MATHEMATICAL MODELS FOR THE DETERMINATION OF EFFECTS OF DIVERSIFICATION OF ASSETS IN PORTFOLIO MANAGEMENT

BY

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A THESIS

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CERTIFICATION

This thesis titled: Mathematical Models for the Determination of Effects of Diversification of Assets in Portfolio Management by Jayeola Dare (M.Tech/SSSE/2004/1197) meets the regulations governing the award of degree of Master of Technology (M. Tech) in Mathematics of the Federal University of Technology Minna and is approved for its contribution to scientific knowledge and literary presentation.

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DEDICATION

This project is dedicated to **CHRIST, THE KING OF KINGS** and **THE LORD OF LORDS.**

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ABSTRACT

Diversification of assets is a method of investing in various assets of portfolio for the purpose of maximizing returns and minimizing risk of the portfolio. This project was carried out to determine the effect of diversification of uncorrelated, perfectly correlated and perfect negatively correlated assets in relation to returns and risk. In view of this, useful data were collected from Investment Banking and Trust Company PLC (IBTC), where the data were carefully computed for uncorrelated, perfectly correlated and perfect negatively correlated assets. The results of the computation were analyzed and simulated, using Microsoft Excel. However, the results indicated that diversification of perfect negatively correlated assets are preferred. The reason is that they generate lowest risk compared to others. This implies that diversifying into risky and riskless assets together minimizes risk of the portfolio. Therefore, it is recommended that before banks or organisation go into diversification, the correlation of the assets, the default correlation of the assets, the default risk of the assets, the volatility of the assets and the market value of the assets, should be taken into consideration.

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CHAPTER ONE

1.0 INTRODUCTION TO PORTFOLIO MANAGEMENT

1.1 Introduction

Corporate liabilities have default risk. There is always a chance that a corporate borrower will not meet its obligations to pay the principal and interest. For the typical high-grade borrower, this risk is small, perhaps 1/10 of 1% per year and for the typical bank borrower the risk is about ½ of 1% Markowitz (1952).

Although these risks do not seem large, they are in fact highly significant. First, they can increase quickly and with little warning. Second, the margins in corporate lending are very tight, and even small miscalculations of default risks can undermine the profitability of lending. But most importantly, many lenders are themselves borrowers, with high levels of leverage. Unexpected realization of default risk have destabilized, de-capitalized and destroyed lenders. Banks, finance companies, insurers, investment banks, and lessor: have not escaped unscathed.

Default risk cannot be hedged out or structured away. The government cannot insure it in companies' future. Various schemes exist, and more are coming, which can shift risk, but in the end, someone must bear this risk. It does not "net out" in the aggregate.

Default risk can be reduced and managed through diversification. Markowitz (1958) Default risk and the rewards for bearing it, will ultimately be owned by those who can diversity it best.

Every lender knows the benefits of diversification. Every lender works to chieve these benefits. However, until recently lenders have been reluctant or nable to implement systems for actually measuring the amount of diversification n a debt portfolio.

Portfolios have "concentration"; export we see them. Ex ante, lenders must ook to models and software to qualify concentrations. Until recently, these types of models have not been generally available. Thus, it should not come as a surprise hat there have not been many unexpected default events in lenders' portfolios in he past.

Quantitative methods for portfolio analysis have developed since Markowitz's pioneering work in 1950. These methods have been applied successfully in a variety of areas of finance, notably to equity portfolio. These methods show the amount of risk reduction achievable through diversification. They measure the amount of risk contributed by an asset, or group of assets, to a portfolio. By extension, they also show the amount of diversification provide by a single asset or group of assets. The aim of these methods is to maximize the return to a portfolio while keeping the risk within acceptable bounds. This minimization requires a balancing of return to risk within the portfolio, asset by asset, group of assets by group of assets.

This logic can be illustrated by imagining that it was not the case. If a low-return-to-return asset is swapped for a high-return-to-risk asset, then the portfolio's return can be improved with no addition to risk Markowitz (1950). The process is equilibrated by changes in risk. As an asset is swapped out of the portfolio, it changes from being a source of concentration to being a source of diversification, that is, its risk contribution falls. The reverse applies as an asset is swapped into the portfolio. Thus, the return-to-risk increases for the low return asset and decreases for the high return asset, until their return-to-risk ratios are equal. At that point, no further swap can raise return without also raising risk. This then characterizes the optimal portfolio or equivalently, the optimal set of holdings.

This conceptual model applies to the default risk of debt as surely as it applies to equities Pratt (1964). Equity practitioners, however, have used the last twenty-five years to develop techniques for measuring the asset attributes that are necessary for an actual portfolio management tool.

The same development has not occurred for debt portfolios because of the greater analytical and empirical difficulties. In particular it is necessary to quantify the level of default risk in a single asset, and to qualify the relationship between the default risks of each pair of assets in the portfolio.

Due to variety of technical development in finance, it has become both possible and feasible to make these measurements. Moody has pioneered the development of these methods for the last twelve years in its practice with commercial banks. The fruits of this development effort are several products designed to address the quantifications and management of credit risk Moody estimates an expected default frequency (EDF) for firms with publicly traded equity and delivers this estimates via a PC- based viewer called credit monitor or an internet-based viewer called Credit Edge. Both of these software products cover nearly 30,000 firms globally and come bundled with a variety of analysis tools. For firms without publicly traded equity Moody offers the private firm model (PFM) which also produces an EDF credit measure. The PFM, EDF values are housed in a software product called the private firm analyst that works in tandem with credit monitor. Moody's EDF values combined with facility-specific data can be used together with Moody's Global correlation model and portfolio manager to analyze and mange portfolios of credit-risky assets. The result is that practical and conceptually sound methods exist for measuring actual diversification, and for determining portfolio holdings to minimize concentrations and maximize return in debt portfolios.

1.2 Background to the Study.

What is portfolio management?

Portfolio management is the process of managing the assets of a mutual fund including choosing and monitoring appropriate investments and allocating fund accordingly Pratt (1964). Also, it is the management of the investment for some business organizations or individuals. The aim of portfolio managers is to optimize portfolio as much as possible. Therefore, portfolio optimization is the process of analyzing a portfolio and managing the assets within it, to obtain the highest return given a level of risk.

The basic portfolio optimization theory hinges on the discrete time, continuous outcome paradigm otherwise known as the mean-variance or

Markowitz paradigm. In 1952, Harry Markowitz introduced this approach, which is widely used in applications involving investment portfolios. Mean-Variance theory assumes that among portfolios with the same standard deviation, the one with the greater expected value is the most efficient Markowitz (1958). Efficient in the sense that for a specified level of expected return, the corresponding risk is minimized; alternatively, for a given level of risk, it yields the highest expected return. He showed how rational investors could build optimal portfolios under conditions of uncertainty by using statistical measures for expectation and variance of return. This set of portfolios is known as the efficient set and can be identified by solving a parametric quadratic program. In the risk-return space, the efficient set forms the so-called efficient frontier.

However, the real world investors are interested in extending the basic mean-variance approach with restrictions such as limiting the number of trades, defining a minimum level of trade for an asset, reducing taxation; e.t.c. Described portfolio optimization is static and useful to build an initial portfolio. Over time, the portfolio is rebalanced to justify the actualities of the situation then.

As a technique for evaluating the quality of the portfolio strategy, back testing is used Konno et al (1991). The essence of the technique is to compare actual trading results with model-generated measures and help to refine portfolio management techniques.

1.3 Statement of the Problems

A corporation has fixed obligations. These may be no more than its trade obligations, although they could just as well include bank loans and public debt. At one time, there was no legal means to escape the fulfillment of such obligations; a defaulter fled or was jailed. Modern treatment allows the defaulter to escape the obligation but only by relinquishing the corporation's assets to the obligee.

Bank assets have a variety of complexities: default risks, correlation, utilization, value correlation and so forth. Some of these complexities will be addressed subsequently in this research work.

1.4 Aims and Objectives

The aim of the study is to present significant basic methods for banks to manage their portfolio for maximum return while maintaining risk at a desirable level. These can be achieved through the

1. Model of Default correlation
2. Model of Value correlation
3. Model of Default risk
4. Optimal Diversification

1.5 Scope and Limitation

Focus point of the study is to formulate methods for banks to manage their portfolio for maximum return while maintaining risk at a desirable level.

Solutions to the problems pose above are: model of default correlation, model of value correlation and measurement of optimal Diversification. It should also be noted that only one bank and two asset will be taken into consideration. This places limitation on our study.

1.6 Justification

This study is based on Bank portfolio management. The study of bank portfolio management contributes or determines the economy of a country. If bank portfolio yields maximum return and minimum risk, this implies positive influence on the economy. But, if the reverse is the case the influence is negative.

However, any study that involves the economy of a country/countries is worth studying.

1.7 Definition of Operational Terms

1.7.1 Share

This is a ratio of sales of a brand to the total sales of that products-type in a defined area (country, continent etc). Share can also be defined as the ratio of sales of a company's entire product.

1.7.2 Asset

Anything owned by a person or organization having monetary value usually its cost or fair market value. An asset may be a specific property, such as title to real estate or other tangible property or enforceable claims against others. Example, land, houses, cars, furniture, cash bank deposit and securities owned are assets.

1.7.3 Investment

An asset or item that is purchased with the hope that it will generate income or appreciate in the future. In an economic sense, an investment is the purchase of goods that are not consumed today but are used in the future to create wealth. In finance, an investment is a monetary asset purchased with the idea that the asset will provide income in the future or appreciate and be sold at higher price.

1.7.4 Mean

The simple mathematical average of a set of two or more numbers. The mean for a given set of numbers can be computed in more than one way, including their arithmetic mean method, which uses the sum of the numbers in the series, and the geometric mean method. However, all of the primary methods for computing a simple result of a normal number series produce the same approximate result most of the time.

1.7.5 Standard Deviation

A measure of a dispersion of a set of data from its mean. The more spread apart the data is, the higher the deviation.

In finance, standard deviation is applied to the annual rate of return of an investment to measure the investment's volatility (risk).

1.7.6 Variance

A measure of the dispersion of a set data points around their mean value. It is a mathematical expectation of the average squared deviation from the mean.

1.7.7 Diversification

A risk management technique that mixes a wide variety of investments within a portfolio. The rationale behind this technique contends that a portfolio of different kinds of investment will on average yield higher returns and pose a lower risk than any individual investment found within the portfolio.

Diversification strives to smoothen out unsystematic risk events in a portfolio so that the positive performance of some investments will neutralize the negative performance of other. Therefore, the benefit of diversification will hold only if the securities in the portfolio are not perfectly corrected.

1.7.8 Covariance

A measure of the degree to which return on two risky assets move in tandem. A positive covariance means that asset returns move together. A negative covariance means returns more inversely. One method of calculating covariance is by looking at return's surprise (deviation from expected return) in each scenario. Another method is to multiply the correlation between the two variables by the standard deviation of each variable.

1.7.9 Return

The gain or loss of a security in a particular period. The returns consist of the income and the capital gain relation on an investment. It is usually quoted as a percentage.

1.7.10 Risk

The chance that an investment's actual return will be different from the expected. This includes the possibility of losing some or all of the original investment.

1.7.11 Volatility

This is the measure of the propensity of the asset value to change within a given period of time.

1.7.12 Default

It means when a borrower borrows and is incapable of paying both the principal and the interest at a specified time.

1.7.13 Default Risk

This is the risk that the lender bears, if he, borrower, is not able to pay back. It can be reduced and managed through diversification.

1.7.14 Portfolio

This is a collection of investments held by an institution or a private individual. In building up an investment portfolio, a financial institution will typically conduct its own investment analysis, whilst a private individual may make use of the services of a financial advisor or a financial institution which offers portfolio management services.

CHAPTER TWO

2.0 BRIEF OVERVIEW OF PORTFOLIO MANAGEMENT

2.1 Brief History of Portfolio Management

Portfolio Management (PM) which is also called "Portfolio Theory" or "Portfolio Management Theory", is a sophisticated investment approach first developed by Professor Harry Markowitz of the University of Chicago, (1952). Thirty-eight years later, in 1990, he shared a Nobel prize in Economics with Merton Miller and William Sharpe for what has become the frame upon which institutions and Sarry investors construct their investment portfolios.

Portfolio Management allows investors to estimate both the expected risks and returns, as measured statistically, for their investment portfolios. In his article "portfolio selection" (in the Journal of Finance, in March 1952), Markowitz described how to combine assets in efficiently diversified portfolios. He demonstrated that investors failed to account correctly for the high correlation among security returns. It was his position that a portfolio's risk could be reduced and the expected rate of return increased, when assets with dissimilar price movements were combined. Holding securities that tend to move in concert with each other does not lower your risk. Diversification, he concluded "reduces risk only when we combine assets whose price move inversely, or at different times, in relation to each other"

Markowitz was among the first to quantify risk and demonstrate quantitatively why and how portfolio diversification can work to reduce risk, and increase returns for investors.

Many investors are under the delusion that their portfolios are diversified if they are in individual stocks, mutual funds, bonds and international stocks. While these are all different investments, they are still in the same asset class and generally move in concert with each other. When the bubble burst in the stock market, this was made painfully clear. Proper diversification according to portfolio management is in different asset classes that move independently from one another.

