STUDY OF THERMODIFFUSION EFFECTS ON MAGNETOHYDRODYNAMICS WITH CONVECTION OF A NANOFLUID FLOW

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ABSRACT

This thesis considered the study of thermodiffusion effects on flow of a nanofluid towards stretching sheet with applied magnetic field and convection. The problems, one dimensional unsteady state with suction and two dimensions steady state are presented. Similarity transformations are applied to reduce the equations that govern the flow to a system of coupled nonlinear ordinary differential equations. The problems are solved using the Adomian decomposition method. The results obtained for skin frictions ($-f^{//}(0)$) and Nusselt number ($-\theta^{/}(0)$) are compared with the existing literatures and a good agreement is established. Concrete graphical analysis is carried out to study the effects of emerging physical parameters such as velocity ratio, magnetic parameter, thermal Grashof number, solutal Grashof number, Prandtl number, Lewis number, Brownian motion and Dufour number. The velocity ratio was kept constant (*A = 0.1*) throughout the study except cases where it was varied. The Grashof numbers are found to enhance the fluid velocity while the magnetic parameter is a reduction agent. Prandtl number and Lewis number are seen as reduction agents to the fluid temperature and solutal concentration profile respectively.

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Abbreviation, Glossaries and Symbols

A	velocity ratio parameter
B_o	applied magnetic field
C_f	skin friction coefficient
C_w	solutal concentration at the surface
T_w	temperature at the surface
φ_w	nanoparticle volume fraction at the surface
C_∞	ambient solutal concentration
T_∞	ambient temperature
φ_∞	ambient nanoparticle volume fraction
U_∞	free stream velocity
u, v	velocity components along x and y directions
M	magnetic parameter
f	dimensionless stream function
s	dimensionless concentration
D_T	thermophoresis diffusion coefficient
D_S	solutal diffusivity
D_B	Brownian motion diffusion coefficient
D_{TC}	Defour diffusivity

D_{CT}	Soret diffusivity
Pr	Prandtl number
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nd	modified Dufour parameter
Le	Lewis number
Ld	Dufour solutal Lewis number
Ln	nano Lewis number
η	dimensionless similarity variable
μ	dynamic viscosity of the fluid
υ	kinematic viscosity of the fluid
$(\rho)_f$	density of the base fluid
$(\rho c)_f$	heat capacity of the base fluid
$(\rho c)_p$	effective heat capacity of the nanoparticle
ψ	stream function
α_m	thermal diffusivity
σ	electric conductivity
θ	dimensionless temperature
β	volumetric coefficient of thermal expansion

g	acceleration due to gravity
t	time
ϕ	dimensionless nanoparticle volume fraction
τ	parameter defined by $(\rho c)p/(\rho c)f$
ADM	Adomian decomposition method

CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

A number of works have been performed to gain an understanding of the heat transfer performance for the practical application to heat transfer enhancement. Thus the advent of high heat flow processes has created significant demand for new technologies to enhance heat transfer. There are several methods to improve the heat transfer efficiency. Some of these methods are utilization of extended surfaces, application of vibration to the heat transfer surfaces, and usage of micro channels. Heat transfer efficiency can also be improved by increasing the thermal conductivity of the working fluid. Commonly used heat transfer fluids such as water, ethylene glycol, and engine oil have relatively low thermal conductivities, when compared to the thermal conductivity of solids (Yusuf *et al.,* 2019).

High thermal conductivity of solids can be used to increase the thermal conductivity of a fluid by adding small solid particles to that fluid. The feasibility of the usage of such suspensions of solid particles with sizes on the order of 2 millimetres or micrometers was previously investigated by several researchers and the following significant drawbacks were observed (Das *et al.*, 2008).

a. The particles settle rapidly, forming a layer on the surface and reducing the heat transfer capacity of the fluid.

b. If the circulation rate of the fluid is increased, sedimentation is reduced, but the erosion of the heat transfer devices increases rapidly.

c. The large size of the particles tends to clog the flow channels, particularly if the cooling channels are narrow.

d. The pressure drop in the fluid increases considerably.

e. Finally, conductivity enhancement based on particle concentration is achieved (the greater the particle volume fraction is, the greater the enhancement and greater the problems, as indicated above).

Thus, the route of suspending particles in liquid was a well known but rejected option for heat transfer applications.

However, the emergence of modern materials technology provided the opportunity to produce nanometer-sized particles which are quite different from the parent material in mechanical, thermal, electrical, and optical properties.

Nanofluid is a new kind of heat transfer medium, containing nanoparticles (1–100 nm) which are uniformly and stably distributed in a base fluid. These distributed nanoparticles, generally a metal or metal oxide greatly enhance the thermal conductivity of the nanofluid, increases conduction and convection coefficients, allowing for more heat transfer (Yusuf *et al.,* 2018).

According to Yusuf *et al.* (2019), nanofluids have been considered for applications as advanced heat transfer fluids for decades. However, due to the wide variety and the complexity of the nanofluid systems, no agreement has been achieved on the magnitude of potential benefits of using nanofluids for heat transfer applications. Compared to conventional solid–liquid suspensions for heat transfer intensifications, nanofluids having properly dispersed nanoparticles possess the following advantages:

-High specific surface area and therefore more heat transfer surface between particles and fluids.

-High dispersion stability with predominant Brownian motion of particles.

-Reduced pumping power as compared to pure liquid to achieve equivalent heat transfer intensification.

-Reduced particle clogging as compared to conventional slurries, thus promoting system miniaturization.

-Adjustable properties, including thermal conductivity and surface wet ability, by varying particle concentrations to suit different applications.

1.2 Statement of the Problem

Khan *et al.* (2014) considered the Thermodiffusion effects on stagnation point flow of a nanofluid towards a stretching surface with applied magnetic field. Similarity transforms were applied to reduce the equations that govern the flow to a system of nonlinear ordinary differential equations. Runge-Kutta-Fehlberg method is applied to solve the system. Results were compared with existing solutions that are special cases to the problem. The researchers did not put into consideration the effect of buoyancy, the method employed did not give solutions at all points, but mesh points and lastly the unsteady case was not considered.

1.3 Justification of the Study

A wide variety of industrial processes involve the transfer of heat energy. Throughout any industrial facility, heat must be added, removed, or moved from one process stream to another and it has become a major task for industrial necessity. These processes provide a source for energy recovery and process fluid heating/cooling (Yusuf *et al.,* 2018). From the literatures available to the researchers, the study of thermo-diffusion effects on magnetohydrodynamics stagnation point flow towards a stretching sheet in a nanofluid with convection has not been explored.

1.4 Aim and Objectives of the Study

1.4.1 Aim of the study

The aim of this study is to carry out the study of thermo-diffusion effects on magnetohydrodynamics stagnation point flow towards a stretching sheet in a nanofluid with convection.

1.4.2 Objectives of the study

The objectives of this present study are to:-

i. transform the Partial differential equation (PDE) formulated to ordinary differential equations (ODE) using the similarity equations.

ii. solve the set of transformed non linear, coupled, ordinary differential equations (ODE) using the Adomian Decomposition Method (ADM).

iii. validate the results obtain with the existing literature.

iv. present and analyse the solutions with the help of the graphical representations.

v. Verify the effects of the various dimensionless parameters that appears in the solutions.

1.5 Scope and Limitation

The Partial Differential Equation (PDE) formulated from the problem is presented in its rectangular coordinate system. The appropriate similarity transformations and stream functions are used to transform the partial differential equations to ordinary differential equations. Non linear coupled ordinary differential equations are derived, corresponding to velocity, temperature, solutal concentration and nanoparticle concentration equations. These equations are solved using Adomian Decomposition Method. The effect of various parameters that appeared are analysed with the help of graphs. This work is limited to incompressible nanofluid dynamics.

1.6 Definition of Terms

Fluid: A substance which deforms continuously when shear stress is applied to it no matter how small, such as liquid or gas which can flow, has no fixed shape and offers little resistance to an external stress.

Nanofluid: it is a fluid containing nanometer-sized particles, called nanoparticles.

Buoyancy: is the force that causes objects to float when submerged in a fluid.

Steady flow: it is when all properties of flow are independent of time.

Unsteady flow: it is when all properties of flow are time-dependent.

Grashof number: is a dimensionless number in fluid dynamics and heat transfer which approximates the ratio of the buoyancy to viscous force acting on a fluid. It frequently arises in the study of situations involving natural_convection. It is named after the German engineer Franz_Grashof.

Prandtl number (Pr): the relationship between the thickness of two boundary layers at a given point along the plate depend on the dimensionless prandtl number which is the ratio of the momentum diffusivity ν or $\frac{\mu}{\rho}$ to the thermal diffusivity $\propto$ or $\frac{k}{\rho Cp}$.

Boundary Layers:- boundary layer is defined as that part of moving fluid in which the fluid motion is influenced by the presence of a solid boundary. As a specific example of boundary layer formation, consider the flow of fluid parallel with a thin plate, when a fluid flows at high Reynolds number past a body, the viscous effects may be neglected everywhere except in a thin region in the vicinity of the walls . This region is termed as the boundary layer.

Magnetohydrodynamics: is the study of magnetic properties and behavior of electrically conducting fluids.

Convection: is the conveying of heat from part of a liquid or gas to another by the movement of heated substances.

Stagnation Point: A point in the flow where the local velocity is zero.

Compressible fluid: change in density of fluid with time

$$\frac{\Delta\rho}{\Delta t} \neq 0$$

Incompressible flow: fluid motion with negligible changes in density ρ

$$\frac{\Delta\rho}{\Delta t} = 0$$

Lewis number (Le): is defined as the ratio of the Schmidt number (Sc) and the Prandtl number (Pr). The Lewis number is also the ratio of thermal diffusivity and mass diffusivity.

$$Le = \frac{\propto}{D} = \frac{Sc}{Pr} = \frac{thermal}{mass}$$

Brownian motion: the erratic random movement of microscopic particles in a fluid, as a result of continuous bombardment from molecules of the surrounding medium.

Skin friction: friction at the surface of a solid and a fluid in relative motion.

Nusselt number (Nu): is defined as the ratio of convection heat transfer to fluid conduction heat transfer under the same conditions.

$$\text{Nu} = \frac{convective\ heat\ transfer}{conductive\ heat\ transfer}$$

Thermodiffusion: is the thermal conductivity divided by density and specific heat capacity at constant pressure. $\alpha = \frac{heat\ conducted}{heat\ stored} = \frac{k}{\rho Cp}$

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Reviews on Fluid Dynamics

Over the years boundary layer flows have been of much interest to researchers because of their real world applications such as engineering melt spinning, manufacturing of rubber sheets, glass fiber production, and so on. One of these boundary layer flows is the stagnation point flow. Cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdowns, etc. are the applications of stagnation flows. Crane (1970) first studied this problem for stagnation point flow towards a solid surface. After this seminar work, many researchers explored various aspects of boundary layer flows incorporating innumerable physical configurations (Banks, 1983).

In real world, most of the fluids such as water, kerosene oil, ethylene, glycol, and others are poor conductors of heat due to their lower values of thermal conductivity. To cope up with this problem and to enhance the thermal conductivity or other thermal properties of these fluids, a newly developed technique is used which includes, addition of nano-sized particles of good conductors such as copper, aluminum, titanium, iron and other oxides to the fluids. Choi (1995) was the first one to come up with this idea and also showed that the thermal conductivity of conventional fluids can be doubled by adding nano particles to base fluids that also incorporate other thermal properties. These enhancements can be used practically in electronic cooling, heat exchangers, double plane windows. Buongiorno (2006) presented a more comprehensive model for the nanotechnology based fluids that unveils the thermal properties superior to base fluids. He discussed all the convective properties of nanofluids by developing a more general model. After these developments in nanofluids, Khan and Pop (2010) were the first to study boundary layer flow over a stretching sheet by using the model of Nield and Kuznetsov (2009). Mustafa *et al.* (2011) presented first study on stagnation point flow of a nanofluid. They presented both Brownian motion and thermophoresis effects on transport equations by reducing them to a nonlinear boundary value problem. One can easily find enough literature on nanofluid flows, however some of the studies are reported in Nadeem and Haq (2013) and references therein. Study of electrically conducting fluids hold importance due to applications in modern metallurgy and metal working processes. Magnetic nanofluids are used to regulate the flow and heat transfer by controlling the fluid velocity. Mahaputra (2002) studied MHD stagnation point flow over a stretching sheet using numerical simulations. MHD stagnation point flow for nanofluid was presented by Ibrahim *et al.* (2013) employing fourth order Runge-Kutta technique. Some other studies regarding magneto-nanofluids are presented in Nadeem and Lee (2012) and references therein. In all presented studies, Soret and Dufour effects were neglected. It is a well-known fact that the temperature and concentration gradients present mass and energy fluxes, respectively. Concentration gradients result in Dufour effect (diffusion-thermo) while Soret effect (thermal-diffusion) is due to temperature gradients. Such effects play a significant role when there are density differences in the flow. For the flows of mixture of gases with light molecular weights and moderate weights, Soret and Dufour effects cannot be neglected. Thermo-diffusion effects on the flow over a stretching sheet are examined by Awad *et al.* (2013).

Ramachandra *et al.* (1988) have investigated the mixed convection flow in the stagnation flow region of a vertical plate. The steady stagnation-point flow towards a permeable vertical surface was investigated by Ishak *et al.* (2008). Li *et al*. (2011) introduced an analysis of the steady mixed convection flow of a viscoelastic fluid stagnating orthogonally on a heated or cooled vertical flat plate. Makinde (2012) examined the hydromagnetic mixed convection stagnation-point flow towards a vertical plate embedded in a highly porous medium with radiation and internal heat generation.

Mabood and Khan (2014) introduced an accurate analytical solution (series solution) for MHD stagnation-point flow in a porous medium for different values of the Prandtl number and the suction/injection parameter. An unsteady boundary layer plays important roles in many engineering problems like a start-up process and a periodic fluid motion. An unsteady boundary layer has different behaviors due to extra time-dependent terms, which will influence the fluid motion pattern and the boundary-layer separation. Kumara *et al.* (1992) have studied the unsteady mixed convection flow of an electrically conducting fluid at the stagnation point of a two-dimensional body and an axisymmetric body in the presence of an applied magnetic field. Seshadri *et al.* (2002) studied the unsteady mixed convection in the stagnation-point flow on a heated vertical plate where the unsteadiness is caused by the impulsive motion of the free stream velocity and by sudden increase in the surface temperature (heat flux). Hassanien *et al.* (2004) analyzed the problem of unsteady free convection flow in the stagnation-point region of a rotating sphere embedded in a porous medium. The unsteady flow and heat transfer of a viscous fluid in the stagnation region of a three-dimensional body embedded in a porous medium was investigated by Hassanien *et al*. (2006). Hassanien and Al-Arabi (2008) studied the problem of thermal radiation and variable viscosity effects on unsteady mixed convection flow in the stagnation region on a vertical surface embedded in a porous medium with surface heat flux. Fang *et al.* (2011) investigated the boundary layers of an unsteady incompressible stagnation-point flow with mass transfer. Shateyi and Marewo (2014) have numerically investigated the problem of unsteady MHD flow near a stagnation point of a two-dimensional porous body with heat and mass transfer in the presence of thermal radiation and chemical reaction. Rosali *et al.* (2014) discussed the effect of unsteadiness on mixed convection boundary-layer stagnation-point flow over a vertical flat surface embedded in a porous medium.

During the past decade, the study of nanofluids has attracted enormous interest from researchers due to their exceptional applications to electronics, automotive, communication, computing technologies, optical devices, lasers, high-power X-rays, scientific measurement, material processing, medicine, and material synthesis, where efficient heat dissipation is necessary. Nanobiotechnology is also a fast-developing field of research and application in many domains, such as in medicine, pharmacy, cosmetics and agro-industry. Nanofluids are prepared by dispersing solid nanoparticles in base fluids such as water, oil, ethylene glycol, or others. According to Yacob *et al.* (2011), nanofluids are produced by dispersing the nanometer-scale solid particles into base liquids with low thermal conductivity such as water and ethylene glycol. Nanoparticles are usually made of metal, metal oxide, carbide, nitride, and even immiscible nanoscale liquid droplets. Congedo *et al.* (2009) compared different models of nanofluids (regarded as a single phase) to investigate the density, specific heat, viscosity, and thermal conductivity, and discussed the water–Al_2O_3 nanofluid in detail by using Computational Fluid Dynamics (CFD). Hamad *et al.* (2011) introduced a one-parameter group to represent similarity reductions for the problem of magnetic field effects on free-convective nanofluid flow past a semi-infinite vertical flat plate following a nanofluid model proposed by Buongiorno (2006). Hady *et al.* (2012a) studied the radiation effect on viscous flow of a nanofluid and heat transfer over a nonlinearly stretching sheet with variable wall temperature. Also, Hady *et al*. (2012b) studied the problem of natural convection boundary-layer flow past a porous plate embedded in a porous medium saturated with a nanofluid using Buongiorno’s model. Further, Abu-Nada and Chamkha (2010) presented the natural convection heat transfer characteristics in a differentially heated enclosure filled with CuO–ethylene glycol (EG)–water nanofluids for different variable thermal conductivity and variable viscosity models.

Rudraswamy and Gireesha (2014) studied the problem of flow and heat transfer of a nanofluid over an exponentially stretching sheet by considering the effect of chemical reaction and thermal radiation. Besthapu and Bandari (2015) presented a study on the mixed convection MHD flow of a Casson nanofluid over a nonlinear permeable stretching sheet with viscous dissipation. Numerical solutions of the natural convection flow of a two-phase dusty nanofluid along a vertical wavy frustum of a cone is discussed by Siddiqa *et al.* (2016a). The bioconvection flow with heat and mass transfer of a water-based nanofluid containing gyrotactic microorganisms over a vertical wavy surface was studied by Siddiqa *et al*. (2016b). Kameswaran *et al.* (2016) studied convective heat transfer in the influence of nonlinear Boussinesq approximation, thermal stratification, and convective boundary conditions on non-Darcy nanofluid flow over a vertical wavy surface.

The effects of radiation on unsteady free convection flow and heat transfer problem have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for design of reliable equipments, nuclear plants, gas turbines and various propulsion devices or aircraft, missiles, satellites and space vehicles. Based on these applications, Cogley *et al.* (1968) showed that in the optically thin limit, the fluid does not absorb its own emitted radiation but the fluid does absorb radiation emitted by the boundaries. Hossain and Takhar (1996) have considered the radiation effects on mixed convection boundary layer flow of an optically dense viscous incompressible fluid along a vertical plate with uniform surface temperature. Makinde (2005) examined the transient free convection interaction with thermal radiation of an absorbing emitting fluid along moving vertical permeable plate. Satter and Hamid (1996) investigated the unsteady free convection interaction with thermal radiation of an absorbing emitting plate.

