation of heat flow in two dimensions in a cylindrical coordinate systems can be written as

$$\frac{\delta T}{\delta t} = \alpha \left(\frac{\delta^2 T}{\delta r^2} + \frac{1}{r} \frac{\delta T}{\delta r} + \frac{\delta^2 T}{\delta y^2} \right) \quad (35)$$

$$\text{and } \alpha = \frac{K_g}{\rho c_p}$$

When α , kg , ℓ , cp and T are the thermal diffusivity, thermal conductivity, bulk density, specific heat capacity and temperature of the stored grain, respectively, α and y are the radial and vertical coordinate of the cylindrical storage silo, respectively, and t is the time. According to Carslaw and Jaeger,(1959); the initial condition for equation (35) is,

$$T(r, y, t) = T_i(r, y) \text{ for } t = 0$$

Term T_i is the initial grain temperature. The boundary condition for equation (35) given by Carslaw and Jaeger(1959) is,

$$K_g \left(\frac{\delta T}{\delta r} + \frac{\delta T}{\delta y} \right) + h_c (T - T_a) = 0 \text{ for } t > 0$$

Considering the boundary surfaces of the silo is subjected to a similar convective heat transfer coefficient and ambient temperature; since the initial and boundary conditions are determined by the varying ambient temperature, it is difficult to develop an analytical solution for equation (35). As an alternative to this, infinite difference methods (such as presented in the next section).

2.5 Method of Solution

Many methods exist, they are analytical, finite difference and finite element. With the reason given above, Finite Difference method was used.

2.5.1 Finite- Difference Method

In equation (35), temperature T is a function of the independent variables of radial length from the centre of the site; axial distance y from the centre of the silo and storage time t . In the r - y plane, any point of coordinate r and y is represented by $r = m\Delta r$ and $y = n\Delta y$, where m and n are integers and $\Delta r = \delta r$ and $\Delta y = \delta y$. the temperature at any point may be denoted by $T_{m,n}$. in addition to being discretized in space, the problem must be discretized in time. The integer P is introduced for this purpose, where $t = p\Delta t$. The subscript P is used to denote the

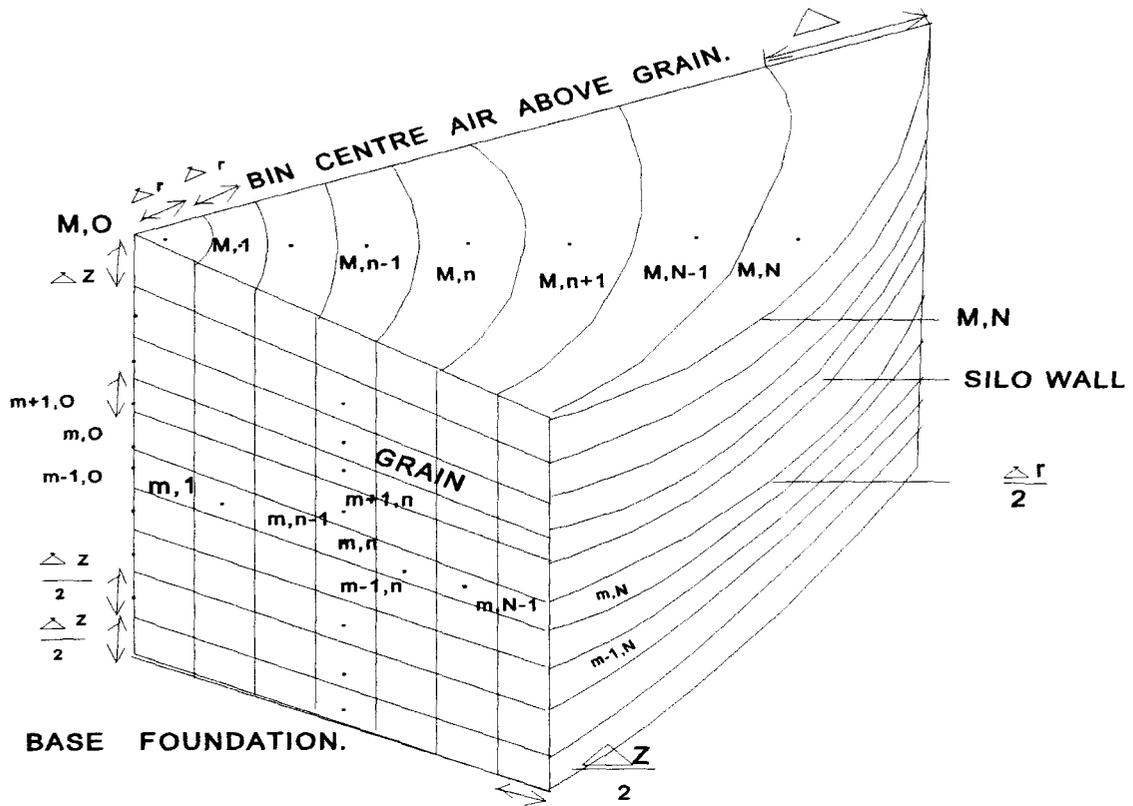


FIGURE 3: A SECTION OF CYLINDRICAL GRAIN BIN DIVIDED INTO A FINITE NUMBER OF SPATIAL ELEMENTS.

time dependence of T, and the time derivative is expressed in terms of the difference in temperatures associated with the new (P+1) and previous (p) times. Hence calculations are performed at successive times separated by interval Δt .

The explicit form of finite difference of equation (35) for the interior mode m, n of the storage silo is

$$\frac{1}{\alpha} \left(\frac{T^{p+1}_{m,n} - T_{m,n}^p}{\Delta t} \right) = \left(\frac{T_{m+1,n}^p + T_{m-1,n}^p - 2T_{m,n}^p}{(\Delta r)^2} \right) + \frac{1}{m\Delta p} \left(\frac{T_{m+1,n}^p - T_{m,n}^p}{\Delta r} \right) + \left(\frac{T_{m,n+1}^p + T_{m,n-1}^p - 2T_{m,n}^p}{\Delta y^2} \right)$$

solving for the nodal temperature at the new (p + 1) time and assuming that $\Delta r = \Delta y$, it follows that

$$T^p_{m,n} = F_0 \left\{ T_{m+1,n}^p \left(1 + \frac{1}{m} \right) + T_{m-1,n}^p + T_{m,n+1}^p + T_{m,n-1}^p \right\} + T_{m,n}^p \left\{ 1 - F_0 \left(4 + \frac{1}{m} \right) \right\}$$

(Basunia et al. 1996).