One of the most uncorrelated and independent investment versus stocks are professionally managed futures. The value of professionally managed futures was thoroughly researched by Dr. John Lintner (1983), "The potential role of managed futures Accounts in portfolios of stocks and Bonds"

Lintner wrote that "the combined portfolios or stocks and bonds) after including judicious investments, in leveraged managed future accounts show substantially less risk at every possible level of expected return than portfolios of stock (or stocks and bonds) alone." Lintner specifically showed how managed futures can decrease portfolio risk, while simultaneously enhancing overall portfolio performance.

In conclusion, according to Markowitz's advice to present days investors that, diversification in different asset classes incorporated in an investment portfolio reduces risk, and increases returns. Also, Lintner concluded that, managed futures is ideal asset class to use in portfolio diversification.

2.2 Efficient Frontier Portfolio

Efficient Frontier Portfolio is one where no added diversification can lower the portfolio's risk for a given return expectation (alternatively, no additional expected return can be gained without increasing the risk of the portfolio). The Markowitz Efficient Frontier is the set of all portfolios that will give the highest expected return for each given level of risk. These concepts of efficiency is essential to the development of the Capital Asset Pricing Model. Graph representing a set of efficient portfolios that maximize expected returns at each level of portfolio risk (or volatility).

According to modern portfolio theory, for any portfolio of assets there exists an efficient frontier, which represents various weighted combinations of the portfolio's assets that yield the maximum possible expected return at any given level of portfolio risk. See graph below:

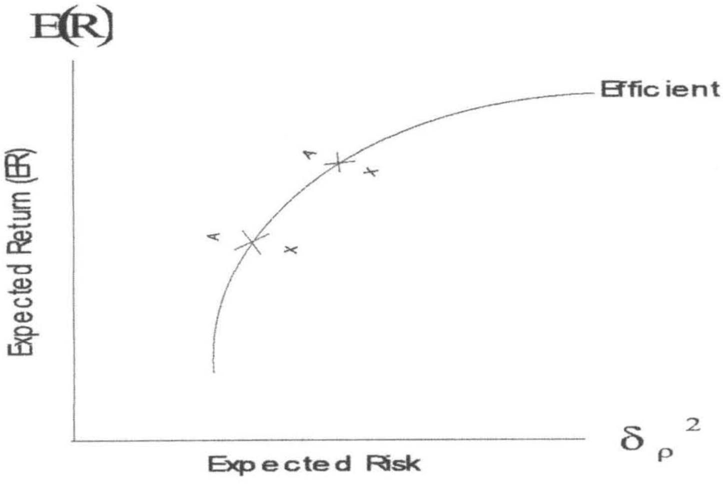


Fig 2.1: Efficient Frontier.

All points lying on the Efficient Frontier (such as A and B) offer the highest Expected Return relative to all other portfolios of comparable risk. Portfolios that lie on the efficient frontier are superior to portfolios located inside the frontier because they have higher risk's: return ratios. Single Asset Portfolios lie within the efficient frontier because they have high level of market and specific Risk. Multi-asset portfolios lie closer to the efficient frontier because diversification causes their specific risk to be reduced by the law of large numbers. Ultimately, portfolios lying on the Efficient Frontier will be those whose specific risks have been eliminated by diversification: they are the efficiently diversified portfolios. The objective of portfolio management is to find the optimal portfolio for an investor. These portfolios share two characteristics:

i Lies on the Efficient frontier

ii Possess only so much risk as the client is willing to assume.

The slope of the efficient frontier at any point depicts how much extra expected return is obtained by taking some more risk. This is called the Return/Risk Trade off.

$$\text{Return / Risk Tradeoff} = \Delta R_p / \Delta \sigma_p \qquad 2.2.1$$

$$where: R_p = \text{Return of the portfolio}$$
$$\sigma_p = \text{Standard deviation of the portfolio}$$

The amount of satisfaction that an investor obtains from his investment can be depicted by a series of indifference curves.

The optimum portfolio for any investor is one that lie on the efficient frontier at the point of Tangency with that indifference curve that represents the highest possible utility for the investors. This point of tangency occurs where the investor's risk-aversion factor (A) equals the slope of the return/risk trade off ratio of the efficient frontier.

$$A = \Delta R_p / \Delta \sigma_p \qquad 2.2.3$$

The more risk-averse an investor is, the lower will be the optimal portfolio on the return/risk spectrum defined by the efficient frontier:

2.2.1 The Capital Asset Pricing Model (CAPM)

The Efficient Frontier depicts the Return-risk relationship for portfolios consisting of Risky Assets. But, there is an alternative to investing in risky assets: It is to invest in a riskless asset that has no standard deviation.

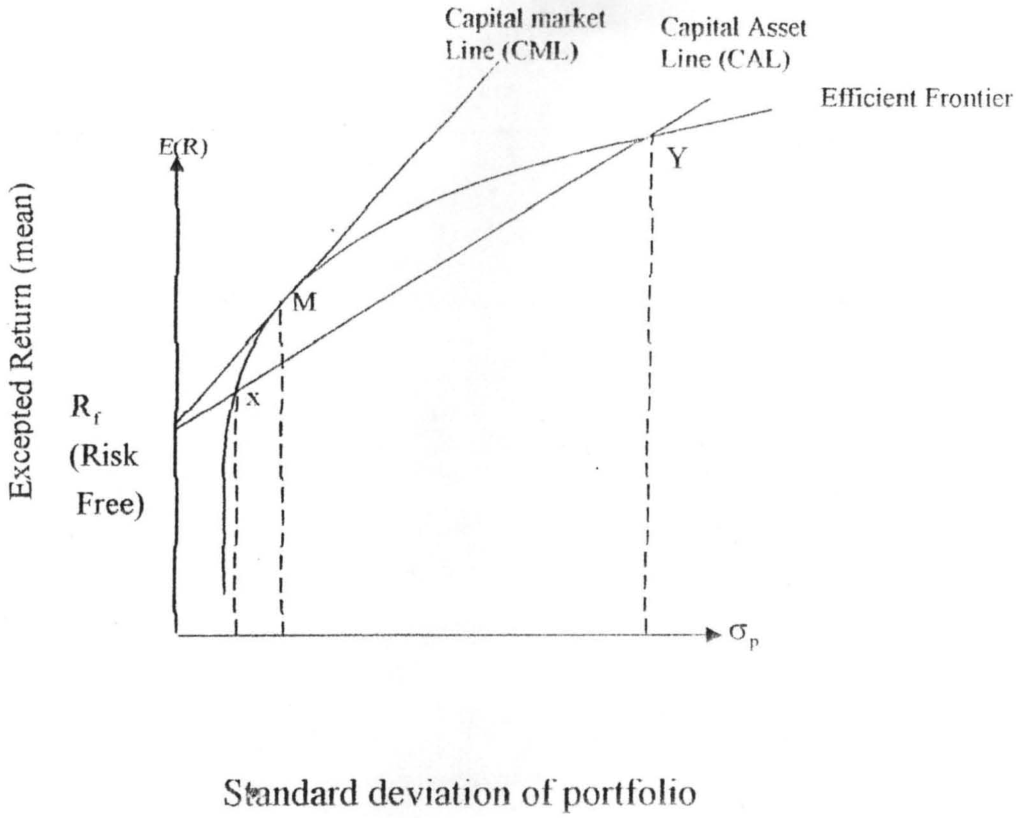


Fig 2.2: Relationship of capital market line, capital Asset line on Efficient frontier.

The capital asset line (CAL) cuts the efficient frontier in two place X and Y. Thus, the CAL represents combinations of portfolios comprised of various mixes of the risk-free portfolio, X and Y. Any portfolio that lies on this particular CAL has the same sharpe ratio. The steeper the CAL, the better the portfolios' (return-variance ratio) that lie on it.

Slope of CAL = sharpe ratio.

Sharpe ratio $= [(R_x - R_f)/\delta_x] = [(R_y - R_f)/\delta_y]$

Where: R_x = Asset return of x

R_y =Asset return of y

R_f = Risk free

δ_x =Standard deviation of the excess return in X

δ_y = Standard deviation of the excess return in Y

The most efficient portfolio is the one that is just tangent to the efficient frontier. This is the market portfolio line and is the capital market line (CML)

$$R_p = R_f + [(R_m - R_f)/\delta_m]\delta p \qquad 2.2.4$$

Where R_p = Return of the portfolio

R_f =Risk free

R_m = Return of the market portfolio

δ_m = Standard deviation of the market portfolio

δ_p = Standard deviation of the portfolio

The expected return of any portfolio which lie on the CML can be calculated from this relationship. As the market portfolio (m) is a completely diversified portfolio, it must have only systematic Risk. Plus, all portfolios, on the CML are perfectly correlated with portfolio (M) since they all have only systematic Risk. The general form of the CAPM is

$$R_p = R_f + [(R_m - R_f)/\delta_m^2] \; Cov_m \qquad 2.2.5$$

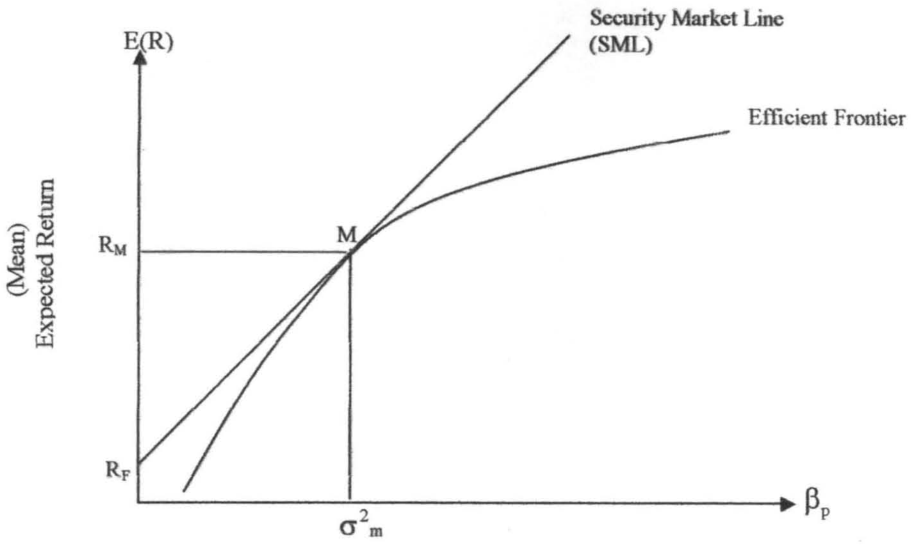
Where: δ_m^2 = Variance of the market portfolio

Cov_m = covariance of the market portfolio

The security market line is based on CAPM and is written as:

$$R_p = R_f + \beta_p(R_m - R_f) \qquad 2.2.6$$

Where β_p = Risk of combining portfolios or securities.



Normalized Risk

Fig 2.4 Relationship between security market and efficient frontier.

The CML and SML are similar, yet different concepts. The CML is the relationship between required Returns on Efficient portfolios (Rp) and their Total Risk (δ_p). The SML is the relationship between the Expected Returns on individual securities or portfolios (R_s or $_p$) and their risk as measured by their covariance with the market portfolio (cov_m) or their normalized risked relative to the market as measured by their Betas (β_p). All fairly priced assets and portfolios should lie on the SML, only efficient portfolios lie on the CML

The linear relationship between the expected or required Return and Risk is called the CAPM. It is a specific form of a general class of models called Risk Premium Model that relate return to risk

Example 2.1

Suppose the risk free rate is 5% and the stock market could decline as much as 30%. An investor does not want to risk more than a 10% loss. What portfolio β, should the investor accept?

We shall use CAPM model

$R_p = R_f + (R_m - R_f)\beta_p$

$R_p = -10$

$R_f = 5$

$R_m = -30$

$-10 = 5 + (-30-5)\beta_p$

$-10 - 5 = -35\beta_p$

$-15 = -35\beta_p$

$\beta_p = \frac{15}{35} = 0.43$

$\beta_p = 0.43$

The ideal β for the investor is 0.43 which means he should invest 43% of his wealth in the market portfolio. This means, for the investor to have efficient portfolio he should invest 43% of his wealth on market portfolio. While, 57% is Risk free Asset.

An Efficient Frontier allows investors to ensure optimal risk-adjusted returns of their portfolios. For a given level of risk portfolios on the efficient frontier will yield the maximum possible expected returns. It can also be a very powerful tool in managing investment portfolio.

2.3 Minimum Variance Portfolio

Suppose an investor desires to invest in a portfolio with the least amount of risk. He does not care about his expected return; he only wants to invest all his money with the lowest possible amount of risk. Because, he will always invest in an efficient portfolio, he will choose the portfolio on the efficient frontier with minimum standard deviation. At this point also the variance is minimal. That is why this portfolio is called the minimum variance portfolio. The graphical representation of the minimum variance portfolio is shown below. This minimum variance portfolio can be calculated by minimizing the variance subject to the necessary constraint that an investor can only invest the amount of capital he has. This called can be budget constraint.