Vasu and Manish (2015) studied the problem of two-dimensional transient hydrodynamic boundary-layer flow of an incompressible Newtonian nanofluid past a cone and plate with constant boundary conditions. Gireesha *et al.* (2015) introduced a numerical solution for hydromagnetic boundary-layer flow and heat transfer past a stretching surface embedded in a non-Darcy porous medium with fluid-particle suspension. The unsteady forced convective boundary-layer flow of an incompressible non-Newtonian nanofluid over a stretching sheet when the sheet is stretched in its own plane is investigated by Gorla and Vasu (2016). Gorla *et al.* (2016) investigated the transient mixed convective boundary-layer flow of an incompressible non-Newtonian quiescent nanofluid adjacent to a vertical stretching surface. The unsteady flow and heat transfer of a nanofluid over a contracting cylinder was studied by Zaimi *et al.* (2014). Srinivasacharya and Surender (2014) studied the effects of thermal and mass stratification on natural convection boundary-layer flow over a vertical plate embedded in a porous medium saturated by a nanofluid.

Abdullah *et al.* (2018) studies the effects of Brownian motion and thermophoresis on unsteady mixed convection flow near the stagnation-point region of a heated vertical plate embedded in a porous medium saturated by a nanofluid. The plate is maintained at a variable wall temperature and nanoparticle volume fraction. The presence of a solid matrix, which exerts first and second resistance parameters, is considered in the study. A suitable coordinate transformation is introduced and the resulting governing equations are transformed and then solved numerically using the local nonsimilarity method and the Runge-Kutta shooting quadrature. The effects of various governing parameters on the flow and heat and mass transfer on the dimensionless velocity, temperature, and nanoparticle volume fraction profiles as well as the skin-friction coefficient, Nusselt number, and the Sherwood number are displayed graphically and discussed to illustrate interesting features of the solutions. The results indicate that as the values of the thermophoresis and Brownian motion parameters increase, the local skin-friction coefficient increases whereas the Nusselt number decreases.

Moreover, the Sherwood number increases as the thermophoresis parameter increases, and decreases as the Brownian motion parameter increases. On the other hand, the unsteadiness parameter and the resistance parameters enhance the local skin-friction coefficient, local Nusselt number, and the local Sherwood number.

Flow of a nanofluid in a boundary layer in an inclined moving sheet at angle Θ is considered analytically by Yusuf *et al.* (2019), the Mathematical formulation consists of the Magnetic parameter, thermophoresis, and Brownian motion. Solutions to momentum, temperature and concentration distribution depends on some parameters. The non linear coupled differential equations were solved using the improved Adomian decomposition method and agreement was established with the numerical method (Shooting technique). The result shows that the velocity and temperature profile of the fluid increases in the thermal Grashof number due to the presence of buoyancy effects.

2.2 Adomian Decomposition Method (ADM)

Begin with an equation $Fu(t) = g(t)$, where F represents a general nonlinear ordinary differential operator involving both linear and nonlinear terms. The linear term is decomposed into $L + R$, where L is easily invertible and R is the remainder of the linear operator. For convenience, L may be taken as the highest order derivative which avoids difficult integrations which result when complicated Green's functions are involved (Adomian, 1994). Thus the equation may be written

$$Lu + Ru + Nu = g \tag{2.1}$$

where Nu represents the nonlinear terms. Solving for Lu,

$$Lu = g - Ru - Nu \tag{2.2}$$

Because L is invertible, an equivalent expression is

$$L^{-1}Lu = L^{-1}g + L^{-1}Ru - L^{-1}Nu \tag{2.3}$$

If this corresponds to an initial-value problem, the integral operator L^{-1} may be regarded as definite integrals from t_0 to t. If L is a second-order operator, L^{-1} is a twofold integration operator and $L^{-1}Lu = u - u\left(t_0\right) - \left(t - t_0\right)u^{/}\left(t_0\right)$. For boundary value problems (and, if desired, for initial-value problems as well), indefinite integrations are used and the constants are evaluated from the given conditions.

$$u = A + Bt + L^{-1}g + L^{-1}Ru - L^{-1}Nu \tag{2.4}$$

The nonlinear term Nu will be equated to $\sum_{n=0}^{\infty} A_n$, where the A_n, are special polynomials to be discussed, and u will be decomposed into $\sum_{n=0}^{\infty} u_n$, with u_0 identified as $A + Bt + L^{-1}g$

$$\sum_{n=0}^{\infty} u_n = u_0 - L^{-1}R\sum_{n=0}^{\infty} u_n - L^{-1}\sum_{n=0}^{\infty} A_n \tag{2.5}$$

Consequently, we can write

$$\left.\begin{aligned}
u_1 &= -L^{-1}Ru_0 - L^{-1}A_0 \\
u_2 &= -L^{-1}Ru_1 - L^{-1}A_1 \\
&. \\
&. \\
&. \\
u_{n+1} &= -L^{-1}Ru_n - L^{-1}A_n
\end{aligned}\right\} \tag{2.6}$$

The polynomials A_n, are generated for each nonlinearity so that A_0, depends only on u_0, A_1, depends only on u_0 ,and u_1, A_2, depends on u_0, u_1, u_2, etc. All of the u_n, components are calculable, and $u = \sum_{n=0}^{\infty} u_n$. If the series converges, the n-term partial sum $\phi_n = \sum_{i=0}^{n-1} u_i$ will be the approximate solution since $\lim_{n\to\infty} \phi_n = \sum_{i=0}^{\infty} u_i = u$ by definition. It is important to emphasize that the A_n can be calculated for complicated nonlinearities of the form $f\left(u,\, u^{/},...\right)$ or $f\left(g\left(u\right)\right)$.

Khan *et al.* (2014) considered an incompressible, Magnetohydrodynamic stagnation point flow of a nanofluid towards a stretching sheet with wall temperature T_w, solutal concentration C_w, nanoparticle concentration ϕ_w and T_∞, C_∞, and ϕ_∞ at larger values of the stretching sheet respectively. The governing equation for continuity, momentum, temperature, solutal and nanoparticle concentrations are written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.7}$$

$$\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = U_\infty \frac{\partial U_\infty}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{\sigma B_0^2}{\rho}\left(U_\infty - u\right) \tag{2.8}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha_m\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\tau\left[D_B\left(\frac{\partial\varphi\partial T}{\partial x\partial x}+\frac{\partial\varphi\partial T}{\partial y\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}\right)^2\right]+ D_{TC}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \tag{2.9}$$

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_S\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+D_{CT}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{2.10}$$

$$\left(u\frac{\partial \varphi}{\partial x}+v\frac{\partial \varphi}{\partial y}\right)=D_B\left(\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{2.11}$$

Subject to the boundary condition:

$$\left.\begin{array}{l} u=U_w=ax,\;\; v=0,\;\; T=T_w,\;\; \mathrm{C}=\mathrm{C}_w,\;\; \varphi=\varphi_w \;\; y=0 \\ u=U_\infty=bx,\;\; T=T_\infty, \mathrm{C}=\mathrm{C}_\infty,\; \varphi=\varphi_\infty \;\; \mathrm{y}\to\infty \end{array}\right\} \tag{2.12}$$

The present work extends Khan *et al.* (2014) by introducing the buoyancy parameter and considered the work in two cases of the extension (one dimensional unsteady state with suction and two dimensions steady state). The analysis is also carried out using the Adomian decomposition method. From the available literatures, this innovation is new.

CHAPTER THREE

3.0 MATERIALS AND METHODS

3.1 Problem Formulation

Considering an incompressible, Magnetohydrodynamic stagnation point flow of a nanofluid towards a stretching sheet with wall temperature T_w, solutal concentration C_w, nanoparticle concentration ϕ_w and T_∞, C_∞, and ϕ_∞ at larger values of the stretching sheet respectively. Following the formulation in Khan *et al.* (2014) with natural convection. The governing equation for continuity, momentum, temperature, solutal and nanoparticle concentrations are written as follows:

$$\frac{\partial v}{\partial y} = 0 \tag{3.1}$$

$$\rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \sigma B_0^2\left(U_\infty - u\right) + g\beta\left(T - T_\infty\right) + g\beta\left(C - C_\infty\right) \tag{3.2}$$

$$\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = \alpha_m\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial\varphi\partial T}{\partial y\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + D_{TC}\frac{\partial^2 C}{\partial y^2} \tag{3.3}$$

$$\left(\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y}\right) = D_S\frac{\partial^2 C}{\partial y^2} + D_{CT}\frac{\partial^2 T}{\partial y^2} \tag{3.4}$$

$$\left(\frac{\partial\varphi}{\partial t} + v\frac{\partial\varphi}{\partial y}\right) = D_B\frac{\partial^2\varphi}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{3.5}$$

Subject to the boundary condition:

$$\left.\begin{array}{l} u = U,\ \ v = v_0,\ \ T = T_w,\ \ C = C_w,\ \ \varphi = \varphi_w\ \ \ y = 0,\ \ t \le 0 \\ u = U_\infty,\ \ T = T_\infty, C = C_\infty,\ \ \varphi = \varphi_\infty\ \ \ y \to \infty,\ \ t \succ 0 \end{array}\right\} \tag{3.6}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.7}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=U_{\infty}\frac{\partial U_{\infty}}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\sigma B_0^2}{\rho}\left(U_{\infty}-u\right)+g\beta\left(T-T_{\infty}\right)+g\beta\left(C-C_{\infty}\right) \tag{3.8}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha_m\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\tau\left[D_B\left(\frac{\partial\varphi\partial T}{\partial x\partial x}+\frac{\partial\varphi\partial T}{\partial y\partial y}\right)+\frac{D_T}{T_{\infty}}\left(\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}\right)^2\right]+ D_{TC}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \tag{3.9}$$

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_S\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+D_{CT}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{3.10}$$

$$\left(u\frac{\partial\varphi}{\partial x}+v\frac{\partial\varphi}{\partial y}\right)=D_B\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}\right)+\frac{D_T}{T_{\infty}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{3.11}$$

Subject to the boundary condition:

$$\left.\begin{array}{l} u=U_w=ax,\ \ v=0,\ T=T_w,\ \mathrm{C}=\mathrm{C}_w,\ \varphi=\varphi_w\ \ y=0 \\ u=U_{\infty}=bx,\ T=T_{\infty},\mathrm{C}=\mathrm{C}_{\infty},\ \varphi=\varphi_{\infty}\ \ \mathrm{y}\to\infty \end{array}\right\} \tag{3.12}$$

where velocity along y axes is u, ρ is the density of the base fluid, υ is the kinematic viscousity, σ is the electrical conductivity, α_m is the heat diffusivity, K_T is the heat-distribution ratio, g acceleration due to gravity, β volumetric coefficient of thermal expansion, D_{TC} is the Duffour diffusivity, B_0 external magnetic field, D_{CT} is the Soret Diffusivity, C_p is the specific heat capacity at constant pressure, D_B is the Brownian diffusion coefficient, D_T is the thermopheric diffusion coefficient and $\tau=\dfrac{(\rho c)_p}{(\rho c)_f}$ is the ratio of heat capacity of the particles to the effective heat capacity of the fluid with D_s as the solutal diffusisvity, U and U_{∞} are the wall velocity and free stream velocity respectively, t is time, v_0 is suction parameter.

Equations (3.1) to (3.5) represent the 1-dimensional unsteady case with suction and (3.7) to (3.11) is the 2-dimensional steady state with (3.6) and (3.12) as the boundary conditions respectively.

In other to reduce the PDEs into ODEs, the following similarity transformational variables are defined as follows:

$$\eta = \frac{y}{2\sqrt{\upsilon t}}, \quad u = Uf\left(\eta\right), \theta = \frac{T - T_\infty}{T_w - T_\infty}, s = \frac{C - C_\infty}{C_w - C_\infty}, \phi = \frac{\varphi - \varphi_\infty}{\varphi_w - \varphi_\infty} \tag{3.13}$$

$$\eta = \sqrt{\frac{a}{\upsilon}}y, \quad u = axf^{/}\left(\eta\right), \quad v = -\sqrt{a\upsilon}f\left(\eta\right), \theta = \frac{T - T_\infty}{T_w - T_\infty}, s = \frac{C - C_\infty}{C_w - C_\infty}, \phi = \frac{\varphi - \varphi_\infty}{\varphi_w - \varphi_\infty} \tag{3.14}$$

From the similarity equation in (3.13)

$$\left.\begin{aligned}
&\eta = \frac{y}{2\sqrt{\upsilon t}}, \eta^2 = \frac{y^2}{4\upsilon t}, 2\eta\frac{\partial \eta}{\partial t} = -\frac{y^2}{4\upsilon t^2} = -\frac{y^2}{4\upsilon t}\frac{1}{t} = -\frac{\eta^2}{t}, \frac{\partial \eta}{\partial t} = \frac{-\eta}{2t}\\
&u = Uf, v = v_0\\
&\frac{\partial u}{\partial y} = \frac{\partial u}{\partial f}\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{Uf^{/}}{2\sqrt{\upsilon t}}, \frac{\partial u}{\partial t} = \frac{\partial u}{\partial f}\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial t} = \frac{-U\eta}{2t}f^{/}\\
&\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left(\frac{Uf^{/}}{2\sqrt{\upsilon t}}\right) = \frac{Uf^{//}}{4\upsilon t}\\
&T = T_\infty + \left(T_w - T_\infty\right)\theta\left(\eta\right), \mathrm{C} = C_\infty + \left(C_w - C_\infty\right)s\left(\eta\right), \varphi = \varphi_\infty + \left(\varphi_w - \varphi_\infty\right)\phi\left(\eta\right)\\
&\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \eta}\frac{\partial \eta}{\partial t} = \left(T_w - T_\infty\right)\theta^{/}\left(\frac{-\eta}{2t}\right) = -\frac{\eta}{2t}\left(T_w - T_\infty\right)\theta^{/}\\
&\frac{\partial C}{\partial t} = \frac{\partial C}{\partial \eta}\frac{\partial \eta}{\partial t} = \left(C_w - C_\infty\right)s^{/}\left(\frac{-\eta}{2t}\right) = -\frac{\eta}{2t}\left(C_w - C_\infty\right)s^{/}\\
&\frac{\partial \varphi}{\partial t} = \frac{\partial \varphi}{\partial \eta}\frac{\partial \eta}{\partial t} = \left(\varphi_w - \varphi_\infty\right)\phi^{/}\left(\frac{-\eta}{2t}\right) = -\frac{\eta}{2t}\left(\varphi_w - \varphi_\infty\right)\phi^{/}\\
&\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{1}{2\sqrt{\upsilon t}}\left(T_w - T_\infty\right)\theta^{/}\\
&\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) = \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left(\frac{1}{2\sqrt{\upsilon t}}\left(T_w - T_\infty\right)\theta^{/}\right) = \frac{1}{4\upsilon t}\left(T_w - T_\infty\right)\theta^{//}\\
&\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) = \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left(\frac{1}{2\sqrt{\upsilon t}}\left(T_w - T_\infty\right)\theta^{/}\right) = \frac{1}{4\upsilon t}\left(T_w - T_\infty\right)\theta^{//}\\
&\frac{\partial C}{\partial y} = \frac{\partial C}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{1}{2\sqrt{\upsilon t}}\left(C_w - C_\infty\right)s^{/}\\
&\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial C}{\partial y}\right) = \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left(\frac{1}{2\sqrt{\upsilon t}}\left(C_w - C_\infty\right)s^{/}\right) = \frac{1}{4\upsilon t}\left(C_w - C_\infty\right)s^{//}\\
&\frac{\partial \varphi}{\partial y} = \frac{\partial \varphi}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{1}{2\sqrt{\upsilon t}}\left(\varphi_w - \varphi_\infty\right)s^{/}\\
&\frac{\partial^2 \varphi}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial \varphi}{\partial y}\right) = \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left(\frac{1}{2\sqrt{\upsilon t}}\left(\varphi_w - \varphi_\infty\right)\phi^{/}\right) = \frac{1}{4\upsilon t}\left(\varphi_w - \varphi_\infty\right)\phi^{//}
\end{aligned}\right\} \quad (3.15)$$