where F_0 is a finite- difference form of the Fourier number

$$F_0 = \frac{\alpha \Delta r}{(\Delta r)^2}$$

Equation (35) is invalid at the centre of the silo where $1/r(\delta T/\delta r)$ becomes an indeterminate quantity

However, using L' Hospital's rule, Equation (35) can be transformed into the following form

$$\lim_{r \rightarrow 0} \left(\frac{1}{r} \frac{\delta T}{\delta r} \right) = \frac{\delta^2 T}{\delta r^2}$$

Now equation (35) for the centre of the silo can be written as

$$\frac{\delta T}{\delta t} = \alpha \left(\frac{\delta^2 T}{\delta r^2} + \frac{\delta^2 T}{\delta y^2} \right) \text{-----} (38)$$

The finite – difference form of equation (38) for the centre of the silo where $r = 0$ ($m = 0, n = 0$) at new time (p+1) is

$$T^{p+1}_{0,0} = F_0 (4T^p_{1,0} + T^p_{0,1} + T^p_{0,-1}) + T^p_{0,0} (1-6F_0) \text{-----} (39)$$

For a constant boundary temperature the solution of equation (35) is simple and can be obtained by equation (36) and (39). But in practice, the surface temperature changes with variation of the outside temperature. For point on such surfaces which are exposed to convective conditions, the finite difference equation must be obtained by applying the energy balance method to a control volume about that node as reported by Frank and David. Thus the finite difference form of the equation (36) for the surface points with specified convective boundary condition at the new time (p+1) is

$$T_{m,n}^{p+1} = F \left(2T_{m-1,n}^p + T_{m,n-1}^p + T_{m,n+1}^p + \frac{2B_i T_a^p}{1 + (h_c L_w / K_w)} \right) + \left(1 - 4F_o - \frac{2F_o B_i}{1 + (h_c L_w / k_w)} \right) T_{M,N}^p$$

Where the finite – difference form of the Biot number is

$$B_i = \frac{h_c \Delta r}{K_g}$$

Terms K_w and L_w are the thermal conductivity and thickness of the silo wall, respectively, and T_a^p is the ambient temperature at time P.

Equations (36, 39 and 40) are explicit because unknown nodal temperatures for the new time are determined exclusively by known nodal temperatures for the previous time. Hence a calculation of the unknown temperatures is straight-forward. Since the temperature of each node is known at $t = 0$ ($p = 0$) from prescribed initial conditions, the calculations begin at $t = \Delta t$ ($p = 1$), where equations (36, 39 and 40) are applied to each interior node, the centre node and to each surface node of the silo, respectively, to determine temperature. With temperatures known for $t = \Delta t$, the appropriate finite-difference equation is then applied at each node to determine its temperature at $t = 2\Delta t$ ($p = 2$). In this way, the transient temperature distribution is obtained by successively incrementing t by Δt . The Gauss-Seidal iteration method is used to solve equations (36, 39 and 40) (Sastry)

2.6 Computer Implementation

2.6.1 Development of Simulation Procedure

In this study, several assumptions were made due to information available to make a more complete model. The assumptions were made such that the model would simulate temperatures higher than might be expected since deterioration of grains normally increase with increase in temperature of the grain and to give some factor of safety in the results. The skeleton flow chart (Fig 7) shows a simplified version of the simulation procedure. After reading the input parameters, simulation begins for each month using the temperature data. Normally simulation begins during the rainy season and continues for duration of five months. Based on input parameter controlling fan operation, the grains conditions are determined using the conduction/forced convection sub routines at the appropriate time interval. The cost and the operating time required to carry out a simulation on the computer varied greatly depending upon the amount of aeration time. Using the Pentium computer at the department in the University to perform the simulations, aeration, as model by the forced convection components, increase computer demands by a factor by nearly 15 when compared with simulations involving no ventilation. The complete model written in C-program notation, and its validation and application as in the programme presented in pages (37-41).

PROGRAM FLOWCHART

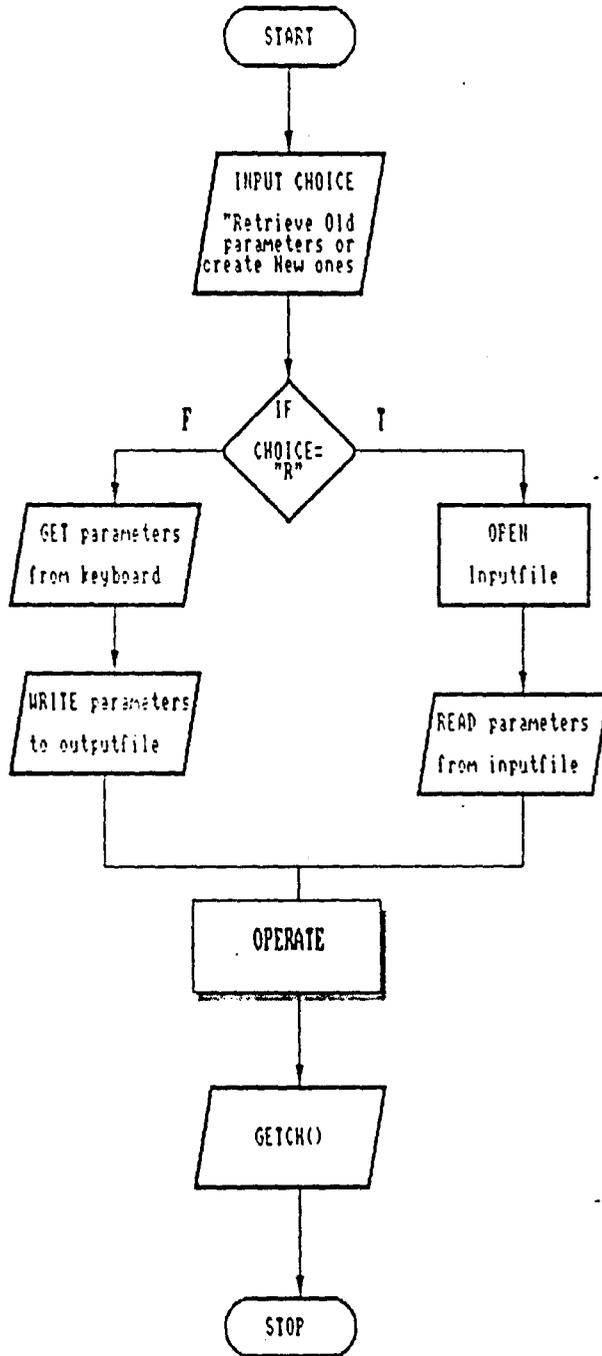
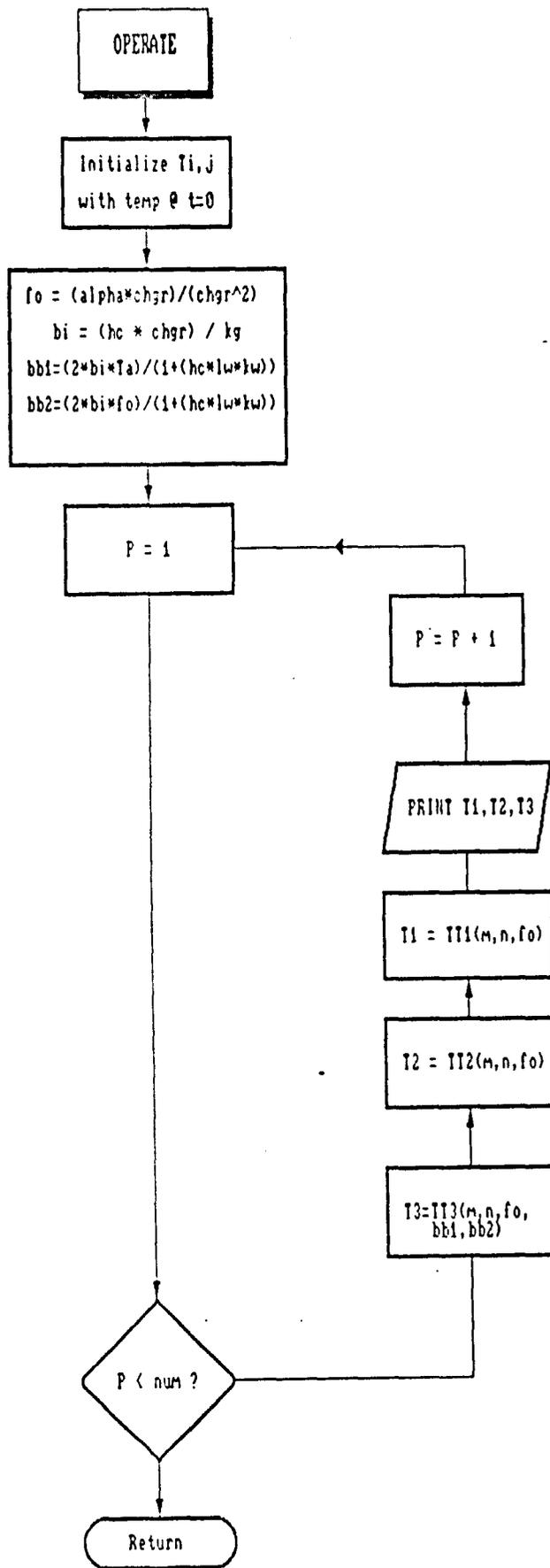