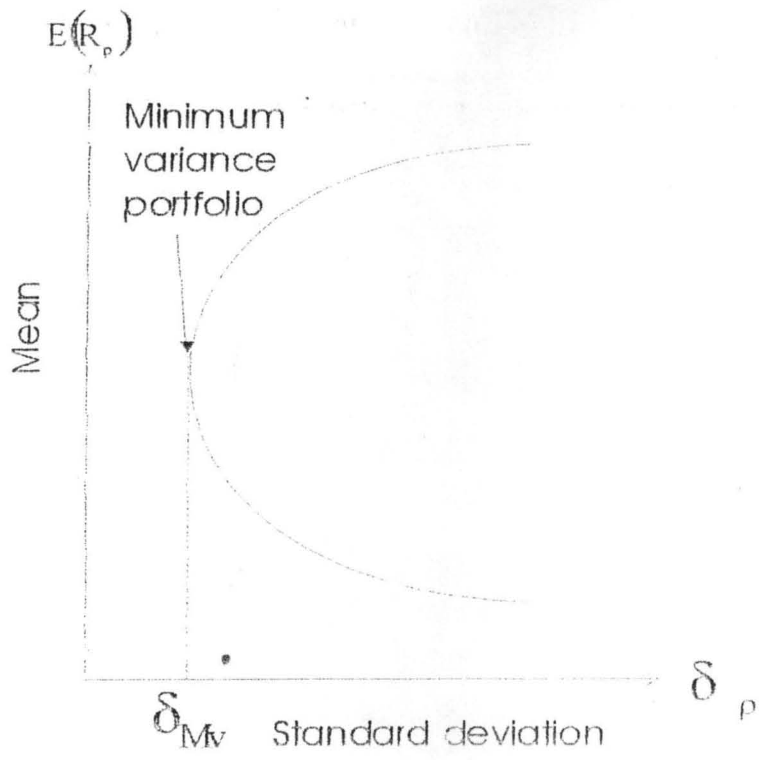


Fig 2.5: Minimum variance portfolio

The curve depicted in the figure above is known as the efficient frontier, which maps the risk and return relationship of a portfolio. It represents the different portfolio that are attainable by an investor. portfolios that lie outside the efficient frontier (as represented by the area to upper left of the efficient frontier) are attainable by the investor.

The risk level of the efficient frontier portfolios changes as its composition changes. Where the risk level of the portfolio bottoms out at a certain combination of portfolio 1 and portfolios 2. The particular combination that causes the risk level of the portfolio to bottom out (that is, lowest level of risk) is known as the minimum variance portfolio. It is not possible to lower the risk of the portfolio any further. If the composition of the portfolio continues to lean towards one portfolio, the risk level of the portfolio will rise. The interesting thing is that it is possible for an investor to create a portfolio that has a risk level that is lower than the individual risk levels of stocks. It is possible for an investor to create a portfolio that has a risk level lower than the individual risk levels of the stocks it contains. This is the power of diversification, which allows an investor to lower the risk level of a portfolio beyond the individual risk levels of the stocks it contains. Minimum variance portfolio is the one that achieves the maximum effect of diversification.

The following formulas will help an investor determine the correct combination of market weight portfolio (W_1 and W_2) that will produce the minimum variance portfolio.

$$W = \frac{\delta_2^2 - \delta_1\delta_2\rho}{\delta_1^2 + \delta_2^2 - 2\delta_1\delta_2\rho} \qquad 2.3.1$$

$$W_2 = 1 - W_1$$

Where:

δ_1 =Standard deviation of portfolio 1

δ_2 =Standard deviation of portfolio 2

$\delta_1{}^2$= Variance of portfolio 1

$\delta_{21}{}^2$= Variance of portfolio 2

$W_1{}^2$ =Market weight portfolio 1

W_2 =Market weight portfolio 2

ρ =Correlation in returns.

2.4 Literature Review.

The history of investments in the United States can be divided into two periods: before and after (1952). That was the year that an economics student at the University of Chicago named Harry Markowitz published his doctoral thesis. His work was the beginning of what is now known as Modern Portfolio Theory or Portfolio Management Theory.

Markowitz (1952) started out with the assumption that all investors would like to avoid risk whenever possible. He defined risk as a standard deviation of expected returns. Rather than look at risk on an individual security level, Markowitz proposes that we measure the risk of an entire portfolio and that when considering a security for a portfolio, we should not base our decision on the amount of risk that carries with it. Instead we should consider how that security contribute to the overall risk of the portfolio.

Markowitz then considered how all the investments in a portfolio can be expected to move together in price under the same circumstances. This is called "correlation", and it measures how much we can expect different securities or asset classes to change in price relative to each other. He gave this example, high fuel prices might be good for oil companies, but bad for airlines who need to buy the fuel. As a result, one might expect that the stocks of companies in these two industries would often move in opposite directions. These two industries have a negative (or low) correlation. One will get better diversification in his portfolio if he owns one airline and one oil companies, rather than two oil companies.

So, it is entirely possible to build a portfolio that has much higher average return than the level of risk it contains. When one builds a diversified portfolio and spread out his investments by asset class, one is really managing risk and return.

The first efficient frontier was discovered by Markowitz (1959), using a handful of stocks from the New York stock Exchange. It has a line going to the origin, because Markowitz was interested in the effects of combining risky assets with a riskless asset.

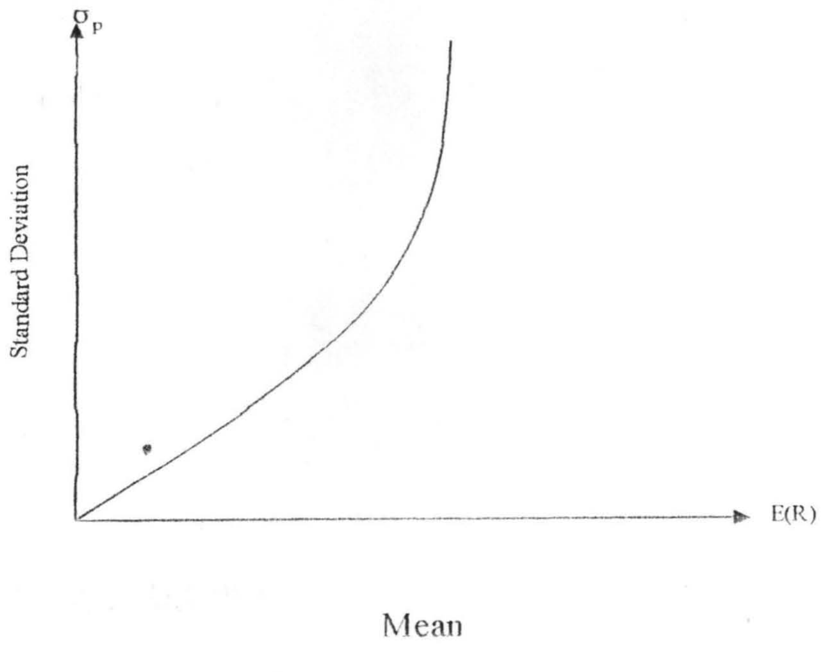


Fig 2.6 First Efficient Frontier

He discovered that, the first efficient frontier will not in anyway yield, efficient portfolio. This now prompted him into the discovery of actual efficient frontier (1970), which is being used up till date.

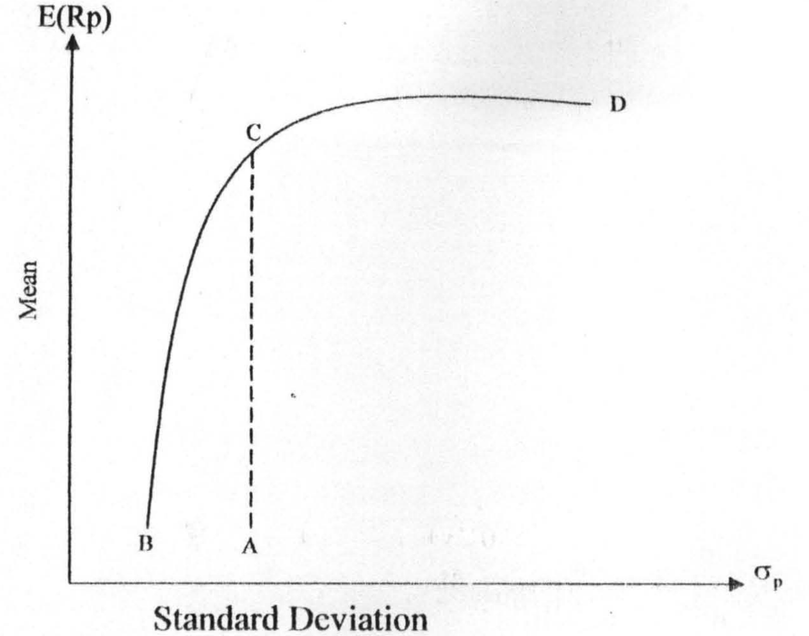


Fig:2.7 Actual Efficient portfolio

Markowitz (1962) observed that T-Bills are often taken to be riskless assets and their return is indicated as R_f, the risk free weight. If the riskless asset is to be combined into a portfolio, the efficient frontier can change. Since, it is riskless, it has no correlation to other securities. Thus, it provides no diversification, per se. It does provide an opportunity to have a low risk portfolio, this diagram of the efficient frontier composed of all the risky assets in the economy, as well as the riskless asset.

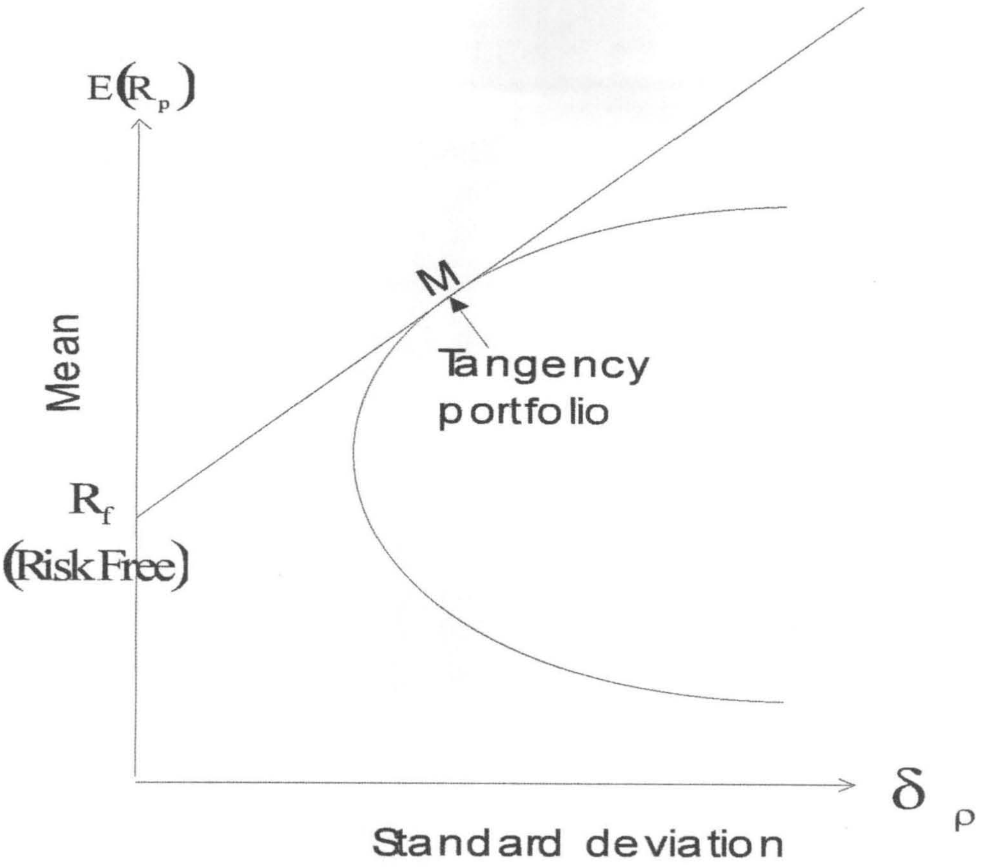


Fig. 2.8: Tangency Portfolio

In fig 2.8 (Tangency portfolio) is a new discovered efficient frontier. Efficient frontier is a ray, extending from risk free (R_f) to the point of tangency (M) with the risky asset efficient frontier and beyond. This line is called the capital market line (CML). It is actually a set of investable portfolio, if one is able to borrow and lend at the riskless rate, all portfolios between R_f and M are generated by borrowing at the riskless rate R_f and investing the proceeds into M.

The Markowitz model was a brilliant innovation in the science of portfolio selection. With almost a disarming slight-of-hand, Markowitz showed that all the information needed to choose the best portfolio for any given level of risk is contained in three simple statistics: Mean, Standard deviation and correlation.

The model requires no information about dividend policy, earning, market share, strategy, quality management. However, Markowitz fundamentally altered how investment decisions were made before 1952. Virtually every major portfolio of manager today consults an optimization program (meant to solve portfolio problems). They use it to evaluate basic risk and return trade-offs.

Von Neumann *et al* (1944) claimed that utility of a lottery could be written as U= $\sum_{x \in \sup p(p)} P(x)U(x)$ 2.4.1

Where they referred to U: $\Delta(x)$ as the expected utility function and $U: X \longrightarrow R$ as the implied elementary utility function. At the risk of confusion on outcomes, $U: X \longrightarrow R$, as the elementary utility function (what is sometimes referred to as "Bernoulli utility function").