Substituting (3.15) into (3.1) to (3.6)

$$\frac{\partial v}{\partial y} = 0$$

$$v = v_0$$

$$\frac{\partial v_0}{\partial y} = 0$$

$$\rho\left(\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+\sigma B_0^2\left(U_\infty-u\right)+g\beta\left(T-T_\infty\right)+g\beta\left(C-C_\infty\right)$$

$$\frac{-U\eta}{2t}f^{/}+v_0\frac{Uf^{/}}{2\sqrt{\upsilon t}}=\upsilon\frac{Uf^{//}}{4\upsilon t}+\frac{\sigma B_0^2}{\rho}\left(U_\infty-Uf\right)+g\beta\left(T_w-T_\infty\right)+g\beta\left(C_w-C_\infty\right)$$

$$-2\eta f^{/}+\frac{2v_0t}{\sqrt{\upsilon t}}f^{/}=f^{//}+\frac{\sigma B_0^2}{\rho}4t\left(\frac{U_\infty}{U}-f\right)+\frac{4tg\beta\left(T_w-T_\infty\right)}{\upsilon}+\frac{4tg\beta\left(C_w-C_\infty\right)}{\upsilon}$$

$$\frac{v_0t}{\sqrt{\upsilon t}}=v_0\sqrt{\frac{t}{\upsilon}}$$

$$f^{//}+2\eta f^{/}-2v_0\sqrt{\frac{t}{\upsilon}}f^{/}+\frac{4t\sigma B_0^2}{\rho}\left(A-f\right)+G_{rT}\theta+G_{rC}s=0$$

$$f^{//}+2\eta f^{/}+2cf^{/}+M\left(A-f\right)+G_{rT}\theta+G_{rC}s=0$$

$$c=-v_0\sqrt{\frac{t}{\upsilon}},M=\frac{4t\sigma B_0^2}{\rho}$$

$$f^{//}+2(\eta+c)f^{/}+M\left(A-f\right)+G_{rT}\theta+G_{rC}s=0$$

$$\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial\varphi\partial T}{\partial y\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+D_{TC}\frac{\partial^2 C}{\partial y^2}$$

$$-\frac{\eta}{2t}\left(T_w-T_\infty\right)\theta^{/}+\frac{v_0}{2\sqrt{\upsilon t}}\left(T_w-T_\infty\right)\theta^{/}=\frac{\alpha_m}{4\upsilon t}\left(T_w-T_\infty\right)\theta^{//}+\frac{\tau D_B}{4\upsilon t}\left(T_w-T_\infty\right)\left(\varphi_w-\varphi_\infty\right)\theta^{/}\phi^{/}+$$

$$\frac{\tau D_B}{T_\infty 4\upsilon t}\left(T_w-T_\infty\right)^2\theta^{/2}+\frac{\tau D_{TC}}{4\upsilon t}\left(C_w-C_\infty\right)s^{//}$$

$$-2\eta\theta^{/}+\frac{v_0}{2\sqrt{\upsilon t}}\theta^{/}=\frac{\alpha_m}{\upsilon}\theta^{//}+\frac{\tau D_B}{\upsilon}\left(\varphi_w-\varphi_\infty\right)\theta^{/}\phi^{/}+\frac{\tau D_B}{T_\infty\upsilon}\left(T_w-T_\infty\right)\theta^{/2}+\frac{\tau D_{TC}}{\upsilon}\frac{\left(C_w-C_\infty\right)}{\left(T_w-T_\infty\right)}s^{//}$$

$$-2\eta\theta^{/}+2v_0\sqrt{\frac{t}{\upsilon}}\theta^{/}=\frac{\theta^{//}}{P_r}+N_b\theta^{/}\phi^{/}+N_t\theta^{/2}+Nds^{//}$$

$$\frac{\theta^{//}}{P_r}+2\eta\theta^{/}-2c\theta^{/}+N_b\theta^{/}\phi^{/}+N_t\theta^{/2}+Nds^{//}=0$$

$$\theta^{//}+2P_r\left(\eta+c\right)\theta^{/}+P_rN_b\theta^{/}\phi^{/}+P_rN_t\theta^{/2}+PNds^{//}=0$$

$$\left(\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}\right)=D_S\frac{\partial^2 C}{\partial y^2}+D_{CT}\frac{\partial^2 T}{\partial y^2}$$

$$-\frac{\eta}{2t}(C_w-C_\infty)s^{/}+\frac{v_0}{2\sqrt{\upsilon t}}(C_w-C_\infty)s^{/}=\frac{D_s}{4\upsilon t}(C_w-C_\infty)s^{//}+\frac{D_{TC}}{4\upsilon t}(T_w-T_\infty)\theta^{//}$$

$$-2\eta s^{/}+2v_0\sqrt{\frac{t}{\upsilon}}s^{/}=\frac{D_s}{\upsilon}s^{//}+\frac{D_{TC}}{\upsilon}\frac{(T_w-T_\infty)}{(C_w-C_\infty)}\theta^{//}$$

$$\frac{s^{//}}{Le}+2(\eta+c)s^{/}+Ld\theta^{//}=0$$

$$s^{//}+2Le(\eta+c)s^{/}+LeLd\theta^{//}=0$$

$$\left(\frac{\partial\varphi}{\partial t}+v\frac{\partial\varphi}{\partial y}\right)=D_B\frac{\partial^2\varphi}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}$$

$$-\frac{\eta}{2t}(\varphi_w-\varphi_\infty)\phi^{/}+\frac{v_0}{2\sqrt{\upsilon t}}(\varphi_w-\varphi_\infty)\phi^{/}=\frac{D_B}{4\upsilon t}(\varphi_w-\varphi_\infty)\phi^{//}+\frac{D_T}{T_\infty 4\upsilon t}(T_w-T_\infty)\theta^{//}$$

$$-2\eta\phi^{/}+2v_0\sqrt{\frac{t}{\upsilon}}\phi^{/}=\frac{D_B}{\upsilon}\phi^{//}+\frac{D_T}{T_\infty\upsilon}\frac{(T_w-T_\infty)}{(\varphi_w-\varphi_\infty)}\theta^{//}$$

$$\frac{\phi^{//}}{Ln}+2(\eta+c)\phi^{/}+\frac{N_t}{N_b}\theta^{//}=0$$

$$\phi^{//}+2Ln(\eta+c)\phi^{/}+Ln\frac{N_t}{N_b}\theta^{//}=0$$

For the boundary conditions

$$\left.\begin{array}{l} u=U,\ v=v_0,\ T=T_w,\ \mathrm{C}=\mathrm{C}_w,\ \varphi=\varphi_w\ y=0,\ t\le 0 \\ u=U_\infty,\ T=T_\infty, \mathrm{C}=\mathrm{C}_\infty,\ \varphi=\varphi_\infty\ \mathrm{y}\to\infty,\ \mathrm{t}\succ 0 \end{array}\right\}$$

$$\eta = \frac{y}{2\sqrt{\upsilon t}}, \quad u = Uf\left(\eta\right), \mathrm{y} = 0, \eta = 0$$

$$T = T_w, T = T_\infty + \left(T_w - T_\infty\right)\theta\left(\eta\right),$$

$$T_w = T_\infty + \left(T_w - T_\infty\right)\theta\left(\eta\right), \left(T_w - T_\infty\right)\theta\left(\eta\right) = \left(T_w - T_\infty\right), \theta\left(\eta\right) = 1$$

$$\theta\left(0\right) = 1, s(0) = 1, \phi(0) = 1$$

$$u = U_w = U, u = Uf\left(\eta\right), Uf\left(\eta\right) = U, f\left(\eta\right) = 1, f\left(0\right) = 1$$

$$U = U_\infty = Uf, f = \frac{U_\infty}{U} = A, f(\infty) = A$$

$$T = T_\infty, T = T_\infty + \left(T_w - T_\infty\right)\theta\left(\eta\right) = T_\infty$$

$$\theta\left(\infty\right) = 0, s\left(\infty\right) = 0, \phi\left(\infty\right) = 0$$

Hence, the equations (3.1) to (3.6) reduces to

$$\left.\begin{array}{l}
f'' + 2\left(\eta + c\right) f' + M\left(A - f\right) + G_{rT}\theta + G_{rC}s = 0 \\
\theta'' + 2P_r\left(\eta + c\right)\theta' + P_r N_b \theta'\phi' + P_r N_t \theta'^2 + P_r N d s'' = 0 \\
s'' + 2Le\left(\eta + c\right)s' + LeLd\theta'' \\
\phi'' + 2Ln\left(\eta + c\right)\phi' + Ln\dfrac{N_t}{N_b}\theta'' = 0 \\
f\left(0\right) = 1, \theta\left(0\right) = 1, s\left(0\right) = 1, \phi\left(0\right) = 1 \\
f\left(\infty\right) = A, \theta\left(\infty\right) = 0, s\left(\infty\right) = 0, \phi\left(\infty\right) = 0
\end{array}\right\} \quad (3.16)$$

where

$$M = \frac{4t\sigma B_0^2}{\rho}, c = -v_0\sqrt{\frac{t}{\upsilon}}, G_{rT} = \frac{4tg\beta\left(T_w - T_\infty\right)}{\upsilon}, G_{rC} = \frac{4tg\beta\left(C_w - C_\infty\right)}{\upsilon}$$

$$A = \frac{U_\infty}{U}, P_r = \frac{\upsilon}{\alpha_m}, \mathrm{N}_b = \frac{\tau D_T\left(\varphi_w - \varphi_\infty\right)}{\upsilon}, \mathrm{N}_t = \frac{\tau D_T}{\upsilon T_\infty}\left(T_w - T_\infty\right), Nd = \frac{D_{TC}\left(C_w - C_\infty\right)}{\upsilon\left(T_w - T_\infty\right)}$$

$$Le = \frac{D_s}{\upsilon}, Ld = \frac{D_{CT}\left(T_w - T_\infty\right)}{\left(C_w - C_\infty\right)}, Ln = \frac{\upsilon}{D_B}$$

are Magnetic parameter, suction parameter, thermal Grashof number, solutal concentration Grashof number, velocity ratio, Prandtl number, Brownian motion, thermophoresis parameter, modified Dufour parameter, Lewis number, Dufour solutal Lewis number, nano Lewis number.

From the similarity equation in (3.14)

$$\left.\begin{aligned}
&\eta=\sqrt{\frac{a}{\upsilon}}y, u=axf^{/}, v=-\sqrt{a\upsilon}f(\eta), \frac{\partial u}{\partial x}=af^{/}, \frac{\partial u}{\partial y}=\frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y}=axf^{//}\sqrt{\frac{a}{\upsilon}}\\
&\frac{\partial v}{\partial y}=\frac{\partial v}{\partial \eta}\frac{\partial \eta}{\partial y}=-\sqrt{a\upsilon}f^{/}\sqrt{\frac{a}{\upsilon}}=-af^{/}\\
&\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}\left(axf^{//}\sqrt{\frac{a}{\upsilon}}\right)=\frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left(axf^{//}\sqrt{\frac{a}{\upsilon}}\right)=\frac{a^2x}{\upsilon}f^{///}\\
&u\frac{\partial u}{\partial x}=axf^{/}af^{/}=a^2xf^{/2}\\
&v\frac{\partial u}{\partial y}=-\sqrt{a\upsilon}f(\eta)a\sqrt{\frac{a}{\upsilon}}xf^{//}=-a^2xff^{//}\\
&U_\infty\frac{\partial U_\infty}{\partial x}=b^2x\\
&T=T_\infty+(T_W-T_\infty)\theta(\eta), \mathrm{C}=C_\infty+(C_W-C_\infty)s(\eta), \varphi=\varphi_\infty+(\varphi_w-\varphi_\infty)\phi(\eta)\\
&\frac{\partial T}{\partial x}=0,\ \frac{\partial T}{\partial y}=(T_W-T_\infty)\frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\theta(\eta)=(T_W-T_\infty)\sqrt{\frac{a}{\upsilon}}\theta^{/}\\
&\frac{\partial C}{\partial x}=0,\ \frac{\partial C}{\partial y}=(C_W-C_\infty)\frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}s(\eta)=(C_W-C_\infty)\sqrt{\frac{a}{\upsilon}}s^{/}\\
&\frac{\partial \varphi}{\partial x}=0,\ \frac{\partial \varphi}{\partial y}=(\varphi_W-\varphi_\infty)\frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\phi(\eta)=(\varphi_W-\varphi_\infty)\sqrt{\frac{a}{\upsilon}}\phi^{/}\\
&\frac{\partial^2 T}{\partial y^2}=\frac{\partial}{\partial y}\left((T_W-T_\infty)\sqrt{\frac{a}{\upsilon}}\theta^{/}\right)=\frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left((T_W-T_\infty)\sqrt{\frac{a}{\upsilon}}\theta^{/}\right)=(T_W-T_\infty)\frac{a}{\upsilon}\theta^{//}\\
&\frac{\partial^2 C}{\partial y^2}=\frac{\partial}{\partial y}\left((C_W-C_\infty)\sqrt{\frac{a}{\upsilon}}s^{/}\right)=\frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left((C_W-C_\infty)\sqrt{\frac{a}{\upsilon}}s^{/}\right)=(C_W-C_\infty)\frac{a}{\upsilon}s^{//}\\
&\frac{\partial^2 \varphi}{\partial y^2}=\frac{\partial}{\partial y}\left((\varphi_W-\varphi_\infty)\sqrt{\frac{a}{\upsilon}}\phi^{/}\right)=\frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left((\varphi_W-\varphi_\infty)\sqrt{\frac{a}{\upsilon}}\phi^{/}\right)=(\varphi_W-\varphi_\infty)\frac{a}{\upsilon}\phi^{//}\\
&v\frac{\partial T}{\partial y}=-\sqrt{a\upsilon}f(\eta)\sqrt{\frac{a}{\upsilon}}(T_w-T_\infty)\theta^{/}=-a(T_w-T_\infty)f\theta^{/}\\
&v\frac{\partial C}{\partial y}=-\sqrt{a\upsilon}f(\eta)\sqrt{\frac{a}{\upsilon}}(C_w-C_\infty)s^{/}=-a(C_w-C_\infty)fs^{/}\\
&v\frac{\partial \varphi}{\partial y}=-\sqrt{a\upsilon}f(\eta)\sqrt{\frac{a}{\upsilon}}(\varphi_w-\varphi_\infty)\theta^{/}=-a(\varphi_w-\varphi_\infty)f\phi^{/}
\end{aligned}\right\} \tag{3.17}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$af^{/}-af^{/}=0$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=U_{\infty}\frac{\partial U_{\infty}}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\sigma B_0^2}{\rho}\left(U_{\infty}-u\right)+g\beta\left(T-T_{\infty}\right)+g\beta\left(C-C_{\infty}\right)$$

$$a^2xf^{/2}-a^2xff^{//}=b^2x\ +\upsilon\left(\frac{a^2x}{\upsilon}f^{///}\right)+\frac{\sigma B_0^2}{\rho}\left(bx-axf^{/}\right)+g\beta\left(T_w-T_{\infty}\right)\theta+g\beta\left(C_w-C_{\infty}\right)s$$

$$f^{/2}-ff^{//}=\frac{b^2}{a^2}\ +f^{///}+\frac{\sigma B_0^2}{\rho}\left(\frac{b}{a^2}-\frac{1}{a}f^{/}\right)+\frac{g\beta\left(T_w-T_{\infty}\right)}{a^2x}\theta+\frac{g\beta\left(C_w-C_{\infty}\right)}{a^2x}s$$

$$f^{///}+ff^{//}-f^{/2}+\frac{b^2}{a^2}+\frac{\sigma B_0^2}{\rho}\left(\frac{b}{a}-f^{/}\right)+G_{rT}\theta+G_{rC}s=0$$

$$f^{///}+ff^{//}-f^{/2}+A^2+M\left(A-f^{/}\right)+G_{rT}\theta+G_{rC}s=0$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha_m\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\tau\left[D_B\left(\frac{\partial\varphi\partial T}{\partial x\partial x}+\frac{\partial\varphi\partial T}{\partial y\partial y}\right)+\frac{D_T}{T_{\infty}}\left(\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}\right)^2\right]+$$

$$D_{TC}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)$$

$$-a\left(T_w-T_{\infty}\right)f\theta^{/}=\alpha_m\left(T_W-T_{\infty}\right)\frac{a}{\upsilon}\theta^{//}+\tau D_B\frac{a}{\upsilon}\left(\varphi_W-\varphi_{\infty}\right)\left(T_w-T_{\infty}\right)\phi^{/}\theta^{/}+$$

$$\frac{\tau D_T}{T_{\infty}}\left(\left(T_W-T_{\infty}\right)\sqrt{\frac{a}{\upsilon}}\theta^{/}\right)^2+D_{TC}\left(C_W-C_{\infty}\right)\frac{a}{\upsilon}s^{//}$$

$$-f\theta^{/}=\frac{\alpha_m}{\upsilon}\theta^{//}+\frac{\tau D_B}{\upsilon}\left(\varphi_W-\varphi_{\infty}\right)\phi^{/}\theta^{/}+\frac{\tau D_T}{T_{\infty}\upsilon}\left(T_W-T_{\infty}\right)\theta^{/2}+\frac{D_{TC}}{\upsilon}\frac{\left(C_W-C_{\infty}\right)}{\left(T_W-T_{\infty}\right)}s^{//}$$

$$\frac{\theta^{//}}{P_r}+f\theta^{/}+N_b\phi^{/}\theta^{/}+N_t\theta^{/2}+Nds^{//}=0$$

$$\theta^{//}+P_rf\theta^{/}+P_rN_b\phi^{/}\theta^{/}+P_rN_t\theta^{/2}+P_rNds^{//}=0$$

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_S\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+D_{CT}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

$$-a\left(C_w - C_\infty\right) fs^{/} = D_S\left(C_W - C_\infty\right)\frac{a}{\upsilon}s^{//} + D_{CT}\left(T_W - T_\infty\right)\frac{a}{\upsilon}\theta^{//}$$

$$-fs^{/} = \frac{D_S}{\upsilon}\left(C_W - C_\infty\right)s^{//} + \frac{D_{CT}}{\upsilon}\frac{\left(T_W - T_\infty\right)}{\left(C_W - C_\infty\right)}\theta^{//}$$

$$\frac{s^{//}}{Le} + fs^{/} + Ld\theta^{//} = 0$$

$$s^{//} + Lefs^{/} + LeLd\theta^{//} = 0$$

$$\left(u\frac{\partial\varphi}{\partial x} + v\frac{\partial\varphi}{\partial y}\right) = D_B\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$-a\left(\varphi_w - \varphi_\infty\right) f\phi^{/} = D_B\left(\varphi_W - \varphi_\infty\right)\frac{a}{\upsilon}\phi^{//} + \frac{D_T}{T_\infty}\left(T_W - T_\infty\right)\frac{a}{\upsilon}\theta^{//}$$

$$-f\phi^{/} = \frac{D_B}{\upsilon}\phi^{//} + \frac{D_T}{T_\infty\upsilon}\frac{\left(T_W - T_\infty\right)}{\left(\varphi_W - \varphi_\infty\right)}\theta^{//}$$

$$\frac{\phi^{//}}{Ln} + f\phi^{/} + \frac{N_t}{N_b}\theta^{//}$$

$$\phi^{//} + Lnf\phi^{/} + Ln\frac{N_t}{N_b}\theta^{//} = 0$$

For boundary condition:

$$\left.\begin{aligned} &u = U_w = ax,\ \ v = 0,\ T = T_w,\ \mathrm{C} = \mathrm{C}_w,\ \ \varphi = \varphi_w\ \ y = 0 \\ &u = U_\infty = bx,\ T = T_\infty, \mathrm{C} = \mathrm{C}_\infty\ ,\ \varphi = \varphi_\infty\ \ \mathrm{y} \to \infty \end{aligned}\right\}$$

$$y = 0, \eta = \sqrt{\frac{a}{v}}y = 0, u = axf^{/}(\eta), u = U_w = ax, ax = axf^{/}(\eta), f^{/}(0) = 1$$

$$v = -\sqrt{a\upsilon}f(\eta), v = 0, -\sqrt{a\upsilon}f(0) = 0, f(0) = 0$$

$$T = T_w, T = T_\infty + \left(T_w - T_\infty\right)\theta(\eta),$$

$$T_w = T_\infty + \left(T_w - T_\infty\right)\theta(\eta), \left(T_w - T_\infty\right)\theta(\eta) = \left(T_w - T_\infty\right), \theta(\eta) = 1$$

$$\theta(0) = 1, s(0) = 1, \phi(0) = 1$$

$$u = U_\infty = bx, u = axf^{/}(\infty), bx = axf^{/}(\infty), f^{/}(\infty) = \frac{b}{a} = A, f^{/}(\infty) = A$$

$$T = T_\infty, T = T_\infty + \left(T_w - T_\infty\right)\theta(\eta) = T_\infty$$

$$\theta(\infty) = 0, s(\infty) = 0, \phi(\infty) = 0$$

Introducing equation (3.17) into (3.7) to (3.12), the equations reduces to

$$
\left.\begin{array}{l}
f^{///} + ff^{//} - f^{/2} + A^2 + M\left(A - f^{/}\right) + G_{rT}\theta + G_{rC}s = 0 \\
\theta^{//} + P_r f\theta^{/} + P_r N_b \theta^{/}\phi^{/} + P_r N_t \theta^{/2} + P_r N d s^{//} = 0 \\
s^{//} + 2Lefs^{/} + LeLd\theta^{//} \\
\phi^{//} + Lnf\phi^{/} + Ln\dfrac{N_t}{N_b}\theta^{//} = 0 \\
f(0) = 0, f^{/}(0) = 1, \theta(0) = 1, s(0) = 1, \phi(0) = 1 \\
f^{/}(\infty) = A, \theta(\infty) = 0, s(\infty) = 0, \phi(\infty) = 0
\end{array}\right\} \qquad (3.18)
$$

where

$$
M = \frac{\sigma B_0^2}{ax\rho}, G_{rT} = \frac{g\beta(T_w - T_\infty)}{a^2 x\upsilon}, G_{rC} = \frac{g\beta(C_w - C_\infty)}{a^2 x\upsilon}
$$

$$
A = \frac{b}{a}, P_r = \frac{\upsilon}{\alpha_m}, \mathrm{N}_b = \frac{\tau D_T(\varphi_w - \varphi_\infty)}{\upsilon}, \mathrm{N}_t = \frac{\tau D_T}{\upsilon T_\infty}(T_w - T_\infty), Nd = \frac{D_{TC}(C_w - C_\infty)}{\upsilon(T_w - T_\infty)}
$$

$$
Le = \frac{\upsilon}{D_s}, Ld = \frac{D_{CT}(T_w - T_\infty)}{(C_w - C_\infty)}, Ln = \frac{\upsilon}{D_B}
$$

are Magnetic parameter, thermal Grashof number, solutal concentration Grashof number, velocity ratio, Prandtl number, Brownian motion, thermophoresis parameter, modified Dufour parameter, Lewis number, Dufour solutal Lewis number, nano Lewis number.