FIG. 4: COMPUTATIONAL FLOW CHART

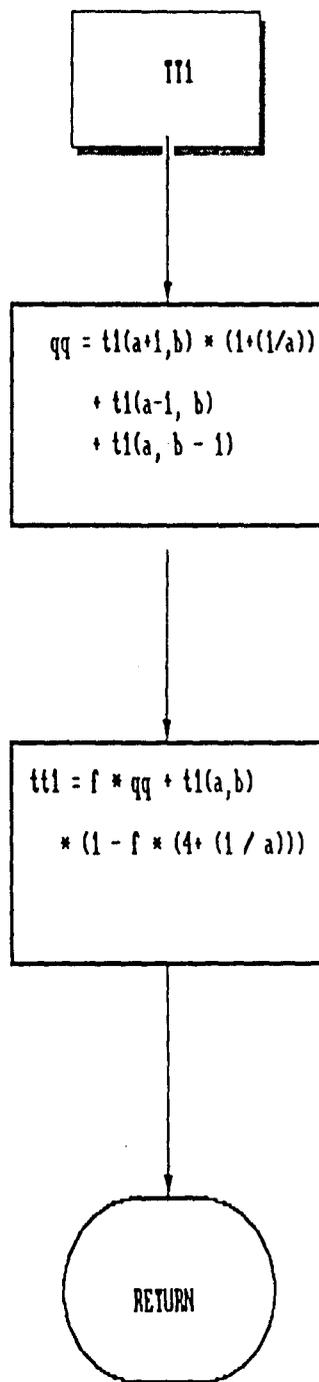


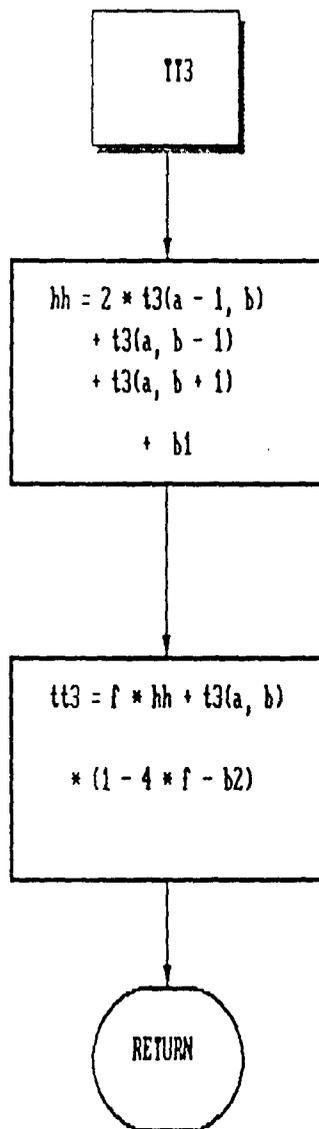
t2

$gg = 4 * t2(a + 1, b)$
 $+ t2(a, b + 1)$
 $+ t2(a, b - 1)$

$tt2 = f * gg + t2(a, b)$
 $* (1 - 6 * f)$

RETURN





2.6 Computer Implementation

2.6.1 Development of Simulation Procedure

In this study, several assumptions were made due to information available to make a more complete model. The assumptions were made such that the model would simulate temperatures higher than might be expected since deterioration of grains normally increase with increase in temperature of the grain and to give some factor of safety in the results. The skeleton flow chart (Fig 7) shows a simplified version of the simulation procedure. After reading the input parameters, simulation begins for each month using the temperature data. Normally simulation begins during the rainy season and continues for duration of five months. Based on input parameter controlling fan operation, the grains conditions are determined using the conduction/forced convection sub routines at the appropriate time interval. The cost and the operating time required to carry out a simulation on the computer varied greatly depending upon the amount of aeration time. Using the Pentium computer at the department in the University to perform the simulations, aeration, as model by the forced convection components, increase computer demands by a factor by nearly 15 when compared with simulations involving no ventilation. The complete model written in C-program notation, and its validation and application as in the programme presented in pages (37-41).