After the axiomatization of the expected utility hypothesis, economists began immediately seeing the potential applications of expected utility to economic issue like portfolio choice, insurance, etc. these simple applications tended to use simple model where outcomes were expressed as a single commodity, "wealth", thus the set of outcomes X, became merely the real line, R. As a result, a "lottery" is now conceived as a random variable Z taking value in R. Consequently, preferences over lotteries can be thought of as preferences over alternative probability distributions. Thus, letting F_z denote the cumulative probability distribution associated with random variable Z where $F_z(x)$

= Prob$\{Z \leq x\}$, then we can think of agents making choices over different F_Z. Accordingly, the preferences over lotteries, $\geq h$, are now defined over the space of cumulative distribution functions. Thus, letting the Von Neumann-Morgenstern utility function U represent preferences over distributions, then lottery F_Z is preferred to f_y, $f_z \geq hf_y$ if and only if $U(F_z) = \int_R U(x)dF_z(x)$. Consequently, the expected utility decomposition of $U(f_z)$ is now:

$$U(F_z) = \int_R U(x)dF_z(x) \qquad 2.4.2$$

Where $U: R \rightarrow R$ is the elementary utility function over outcomes. Naturally, if Z only takes a finite number of values and thus there were a finite number of probabilities, then this becomes more familiar

$$U(F_z) = \sum xP(x)U(x) \qquad 2.4.3$$

Friedman and Savage (1948) constructed the concept of univariate "risk aversion" which, intuitively, implies that when facing choices with comparable returns, agents tends to choose the less-risky alternative; we can visualize the problem as in figure 2.9 below. Let Z be a random variable which can take on two values, $\{Z_1 Z_2\}$, and let p be the probability that Z_1 happens and (1-p) the probability that Z_2 happens. Consequently, expected outcome, or E(z)= $Pz_1 + (1-p)Z_2$ which is shown in figure 2.9 on the horizontal axis as the convex combination of Z_1 and Z_2. Let $U: R \rightarrow R$ be the elementary utility function depicted in figure 2.9 as concave. Thus, expected utility $E(U) = PU(Z_1) +$ $(1-p)U(Z_2)$, as shown in figure below by point E on the chord connecting

$A=\{Z_1, U(Z_1)\}$ and $B=\{Z_2, U(Z_2)\}$. The position of E on the chord depends, on the probability p.

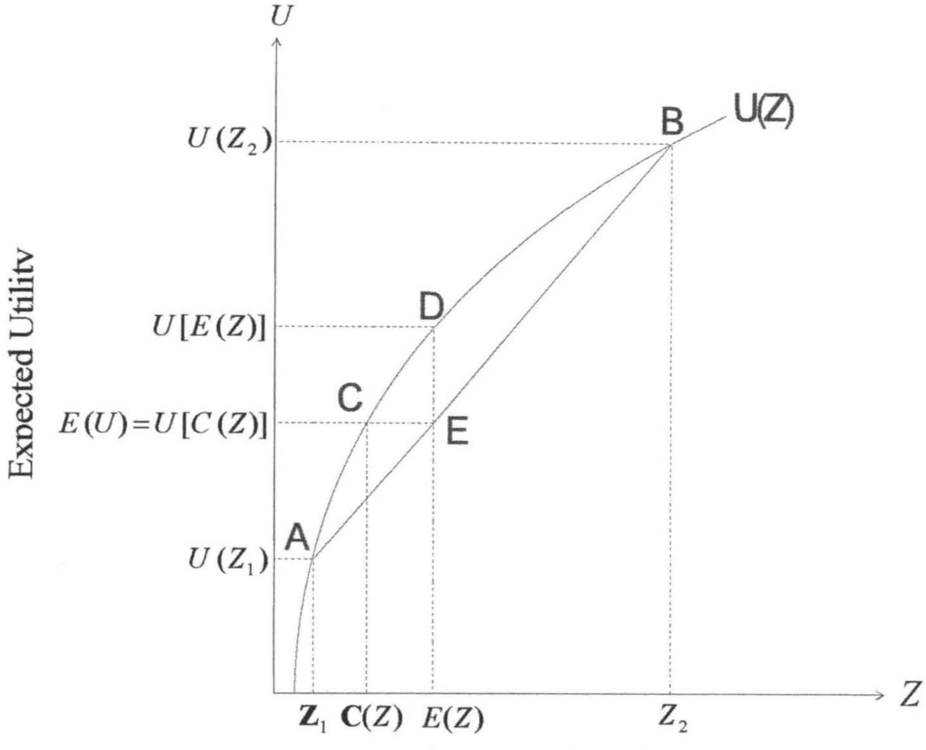


Fig 2.9 Risk- Aversion and certainty Equivalence

We note that by comparing points D and E in fig. 2.9 above that the concavity of the elementary utility function implies that the utility of expected income, $U[E(Z)]$ is greater than expected utility E(U), that is, $U[PZ_1 + (1-P)Z_2] > PU(Z_1) + (1-P)U(Z_2)$. This represents the utility- decreasing aspects of pure risk-bearing. Suppose, there are two lotteries, one that pays $E(Z)$ with certainty and another that pays $Z_1 \text{or} Z_2$ with probabilities $(p, 1-p)$ respectively. Reverting to Neumann-Morgenstern (1948) notation, the utility of the second lottery is

$U(Z_1, Z_2; p, 1-p) = PU(Z_1) + (1-p)U(Z_2)$. Now, the expected income in both lotteries is the same, yet it is obvious that if an agent is generally averse to risk, he would prefer E(Z) with certainty than E(Z) with uncertainty, that is, he would choose the first lottery over the second. This is what is captured in figure 2.9 as $U[E(Z)] > E(U)$.

Another way to capture this effect is by finding a "certainty-equivalent" allocation. In other words, consider a third lottery which yields the income C (Z) with certainty. As it is obvious from figure 2.9, the utility of this allocation is equal to the expected utility of the random prospect, that is, $U(C(Z)) = E(U)$. Thus, lottery C (Z) with certainty is known as the certainty-equivalent lottery, that is, the sure-thing lottery which yields the same utility as the random lottery. However, notice that the income C (Z) is less than the expected income, C (Z) <E (Z). Yet we know that an agent would be indifferent between receiving C (Z) with certainty and E (Z) with uncertainty. This difference, which is denoted, $\pi(Z) = E(Z) - C(Z)$ is known as the risk-premium, that is, the maximum amount of income that allocation without risk Pratt (1964).

Turning to generalities, letting $U: R \to R$ be an elementary utility function, Z be a random variable with cumulative distribution function $F_z, so\ F_Z(X) = P\{Z \leq X\}$ Denote by M the set of all random variables. For a particular random variable $Z \in M$, the expected Z is $E(Z) = \int_R X dF_Z(x)$ and the expected utility is $E(U(Z)) = \int_R U(x) dF_Z(x)$. Let $C^U(Z)$ denote the certainty-equivalent allocation, that is, $C^{U}(Z) \sim hZ$ and the risk premium as that certainty equivalence and risk-premium are dependent on the form of the elementary utility function. Then, we define risk aversion as follows:

Risk Aversion: An agent is "risk-averse" if

$C^{U}(Z) \leq E(Z) or \pi^{u}(Z) \geq o$ For all $Z \in M$.

This just formalizes the notion that we had figure 2.9. of course, we can easily visualize that if an agent is not risk averse, for instance he does not care about risk, then we should expect that receiving E (Z) with certainty or uncertainty should not matter to him, thus $U(E(Z) = E(U)$. In terms of figure 2.9, this would require that the elementary utility function U(Z) be a straight line so that point D and E coincide. It is obvious in this case that $C^{U}(Z) = E(Z)$ and $\pi^{U}(Z) = o$ thus:

Risk Neutral: an agent is "risk-neutral" if

$C^{U}(Z) = E(Z) or \pi^{U}(Z) = o$ for all $Z \in M$.

If we have a risk-loving agent, we should expect that he would prefer receiving E(Z) with uncertainty than receiving it with certainty, thus $U(E(Z)) < E(U)$. In this case, his utility function would have to be one where point E lies above D. This will be the case if the elementary utility functions $U: R \rightarrow R$ is a convex function. It is easy to visualize that, in such case, he would pay a premium to take on the risk or, equivalently, one would have pay him to move to a certainty-equivalent allocation, thus $C^{U}(Z) > E(Z)$ and $\pi^{U}(Z) < o$. Thus:

Risk-Proclivity: an agent has "risk-proclivity" (or is risk-loving) if $C^U(Z) > E(Z) or \pi^U(Z) < 0$ for all Z∈M.

Now, we have appealed to the ideas of concave, linear and convex utility functions to represent risk-aversion, risk-neutrality and risk-proclivity.

As Friedman and Savage (1948) indicated, it is not necessarily true that an individual's utility function has the same kind of curvature everywhere: there may be levels of wealth, for instance, when he is a risk-lover and levels of wealth when he is risk-neutral. We can see this in the famous Friedman-Savage double infection utility function in figure 2.10. Obviously, U (Z) is concave up until inflection point B and then becomes convex until inflection C after which it becomes concave again. Thus, at low income levels (between the origin and Z_B) agents exhibit risk-averse behaviors; similarly, they are also risk averse at very high incomes levels (above Z_C). However, between the inflection point B and C, agents are risk-loving.

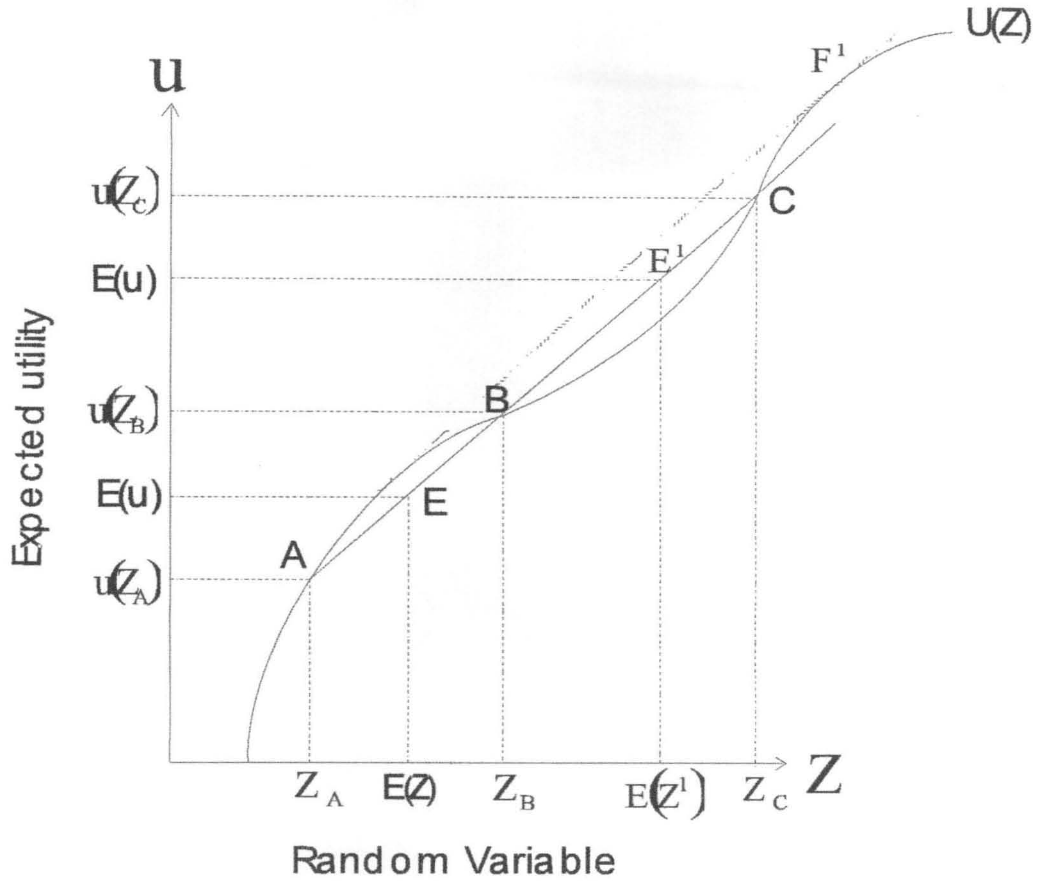


Fig 2.10: Friedman-Savage Double-inflection utility function

Friedman and Savage (1948) tried to use this to explain why people may take low probability, high-pay off risks (e.g lottery tickets) while at the same insuring against mild risks with mild pay offs (e.g flight insurance). To see this, presume one is at point B, on the inflection between risk-aversion and risk-loving. Suppose, one faces two lotteries, one yielding A or B another yielding B or C. there lotteries are captured by the solid-line chords between the respective payoffs AB and BC. Expected utility of the first gamble is noted to be E(U) and is depicted in figure 2.10 at point E-where, obviously, E(U) is less than the utility of the expected outcome of the first gamble, U(E(Z). consequently, a risk averse agent would pay a premium to avoid it. The second gamble yields expected utility $E(U^1)$- at point E^1 on the BC chord-which is greater than the utility of the expected outcome $U(E(Z^1))$. A risk-loving agent would pay a premium to under take this gamble. Thus, we can view risk-averse behavior with regard to AB as a case of insurance against small losses and the risk-loving behavior with regard to BC as a case of purchasing lottery tickets.