Let $\dfrac{d^3}{d\eta^3} = L_1$ and $\dfrac{d^2}{d\eta^2} = L_2$ and from equation (3.16) we have

$$
\left.\begin{array}{l}
f^{//} = -2(\eta + c)f^{/} - M(A - f) - G_{rT}\theta - G_{rC}s \\
\theta^{//} = -2P_r(\eta + c)\theta^{/} - P_r N_b \theta^{/}\phi^{/} - P_r N_t \theta^{/2} - P_r N d s^{//} \\
s^{//} = -2Le(\eta + c)s^{/} - LeLd\theta^{//} \\
\phi^{//} = -2Ln(\eta + c)\phi^{/} - Ln\dfrac{N_t}{N_b}\theta^{//}
\end{array}\right\} \qquad (3.19)
$$

Introducing the operators into equations (3.19) we have

$$\left.\begin{aligned}
&L_2^{-1}L_2\left[f(\eta)\right]=L_2^{-1}\left[-2(\eta+c)f^{/}-M(A-f)-G_{rT}\theta-G_{rC}s\right]\\
&L_2^{-1}L_2\left[\theta(\eta)\right]=L_2^{-1}\left[-2P_r(\eta+c)\theta^{/}-P_rN_b\theta^{/}\phi^{/}-P_rN_t\theta^{/2}-P_rNds^{//}\right]\\
&L_2^{-1}L_2\left[s(\eta)\right]=L_2^{-1}\left[-2Le(\eta+c)s^{/}-LeLd\theta^{//}\right]\\
&L_2^{-1}L_2\left[\phi(\eta)\right]=L_2^{-1}\left[-2Ln(\eta+c)\phi^{/}-Ln\frac{N_t}{N_b}\theta^{//}\right]
\end{aligned}\right\} \qquad (3.20)$$

where $L_1^{-1}=\int\int\int(\bullet)\,d\eta d\eta d\eta$ and $L_2^{-1}=\int\int(\bullet)\,d\eta d\eta$ (3.21)

Introducing the Adomian polynomials into (3.20) we have

$$\left.\begin{aligned}
&\sum_{n=0}^{\infty}f_n=-2(\eta+c)L_2^{-1}\sum_{n=0}^{\infty}f_n^{/}-ML_2^{-1}\sum_{n=0}^{\infty}(A-f_n)-G_{rT}L_2^{-1}\sum_{n=0}^{\infty}\theta_n-G_{rC}L_2^{-1}\sum_{n=0}^{\infty}s_n\\
&\sum_{n=0}^{\infty}\theta_n=-2P_r(\eta+c)L_2^{-1}\sum_{n=0}^{\infty}\theta_n^{/}-P_rN_bL_2^{-1}\sum_{n=0}^{\infty}A_n-P_rN_tL_2^{-1}\sum_{n=0}^{\infty}B_n-P_rNdL_2^{-1}\sum_{n=0}^{\infty}s_n^{//}\\
&\sum_{n=0}^{\infty}s_n=-2Le(\eta+c)L_2^{-1}\sum_{n=0}^{\infty}s_n^{/}-LeLdL_2^{-1}\sum_{n=0}^{\infty}\theta^{//}{}_n\\
&\sum_{n=0}^{\infty}\phi_n=-2Ln(\eta+c)L_2^{-1}\sum_{n=0}^{\infty}\phi_n^{/}-Ln\frac{N_t}{N_b}L_2^{-1}\sum_{n=0}^{\infty}\theta_n^{//}\\
&where\ A_n=\theta_n^{/}\phi_{n-k}^{/},\ \mathrm{B}_n=\theta_n^{/}\theta_{n-k}^{/}
\end{aligned}\right\} \qquad (3.22)$$

$$\left.\begin{aligned}
&f_{n+1}=-2L_2^{-1}(\eta+c)f_n^{/}+ML_2^{-1}f_n-G_{rT}L_2^{-1}\theta_n-G_{rC}L_2^{-1}s_n\\
&\theta_{n+1}=-2P_rL_2^{-1}(\eta+c)\theta_n^{/}-P_rN_bL_2^{-1}\sum_{k=0}^{n}\theta_n^{/}\phi_{n-k}^{/}-P_rN_tL_2^{-1}\sum_{k=0}^{n}\theta_n^{/}\theta_{n-k}^{/}-P_rNdL_2^{-1}s_n^{//}\\
&s_{n+1}=-2LeL_2^{-1}(\eta+c)s_n^{/}-LeLdL_2^{-1}\theta_n^{//}\\
&\phi_{n+1}=-2LnL_2^{-1}(\eta+c)\phi_n^{/}-Ln\frac{N_t}{N_b}L_2^{-1}\theta_n^{//}
\end{aligned}\right\} \qquad (3.23)$$

Where

$$\left.\begin{aligned} f_0(\eta) &= 1 + (\alpha_1 + MA)\frac{\eta^2}{2} \\ \theta_0(\eta) &= 1 + \eta\alpha_2 \\ s_0(\eta) &= 1 + \eta\alpha_3 \\ \phi_0(\eta) &= 1 + \eta\alpha_4 \end{aligned}\right\} \quad (3.24)$$

are the initial guesses.

Using maple18 to evaluate the integrals we have the final solutions as:

$$\left.\begin{aligned} f(\eta) &= \sum_{n=0}^{5} f_n(\eta) \\ \theta(\eta) &= \sum_{n=0}^{5} \theta_n(\eta) \\ s(\eta) &= \sum_{n=0}^{5} s_n(\eta) \\ \phi(\eta) &= \sum_{n=0}^{5} \phi_n(\eta) \end{aligned}\right\} \quad (3.25)$$

Similarly from Equation (3.18), we have

$$\left.\begin{aligned} f^{///} &= -ff^{//} + f^{/2} - M\left(A - f^{/}\right) - A^2 - G_{rT}\theta - G_{rC}s \\ \theta^{//} &= -P_r f\theta^{/} - P_r N_b\theta^{/}\phi^{/} - P_r N_t\theta^{/2} - P_r Nds^{//} \\ s^{//} &= -Lefs^{/} - LeLd\theta^{//} \\ \phi^{//} &= -Lnf\phi^{/} - Ln\frac{N_t}{N_b}\theta^{//} \end{aligned}\right\} \quad (3.26)$$

Introducing the operators into equations (3.26) we have

$$\left.\begin{aligned} L_1^{-1}L_1\left[f(\eta)\right] &= L_1^{-1}\left[-ff^{//} - A^2 - M\left(A - f^{/}\right) - G_{rT}\theta - G_{rC}s\right] \\ L_2^{-1}L_2\left[\theta(\eta)\right] &= L_2^{-1}\left[-P_r f\theta^{/} - P_r N_b\theta^{/}\phi^{/} - P_r N_t\theta^{/2} - P_r Nds^{//}\right] \\ L_2^{-1}L_2\left[s(\eta)\right] &= L_2^{-1}\left[-Lefs^{/} - LeLd\theta^{//}\right] \\ L_2^{-1}L_2\left[\phi(\eta)\right] &= L_2^{-1}\left[-Lnf\phi^{/} - Ln\frac{N_t}{N_b}\theta^{//}\right] \end{aligned}\right\} \quad (3.27)$$

Introducing the Adomian polynomials into (3.27) we have

$$
\left.\begin{aligned}
&\sum_{n=0}^{\infty} f_n = -L_1^{-1}\sum_{n=0}^{\infty} A_n + L_1^{-1}\sum_{n=0}^{\infty} B_n - ML_1^{-1}\sum_{n=0}^{\infty}\left(A - f_n^{/}\right) - G_{rT}L_1^{-1}\sum_{n=0}^{\infty}\theta_n - G_{rC}L_1^{-1}\sum_{n=0}^{\infty} s_n - A^2\\
&\sum_{n=0}^{\infty}\theta_n = -P_rL_2^{-1}\sum_{n=0}^{\infty} C_n - P_rN_bL_2^{-1}\sum_{n=0}^{\infty} D_n - P_rN_tL_2^{-1}\sum_{n=0}^{\infty} E_n - P_rNdL_2^{-1}\sum_{n=0}^{\infty} s_n^{//}\\
&\sum_{n=0}^{\infty} s_n = -LeL_2^{-1}\sum_{n=0}^{\infty} F_n - LeLdL_2^{-1}\sum_{n=0}^{\infty}\theta^{//}{}_n\\
&\sum_{n=0}^{\infty}\phi_n = -LnL_2^{-1}\sum_{n=0}^{\infty} G_n - Ln\frac{N_t}{N_b}L_2^{-1}\sum_{n=0}^{\infty}\theta_n^{//}\\
&where\ A_n = f_nf_{n-k}^{//}, B_n = f_n^{/}f_{n-k}^{/}, \mathrm{C}_n = f_n\theta_{n-k}^{/}, D_n = \theta_n^{/}\phi_{n-k}^{/},\ \mathrm{E}_n = \theta_n^{/}\theta_{n-k}^{/}\\
&F_n = f_ns_{n-k}^{/}, G_n = f\phi_{n-k}^{/}
\end{aligned}\right\} \quad (3.28)
$$

$$
\left.\begin{aligned}
&f_{n+1} = -L_1^{-1}\sum_{k=0}^{n} f_nf_{n-k}^{//} + L_1^{-1}\sum_{k=0}^{n} f_n^{/}f_{n-k}^{/} + ML_1^{-1}f_n^{/} - G_{rT}L_1^{-1}\theta_n - G_{rC}L_1^{-1}s_n\\
&\theta_{n+1} = -P_rL_2^{-1}\sum_{k=0}^{n} f_n\theta_{n-k}^{/} - P_rN_bL_2^{-1}\sum_{k=0}^{n}\theta_n^{/}\phi_{n-k}^{/} - P_rN_tL_2^{-1}\sum_{k=0}^{n}\theta_n^{/}\theta_{n-k}^{/} - P_rNdL_2^{-1}s_n^{//}\\
&s_{n+1} = -LeL_2^{-1}\sum_{k=0}^{n} f_ns_{n-k}^{/} - LeLdL_2^{-1}\theta_n^{//}\\
&\phi_{n+1} = -LnL_2^{-1}\sum_{k=0}^{n} f_n\phi_{n-k}^{/} - Ln\frac{N_t}{N_b}L_2^{-1}\theta_n^{//}
\end{aligned}\right\} \quad (3.29)
$$

Where
$$
\left.\begin{aligned}
&f_0(\eta) = \eta + \frac{\alpha_1\eta^2}{2} - \left(MA + A^2\right)\frac{\eta^3}{6}\\
&\theta_0(\eta) = 1 + \eta\alpha_2\\
&s_0(\eta) = 1 + \eta\alpha_3\\
&\phi_0(\eta) = 1 + \eta\alpha_4
\end{aligned}\right\} \quad (3.30)
$$

are the initial guesses.

Similarly, using maple18 to evaluate the integrals we have the final solutions as:

$$\left.\begin{aligned}
f(\eta) &= \sum_{n=0}^{5} f_n(\eta) \\
\theta(\eta) &= \sum_{n=0}^{5} \theta_n(\eta) \\
s(\eta) &= \sum_{n=0}^{5} s_n(\eta) \\
\phi(\eta) &= \sum_{n=0}^{5} \phi_n(\eta)
\end{aligned}\right\} \tag{3.31}$$

The solution obtained from equations (3.25) and (3.31) are presented in the Appendices A and B respectively.

CHAPTER FOUR

4.0 RESULTS AND DISCUSSION

4.1 Results

In this chapter, Tables showing the comparison of the results of the present method and those in the literature such as Khan *et al.* (2014) and Ibrahim *et al.* (2013) are presented and the variation of each dimensionless property that appear such as thermal Grashof number, Lewis number, Prandtl number and Magnetic parameter are also presented graphically with the aid of Maple 18.

Table 4.1: Comparison of values of $f^{//}(0)$ with existing solutions for $M = G_{rC} = G_{rT} = 0$

A	Present Results	Khan *et al.* (2014)	Ibrahim *et al.* (2013)	Mustafaa *et al.* (2011)
0.01	-1.0534	-0.998	-0.998	-0.998
0.1	-0.9774	-0.9694	-0.9694	-0.9694
0.2	-0.9206	-0.9181	-0.9181	-0.9181
0.5	-0.6608	-0.6673	-0.6673	-0.6673
2	2.0223	2.0175	2.0175	2.0175

Table 4.2: Comparison of values for local Nusselt number $-\theta^{/}(0)$ with existing solutions for $N_b = N_t = Nd = 0$

Pr	A	Present Results	Khan *et al.* (2014)	Ibrahim *et al.* (2013)	Mustafaa *et al.* (2011)
1	0.1	0.6329	0.6021	0.6022	0.603
1	0.3	0.6564	0.6244	0.6255	0.625
1	0.5	0.6856	0.6924	0.6924	0.692
1.5	0.1	0.6965	0.7768	0.7768	0.777
1.5	0.3	0.7412	0.7971	0.7971	0.797
1.5	0.5	0.8124	0.8647	0.8648	0.863

Tables 4.1 and 4.2 above shows the comparison of the present work and previous works published in the literature and agreement is observe between the present method and the

previously published works.

4.2 Presentation of Graphical Results for Unsteady One Dimensional with Suction

The graphical results for unsteady one dimensional with suction are presented and discussed in this section.

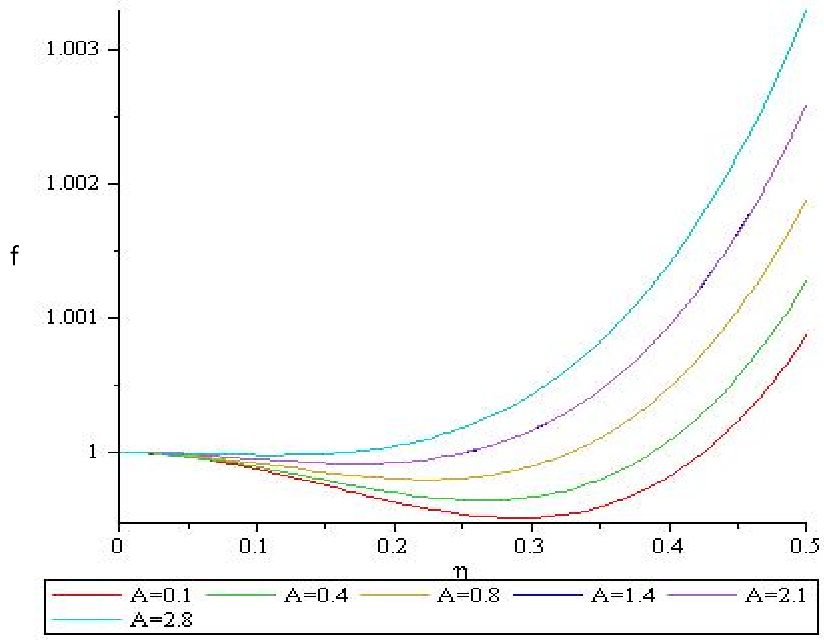

Figure 4.1: Variation of Velocity ratio on the velocity profile for one dimension

Figure 4.1 shows the variation of the velocity ratio on the velocity profile and it is observe that as the velocity ratio increases, the velocity boundary layer dropped. This is as a result of the increase in the velocity ratio which rises above the boundary layer at higher values.

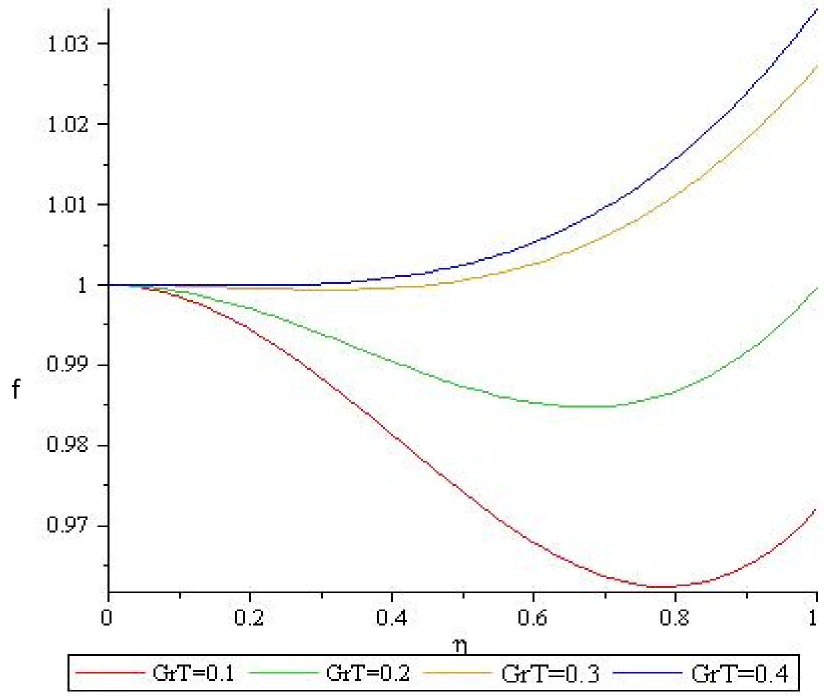

Figure 4.2: Variation of thermal Grashof number on velocity profile for one dimension

Figure 4.2 depicts the effects of thermal Grashof number on the velocity profile. It is observe that as the Grashof number rises, the fluid velocity rises above the velocity boundary layer.