2.6.3 Programme Listing

```
/* THIS PROGRAM COMPUTES INTERIOR, CENTRE AND SURFACE
TEMPERATURE */
#include <stdlib.h>
#include <math.h>
#include <stdio.h>
#include <conio.h>

void GETVALS(void);
void READFL(void);
void WRITEFL(void);
void OPERATE(int NN);

float TEMP, ALPHA, CHGR, HC, KG, KW, LW, TA;
int N, M, NUM;

float T1[20][20];
float T2[20][20];
float T3[20][20];

float TT1(int A, int B, float F);
float TT2(int A, int B, float F);
float TT3(int A, int B, float F, float B1, float B2);

main()
{
    char CH;
    clrscr();
    printf("ENTER (R) TO RETRIEVE EXISTING PARAMETERS \n");
    printf("ENTER (N) TO CREAT NEW PARAMETERS ");
        scanf("%c",&CH);
        printf("\n ENTER VALUE OF M"); scanf("%d",&M);
        printf("\n ENTER VALUE OF N"); scanf("%d",&N);
        printf("\n ENTER NUMBER OF ITERATIONS EXPECTED
");scanf("%d",&NUM);
        printf("\n ENTER TEMPERATURE VALUE AT T = 0
");scanf("%f",&TEMP);
        if ((CH=='R') || (CH == 'r'))
            {
                writefl();
                operate(NUM);
            }
        if ((CH=='N') || (CH == 'n'));
            {
                getvals();
                readfl();
                operate(NUM);
            }
}
```

```

    }
    SCANF("%C",&CH);
    CLRSCR();
    RETURN 0;
}

```

```

VOID GETVALS(VOID)

```

```

{
    PRINTF("\n ENTER THERMAL DIFFUSIVITY ");SCANF("%F",&ALPHA);
    PRINTF("\n ENTER CHANGE IN RADIAL LENGTH ");SCANF("%F",&CHGR);
    PRINTF("\n ENTER CONVECTIVE HEAT TRANSFER ");SCANF("%F",&HC);
    PRINTF("\n ENTER THERMAL CONDUCTIVITY ");SCANF("%F",&KG);
    PRINTF("\n ENTER BIN WALL THICKNESS ");SCANF("%F",&LW);
    PRINTF("\n ENTER AMBIENT TIME AIR TEMPERATURE
");SCANF("%F",&TA);
    PRINTF("\n ENTER THERMAL CONDUCTIVITY OF BIN WALL MATERIAL
");SCANF("%F",&KW);
    RETURN;
}

```

```

VOID READFL (VOID)

```

```

{
    FILE * INPUTFILE;
    IF ((INPUTFILE = FOPEN("VALUES.DOC","W"))==NULL)
    PRINTF("FILE COULD NOT BE OPENED\n");
    ELSE
    {
        FPRINTF(INPUTFILE,"%F %F %F %F %F",ALPHA,CHGR,HC,KG,KW);
        FPRINTF(INPUTFILE,"%F %F",LW,TA);
    }
    FCLOSE(INPUTFILE);
    RETURN;
}

```

```

VOID WRITEFL (VOID) {

```

```

    FILE* OUTPUTFILE;
    IF((OUTPUTFILE =FOPEN("VALUES.DOC","R"))==NULL)
    PRINTF("FILE COULD NOT BE OPENED\n");
    ELSE
    {
        FSCANF(OUTPUTFILE,"%F %F %F %F %F",&ALPHA,&CHGR,&HC,&KG,&KW);
        FSCANF(OUTPUTFILE,"%F %F",&LW,&TA);
    }
    FCLOSE(OUTPUTFILE);
    RETURN;
}

```

```

VOID OPERATE(INT NN)
{
FILE * RESULTFILE;
INT I,J,P;
FLOAT FO, BI, BB1, BB2, AVE1, AVE2, AVE3, SUM1, SUM2, SUM3;
CLRSCR();
SUM1 =0; SUM2=0; SUM3=0;
IF((RESULTFILE=FOPEN("CON","W"))==NULL)
PRINTF("FILE COULD NOT BE OPENED\n");
ELSE
{
FOR (I=1; I<=20; I++)
{FOR (J=1; J<=20; J++)
{
T1[I][J] = TEMP;
T2[I][J] = TEMP;
T3[I][J] = TEMP;
}
}
FO = (ALPHA * CHGR) / (CHGR * CHGR);
BI = (HC * CHGR) / KG;
BB1 = (2 * BI * TA) / (1 + (HC * LW / KW));
BB2 = (2 * BI * FO) / (1 + (HC * LW / KW));

FPRINTF(RESULTFILE, "*****\n
");
FPRINTF(RESULTFILE, "\n          TEMPERATURES \n");
FPRINTF(RESULTFILE, "\n T          INTERIOR    CENTRE    SURFACE \n");

FPRINTF(RESULTFILE, "*****\n
");
FOR(P=1;P<=NN;P++)
{
IF (P == 1)
{
T1[M][N] = TT1(M,N,FO);
T2[M][N] = TT2(M,N,FO);
T3[M][N] = TT3(M,N,FO,BB1,BB2);
}
ELSE
{

T1[M+1][N] = TT1(M+1,N,FO);
T1[M-1][N] = TT1(M-1,N,FO);
T1[M][N+1] = TT1(M,N+1,FO);
T1[M][N-1] = TT1(M,N-1,FO);
T1[M][N] = TT1(M,N,FO);
}
}
}

```

```

T2[M+1][N] = TT2(M+1,N,FO);
T2[M-1][N] = TT2(M-1,N,FO);
T2[M][N+1] = TT2(M,N+1,FO);
T2[M][N-1] = TT2(M,N-1,FO);
T2[M][N] = TT2(M,N,FO);

T3[M-1][N] = TT3(M-1,N,FO,BB1,BB2);
T3[M][N+1] = TT3(M,N+1,FO,BB1,BB2);
T3[M][N-1] = TT3(M,N-1,FO,BB1,BB2);
T3[M][N] = TT3(M,N,FO,BB1,BB2);
}
SUM1 = SUM1 + T1[M][N];
SUM2 = SUM2 + T2[M][N];
SUM3 = SUM3 + T3[M][N];
FPRINTF(RESULTFILE," %4D %14.5F %14.5F %14.5F
\n",P,T1[M][N],T2[M][N],T3[M][N]);
}
}
AVE1 = SUM1 / NN;
AVE2 = SUM2 / NN;
AVE3 = SUM3 / NN;

FPRINTF(RESULTFILE,"*****\n
");
FPRINTF(RESULTFILE," %14.5F %14.5F %14.5F\n",AVE1,AVE2,AVE3);
RETURN;
}