Harry Markowitz (1952), however, disputed the Friedman-Savage conjective that people, or at least the population in aggregate, have such doubly infected utility curves. Specifically, Markowitz noted that a person at point F would also accept a gamble that might take them to F'. conversely, a person at F' or slightly below it will not pay a premium against being taken down to F, for instance, that is, he will not take insurance against situations of huge losses with low probability. Finally, people above F', that is the very rich, will never take a fair bet-a phenomenon that does not seem compatible with empirical phenomena such as, well, Monte Carlo casinos. What Markowitz (1952) proposed, instead, was that the Z^1s be considered not "income levels" as Friedman and Savage proposed, but rather "changes in income" and added an additional inflection point at the bottom. People's "normal" income-whether rich, poor or moderate, and controlling for the utility derived from the recreational pleasure of gambling would all a point such as B and the rest would reflect deviations from this average income. In this manner, the apparent lottery-insurance paradox is resolved without invoking the strange implications of the original Friedman-Savage hypothesis.

Actually, we want to say that little is known about the distribution of the minimum variance portfolio. Dickinson (1974) calculates the unconditional distributions of the portfolio weights in the special case of two uncorrelated assets.

Jorion (1991) and Chopra et al (1993) suggested that the tangency portfolio is the only efficient stock portfolio. However, many empirical studies show that an investment in the minimum variance portfolio often yields better out-of-sample results than does an investment in the tangency portfolio. This result is typically attributed to the high estimation risk associated with expected returns. However, Ledoit and Wolf (2003) and Jagannathan et al (2003) recently confirmed that investing into the minimum variance portfolio yields more returns than tangency portfolio.

In conclusion, it is worth mentioning the on going research about the conditional distribution of the estimated weights of the minimum variance portfolio by Okhrin and Schmid (2005) to support the development made by Dickinson (1974). Estimates for expected return and the return variance of the minimum variance portfolio are obtained. If conditional distributions are known, it will lead to great development in assets management

2.5 Diversification

Diversification involves spreading investments around into many types of investments, including stocks mutual funds, bonds and cash. Money can also be diversified into different mutual fund investment strategies, including growths funds, balance funds, index funds, and sector-specific funds. Geographic diversification involves a mixture of domestic and international investment.

Diversification reduces the risk of a portfolio. It does not necessarily reduce the returns. This is why diversification is referred to as the only free lunch in finance.

Diversification can be quantified as the intra-portfolio correlation. This is a statistical measurement from negative one to one that measures the degree to which the various assets in a portfolio can be expected to perform in a similar fraction or not. Portfolio balance occurs as the sum of all intra-portfolio correlations approaches negative one. Diversification is thus defined as the intra-portfolio correlation or, more specifically, the weighted average intra-portfolio correlation. Maximum diversification occurs when the intra-portfolio correlation is minimized. intra-portfolio correlation may be an effective risk management measurement. The computation may be expressed as:

$$Q = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j P_{ij}}{\sum_{j=1}^{n}\sum_{j=1}^{n} X_j X_j} \qquad 2.5.1$$

Where Q is the intra-portfolio correlation, X_i is the fraction invested in asset i, X_j is the fraction invested in asset j, P_{ij} is the correlation between assets i and j, and n is the number of different assets.

Types of Diversification

(i) Horizontal Diversification: Is when you diversify between same-type investments. It can be a broad diversification (like investing in several companies) or more narrowed (investing in several stocks of the same branch or sector).

(ii) Vertical Diversification: Is investing between different types of investment. Again, can be a very broad diversification, like diversifying between bonds and stocks, or a more narrowed diversification, like diversifying between stocks of different branches.

While horizontal diversification lessons the risk of just investing al-in-one, a vertical diversification goes far beyond that and insures you against market and economical changes. Furthermore, the broader the diversification the lesser the risk.

2.5.1 Measurement of Portfolio Diversification

Defaults translate into losses. The loss associated with a single default depends on the amount recovered. We assume that the recovery in the event of default is known, and that this recovery is net of the expenses of collection including the time value of the recovery process. Thinking of the recovery as a percent of the face value of the loan, we can also specify the "loss given default" as one minus the expected recovery.

Using this structure, the expected loss for a single borrowing is the probability of default times the loss given default the unexpected loss depends on the same variables as the expected loss. (It equals the loss given default times the square root of the product of the probability of default times one minus the probability of default). The unexpected loss represents the volatility, or standard deviation of loss. This approach raises the question of how to deal with instruments of different maturities. The analysis here uses a single time horizon for measuring risk. Establishing one horizon for analysis forms the basis of a frame work for comparing the attractiveness of different types of credit exposures on the same scale. The risk at the horizon has two parts: the risk due to possible default, and the risk of loss of value due to credit deterioration. Instruments of the same borrower with different maturities (as long as the maturity is at or beyond the horizon) have the same default risk at the horizon, but the value risk (that is, uncertainty around the value of the instrument as horizon) depends upon the remaining time to maturity. The longer the remaining time, the greater the variation in value due to credit quality changes.

$$EL \equiv \text{Expected loss} = \text{EDF XLGD}$$

$$UL \equiv \text{Unexpected loss} = \text{LGD}\sqrt{\text{EDF}(1-\text{EDF})}$$

Where

EDF ≡Probability of default (Expected Default Frequency)

LGD ≡Loss given default, (in percentage)

Measuring the diversification of a portfolio means specifying the range and likelihood of possible losses associated with the portfolio. All else equal, a well diversified portfolio is one that has a small likelihood of generating large losses.

The average expected loss for a portfolio is the average of the expected losses of the assets in the portfolio. It is not a simple average but a weighted average, with the weights equal to each exposure amount as a percent of the total portfolio exposure. It would be convenient if the volatility, or unexpected loss, of the portfolio were simply the weighted average of the unexpected losses of the individual assets, but it is not. The reason is that portfolio losses depend also on the relationship (correlation) between possible defaults.

Now let us extend this notion of diversification to the more general case of a portfolio with multiple risky securities.

Let use these representation:

$X_i \equiv$ Face value of security i

$P_i \equiv$ Price of security i

$V_p \equiv$ Portfolio value $= P_1X_1 + P_2X_2 + ... + P_nX_n$

$W_i \equiv$ Value proportion of security i in portfolio

("weight") $= P_iX_i / V_p$

$P_{ij} \equiv$ Loss correlation between security i and security j Note that $W_1 + W_2 + ... + W_n = 1$

$EL_i \equiv$ Expected loss for security i

$EL_i \equiv$ Portfolio expected loss $\equiv W_1ELi + W_2EL_2 + ... + W_nEL_n$

$UL_i \equiv$ Unexpected loss for security i

$UL_p \equiv$ Unexpected loss for portfolio

Therefore, the portfolio loss measures can be calculated as follows:

$$= \sqrt{\begin{array}{l} W_1W_1UL_1\rho_u + W_1W_2UL_2\rho_{12} + \ldots\ldots\ldots\ldots + W_1W_2UL_1UL_n\rho_{in} + \\ W_2W_1UL_2UL_1\rho_{21} + W_2W_2UL_2\rho_{22}\ldots\ldots\ldots\ldots + W_2W_nUL_2UL_2UL_n\rho_{2n} \\ + \ldots\ldots\ldots + W_nW_1UL_nUL_1\rho_{ni} + W_nW_2UL_2\rho_{n2} + \ldots\ldots\ldots\ldots + \\ W_nW_nUL_n\rho_{nn} \end{array}}$$

Note that, $\rho_{ij} = 1$ i=j and $\rho_{ij} = \rho_{ji}$

The portfolio expected loss is the weight average of the expected losses of the individual securities, where the weights are the value of proportions. On the other hand, the portfolio's unexpected loss is a more complex function of the ULs of the individual securities, the portfolio weights and the pair wise loss correlation between securities. In practice, actual defaults are positively but not perfectly positively correlated. Diversification, while not perfect, conveys significant benefits. Unfortunately negative default correlations are rare to non-existent.

Calculating portfolio diversification means determining the portfolio's unexpected loss. To do this, default correlation and ultimately, correlation in instrument values are required.

CHAPTER THREE

3.0 THE PORTFOLIO MANAGEMENT MODELS

3.1 Default Risk

A corporation has fixed obligations. These may be no more than its trade obligations, although they could just as well include bank loans and public debt. At one time, there was no legal means to escape the fulfillment of such obligations, if a defaulter fled or was jailed. Modern treatment allows the defaulter to escape the obligation but only by relinquishing the corporation's assets to the obligee. In other words, a firm owing a single creditor #75 million fulfils the obligation by either paying the #75 million or by transferring the corporation's assets to the lender.

Any action the borrower will take is an economic decision. The economic answer will be: if the corporate assets are worth more than #75 million, the borrower will meet the obligation, if they are worth less the borrower will default. The critical point is that the action depends on the market value of assets.

Note that the option to default is valuable. Without it, the corporation could be forced to raise additional capital with the benefit accruing not to its owners but instead to its prior leader.

A lender purchasing a corporations note can be thought of as engaging in two transactions. In the first it is purchasing an "inescapable" debt obligation, that is, one which cannot be defaulted on. In the second, it is selling a "put" option to the borrower that states that the lender will buy the corporation's assets for the amount of the note at the option of the borrower. In the event the assets turn out to be worth less than the amount of the note, the borrower "put" the assets to the lender and uses the proceeds to pay the note.

The lender owns a risk-free note and is "short" the default option. The probability of default is the same as the probability of the option being exercised. If the probability of default goes up, the value of the option goes up, and the value of the lender's position (because it is "short" the option) goes down.

The probability of exercising the default option can be determined by the application of option valuation methods. Assume for a moment that the market value of the corporation's assets is known, as well as the volatility of that value. The volatility measures the propensity of the asset value to change within a given time period. This information determines the probability of default, given the corporation's obligation. For instance, if the current asset market value is #150 million and the corporations debt is #75 million and is due in one year, then default will occur if the asset value turns out to be less than # 75 million in one year.

If the firm's asset volatility is 17% per year, then a fall in value from #150 million to # 75million is a three standard deviation event with a probability of 0.3%. Thus the firm has a default probability of 0.3% (17% of

150 is 25). This is the amount of a one standard deviation more. The probability calculation assumes that the asset have a lognormal distribution.

3.1.1 Asset Market Value and Volatility

Just as the firm's default risk can be derived from the behaviour of the firm's asset value and the level of its obligation, the firm's equity behaviour can be similarly derived. The shareholders of the firm can be viewed as having a call option on the firm's asset value, where the exercise price is equal to the firm's obligations. If the market asset value exceeds the obligation amount at the maturity date, then the shareholder will exercise their option by paying off the obligation amount. If the asset value is less, they will prefer to default on the obligation and relinquish the remaining asset value to the lender.

Using this framework, the equity value and volatility can be determined from the asset value, asset volatility, and the amount and maturity of obligation. What is actually more important is that the converse is also true: the asset value and volatility can be inferred from the equity value, equity volatility and the amount and maturity of obligations. This process enables us to determine the market characteristics of a firm's assets from directly observable equity characteristics.

Knowing the market value and volatility of the firm's asset is critical, as we have seen, to the determination of the probability of default. With it we can also determine the correlation of two assets values. These correlations play an important role in the measurement of portfolio diversification.

3.2 Model of Default Correlation

Default correlation measures the strength of the default relationship between two assets in a firm. If there is no relationship, then the defaults are independent and the correlation is zero. In such a case, the probability of both assets being default at the same time is the product of their individual probabilities of default.

When two assets are correlated this means that the probability of both defaulting at the same time is higher than it would be if they were completely independent. In fact, the correlation is just proportional to this difference. Thus, holding their individual default probabilities fixed, it is equivalent to say either that two assets are highly correlated or that they have a relatively high probability of defaulting in the same time period.

The basic default model says that asset will default when its market value falls below the face value of obligation (the "default point"). This means that the joint probability of default is the likelihood of both assets market value being below their respective default point in the same time period.

This probability can be determined quite readily from knowing:

i. the assets' current market values

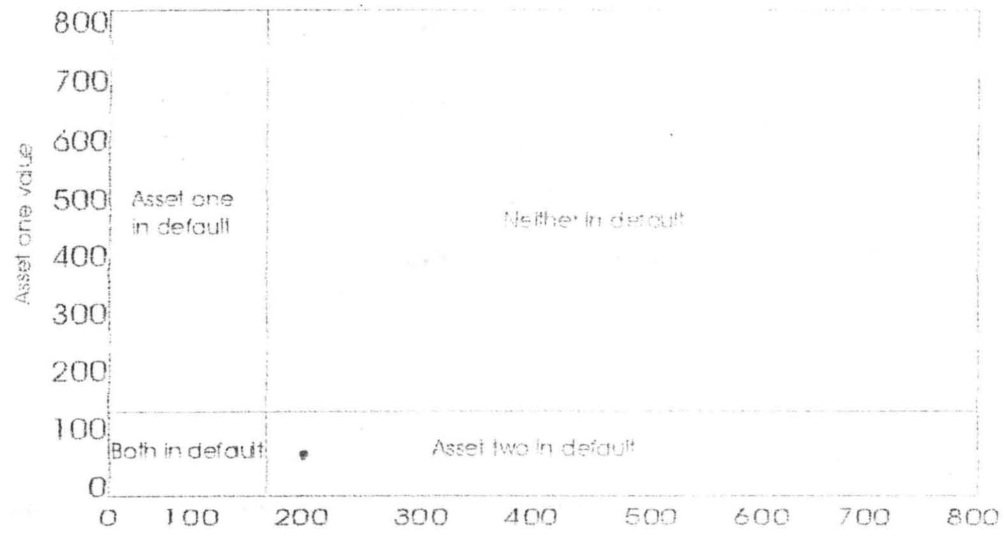
ii. their assets' volatilities and

iii. the correlation between the two assets' market values.