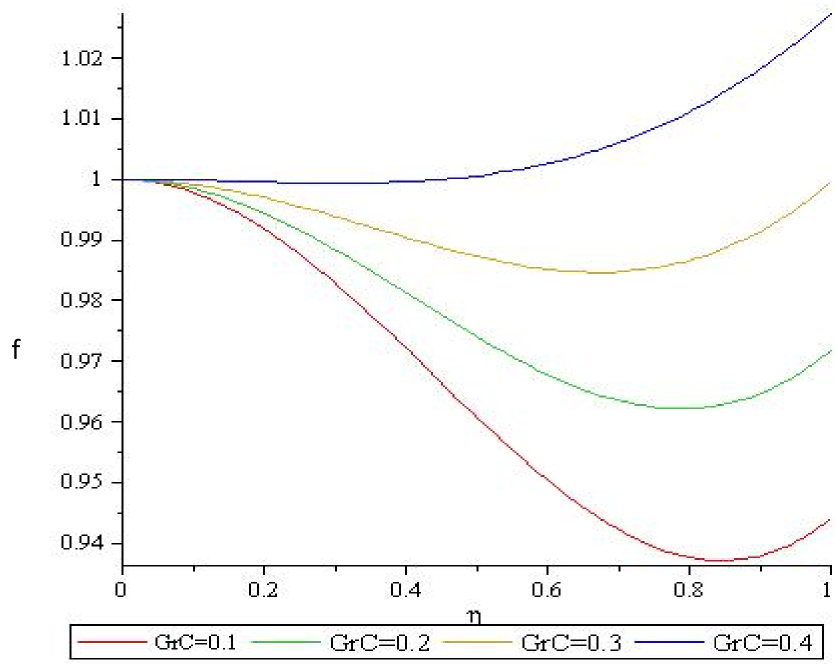

Figure 4.3: Variation of solutal Grashof number on velocity profile for one dimension

Figure 4.3 depicts the effects of solutal Grashof number on the velocity profile. It is observe that as the Grashof number rises, the fluid velocity rises above the velocity boundary layer.

It is also observe clearly that the thermal Grashof number increases the fluid velocity at a faster rate with same values than the solutal Grashof number.

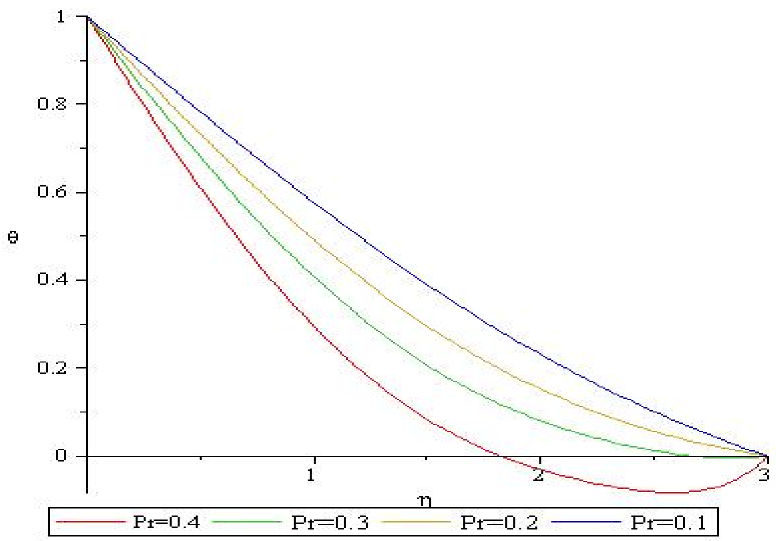

Figure 4.4: Variation of Prandtl number on temperature profile for one dimension

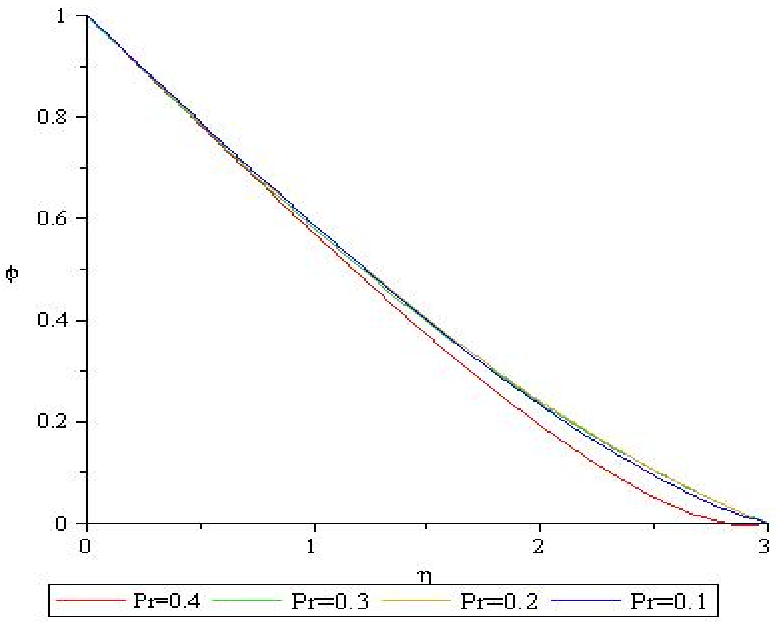

Figure 4.5: Variation of Prandtl number on nanoparticle profile for one dimension

Figures 4.4 to 4.5 presents the variation of Prandtl number on temperature profile and nanoparticle concentration profiles. It is observe that as the Prandtl number increases both the fluid temperature and nanoparticle concentration profile reduces.

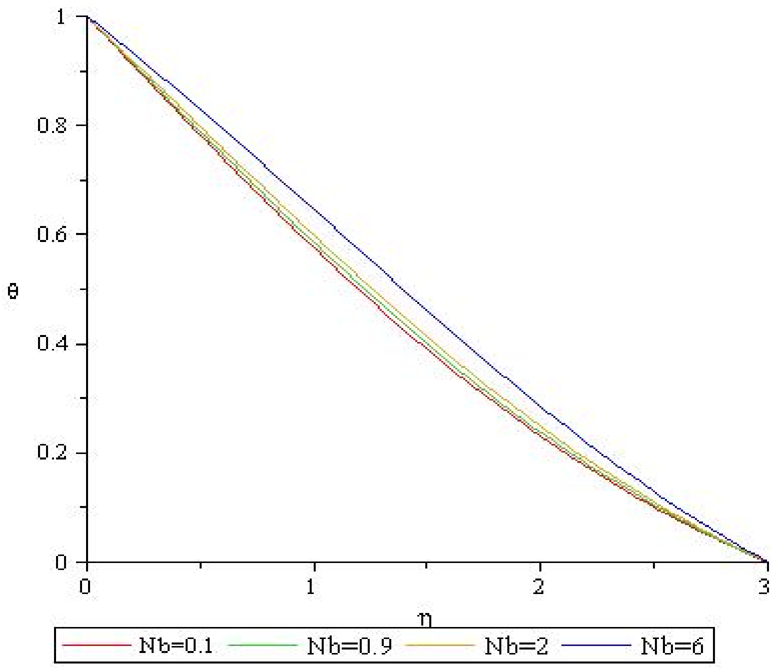

Figure 4.6: Variation of Brownian motion on temperature profile for one dimension

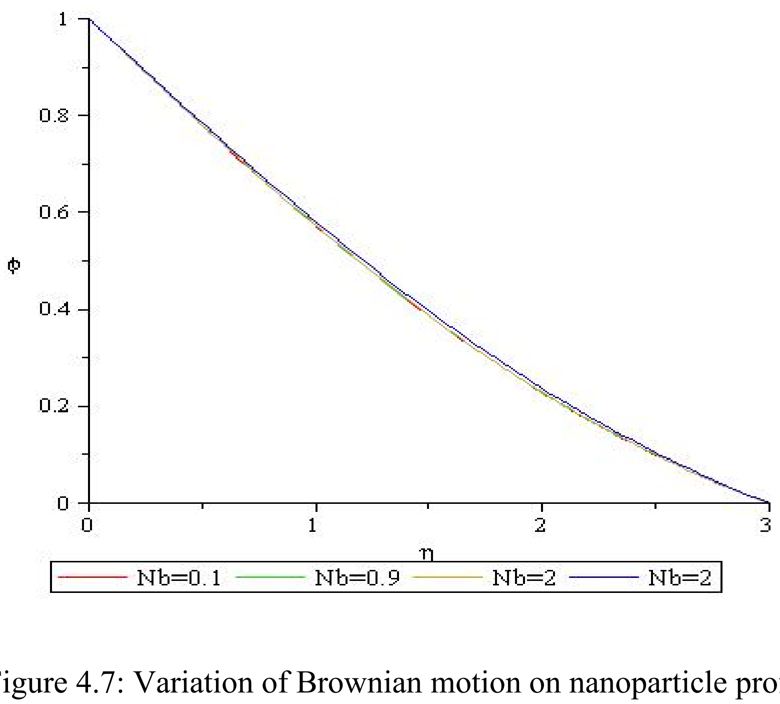

Figure 4.7: Variation of Brownian motion on nanoparticle profile for one dimension

Figure 4.6 to 4.7 depicts the effects of Brownian motion on temperature and nanoparticle concentration profile respectively. The Brownian motion is seen to enhance the both temperature and nanoparticle concentration profile. The rate of increase of the nanoparticle is observed to be low.

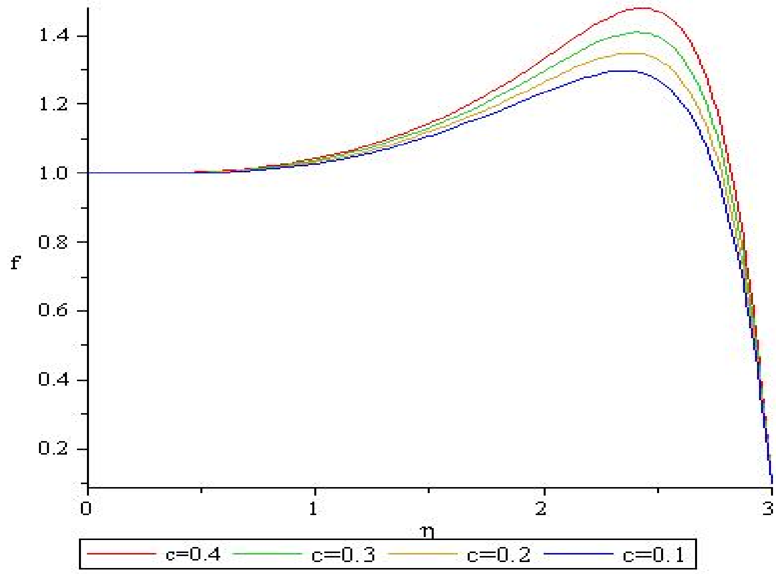

Figure 4.8: Variation of suction parameter on velocity profile for one dimension

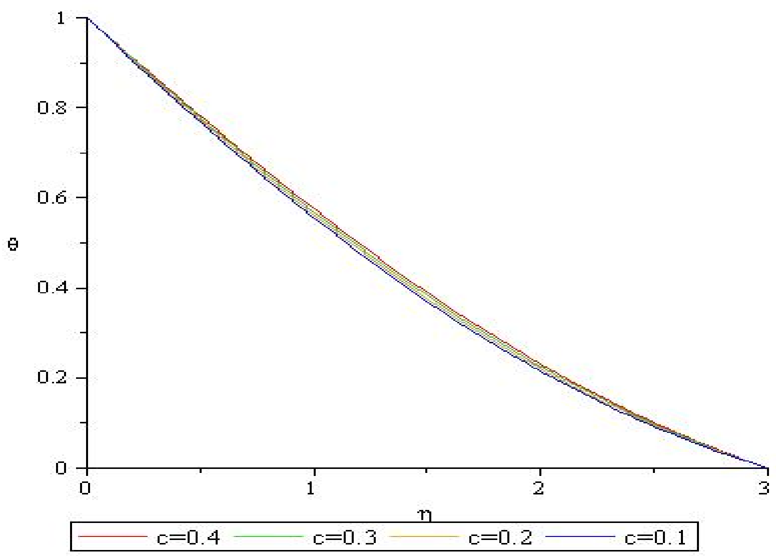

Figure 4.9: Variation of suction parameter on temperature profile for one dimension

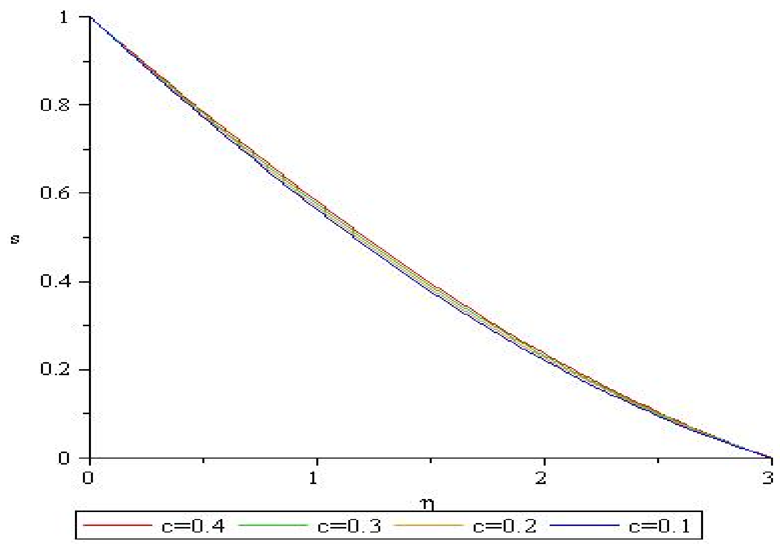

Figure 4.10: Variation of suction parameter on solutal concentration for one dimension

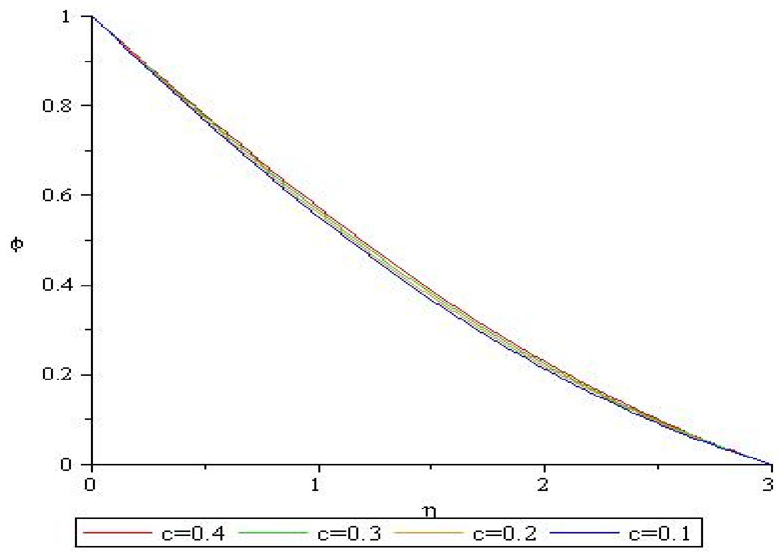

Figure 4.11: Variation of suction parameter on nanoparticle for one dimension

Figures 4.8 to 4.11 display the effects of suction parameter on fluid velocity, temperature, solutal and nanopartcle concentrations respectively. It is observe that as the suction parameter increases, the fluid velocity, temperature, solutal and nanoparticle concentration all dropped.

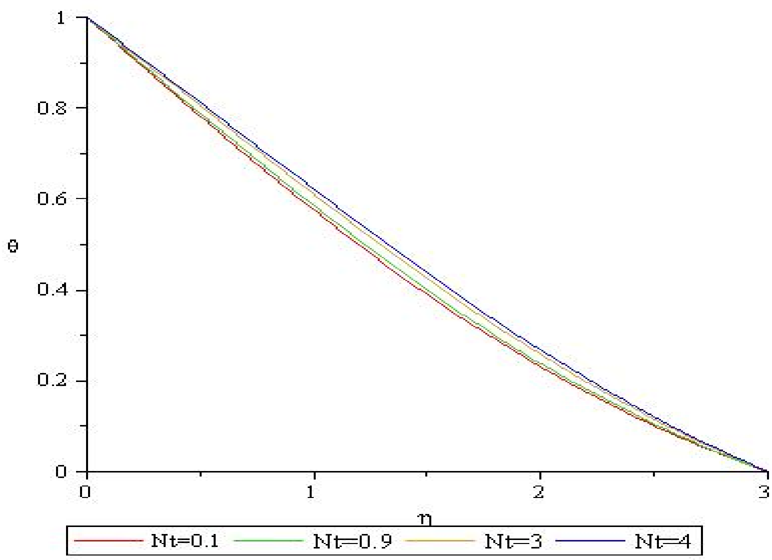

Figure 4.12: Variation of thermophoresis parameter on temperature for one dimension

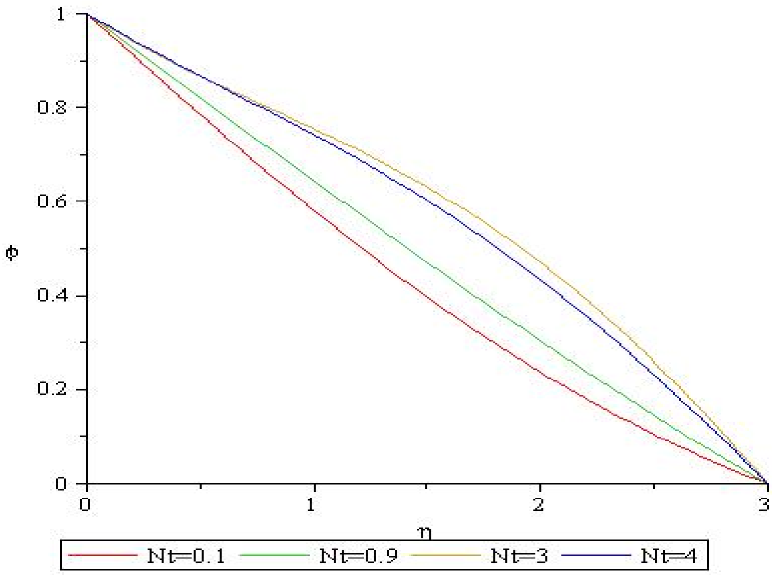

Figure 4.13: Variation of thermophoresis parameter on nanoparticle for one dimension

Figure 4.12 to 4.13 presents the effect of thermophoresis parameter on temperature and nanoparticle concentration profiles. It is observe that as the parameter increases the fluid temperature and nanoparticle concentration also increases.