```

```

FLOAT TT1(INT A, INT B, FLOAT F)
{
FLOAT QQ, QQ1;
QQ = T1[A+1][B]*(1+(1/A))+T1[A-1][B]+T1[A][B+1]+T1[A][B-1];
QQ1 = F * QQ + T1[A][B] * (1-F*(4+(1/A)));
RETURN(QQ1);
}

```

```

FLOAT TT2(INT A, INT B, FLOAT F)
{
FLOAT GG, GG2;
GG = 4 * T2[A+1][B]+T2[A][B+1]+T2[A][B-1];
GG2 = F * GG + T2[A][B] * (1-6*F);
RETURN(GG2);
}

```

```

FLOAT TT3(INT A, INT B, FLOAT F, FLOAT B1, FLOAT B2)
{
FLOAT HH, HH3;

```

```
HH = 2 * T3[A-1][B]+T3[A][B-1]+T3[A][B+1] + B1;  
HH3 = F * HH + T3[A][B] * (1-4 * F - B2);  
RETURN(HH3);  
}
```

2.7 VALIDATION OF MODEL

2.7.1 Grain Aeration Test

Validation of the mathematical model by comparing predicted output with experimentally determined data is required to assure reasonable accuracy. The critical parameter requiring validation are the predictions of grain temperature. Experimental values were obtained for maize stored in aeration from silos. These were compared with values predicted by the computer model.

Two 19.0m – diameter grain silo with partially perforated ventilation floor located at Strategic Grains Reserve, Minna silo complex were used. Two each 1Kw 30cm nominal diameter fans were used to provide forced air ventilation. 6-grids of 18 copper –constantan thermocouples on the inside radius were installed for temperature measurement (fig.4)

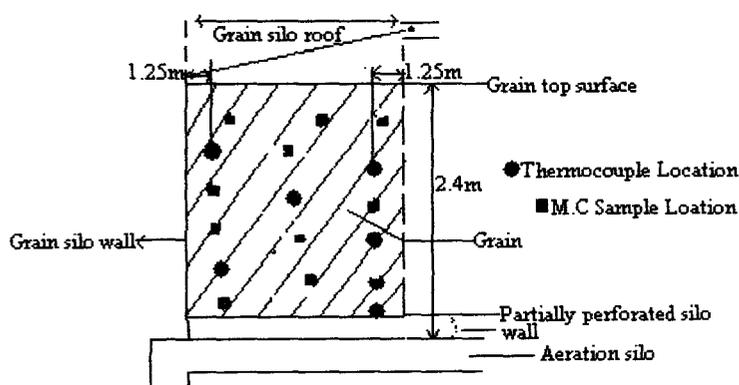


Fig. 5. Cross section of experimental aeration silo showing the thermocouple and grain sampling locations

Temperature were measured by electronic recorder attached to the operation panel (accuracy $\pm 10^0\text{C}$) was used to record grain temperatures at the two silos.

Moisture contents determination of maize samples were made according to the oven-drying method: ASAE standard s 352 (American Society of Agricultural Engineers 1906). The accuracy of this method is 0.5 percentage points.

In Figure 5, the moisture content with the temperature of the grains were measured at time interval with respect to the radius and height which are depended on the cylindrical shape of the metal silo.

Two-dimensional conduction heat transfer equation in radial dimensions for cylindrical storage silos were formulated and solved numerically to predict moisture content with temperature changes. It was assumed that a true equilibrium is obtained between the air and the grain during ambient temperature aeration with low air flow rates.

Grain depth were at 200cm and 250cm from the centre of the silos along the four radii to be measured. Samples were collected for the moisture content determination at reception, floor of the silos at 0, 60, 120, 180, 210cms respectively from the silo centre line along four radii at 90o apart using tube probe. The probe was spaced (i.e. 1x10 cm) 40 cm apart with the centre of opening as first at about 21cm from the point end of probe. The total length of the probe is 1.8m. The probe is always inserted into the grain until the pointed end of the probe struck the silo floor.

The experiments were done once in a week for five months, for moisture content and temperature of grains stored (i.e. maize). Dates were taken for the temperature measured at different depths and samples were taken. The samples taken were measured with moistometer and compared with oven method which sample was dried to 130^o C for 10 hours in convectional oven. Bulk densities were determined and predicted.

3.0 RESULTS AND DISCUSSION

There are not much difference between the measured and Simulated temperature in Figs (6-8) for silo 6 with a difference of about 1-2^o C while the ambient temperature remain higher than both the measured and the simulated temperatures. Therefore, there are no discrepancy within and they are in agreement with each other. For Figs (9-11) of silo 8 follow the same pattern as Figs (6-8) but the month of July have the same measure and simulated temperature and these differs between 1.5 to 4^o C from the other months. The interior temperatures in Figs (9-11) are higher due to the closeness to the walls of the silos.

Temperature Variations – Variation of dry – bulb temperatures of the ambient air with storage are shown in the graph (Figs 6-11) for silo 6 and silo 8). The initial dry - bulb temperatures of the grain at the center of silo 6 was 22.5^o C and silo 8 was 22.4^oC. They were 2.5^oC and 3.0^oC lower than dry-bulb temperature of the ambient air temperature: immediately, after filling the silo with cleaned maize. And almost similar difference between the dry – bulb temperatures of the grain and the ambient air were observed for all the location of both silos, at the beginning of the study. These indicate that initial grain temperature was uniform throughout the grain silo.

The finite-difference of heat transfer model is coded in C- Programming to predict the temperature distribution in the storage silo. The simulated temperature shown in Figs. (6-11) for $\Delta r = \Delta y$ which is equal to 1.085metres and $\Delta t = 8$ hours. The physical and thermal properties used for the cleaned maize were specific heat, 0.2265KJ/Kg.K, thermal conductivity, 0.28W/m. K and bulk density, 780Kg/m³ (ASAE Standard 1991). Thermal and physical properties of silo wall materials are shown in table 1.