In other words the derivatives framework enables us to use the asset correlation to obtain asset default correlation.

The derivatives approach enables us to measure the default correlation between two assets using their asset correlation and their individual probabilities of default.

Default and Non-Default Ranges



Asset two value

Table 3.2.1 Default and Non-Default Ranges

The figure above illustrates the ranges of possible future asset values for a firm. The two intersection lines represent the default points for the two asset's. For instance, if asset's one value ends up being below #180 million (the point represented by the vertical line), then asset one will default.

The intersecting lines divide the range of possibilities into four regions. The upper right region represents those asset values for which neither asset one nor asset two will default. The lower left region represents those asset values for which both assets will default. The probabilities of all these region taken together must equal one. If the asset values of the two assets were independent, the probabilities of the region could be determined simply by multiplying the individual probabilities of default and non-default for the two assets. For instance, suppose that asset one's default probability is 0.6% and asset two's is 0.3%. The probability of both defaulting, if they are independent, is the product of the default probabilities, or 0.0018%.

If the two asset values are positively correlated, then the probability of both asset values being high or low at the same time is higher than if they were independent and the probability of one being high and the other low is lower. For instance, using the previous default probabilities, the probability of both defaulting might now be 0.01%, if their asset values are positively correlated.

By knowing the individual assets' default probabilities, and knowing the correlation of their asset values, the likelihood of both defaulting at the same time can be calculated. The time series of a firm's asset values can be determined from its equity values. The correlation between two assets values can be calculated from their respective time series.

We can calculate default correlation as follows:

ρ_D=default correlation for asset 1 and asset 2

$$= \frac{JDF - EDF_1 EDF_2}{\sqrt{EDF_1(1 - EDF_1)EDF_2(1 - EDF_2)}} \qquad 3.2.1$$

Where:

JDF= Joint default frequency of asset 1 and asset 2 that is, actual probability of both assets defaulting together.

EDF_1 =Expected default frequency of asset 1

EDF_2= Expected default frequency of asset2

The numerator of this model represents the difference of the actual probability of both assets defaulting and the probability of both defaulting if they were independent. Note that if the asset values are independent, then the default correlation is zero.

3.3 Model of Value Correlation

An important strength of the structural model of default presented here has ability to generalize relationships in a way to create a comprehensive credit portfolio model. In addition to the EDF values for each asset, the joint default frequency (JDF) must be calculated to determine a value correlation. The JDF can be calculated by focusing on the relationship between an asset's market value and its respective default point. EDF values embed this information on an individual firm level. The remaining piece of the puzzle is the correlation between each asset's market value.

We can write down the following function for the JDF.

$$JDF=N_2[N^{-1}(EDF_1), N^{-1}(EDF_2), \rho_A] \qquad 3.3.1$$

Where

$N_2 \equiv$ bivariate normal distribution function

$N^{-1} \equiv$ inverse normal distribution function

$\rho_A \equiv$ correlation between asset 1's return and asset 2's return.

We turn to factor modelling to calculate correlation. A factor model relates the systematic or non-diversifiable components of the economy that drive changes in asset value. For example, the entire economy may follow a business cycle which affects most companies prospects. The impact may differ from company to company, but they are affected nonetheless. Determining the sensitivity of changes in asset values to changes in a particular economic factor provides the basis for estimating asset correlation.

Changes in a firms asset value constitutes an asset value return. We can decompose this return as follows:

[Asset return] = [Composite Factors Return] + [Asset Specific Effects]

We can further decompose this composite factor return as follows

[Composite Factor Return] = [Country Factor Returns] + [Firm Factor Return]

[Country Factor Return] = [Global Economic Effect] + [Regional Factor Effect] + [Sector Factor Effect] + [Country Specific Effect]

[Firm Factor Return] = [Global Economic Effect] + [Regional Factor Effect] + [Sector Factor Effect] + [Firm Specific Effect]

Asset correlation can then be calculated from each assets systematic or composite factor return.

The following relationship is constructed as follows:

$$\phi_k = \sum_{c=1}^{c} \omega_{kc}\gamma_c + \sum_{f=1}^{f} \omega_{kf}\gamma_f \qquad 3.3.2$$

Where:

$\omega_{kc} \equiv$ weight of asset k in country c

$\omega_{kf} \equiv$ weight of asset k in firm f

$\gamma_c \equiv$ return index for country c

$\gamma_f \equiv$ return index for firm f

$\phi_k \equiv$ composite market factor index for asset k.

Once the composite index is calculated for a particular asset, the sensitivity (that is, beta) to the market factor reflected in this index can be estimated. The relationship used for this estimation is written as follows:

$$\gamma_k = \beta_k \phi_k + \varepsilon_k \qquad 3.3.3$$

Where :

$\gamma_k \equiv$ return for asset K

$\beta_k \equiv$ beta for asset k

$\varepsilon_k \equiv$asset-specific component of return for asset k

We can similarly estimate the sensitivity or beta ($\beta_{\text{country,common factor}}$ and $\beta_{\text{firm,ommon factor}}$) of countries and firms on factors we specify.

An example of calculating the sensitivity of asset k to a global factor is written as

$$\beta_{kG} = \beta_k \left(\sum_{c=1}^{\bar{c}} \omega_{kc} \beta_{cG} + \sum_{f=1}^{\bar{f}} \omega_{kf} \beta_{fG} \right) \qquad 3.3.4$$

These calculations produce the parameters necessary to estimate firm asset value correlation. We construct this calculation as follows.

$$\delta(j,k) = \sum_{G=1}^{\bar{G}} \beta_{jG}\beta_{kG}\delta_G^2 + \sum_{R=1}^{\bar{R}} \beta_{jR}\beta_{kR}\delta_R^2 + \sum_{S=1}^{\bar{S}} \beta_{jS}\beta_{kS}\delta_S^2 + \sum_{F=1}^{\bar{F}} \beta_{jF}\beta_{kF}\varepsilon_F^2 + \sum_{C=1}^{\bar{C}} \beta_{jC}\beta_{kC}\varepsilon_C^2 \qquad 3.3.5$$

Where

$\delta(j,k) \equiv$ Covariance between asset's and asset k

$$\rho_{jk} \equiv \frac{\delta(j,k)}{\delta_j \delta_k} \qquad 3.3.6$$

Where

$\rho_{jk} \equiv$ Correlation between asset j's and asset k's value return

$\delta_j \equiv$ S tan dard deviation of asset j's value return

$\delta_k \equiv$ S tan dard deviation of asset k's value return

The covariance depends on the sensitivity or betas $(\beta_{firm,\,factor})$ for each asset combined with the factor variances (δ^2_{factor}). To arrive at the correlation we must scale the covariance by the standard deviation of the returns as shown in equation 3.3.6 above.

Example 3.3.1

Assume we are analysing a portfolio of three loans to three assets in a firm. We determine that the asset values of asset A and asset B increase (decrease) whenever interest rate decline (rise). Asset C is unaffected by changes in interest rates. In this economy, we have only one factor-interest rate movement. We then simulate this one factor. Whenever this interest rate factor is high, A's and B's values are small. These low asset values result in the loans to A and B being valued at a discount; C's loan value is unchanged, since c is not affected by the interest-rate factor. If the interest rate factor is low, A's and B's loans will be valued at a premium. The key to the correlation arises from the similar behaviour in loan value whenever a particular factor level is drawn.

Clearly, the movement in the value of A's and B's loans are correlated while C's loans are uncorrelated with the rest of the portfolio.

3.3.1 Probability of Losses

If portfolio losses had a bell shaped distribution we could accurately specify the likelihood of losses simply knowing the expected and unexpected loss for the portfolio. The problem is that individual debt assets have very "skewed" loss probabilities. Most of the time the asset does not default and the loss is zero. However when default occurs, the loss is usually substantial.

Given the positive correlation between default, this unevenness of loss never fully "smoothes out", even in very large portfolio. There is always a large probability of relatively small losses, and a small probability of rather large losses.

This "skewness" leads to an unintuitive result a very high percentage of the time (around 80%), the actual losses will be less than the average loss. The reason is that the average is pulled upwards by the potential for large losses. There is a danger of being "silenced" by a string of low losses into believing that the portfolio is much better diversified than in fact is it.

The frequency distribution of portfolio losses can be determined using the information we have already discussed. Knowing this distribution for a given portfolio gives an alternative characterization of diversification. Portfolio A is better diversified than portfolio B if the probability of loss exceeding a given percent is smaller for A than for B, and both portfolios have the same expected loss.

The view of diversification has an immediate concrete implication for capital adequacy. Given the frequency distribution of loss, we can determine the likelihood of losses which exceed the amount of capital held against the portfolio. This probability can be set to the desired level by varying the amount of capital.

This can be done in practice, since it is necessary to consider the market value, rather than the book value of the portfolio. To do that, we need to be able to determine the market value of an asset. Consequently, we must rely on models to determine a mark-to-market value. Let us consider the market value of a loan.

3.4 Model of Valuation

It should be noted at this juncture that this work is based on return and risk of assets to portfolio. In view of this, it is expedient we look at value of loans spent on assets. This value of loan would stand as a benchmark at which the assets under consideration would either default or yield a reasonable return.

The market value of a loan is simply the price for which it can be bought or sold. Although, there is a loan sales market, loans by and large are not actively transacted for extended periods. The result is that current market prices do not exist for most loans. The objective of valuation is to determine what a loan should sell for, were it to trade. The value cannot be determined in the abstract or in some absolute sense, but only by comparison to the market prices of financial instruments that are traded. Valuation consists of extrapolating actual market prices to non-traded assets, based on the relationship between their characteristics.

The so called "pricing" on a loan is the set of fees and spreads which determines the promised cash flows between asset and manager (as case of this research work). This is the equivalent of the coupon rate on a bond. The value of the loan is obtained by discounting the loan cash flows by an appropriate set of discount rates. The discount rates in the absence of default risk, would simply differ by increments of term, according to the current term structure.

In the presence of default risk, the discount rates must contain two additional elements. The first is the expected loss premium. This reflects an adjustment to the discount rate to account for the actuarial expectation of loss. It is based on the probability of default and the loss first given default. The Second is the risk premium. This is compensation for the non-diversifiable loss risk in the loan.

If the loan does not contain a risk premium, then on average it would only return the risk-free base rate. The key point is the qualifier: "on average". If default does not occur, the asset would return a little more due to the expected loss premium. However, if default occurs, it would return much less.

An investor could obtain the risk free base rate not just "on average", but all the time by buying the risk free asset, the risky asset must provide additional compensatory return. Actually, this would not be the case if dafault risk was completely diversifiable. The market will provide compensation for unavoidable risk bearing, that is the portion of the loan;s loss risk that cannot be eliminated through diversification.

The amount of non-diversifiable risk can be determined from knowing the asset's probability of default and the risk characteristics of the assets. The market price for risk bearing can be determined from the equity and fixed income markets.

There are only two possible outcomes for an asset. Either it yields return, or the asset defaults. The loss distribution for a single loan on asset is given as follows:

Event	Probability
Default	EDF
No default	1-EDF

Table 3.4.1: loss distribution

In the event of default, the asset is expected to lose a percentage of the face value of the loan, which is the loss given default (L G D). If the yield on the loan of the asset is Y and the risk-free base rate is Rf, then the return distribution can be characterized as follows:

Event	Probability	Return
Default	EDF	Rf-LGD
No default	1-EDF	Y

Table 3.4.2: return distribution

The expected return is the probability weighted average of the return

$$E(R) = EDF(Rf - LGD) + (1 - EDF)Y \qquad 3.4.1$$

The required compensation for the actuarial risk of default:

$$= \frac{LGD \times EDF}{1 - EDF} \qquad 3.4.2$$

This is called the expected loss premium. If the loan yield equaled the risk-free base rate plus the expected loss premium, then.

$$Y = R_f + \frac{LGD \times EDF}{(1 - EDF)} \text{ And} \qquad 3.4.3$$

$$E(R) = EDF(R_f - LGD) + (1 - EDF)\left[R_f + \frac{LGD \times EDF}{(1 - EDF)}\right] \qquad 3.4.3$$

$$E(R) = R_f \qquad 3.4.4$$

The expected loss premium provides just enough additional return when the asset does not default to compensate for the expected loss when the asset does default.

The model above shows that if the only additional compensation were the expected loss premium, then the asset on average would receive only the risk free base: Base on the model above, it would be much better for a portfolio to use risk free base loan on asset, since it would get the same average return and would incur no default risk. But it has been noticed that there must be additional compensation for the fact that the realized return is risky even for a large, well-diversified portfolio of loan on asset. That additional compensation is called the risk premium.

The required pricing on a loan is thus the risk-free base rate plus the expected premium plus the risk premium.

$Y = R_f + EL$ premium + risk premium

The required risk premium in the market can be determined by taking the credit spread on debt securities and subtracting the appreciate expected loss premium. The remainder is the market risk premium.

Now, looking at the yield or return on a loan in asset to portfolio as being an average of those various discount rates, then the value of the loan is its promised cash flows discounted at its yield. If the yields exceed the loan rate, the loan will be at a discount rate, and then asset yields a reasonable return to portfolio. But, if other wise asset defaults.