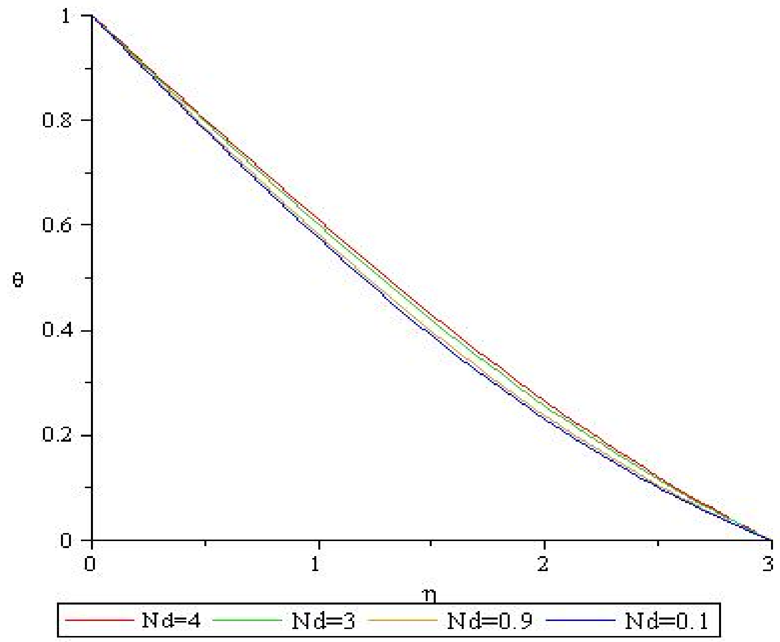

Figure 4.14: Variation of modify Dufour parameter on temperature for one dimension

Figure 4.14 shows the effects of modify Dufour number on temperature parameter and it is observe that as the dufour number increases the fluid temperature also increases.

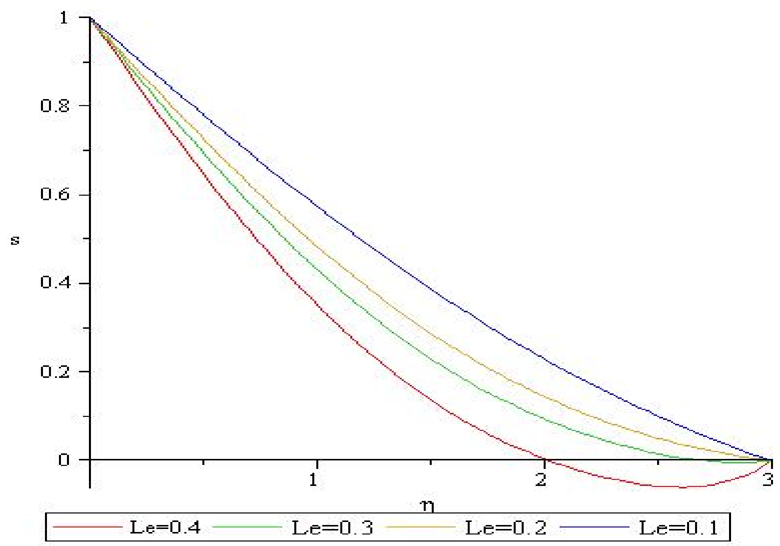

Figure 4.15: Variation of Lewis number on solutal concentration for one dimension

Figure 4.15 shows the effects of Lewis number on the solutal concentration profile and it is observe that as the Lewis number increases, the solutal concentration decreases.

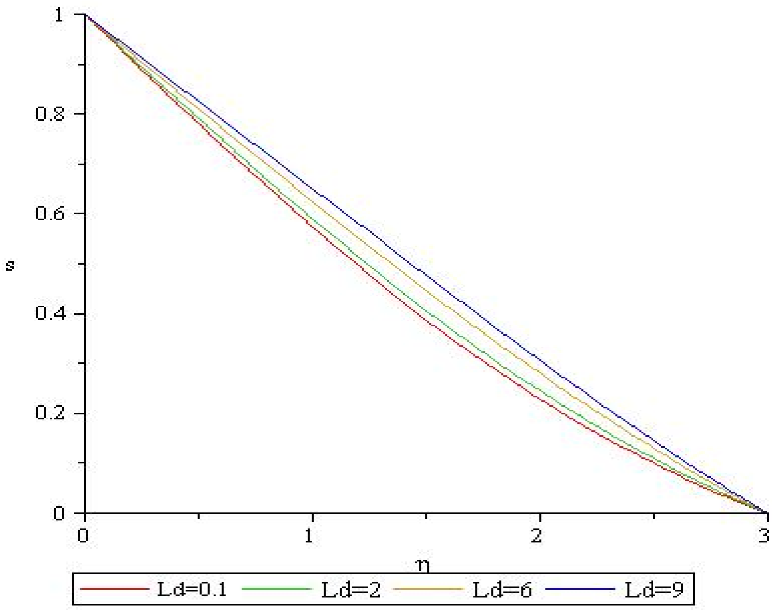

Figure 4.16: Variation of Dufour solutal Lewis number on solutal concentration for one dimension

Figure 4.16 depicts the effect of Dufour solutal Lewis number on the solutal concentration profile. It is seen that as the Dufour solutal Lewis number increases, the solutal concentration boundary layer also increases.

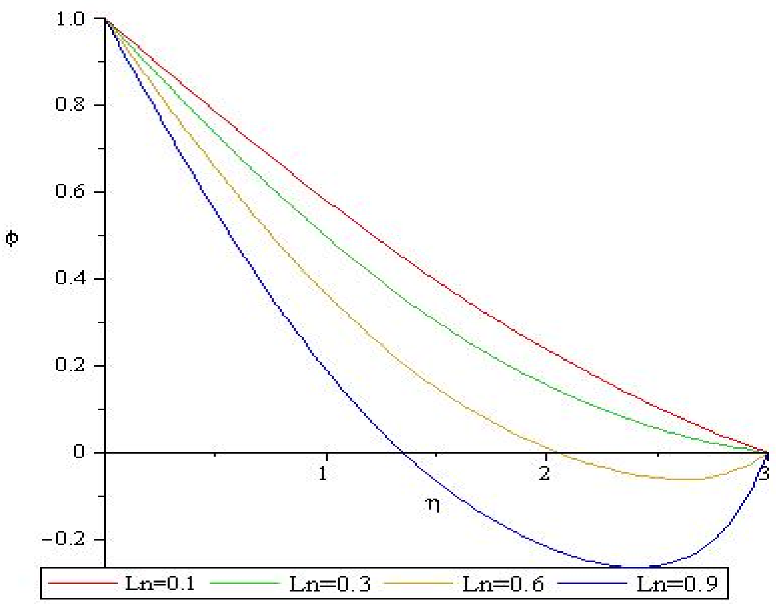

Figure 4.17: Variation of nano Lewis number on nanoparticle for one dimension

Figure 4.17 display the effects of nano Lewis number on nanoparticle concentration profile. As the nano Lewis number increases, the nanoparticle concentration profile dropped.

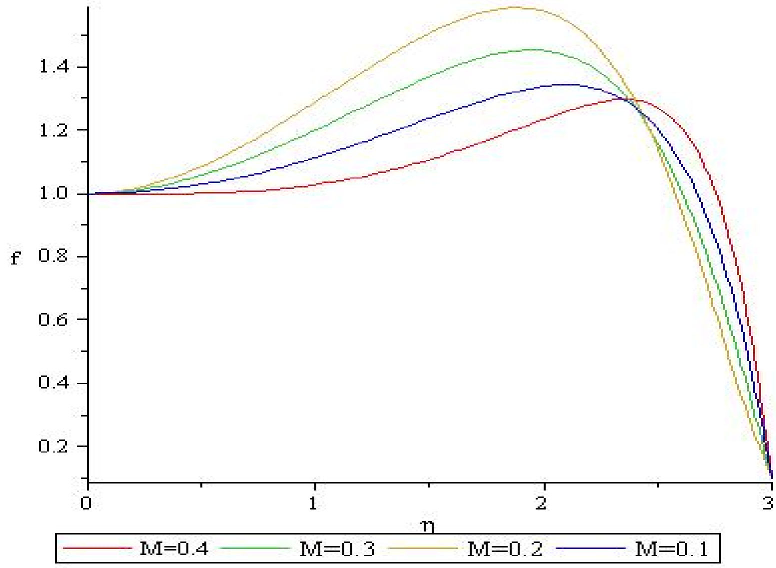

Figure 4.18: Variation of magnetic parameter on velocity profile for one dimension

Figure 4.18 display the effect of magnetic parameter on the fluid velocity and it is clearly seen that the magnetic parameter is a decreasing agent of the fluid velocity due to the Lorentz force that is produces.

4.4 Presentation of Graphical Results for Steady State Two Dimensions

The graphical results for steady state two dimensions are presented and discussed in this section.

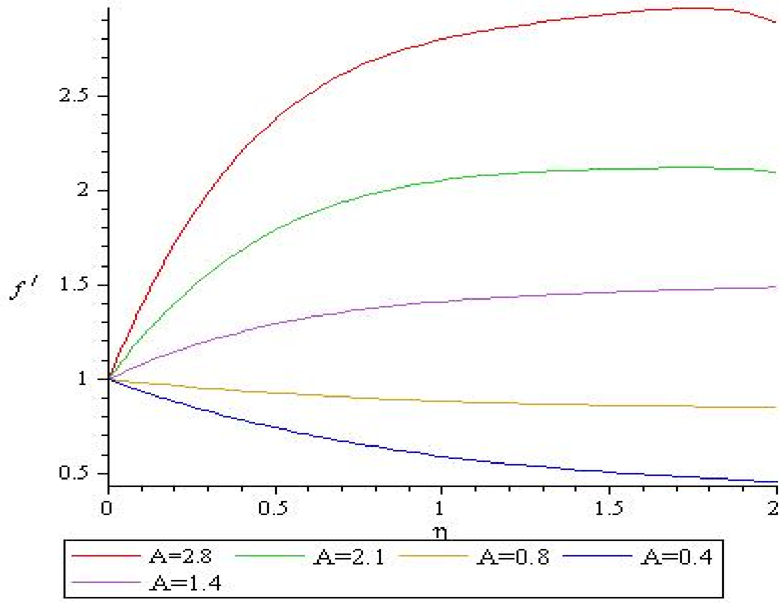

Figure 4.19: Variation of velocity ratio on velocity profile for two dimensions

Figure 4.19 shows the effect of velocity ratio on the velocity profile. It is observer that when the free stream velocity is lower than the stretching sheet (A=0.8, 0.4), the velocity dropped below 1 and rises above when otherwise (A= 2.8, 2.1, 1.4).

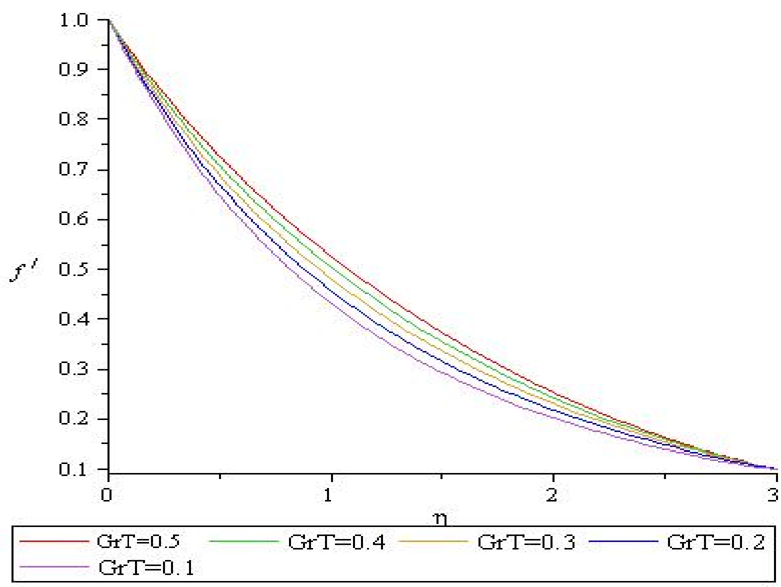

Figure 4.20: Variation of thermal Grashof number on velocity profile for two dimensions

Figure 4.20 shows the effects of thermal Grashof number on the velocity profile and it is observe that as Grashof number increases, the velocity also increases due to the buoyancy effect that is possess.

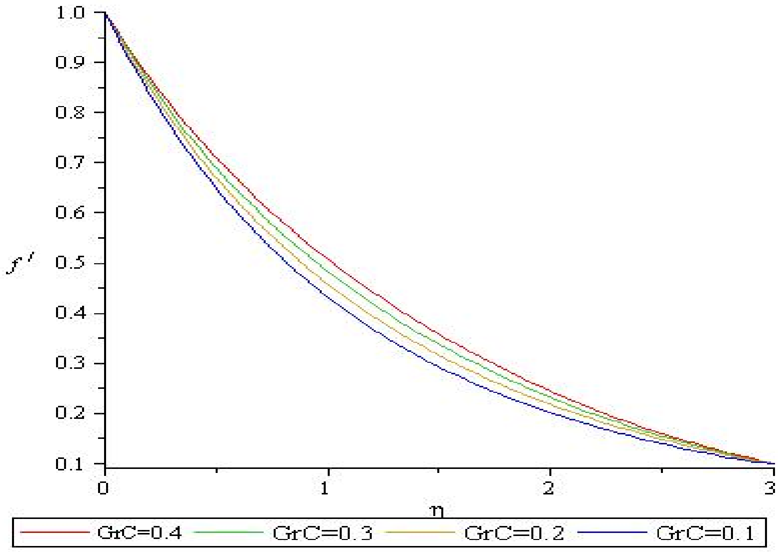

Figure 4.21: Variation of concentration Grashof number on velocity profile for two dimensions

Figure 4.21 is the graph depicting the effects of solutal grashof number on the velocity profile. It is observe that as the solutal Grashof number is enhancing the velocity profile is also increasing.

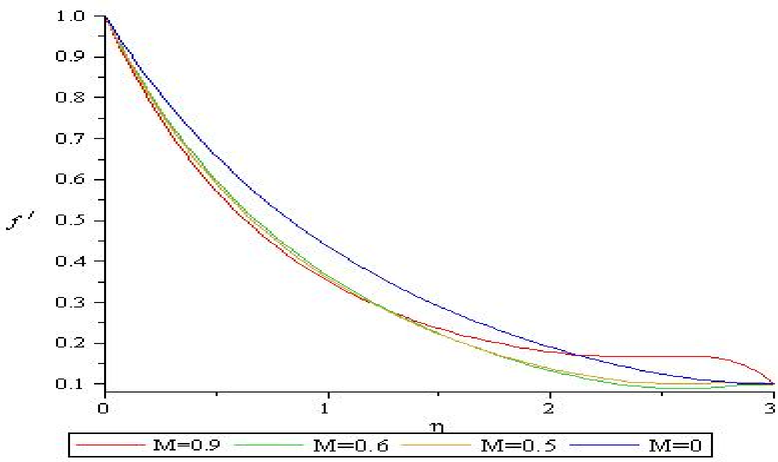

Figure 4.22: Variation of magnetic parameter on velocity profile for two dimensions

Figure 4.22 is the variation of magnetic parameter on the velocity profile and it is observe that as the magnetic parameter increases, the velocity profile reduces due to the drag like force present in the magnetic field.

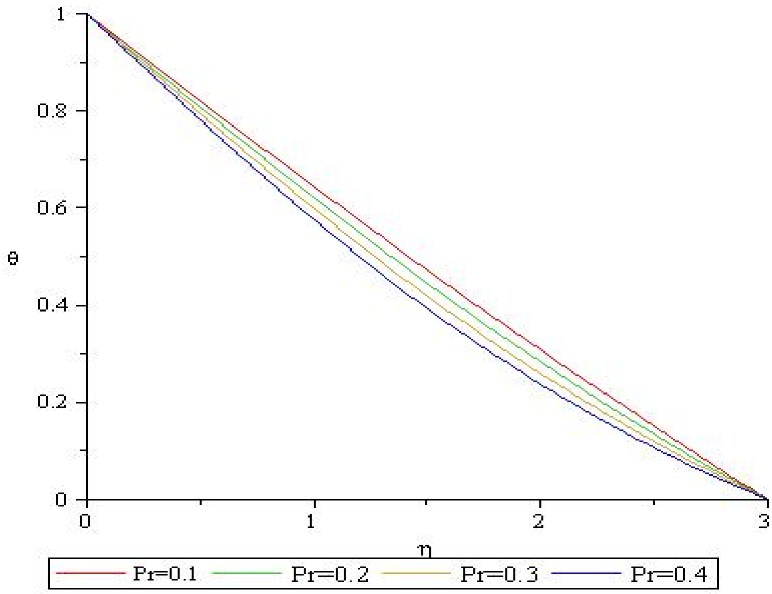

Figure 4.23: Variation of Prandtl number on temperature profile for two dimensions

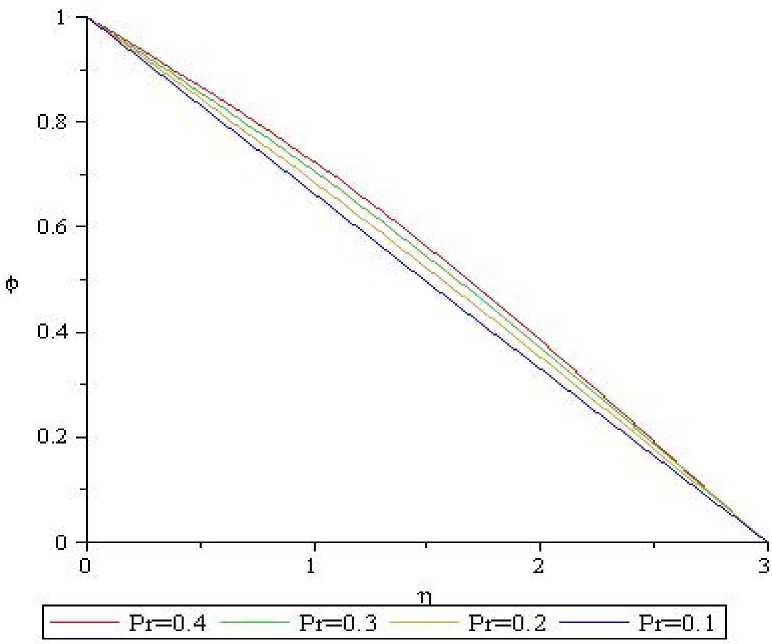

Figure 4.24: Variation of Prandtl number on nanoparticle profile for two dimensions

Figure 4.23 to 4.24 shows the effects of Prandtl number on temperature and nanoparticle concentration profiles. It is observe that as the Prandtl number increases, the temperature of the fluid reduces while the nanoparticle concentration is enhancing.