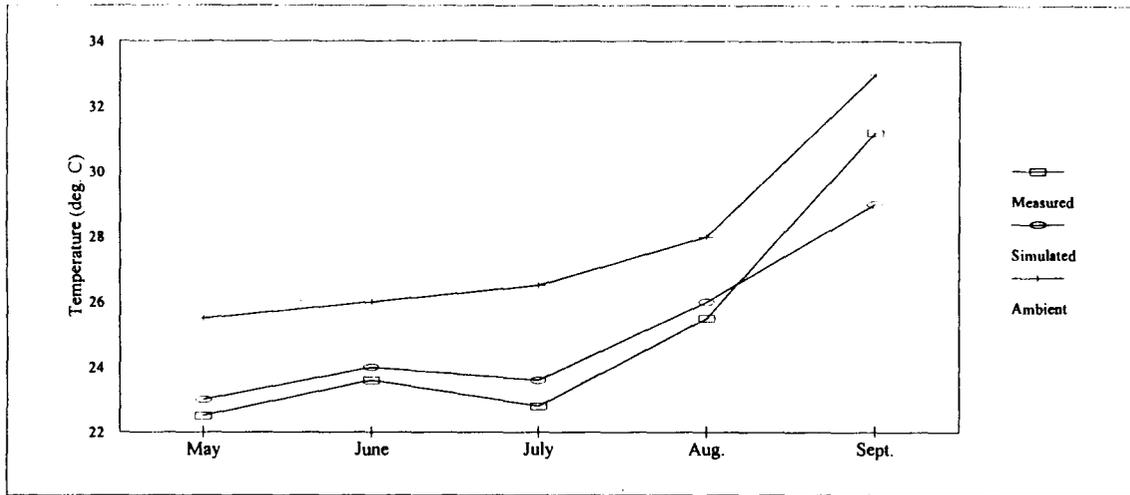


Fig. 6: Measured and simulated temperature (dry bulb) at centre at a depth of 1.2m from the surface of the grain bulk and dry bulb temperature of the ambient temp.

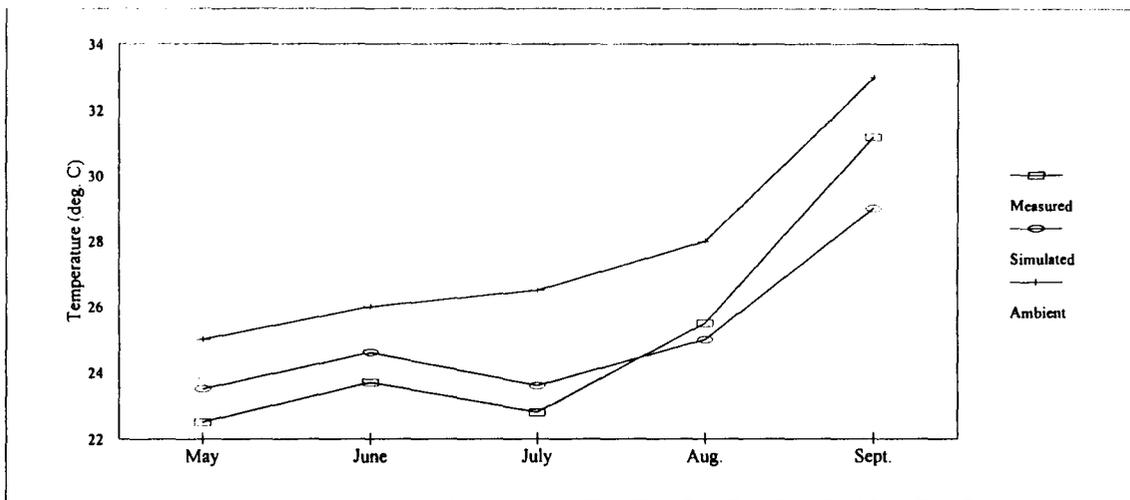


Fig. 7: Measured and simulated temperature (dry bulb) at a radius 3.25m from the centre of the metal silo at a depth of 1.2m from the surface of the grain bulk and dry bulb temperature of the ambient temp.

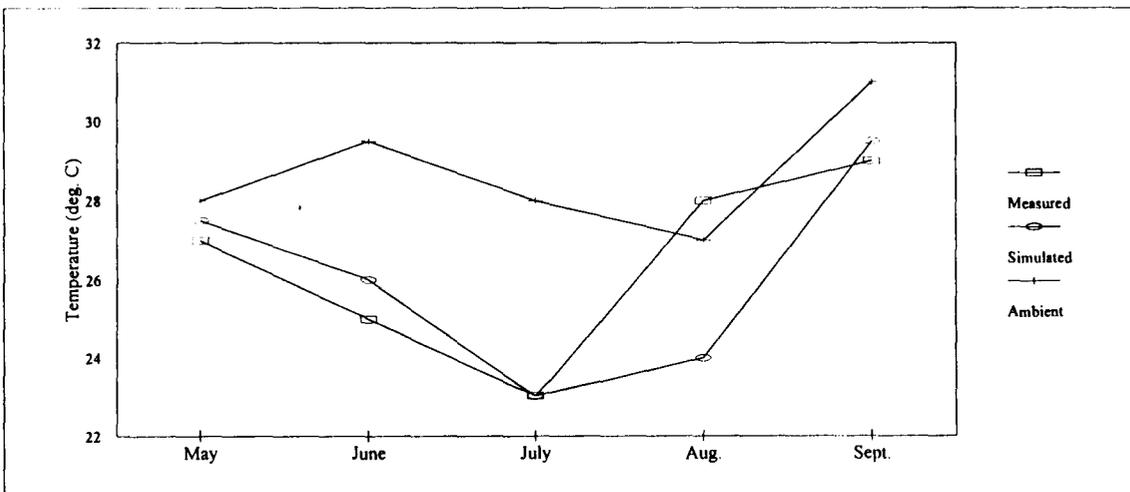


Fig. 8: Measured and simulated temperature (dry bulb) at a radius of 8.25m from the centre of the metal silo at a depth of 1.2m from the surface of the grain bulk and dry bulb temperature of the ambient temp.

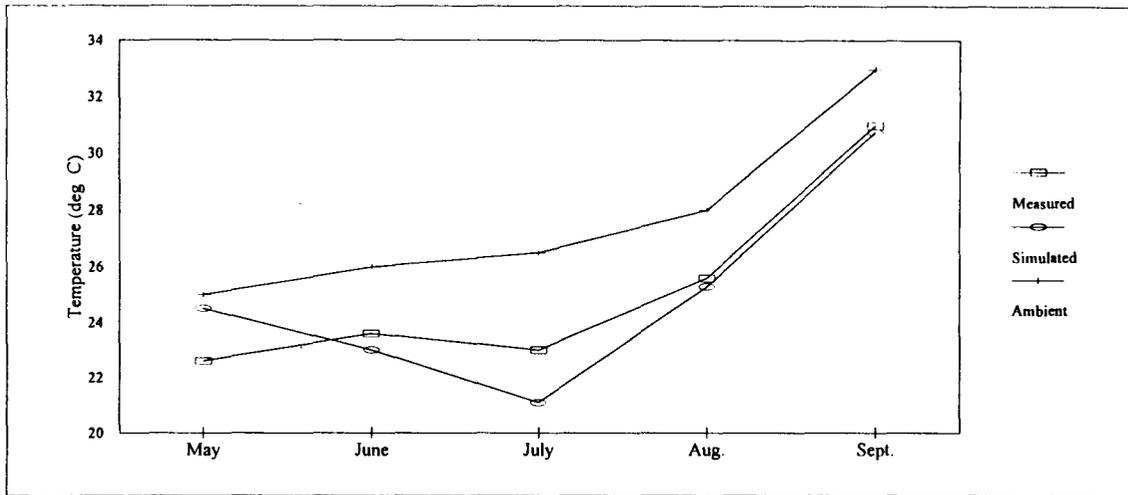


Fig. 9 Measured and simulated temperature (dry bulb) at centres at a depth of 1.2m from the surface of the grain bulk and dry bulb temperature of the ambient temp.