In conclusion, the models described in this section are sufficient to measure or monitor the performance of assets in a portfolio. With this, we would know whether the assets are yielding returns or defaulting, and if defaulting, so that different approach can be applied. But, it should be noted that research still continues in an attempt to find direct way of estimating default correlation and default probability of asset.

CHAPTER FOUR

4.0 COMPUTATION OF RESULTS

4.1 Numerical Computation

As it was mentioned in the aims and objectives of this work that, we wish to construct models to determine the returns and risk of an organization. Hence, in this section we shall construct models to evaluate the returns and risk of the organization under consideration.

We shall construct models for two, three and four assets. Thereafter, we shall work with the two assets models and this would serve for our limitation.

Two Assets Model

$$Rp = w_1 R_1 + W_2 R_2$$

$$\delta^2 p = w_1^2 \delta_1^2 + W_2^2 \delta_2^2 + 2 w_1 w_2 \operatorname{cov}_{1,2}$$

$$\operatorname{cov}_{1,2} = r_{1,2} \delta_1 \delta_2$$

$$\delta^2 p = w_1^2 \delta_1^2 + W_2^2 \delta_2^2 + 2 w_1 w_2 r_{1,2} \delta_1 \delta_2$$

$$\delta p = \sqrt{\delta^2 p}$$

Three Assets Model

$$Rp = w_1 R_1 + W_2 R_2 + W_3 R_3$$

$$\delta^2 p = w_1^2 \delta_1^2 + W_2^2 \delta_2^2 + W_3^2 \delta_3^2 + 2 w_1 w_2 r_{1,2} \delta_1 \delta_2 + 2 w_1 w_3 r_{1,3} \delta_1 \delta_3 + 2 w_2 w_3 r_{2,3} \delta_2 \delta_3$$

$$\delta p = \sqrt{\delta^2 p}$$

Four Assets Model

$$Rp = w_1R_1 + W_2R_2 + W_3R_3 + W_4R_4$$

$$\delta^2 p = w_1^2\delta_1^2 + W_2^2\delta_2^2 + W_3^2\delta_3^2 + W_4^2\delta_4^2 + 2w_1w_2r_{1,2}\delta_1\delta_2$$

$$+ 2w_1w_3r_{1,3}\delta_1\delta_3 + 2w_1w_4r_{1,4}\delta_1\delta_4 + 2w_2w_3r_{2,3}\delta_2\delta_3$$

$$+ 2w_2w_4r_{2,4}\delta_2\delta_4 + 2w_3w_4r_{3,4}\delta_3\delta_4$$

$$\delta p = \sqrt{\delta^2 p}$$

Where:

r=Correlation coefficient between assets

R_p=returns of the portfolio

δ_p^2=variance of the portfolio

δ_p=standard deviation of the portfolio (RISK)

W=weight or value of the asset.

Thus, for any n-asset portfolio, as long as we know the following parameters, one can determine the return and risk characteristics; $R_{A,}$ δ_A, $cov_{A,I}$ or $\delta r_{A,I}$ .With N assets, there are N (N-1)/2 such pairs, and weightings of the portfolio.

For the purpose of the research work, we shall work with two assets models in three focuses. We want to see the effect of diversification on uncorrelated assets, perfectly correlated assets and perfect negatively correlated assets.

4.2 Data Analysis

For the analysis of this research work, we collected useful data from Investment Banking and Trust Company PLC (IBTC). As, it has been mentioned earlier that the research work would be limited to two assets therefore we only collected data for those two assets. The assets are equipment on lease and fixed assets from 2001 to 2005. Hence, we shall use the data to see the effect of these assets on percentage return and risk to the portfolio with in this period.

Year	**Rp_1 on equipment on lease (%)**	δ_1 (%)	**Rp_2 on fixed Assets (%)**	δ_2 (%)	**W (%)**
2001	0.2	9.9	5.0	0.8	50.0
2002	0.8	5.5	4.3	7.7	50.0
2003	0.8	10.7	0.3	50.0	50.0
2004	0.5	61.5	2.7	10.6	50.0
2005	0	12.5	3.6	31.4	50.0

Tables 4:1 Data for Equipment on lease and Fixed assets

Where:

$Rp_1 \equiv$ Return of equipment on lease to portfolio

$Rp_2 \equiv$ Return of fixed assets to portfolio

$\delta_1 \equiv$ Standard deviation (risk) of equipment on lease to portfolio

$\delta_2 \equiv$ Standard deviation (risk) of fixed assets to portfolio

W ≡ Weighting or value of the assets

$$Rp = w_1R_1 + W_2R_2$$
$$\delta^2 p = w_1^2\delta_1^2 + W_2^2\delta_2^2 + 2w_1w_2r_{1,2}\delta_1\delta_2$$
$$\delta p = \sqrt{\delta^2 p}$$

For year 2001

When rates of returns on assets are uncorrelated ($r_{1,2} = 0$)

$$Rp = (0.5)(0.002) + (0.5)(0.05) = 2.6\%$$
$$\delta^2 p = (0.25)(97) + (0.25)(0.64) + 0 = 24.4$$
$$\delta p = \pm 4.9\%$$

When rates of returns on the assets are perfectly correlated ($r_{1,2} = 1$)

$$Rp = (0.5)(0.002) + (0.5)(0.05) = 2.6\%$$
$$\delta^2 p = (0.25)(97) + (0.25)(0.64) + (2)(0.5)(0.5)(1)(9.85)(0.8) = 28.35$$
$$\delta p = \pm 5.3\%$$

When rates of return on assets are perfect negatively correlated ($r_{1,2} = -1$)

$$Rp = (0.5)(0.002) + (0.5)(0.05) = 2.6\%$$
$$\delta^2 p = (0.25)(97) + (0.25)(0.64) + (2)(0.5)(0.5)(-1)(9.85)(0.8) = 20.47$$
$$\delta p = \pm 4.52\%$$

For year 2002

When rate of return on assets are uncorrelated ($r_{1,2} = 0$)

$$Rp = (0.5)(0.008) + (0.5)(0.043) = 2.6\%$$
$$\delta^2 p = (0.25)(29.70) + (0.25)(59.29) = 22.2$$
$$\delta p = \pm 4.72\%$$

When rate of return on assets are perfectly correlated ($r_{1,2} = 1$)

$$Rp = (0.5)(0.008) + (0.5)(0.043) = 2.6\%$$
$$\delta^2 p = (0.25)(29.70) + (0.25)(59.29) + (2)(0.5)(0.5)(1)(5.45)(7.7) = 43.23$$
$$\delta p = \pm 6.57\%$$

When rate of return on assets are perfect negatively correlated ($r_{1,2} = -1$)

$$Rp = (0.5)(0.008) + (0.5)(0.043) = 2.6\%$$
$$\delta^2 p = (0.25)(29.70) + (0.25)(59.29) + (2)(0.5)(0.5)(-1)(5.45)(7.7) = 1.27$$
$$\delta p = \pm 1.12\%$$

For year 2003

When rate of return on assets are uncorrelated $r_{1,2} = 0$

$$Rp = (0.5)(0.008) + (0.5)(0.003) = 0.5\%$$
$$\delta^2 p = (0.25)(114.5) + (0.25)(2500) + 0 = 653.63$$
$$\delta p = \pm 25.56\%$$

When rate of return on assets are perfectly correlated $r_{1,2} = 1$

$$Rp = (0.5)(0.08) + (0.5)(0.003) = 0.5\%$$
$$\delta^2 p = (0.25)(14.5) + (0.25)(2500) + (2)(0.5)(0.5)(1)(10.70)(50) = 921.13$$
$$\delta p = \pm 30.35\%$$

When rate of return on assets are perfect negatively correlated $r_{1,2} = -1$

$$Rp = (0.5)(0.08) + (0.5)(0.003) = 0.5\%$$
$$\delta^2 p = (0.25)(114.5) + (0.25)(2500) + (2)(0.5)(0.5)(-1)(10.70)(50) = 118.63$$
$$\delta p = \pm 10.89\%$$

For year 2004

When rate of return on assets are uncorrelated $r_{1,2} = 0$

$$Rp = (0.5)(0.005) + (0.5)(0.027) = 1.6\%$$
$$\delta^2 p = (0.25)(3782.3) + (0.25)(112.4) + 0 = 973.68$$
$$\delta p = \pm 31.20\%$$

When rate of return on assets are perfectly correlated ($r_{1,2} = 1$)

$$Rp = (0.5)(0.005) + (0.5)(0.027) = 1.6\%$$
$$\delta^2 p = (0.25)(3782.3) + (0.25)(112.4) + (2)(0.5)(0.5)(1)(61.5)(10.6) = 1299.6$$
$$\delta p = \pm 36\%$$

When rate of return on assets are perfect negatively correlated ($r_{1,2} = -1$)

$$Rp = (0.5)(0.005) + (0.5)(0.027) = 1.6\%$$
$$\delta^2 p = (0.25)(3782.3) + (0.25)(112.4) + (2)(0.5)(0.5)(-1)(61.5)(10.6) = 329.28$$
$$\delta p = \pm 18.15\%$$

For Year 2005

When rate of return on assets are uncorrelated ($r_{1,2} = 0$)

$$Rp = (0.5)(0) + (0.5)(0.036) = 1.8\%$$
$$\delta^2 p = (0.25)(156.25) + (0.25)(985.96) + 0 = 285.55$$
$$\delta p = \pm 16.9\%$$

When rate of return on assets are perfectly correlated $r_{1,2} = 1$

$$Rp = (0.5)(0) + (0.5)(0.036) = 1.8\%$$
$$\delta^2 p = (0.25)(156.25) + (0.25)(985.96) + (2)(0.5)(0.5)(1)(12.6)(31.4) = 483.37$$
$$\delta p = \pm 21\%$$

When rate return on assets are perfect negatively correlated $r_{1,2} = -1$

$$Rp = (0.5)(0) + (0.5)(0.036) = 1.8\%$$
$$\delta^2 p = (0.25)(156.25) + (0.25)(985.96) + (2)(0.5)(0.5)(-1)(12.6)(31.4) = 87.7$$
$$\delta p = \pm 9.37\%$$

We shall now tabulate our result for further comment.

Year	Uncorrelated Assets		Perfectly correlated Assets		Perfect negatively correlated Assets	
	Rp (%)	δ (%)	Rp (%)	δ (%)	Rp (%)	δ (%)
2001	2.6	4.9	2.6	5.3	2.6	4.5
2002	2.6	4.7	2.6	6.6	2.6	1.1
2003	0.5	25.6	0.5	30.4	0.5	10.9
2004	1.6	31.2	1.6	36.0	1.6	18.2
2005	1.8	16.9	1.8	21.0	1.8	9.4
TOTAL	9.1	83.3	9.1	99.3	9.1	44.1

Table 4.2: Result of the computed data

Where:

Rp = Result of Returns to the portfolio

δ = Risk to the portfolio

In the above analysis, we tested for the effect of diversification of uncorrelated, perfectly correlated and perfect negatively correlated assets on returns and risk in the portfolio.

From the tables above, we discovered that diversification of uncorrelated, perfectly correlated and perfect negatively correlated assets yielded the same returns. But, diversification of perfect negatively correlated assets yielded minimum risk to the portfolio.

Implication of the table

In this section, we shall discuss the results of the table above. Uncorrelated assets means diversifying two assets that are not similar in characteristics. From the table, the result yielded minimum return and high risk but the risk is not as high as perfectly correlated assets. The diversification of uncorrelated assets is not too effective.

Moreover, diversifications of perfectly correlated assets imply diversifying two assets with similar characteristics. From the table, the result yielding the same return with uncorrelated and perfect negatively correlated assets but with highest risk. This means that diversification of such is not effectives at all.

However, diversification of perfect negatively correlated assets means diversifying both risky and risk-free assets together. As a result, the volatility of the two assets cancelled one another completely and this leads to a situation where portfolio has little or no volatility at all. In this case diversification of perfectly negatively correlated assets is very effective.

4.3 Simulation.

In this section, we shall use Microsoft Excel to simulate the results of the above table. In view of this we shall use line graph to see the effect of diversification of uncorrelated, perfectly correlated and perfect negatively correlated.