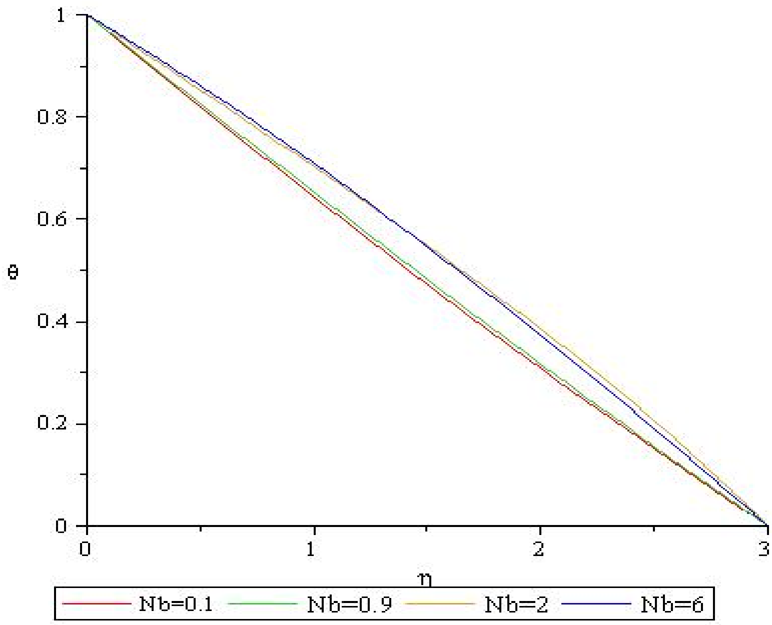

Figure 4.25: Variation of Brownian motion on temperature profile for two dimensions

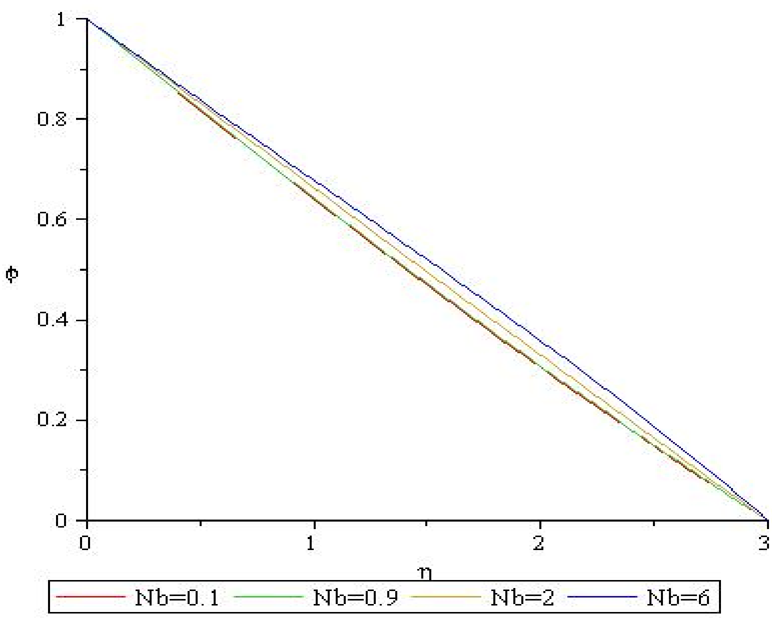

Figure 4.26: Variation of Brownian motion on nanoparticle profile for two dimensions

Figure 4.25 to 4.26 are the graphs of Brownian motion parameter on the fluid temperature and nanoparticle concentration profiles. It is seen that as the Brownian motion parameter increases both fluid temperature and nanoparticle concentration also increase.

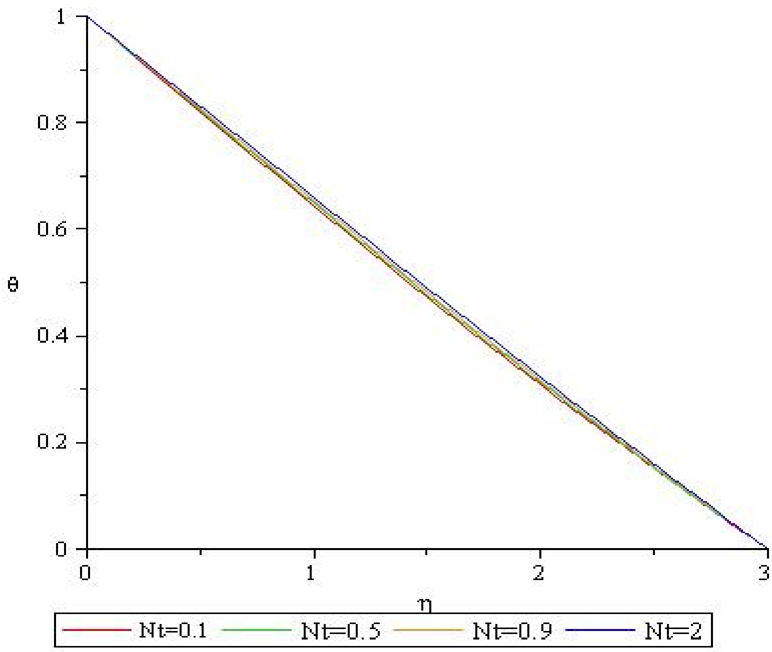

Figure 4.27: Variation of thermopherosis parameter on temperature for two dimensions

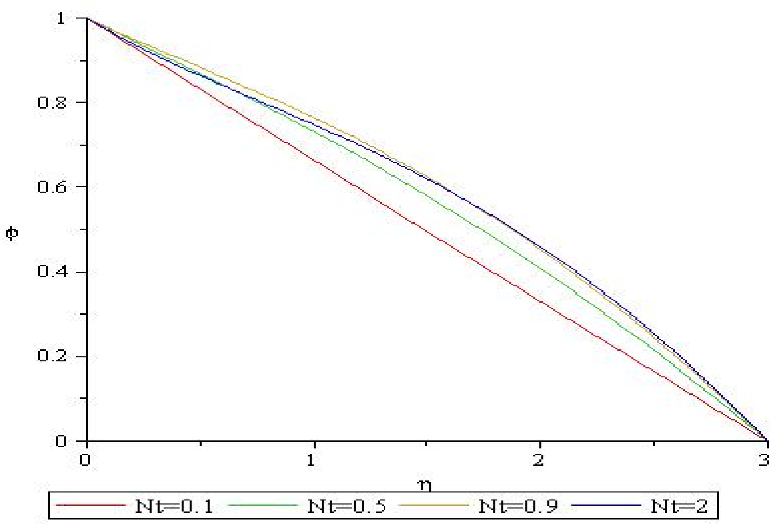

Figure 4.28: Variation of thermopherosis parameter on nanoparticle for two dimensions

Figure 4.27 to 4.28 are the graphs of thermopherosis parameter on the fluid temperature and nanoparticle concentration profiles. It is seen that as the thermopherosis parameter increases both fluid temperature and nanoparticle concentration also increase.

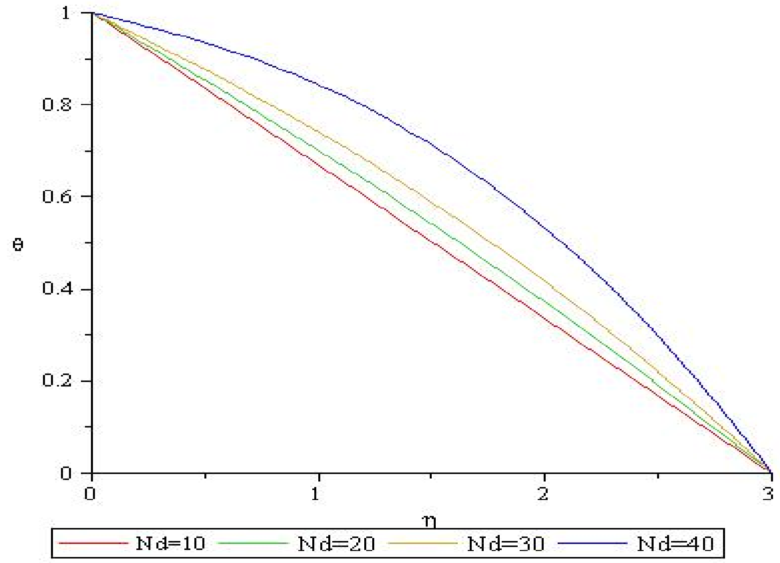

Figure 4.29: Variation of modify Dufour parameter on temperature for two dimensions

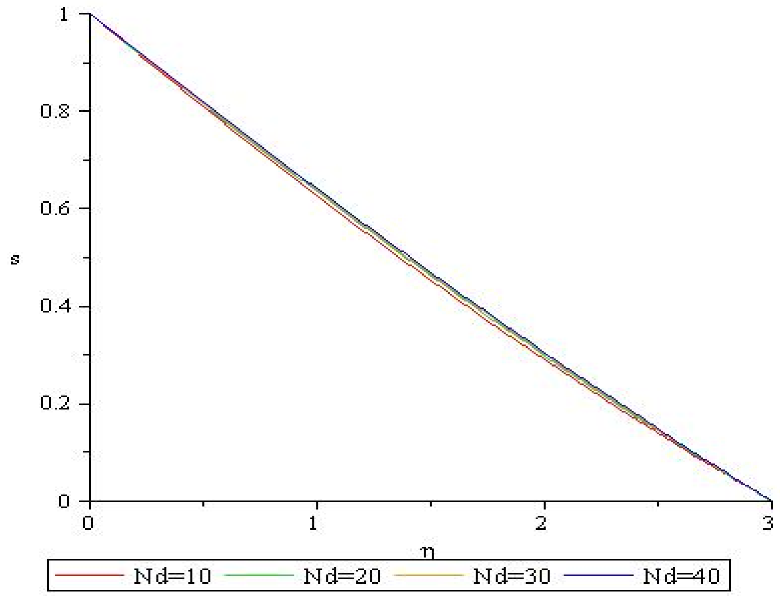

Figure 4.30: Variation of modify Dufour parameter on solutal for two dimensions

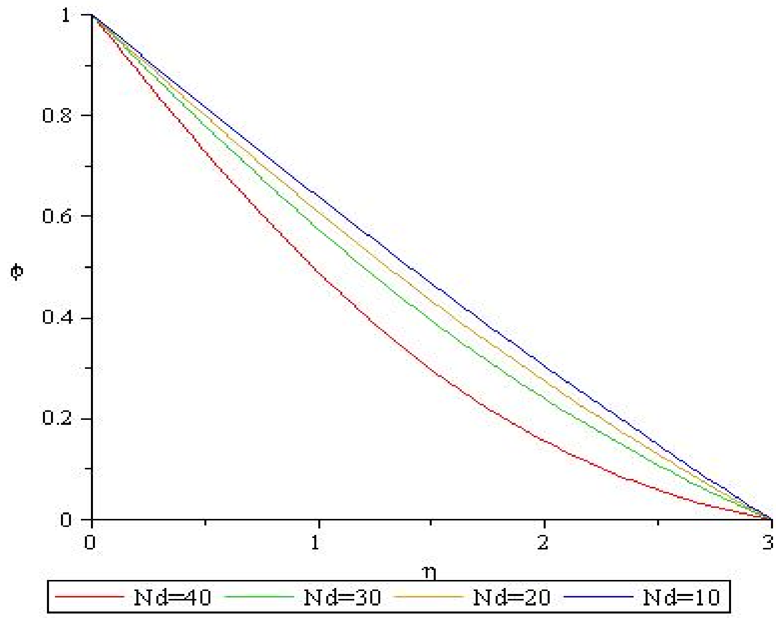

Figure 4.31: Variation of modify Dufour parameter on nanoparticle for two dimensions

Figure 4.29 to 4.31 display the effects of modify Dufour number on the fluid temperature, solutal and nanoparticle concentrations respectively. It is observe that as the modify Dufour number is increasing, the temperature and the solutal concentration are increasing agent while the nanoparticle concentration profile is a reduction agent.

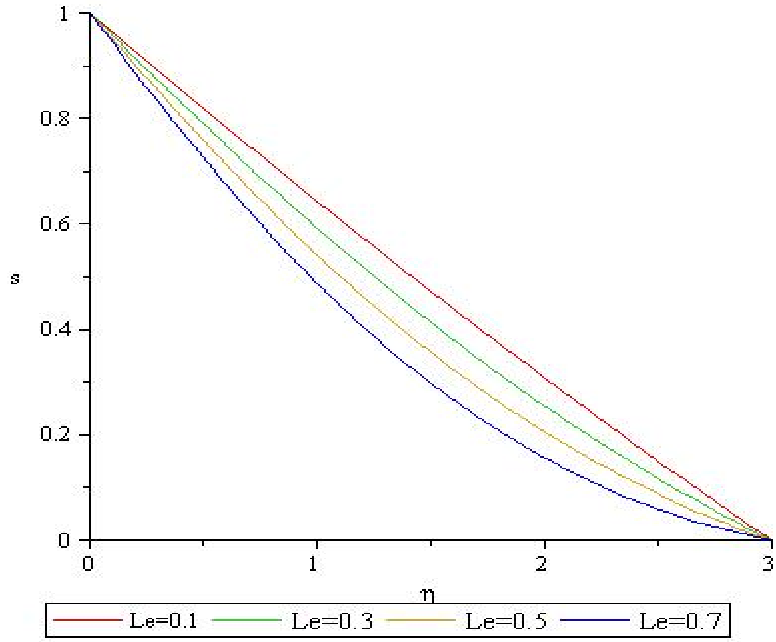

Figure 4.32: Variation of Lewis number on solutal concentration for two dimensions

Figure 4.32 shows the effects of Lewis number on solutal concentration profile and it is observe that as the Lewis number increases, solutal concentration profile reduces.

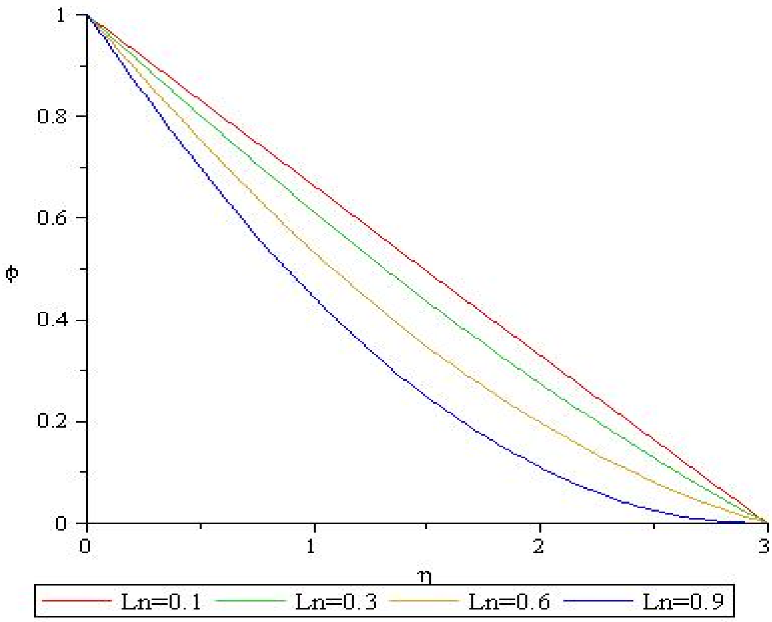

Figure 4.33: Variation of nano Lewis number on nanoparticle for two dimensions

Figure 4.33 shows the effects of nano Lewis number on nanoparticle concentration profile and it is observe that as the Lewis number increases, nano particle concentration profile reduces.

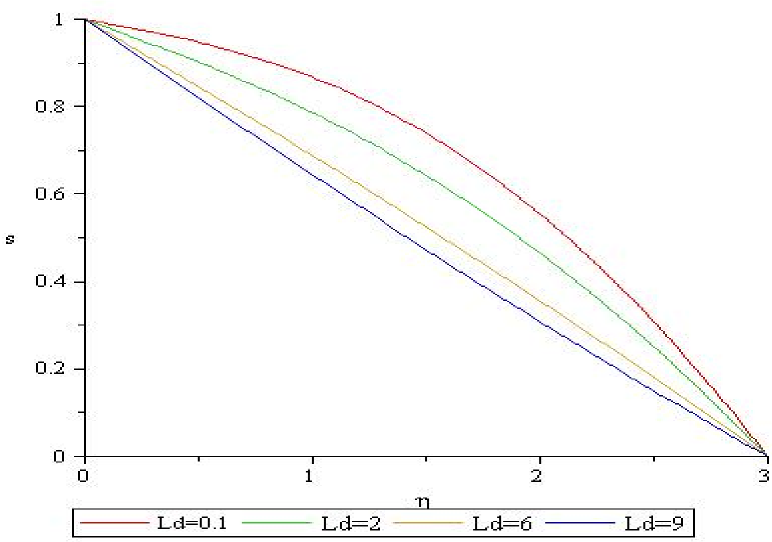

Figure 4.34: Variation of Dufour solutal Lewis number on solutal for two dimensions

Figure 4.34 presents the variation of Dufour solutal Lewis number on the solutal concentration profile. It was observed that as the Dufour solutal Lewis number increases the solutal concentration profile also increases.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This present study extends the work of Khan *et al*. (2014) by considering the natural convection. The PDEs formulated in rectangular system was reduced to ODEs using some transformational variables. The non linear coupled ODEs depends on some physical parameters such as magnetic parameter, Lewis number, and Grashof number. The following observations were made:-

1. The graphs presented in this work clearly satisfy the boundary conditions, which imply that the problem is well posed.
2. The larger values of the dimensionless distance is choosing to be at $\eta = 3$.
3. The results presented in this work were compared with the results of the existing literatures as seen in Table 4.1 and 4.2 and a good agreement was established which justify the choice of method.
4. The value of the velocity ratio was kept at A=0.1, except where it was varied.
5. All the parameters that were varied in the work have same effects on both one dimensional and two dimensional cases considered, but the boundary layer thickness varies.
6. This study presents the results of the problems considered at all point (semi-analytically) unlike the work of Khan *et al.* (2014) which results were at mesh points (Numerically).

5.2 Recommendations

1. Researchers are encouraged to extend the present study by adding more parameters and obtain the result of the problem using a different approach other than ADM.
2. Researchers are also advised to consider the work in three dimensions for both steady and unsteady state.