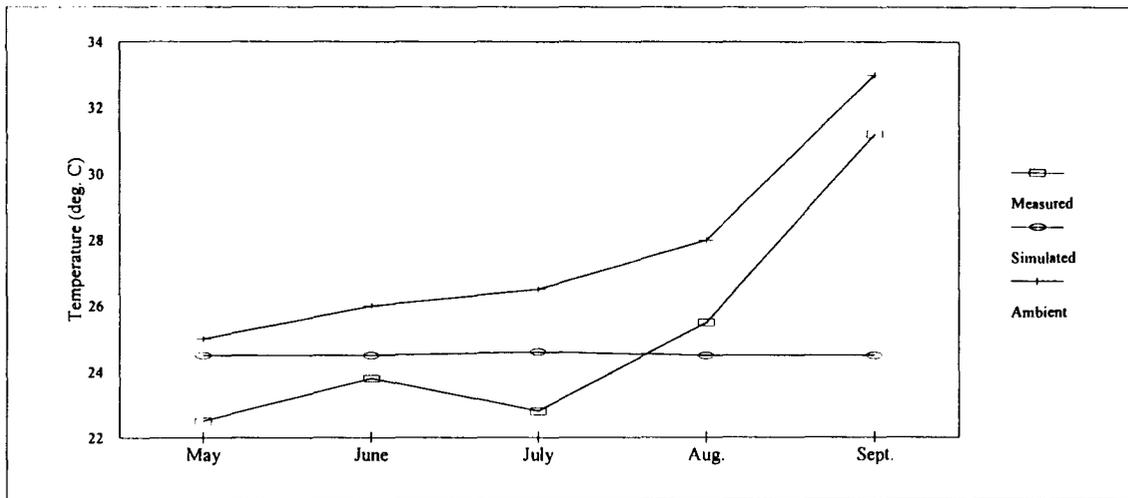


Fig. 10: Measured and simulated temperature (dry bulb) at 3.25m from the centre of the metal silo at a depth of 1.2m from the surface of the grain bulk and dry bulb temperature of the ambient temp.

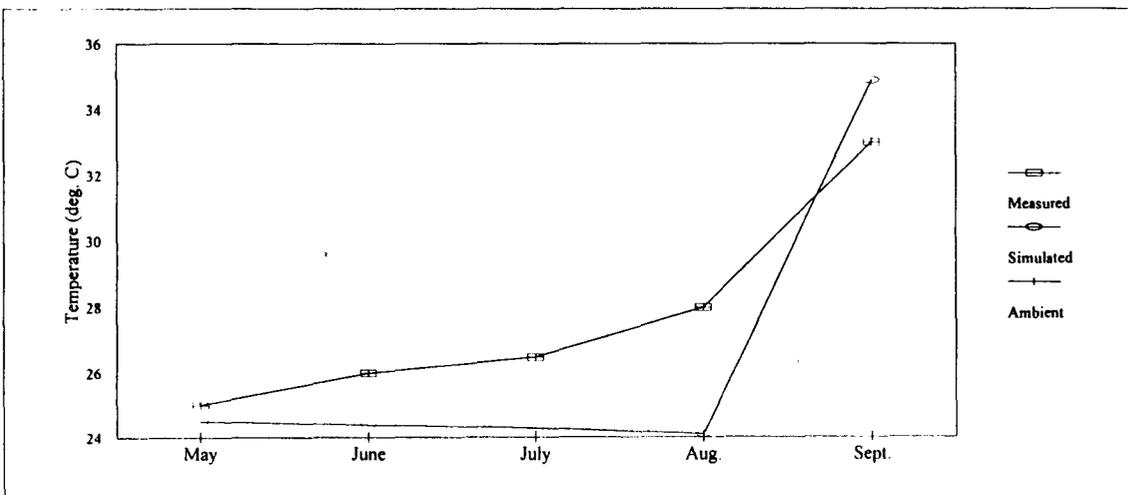


Fig. 11 Measured and simulated temperature (dry bulb) at a radius of 8.25m from the centre of the metal silo at a depth of 1.2m from the surface of the grain bulk and dry bulb temperature of the ambient temp.

The result of the explicit finite-difference technique has been found to be stable and convergent for spatial element sizes in the range of $\Delta r = \Delta y = 1.1$ to 2.0 meters and time steps of 8-24hours. The stability criterion was calculated from equation (36). The temperature predicted by the two dimensional finite-difference heat transfers model closely followed the observed values at six (6) locations of each of the storage silos.

5.0 CONCLUSIONS

Based on the results of this simulation study, the following conclusion can be drawn:

- 1 There is a time lag between the grain bulk center temperature and the average seasonal ambient temperature; and due to the large diameter of the silo the center maintained high temperature.
- 2 The ranges of seasonal temperature in the grain decrease as the distance of the grain from the expose wall increase.
- 3 The average number of unfavourable days during the study for storage increased as the initial temperature increase.

The above conclusions are in agreement with reports in the literature insofar as the radial direction is concerned. This indicates that at least in qualitative sense, the validity of the simulation method. The analysis shows that the grains most susceptible to damage are located at or near the walls where the standard deviation is the largest. It can be expected that the variations at the top layer of the bulk grain will be greater at the wall, since convection effects are more sensitive than the conduction-convection effects to changes in ambient conditions. Therefore, it can be concluded that spoilage is mostly likely to occur just below the grain surface at the top, as has been experienced.

The analyses of two-dimensional problems require specific ventilation condition. The present method can be extended to stimulate such problems provided specific boundary conditions are known. Furthermore, the effects due to grain respiration and micro- organism changes can also be studied once functional relation and moisture levels become available.