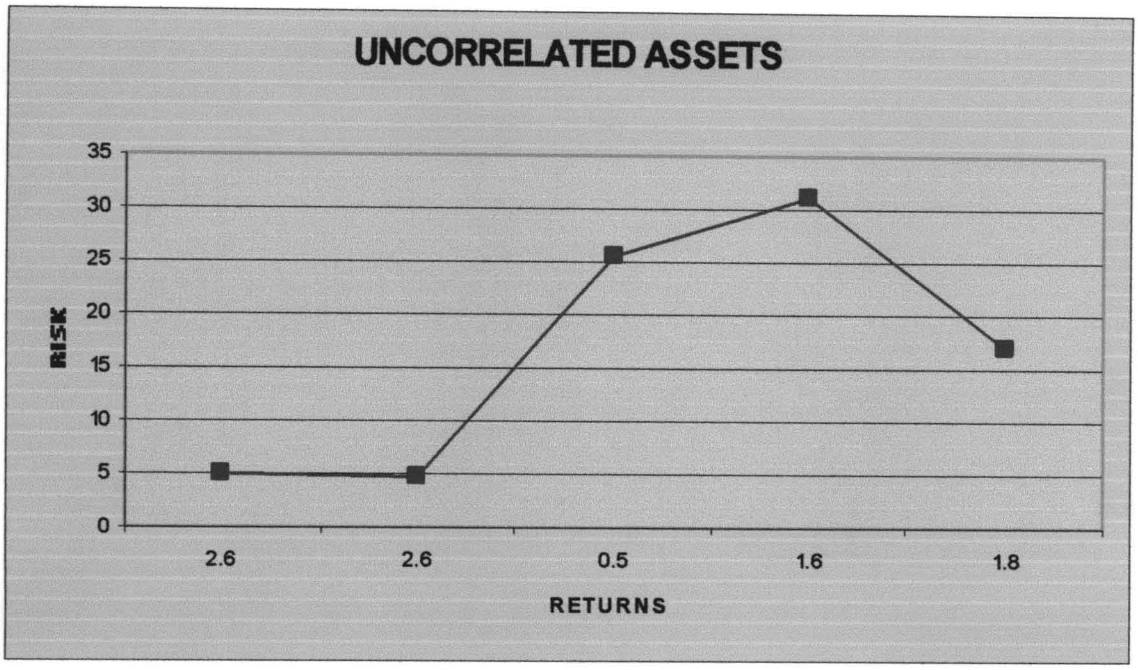


Fig. 4.1: showing the effect of diversification of uncorrelated assets

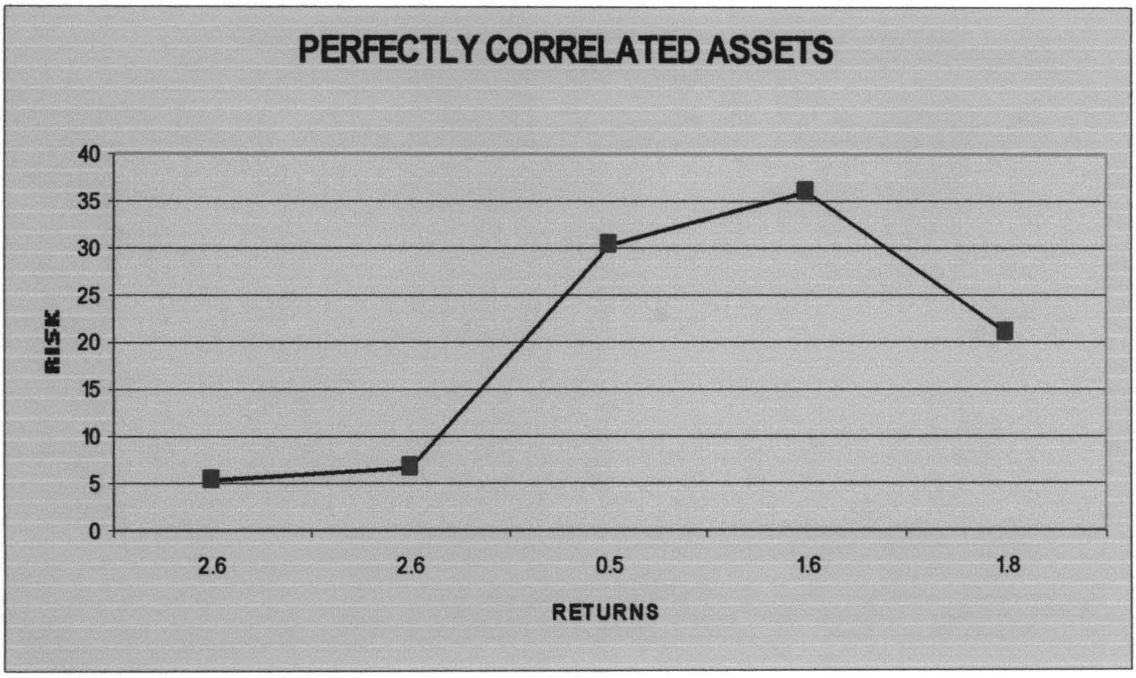


Fig. 4.2: showing the effect of diversification of perfectly correlated assets

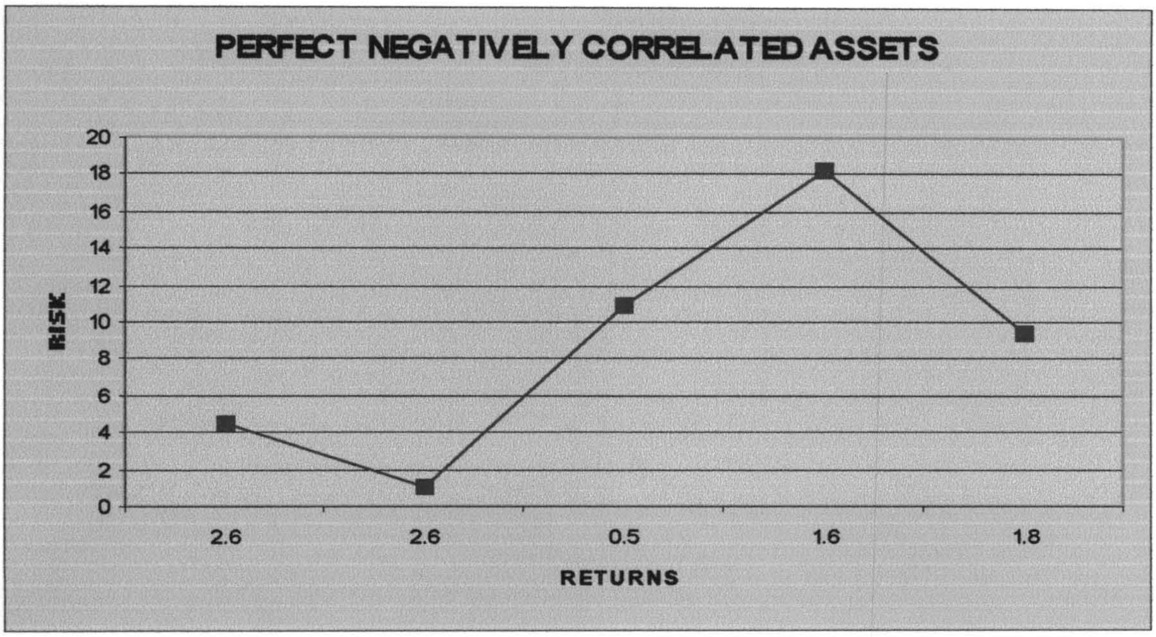


Fig. 4.3: showing the effect of diversification of perfect negatively correlated assets

In each of the figures above indicates the risk generated for diversifying into uncorrelated, perfectly correlated, and perfect negatively correlated.

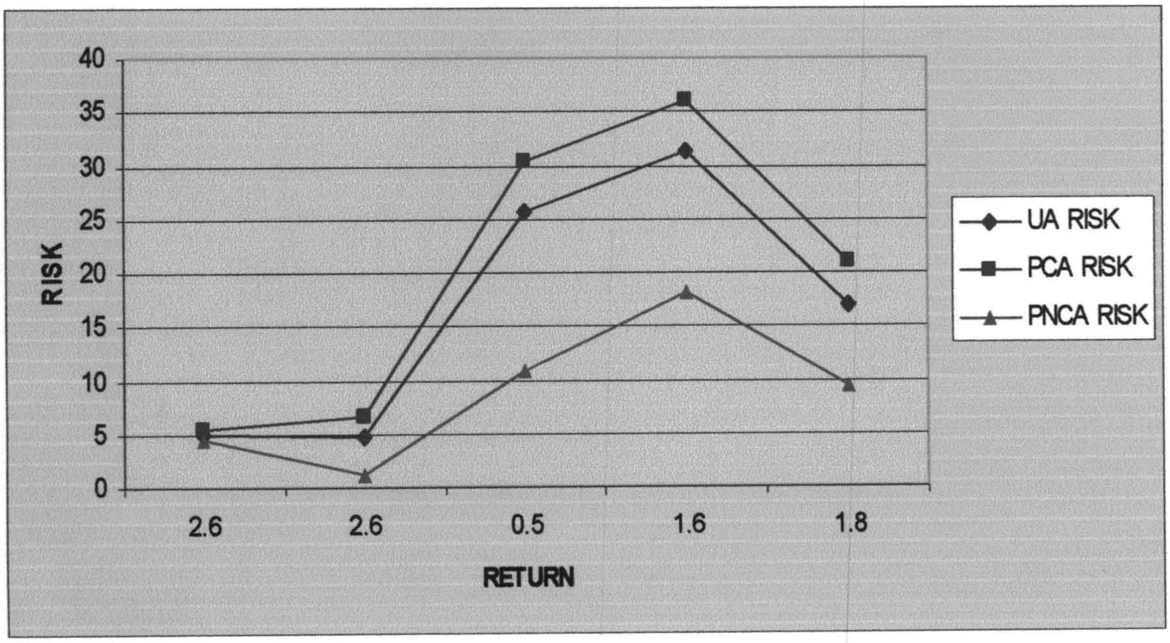


Fig. 4.4: showing the relationship between risks for diversifying into uncorrelated, perfectly correlated and perfect negatively correlated assets.

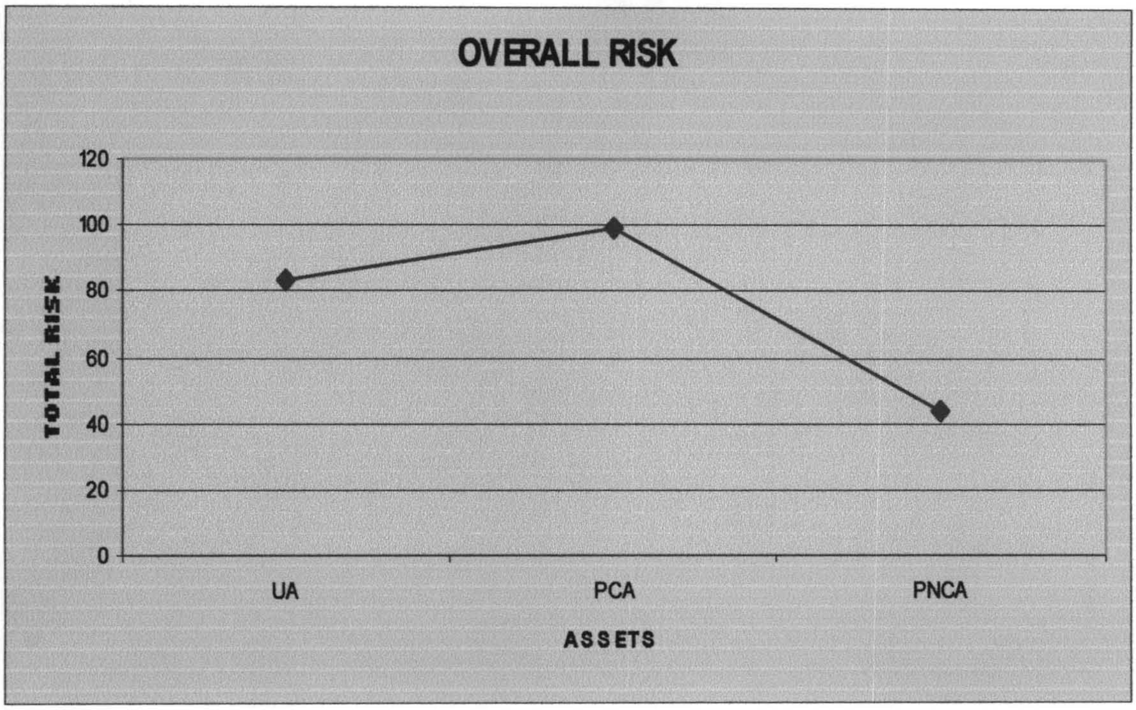


Fig. 4.5: showing the total risk for diversifying into uncorrelated, perfectly correlated and perfect negatively correlated assets.

CHAPTER FIVE

5.0 SUMMARY CONCLUSION AND RECOMMENDATION

5.1 Summary

This study was carried out to determine the returns and risk of banking system. Test was carried out in three perspectives: Uncorrelated assets, perfectly correlated assets and perfect negatively correlated assets.

Before this was done, useful data was collected for two assets from Investment Banking and Trust Company PLC (IBTC). This data was analyzed and the results of the previous section was gotten.

Moreover, statistical analysis was carried out using Microsoft Excel to show the results of diversification of the uncorrelated, perfectly correlated and perfect negatively correlated assets for the assets.

5.2 Conclusion

This study was carried out to know the causes and reasons why our recent banks often go distress after few years in operation.

Actually, before now banking system only engage in banking and giving of loans but nowadays banks have diversified into many professions like agriculture, building, buying and selling of stocks etc. but, despite the diversification the rate at which they close down is alarming. The question now is why and what is responsible for this?

The analysis was carefully carried out and discovered that, though these banks really go into diversification which is a good area of improvement, but may be, they are not diversifying into relevant assets.

However, before banks or organizations go into diversification it is expedient to know the effects of diversification on the assets they are diversifying into.

5.3 Suggestions and Recommendations

Before banks or organizations go into diversification the following recommendations are suggested based on the findings as a result of this study: the management should endeavour to find out:

i. The correlation of the assets
ii. The default correlation of the assets
iii. The default risk of the assets
iv. The volatility of the assets
v. The market value of the assets

At this juncture, we wish to suggest that banks or organizations may invest more into assets that are perfectly negatively correlated. That is, they should diversify more into risk- free (riskless) and risky assets.

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