REFERENCES

1. Abbouda, S.K. (1984). Heat And Mass Transfer In Storage Grain. Ph.D. Diss. Kansas State University, Manhattan.
2. Abe, T. Basunia, M.A. (1996). Simulation Of Temperatures Of Stored Grain Bulks. Canadian Agricultural Engineering Vol. 36, No. 4
3. Ajisegiri, E.S.A. (1987). Sorption Phenomena and Storage Stability of Some Tropical Agricultural Grains. Ph.D. Thesis, U.I., Ibadan, Nigeria.
4. ASAE Standards, 36th Ed. (1989). Thermal Properties of Grain and Grain Products, 328.St.Joseph, MI:ASAE
5. Bhamidipati, S.R., And K. Singh,(1995). Determination of Fluid Particles Convective Heat Coefficient. Transaction of The ASAE Vol.38 (3): 857-862.
6. Broker, D.B., Bakker-Arkema, F.W. Hall, C.W., (1979). Drying of Cereal Grains. The Avi Publish Company, Inc. Westport, Connecticut.
7. Carslaw H.S., Jaeger, J.C. Conduction of Heat in Solids, 2nd Edition, London. Oxford University Press, 1959.
8. Chang, C.S., H.H. Covers, J.L. Steele (1993). Modelling of Temperature of Grain During Storage With Aeration. ASAE Vol. 36(2) 509-519.
9. Converse, H.H., A.H. Graves And D.S. Chang (1969). Transient Heat Transfer Within Wheat Stored in Cylindrical Bin. ASAE, St. Louis Paper No. 69-855 P24.
10. Ferzger, J.H. (1988). Numeric Method for Engineering Applications, Wiley Interscience, Pp248-265.
11. Gokhale, M.Y. (1990). Finite Difference Analysis of The Transient Free Convection on a Isothermal Plate Dependent Heat Source Pp66-74.

12. Henderson, S.M. And Perry, R.L. (1976). "Agricultural Processing Engineering" 3rd Edition, The Avi Publishing Company, Inc, Westport, Connecticut.
13. Igbeka, J.C. (1992). Interrelations of Variable Factors of Deterioration in Stored Grains. Lecture Delivered at an Induction Course for Engineers and Pest Control Officers of NSGRD/FMA held at Minna Silo Complex, December 1994.
14. D.S. Jayas, K. Alagusundanam, G. Shunmugam, W.E.Muir and N.D.G. White (1994). Simulation Temperatures of Stored Grain Bunks. Canadian Agricultural Engineering Vol.36, No.4.Pp239-245..
15. Kreith, F. (1965). Principles of Heat Transfer. Scranton, International Textbooks Company. Pp619.
16. Metzger, J.F. And W.E. Muir, (1993). Computer Model of Two-Dimensional Conduction and Forced Convection in Stored Grains. can. Agric. Engng. 25:119-337.
17. National Strategy Grains Reserve Storage Division, (NSGRSD) Minna Silo Complex (1997). Quality Control Laboratory.
18. Obaldo L.G., J.P. Harner And H.H. Converse, (1990). Predicting Temperature Changes In Stored Corn. ASAE Paper No. 90-6067. St. Joseph , Mi: ASAE ,1984
19. Sachdeva, R.C.. Fundamentals of Engineering Heat and Mass Transfer. India. Wiley Eastern Limited 1993.
20. Sokhansanjis And Bruce M.D. (1987). Conduction Model to Predict Grain Temperatures In Grain Drying Simulation. Transactions of The ASAE 30(4): 1181-4.
21. Thompson, T.L. 91972). Temporary Storage of High Moisture Shelled Corn Using Continuous Aeration. Transactions of The Asae 15(2): 1333-337.
22. Welty, J.R., (1974). Engineering Heat Transfer, Wiley, New York, Pp39-115.

23. Woods J.L. (1978). Heat And Mass Transferring Drying And Cooling of Crops Pp. 203-229.
24. Yaciuk, G, W.E. Muir And R.N. Sinha (1975). A Simulation Model of Temperature in Stored Grains. Journal of Agric. Engineering Research 20,245-258.

Δt (s)

Δt =time increment(s)

β =bin parameter (m)

B_i =biot number for wall surface

B_g =biot number for grain surface

C_a =specific heat of air (j/kg.k)

C_g =specific heat of grain (j/kg.k)

D =silo diameter (m)

F_o =fourier number

F_b =shape factor for silo root and grain surface

H =convective heat transfer coefficient at exterior wall (W/m^2K)

H_b = convective heat transfer coefficient at top grain surface (W/m^2K)

H_c = convective heat transfer coefficient (W/m^2K)

I_b =hourly beam radiation (W/m^2)

$I_{b,h}$ =component of I_b on horizontal surface (W/m^2)

$I_{b,v}$ =component of I_b on a vertical surface (W/m^2)

I_d =diffuse radiation (W/m^2)

∇_j =spatial operator (m)

J_d =day of the year

K =thermal conductivity of the grain (W/mk)

K_a =thermal conductivity of air (W/mk)

K_w =thermal conductivity of bin wall material (W/mk)

L =length, variation $=z$ (m)

L_w =bin wall thickness (m)

M =number of spatial element (grain) in vertical direction

M =number of spatial element at the grain surface.

N =number of spatial element (grain) in radial direction.
 N =number of spatial element at the silo wall
 P =surface temperature phase angle (rad)
 Q =heat generated (w)
 R =radial coordinate (m)
 R_h =relative humidity, decimal
 R =total radius (m)
 S =storage parameter
 T =temperature (k)
 T =time(s)
 T_i =initial temperature (k)
 T_{i+1} =temperature increment ($^{\circ}\text{C}$)
 T_a =temperature of ambient air ($^{\circ}\text{C}$)
 T_{a_s} =absolute temperature of ambient air (k)
 T_{a_b} =absolute temperature of air above the grain surface (k)
 T_{a_r} =absolute temperature of silo roof (k)
 T_{a_s} =absolute temperature of the grain surface (k)
 T_{a_w} =absolute temperature of silo wall (k)
 T_b =temperature of air above the grain surface ($^{\circ}\text{C}$)
 T_s =temperature of grain surface ($^{\circ}\text{C}$)
 T_w =temperature of silo wall ($^{\circ}\text{C}$)
 $T_{m,n}$ =temperature of grain element m,n at time t ($^{\circ}\text{C}$)
 T_{pa} =ambient temperature at any time p ($^{\circ}\text{C}$)
 T^{p+1}_a = ambient temperature at any time $p+1$, ($^{\circ}\text{C}$)
 T_{db} =dry-bulb temperature of grain inside the grain bin, ($^{\circ}\text{C}$)
 T_{wb} =wet-bulb temperatuere of grain inside grain bin , ($^{\circ}\text{C}$)
 $T^p_{m,n}$ =temperature at node m,n at any time p , ($^{\circ}\text{C}$)
 $T^{p+1}_{m,n}$ =temperature at node m,n at any time $p+1$, ($^{\circ}\text{C}$)