Contribution to Knowledge

The following contributions were made to knowledge:

- The present work extends the work of Khan *et al.* (2014) by introducing the buoyancy parameter and considered the work in two cases of the extension (one dimensional unsteady state with suction and two dimensions steady state).
- This study present the results of the problems considered at all point (analytically) unlike the work of Khan *et al.* (2014) which results were at mesh points (Numerically).
- The study reveals that Prandtl number (0.1, 0.2, 0.3, 0.4), Lewis number (0.1, 0.2, 0.3, 0.4) and Magnetic parameter (0, 0.5, 0.6, 0.9) are seen as reduction agents to the fluid temperature, solutal concentration profile and fluid velocity respectively. Also, the Grashof number (0.1, 0.2, 0.3, 0.4) are found to enhance the fluid velocity.

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APPENDIX A: SOLUTION TO EQUATION (3.25)

$$
\begin{aligned}
f(\eta) :=& \frac{1}{2} a \eta^2 + 1 - \frac{1}{2} M A \eta^2 - \frac{1}{6} (a - M A) \eta^4 - \frac{1}{3} c (a \\
& - M A) \eta^3 + 2 M \left(\frac{1}{24} a \eta^4 + \frac{1}{2} \eta^2 - \frac{1}{24} M A \eta^4 \right) \\
& - GrT \left(\frac{1}{2} \eta^2 + \frac{1}{6} b \eta^3 \right) - GrC \left(\frac{1}{2} \eta^2 + \frac{1}{6} c1 \eta^3 \right) \\
& - \frac{2}{15} \left(-\frac{2}{3} a + \frac{2}{3} M A + M \left(\frac{1}{6} a - \frac{1}{6} M A \right) \right) \eta^6 \\
& - \frac{1}{5} \left(c \left(-\frac{2}{3} a + \frac{2}{3} M A + M \left(\frac{1}{6} a - \frac{1}{6} M A \right) \right) - c (a \right. \\
& \left. - M A) - \frac{1}{2} GrC\, c1 - \frac{1}{2} GrT\, b \right) \eta^5 - \frac{1}{3} \left(c \left(-c (a \right. \right. \\
& \left. \left. - M A) - \frac{1}{2} GrC\, c1 - \frac{1}{2} GrT\, b \right) - GrT + M - GrC \right) \eta^4 \\
& - \frac{2}{3} c (-GrT + M - GrC) \eta^3 - GrT \left(-\frac{1}{24} P \left(Nb\, c1\, b \right. \right. \\
& \left. \left. + Nt\, b^2 \right) \eta^4 - 2 P b \left(\frac{1}{120} \eta^5 + \frac{1}{24} c \eta^4 \right) \right) \\
& + 2\, GrC\, Ln\, c1 \left(\frac{1}{120} \eta^5 + \frac{1}{24} c \eta^4 \right) + M \left(-\frac{1}{840} \left(-\frac{2}{3} a \right. \right. \\
& \left. \left. + \frac{2}{3} M A + M \left(\frac{1}{6} a - \frac{1}{6} M A \right) \right) \eta^8 - \frac{1}{420} \left(c \left(-\frac{2}{3} a \right. \right. \right. \\
& \left. + \frac{2}{3} M A + M \left(\frac{1}{6} a - \frac{1}{6} M A \right) \right) - c (a - M A) \\
& \left. - \frac{1}{2} GrC\, c1 - \frac{1}{2} GrT\, b \right) \eta^7 - \frac{1}{180} \left(c \left(-c (a - M A) \right. \right. \\
& \left. \left. - \frac{1}{2} GrC\, c1 - \frac{1}{2} GrT\, b \right) - GrT + M - GrC \right) \eta^6 - \frac{1}{60} c (\\
& -GrT + M - GrC) \eta^5 + M \left(\frac{1}{720} a \eta^6 + \frac{1}{24} \eta^4 \right. \\
& \left. - \frac{1}{720} M A \eta^6 \right) - GrT \left(-\frac{1}{720} P \left(Nb\, c1\, b + Nt\, b^2 \right) \eta^6 \right. \\
& \left. - 2 P b \left(\frac{1}{5040} \eta^7 + \frac{1}{720} c \eta^6 \right) \right) + 2\, GrC\, Ln\, c1 \left(\frac{1}{5040} \eta^7 \right. \\
& \left. \left. + \frac{1}{720} c \eta^6 \right) \right) - GrT \left(-P \left(Nb\, c1 \left(-\frac{1}{120} P \left(Nb\, c1\, b \right. \right. \right. \right. \\
& \left. \left. + Nt\, b^2 \right) \eta^5 - 2 P b \left(\frac{1}{720} \eta^6 + \frac{1}{120} c \eta^5 \right) \right) + 2\, Nt\, b \left(\right. \\
& -\frac{1}{120} P \left(Nb\, c1\, b + Nt\, b^2 \right) \eta^5 - 2 P b \left(\frac{1}{720} \eta^6 \right. \\
& \left. \left. + \frac{1}{120} c \eta^5 \right) \right) - 2\, Nb\, Ln\, c1\, b \left(\frac{1}{720} \eta^6 + \frac{1}{120} c \eta^5 \right) \Big) \\
& - 2 P \left(-\frac{1}{840} P b \eta^7 + \frac{1}{360} \left(-3 P b c - P \left(Nb\, c1\, b \right. \right. \right. \\
& \left. \left. + Nt\, b^2 \right) \right) \eta^6 + \frac{1}{120} c \left(-P \left(Nb\, c1\, b + Nt\, b^2 \right) - 2 P b c \right) \eta^5 \Big) \\
& + 2 P\, Nd\, Le\, d1 \left(\frac{1}{120} \eta^5 + \frac{1}{24} c \eta^4 \right) \Big) + GrC\, Ln \Big(\\
& -4\, Ln\, c1 \left(\frac{1}{1680} \eta^7 + \frac{1}{240} c \eta^6 + \frac{1}{120} c^2 \eta^5 \right) \\
& + \frac{1}{Nb} \left(Nt \left(-\frac{1}{24} P \left(Nb\, c1\, b + Nt\, b^2 \right) \eta^4 - 2 P b \left(\frac{1}{120} \eta^5 \right. \right. \right. \\
& \left. \left. \left. + \frac{1}{24} c \eta^4 \right) \right) \right) \Big) \Big)
\end{aligned}
$$

$$\theta(\eta) := 1 + b\,\eta - \frac{1}{2} P \left(Nb\,c1\,b + Nt\,b^{2}\right) \eta^{2} - 2\,P\,b\,\Big(\frac{1}{6}\,\eta^{3}$$
$$+ \frac{1}{2}\,c\,\eta^{2}\Big) - P\,\Big(Nb\,c1\,\Big(-\frac{1}{6}\,P\,\left(Nb\,c1\,b + Nt\,b^{2}\right)\eta^{3}$$
$$- 2\,P\,b\,\Big(\frac{1}{24}\,\eta^{4} + \frac{1}{6}\,c\,\eta^{3}\Big)\Big) + 2\,Nt\,b\,\Big(-\frac{1}{6}\,P\,\big(Nb\,c1\,b$$
$$+ Nt\,b^{2}\big)\,\eta^{3} - 2\,P\,b\,\Big(\frac{1}{24}\,\eta^{4} + \frac{1}{6}\,c\,\eta^{3}\Big)\Big)$$
$$- 2\,Nb\,Ln\,c1\,b\,\Big(\frac{1}{24}\,\eta^{4} + \frac{1}{6}\,c\,\eta^{3}\Big)\Big) - 2\,P\,\Big(-\frac{1}{20}\,P\,b\,\eta^{5}$$
$$+ \frac{1}{12}\,\left(-3\,P\,b\,c - P\,\left(Nb\,c1\,b + Nt\,b^{2}\right)\right)\eta^{4} + \frac{1}{6}\,c\,\big($$
$$-P\,\left(Nb\,c1\,b + Nt\,b^{2}\right) - 2\,P\,b\,c\big)\,\eta^{3}\Big) + 2\,P\,Nd\,Le\,d1\,\Big(\frac{1}{6}\,\eta^{3}$$
$$+ \frac{1}{2}\,c\,\eta^{2}\Big) - P\,\Big(Nb\,c1\,\Big(-P\,\Big(Nb\,c1\,\Big(-\frac{1}{24}\,P\,\big(Nb\,c1\,b$$
$$+ Nt\,b^{2}\big)\,\eta^{4} - 2\,P\,b\,\Big(\frac{1}{120}\,\eta^{5} + \frac{1}{24}\,c\,\eta^{4}\Big)\Big) + 2\,Nt\,b\,\Big($$
$$-\frac{1}{24}\,P\,\left(Nb\,c1\,b + Nt\,b^{2}\right)\eta^{4} - 2\,P\,b\,\Big(\frac{1}{120}\,\eta^{5} + \frac{1}{24}\,c\,\eta^{4}\Big)\Big)$$
$$- 2\,Nb\,Ln\,c1\,b\,\Big(\frac{1}{120}\,\eta^{5} + \frac{1}{24}\,c\,\eta^{4}\Big)\Big) - 2\,P\,\Big(-\frac{1}{120}\,P\,b\,\eta^{6}$$
$$+ \frac{1}{60}\,\left(-3\,P\,b\,c - P\,\left(Nb\,c1\,b + Nt\,b^{2}\right)\right)\eta^{5} + \frac{1}{24}\,c\,\big($$
$$-P\,\left(Nb\,c1\,b + Nt\,b^{2}\right) - 2\,P\,b\,c\big)\,\eta^{4}\Big) + 2\,P\,Nd\,Le\,d1\,\Big(\frac{1}{24}\,\eta^{4}$$
$$+ \frac{1}{6}\,c\,\eta^{3}\Big)\Big) + 2\,Nt\,b\,\Big(-P\,\Big(Nb\,c1\,\Big(-\frac{1}{24}\,P\,\big(Nb\,c1\,b$$
$$+ Nt\,b^{2}\big)\,\eta^{4} - 2\,P\,b\,\Big(\frac{1}{120}\,\eta^{5} + \frac{1}{24}\,c\,\eta^{4}\Big)\Big) + 2\,Nt\,b\,\Big($$
$$-\frac{1}{24}\,P\,\left(Nb\,c1\,b + Nt\,b^{2}\right)\eta^{4} - 2\,P\,b\,\Big(\frac{1}{120}\,\eta^{5} + \frac{1}{24}\,c\,\eta^{4}\Big)\Big)$$
$$- 2\,Nb\,Ln\,c1\,b\,\Big(\frac{1}{120}\,\eta^{5} + \frac{1}{24}\,c\,\eta^{4}\Big)\Big) - 2\,P\,\Big(-\frac{1}{120}\,P\,b\,\eta^{6}$$
$$+ \frac{1}{60}\,\left(-3\,P\,b\,c - P\,\left(Nb\,c1\,b + Nt\,b^{2}\right)\right)\eta^{5} + \frac{1}{24}\,c\,\big($$
$$-P\,\left(Nb\,c1\,b + Nt\,b^{2}\right) - 2\,P\,b\,c\big)\,\eta^{4}\Big) + 2\,P\,Nd\,Le\,d1\,\Big(\frac{1}{24}\,\eta^{4}$$
$$+ \frac{1}{6}\,c\,\eta^{3}\Big)\Big) - 2\,Nb\,Ln\,c1\,\Big(-\frac{1}{60}\,P\,b\,\eta^{6} + \frac{1}{20}\,\Big(-2\,P\,b\,c$$
$$- \frac{1}{2}\,P\,\left(Nb\,c1\,b + Nt\,b^{2}\right)\Big)\,\eta^{5} + \frac{1}{12}\,c\,\big(-P\,\big(Nb\,c1\,b$$

$$\phi(\eta) := 1 + c1\,\eta - 2\,Ln\,c1\left(\frac{1}{6}\eta^3 + \frac{1}{2}c\,\eta^2\right) - Ln\Bigg($$

$$-4\,Ln\,c1\left(\frac{1}{40}\eta^5 + \frac{1}{8}c\,\eta^4 + \frac{1}{6}c^2\eta^3\right)$$

$$+ \frac{1}{Nb}\Bigg(Nt\bigg(-\frac{1}{2}P\left(Nb\,c1\,b + Nt\,b^2\right)\eta^2 - 2\,P\,b\left(\frac{1}{6}\eta^3\right.$$

$$\left.\left. + \frac{1}{2}c\,\eta^2\right)\bigg)\Bigg)\Bigg) - Ln\Bigg(-2\,Ln\bigg(-\frac{1}{84}Ln\,c1\,\eta^7$$

$$-\frac{1}{12}Ln\,c1\,c\,\eta^6 + \frac{1}{20}\left(-4\,Ln\,c1\,c^2 - \frac{Nt\,P\,b}{Nb}\right)\eta^5$$

$$+ \frac{1}{12}\Bigg(c\left(-2\,Ln\,c1\,c^2 - \frac{Nt\,P\,b}{Nb}\right)$$

$$+ \frac{Nt\left(-P\left(Nb\,c1\,b + Nt\,b^2\right) - 2\,P\,b\,c\right)}{Nb}\Bigg)\eta^4$$

$$+ \frac{1}{6}\,\frac{c\,Nt\left(-P\left(Nb\,c1\,b + Nt\,b^2\right) - 2\,P\,b\,c\right)\eta^3}{Nb}\bigg)$$

$$+ \frac{1}{Nb}\Bigg(Nt\Bigg(-P\Bigg(Nb\,c1\left(-\frac{1}{6}P\left(Nb\,c1\,b + Nt\,b^2\right)\eta^3\right.$$

$$\left.\left. - 2\,P\,b\left(\frac{1}{24}\eta^4 + \frac{1}{6}c\,\eta^3\right)\right)\right) + 2\,Nt\,b\left(-\frac{1}{6}P\left(Nb\,c1\,b\right.\right.$$

$$\left.\left. + Nt\,b^2\right)\eta^3 - 2\,P\,b\left(\frac{1}{24}\eta^4 + \frac{1}{6}c\,\eta^3\right)\right)\Bigg)$$

$$- 2\,Nb\,Ln\,c1\,b\left(\frac{1}{24}\eta^4 + \frac{1}{6}c\,\eta^3\right)\Bigg) - 2\,P\,\Bigg(-\frac{1}{20}P\,b\,\eta^5$$

$$+ \frac{1}{12}\left(-3\,P\,b\,c - P\left(Nb\,c1\,b + Nt\,b^2\right)\right)\eta^4 + \frac{1}{6}c\,\Big($$

$$-P\left(Nb\,c1\,b + Nt\,b^2\right) - 2\,P\,b\,c\Big)\eta^3\Bigg) + 2\,P\,Nd\,Le\,d1\left(\frac{1}{6}\eta^3\right.$$

$$\left. + \frac{1}{2}c\,\eta^2\right)\Bigg)\Bigg)\Bigg)$$

$$S(\eta) := 1 + d1\,\eta - 2\,Le\,d1\left(\frac{1}{6}\eta^3 + \frac{1}{2}c\,\eta^2\right) - Le\Bigg(
-4\,Le\,d1\left(\frac{1}{40}\eta^5 + \frac{1}{8}c\,\eta^4 + \frac{1}{6}c^2\eta^3\right) + Ld\Bigg(
-\frac{1}{2}P\left(Nb\,c1\,b + Nt\,b^2\right)\eta^2 - 2\,P\,b\left(\frac{1}{6}\eta^3 + \frac{1}{2}c\,\eta^2\right)\Bigg)\Bigg)
- Le\Bigg(-2\,Le\Bigg(-\frac{1}{84}Le\,d1\,\eta^7 - \frac{1}{12}Le\,d1\,c\,\eta^6 + \frac{1}{20}\big(
-4\,Le\,d1\,c^2 - Ld\,P\,b\big)\eta^5 + \frac{1}{12}\big(c\left(-2\,Le\,d1\,c^2 - Ld\,P\,b\right)
+ Ld\left(-P\left(Nb\,c1\,b + Nt\,b^2\right) - 2\,P\,b\,c\right)\big)\eta^4 + \frac{1}{6}c\,Ld\big(
-P\left(Nb\,c1\,b + Nt\,b^2\right) - 2\,P\,b\,c\big)\eta^3\Bigg) + Ld\Bigg(-P\Bigg(Nb\,c1\Bigg(
-\frac{1}{6}P\left(Nb\,c1\,b + Nt\,b^2\right)\eta^3 - 2\,P\,b\left(\frac{1}{24}\eta^4 + \frac{1}{6}c\,\eta^3\right)\Bigg)
+ 2\,Nt\,b\Bigg(-\frac{1}{6}P\left(Nb\,c1\,b + Nt\,b^2\right)\eta^3 - 2\,P\,b\Bigg(\frac{1}{24}\eta^4
+ \frac{1}{6}c\,\eta^3\Bigg)\Bigg) - 2\,Nb\,Ln\,c1\,b\left(\frac{1}{24}\eta^4 + \frac{1}{6}c\,\eta^3\right)\Bigg) - 2\,P\Bigg(
-\frac{1}{20}P\,b\,\eta^5 + \frac{1}{12}\left(-3\,P\,b\,c - P\left(Nb\,c1\,b + Nt\,b^2\right)\right)\eta^4
+ \frac{1}{6}c\left(-P\left(Nb\,c1\,b + Nt\,b^2\right) - 2\,P\,b\,c\right)\eta^3\Bigg)
+ 2\,P\,Nd\,Le\,d1\left(\frac{1}{6}\eta^3 + \frac{1}{2}c\,\eta^2\right)\Bigg)\Bigg)$$

APPENDIX B: SOLUTION TO EQUATION (3.31)

$$
\begin{aligned}
f(\eta) := {} & \eta + \frac{1}{2} a \eta^2 - \frac{1}{6} (M A + A^2) \eta^3 - \frac{1}{720} \Big(\frac{1}{2} a \Big(\frac{1}{5} \Big(\\
& -\frac{1}{2} M A - \frac{1}{2} A^2 \Big)^2 - \frac{1}{5} \Big(-\frac{1}{6} M A - \frac{1}{6} A^2 \Big) (-M A \\
& - A^2) \Big) + \Big(-\frac{1}{6} M A - \frac{1}{6} A^2 \Big) \Big(\frac{1}{2} a \Big(-\frac{1}{2} M A - \frac{1}{2} A^2 \Big) \\
& - \frac{1}{8} a (-M A - A^2) - \frac{1}{4} \Big(-\frac{1}{6} M A - \frac{1}{6} A^2 \Big) a \Big) \Big) \eta^{10} \\
& - \frac{1}{504} \Big(\frac{1}{5} \Big(-\frac{1}{2} M A - \frac{1}{2} A^2 \Big)^2 - \frac{1}{5} \Big(-\frac{1}{6} M A \\
& - \frac{1}{6} A^2 \Big) (-M A - A^2) + \frac{1}{2} a \Big(\frac{1}{2} a \Big(-\frac{1}{2} M A - \frac{1}{2} A^2 \Big) \\
& - \frac{1}{8} a (-M A - A^2) - \frac{1}{4} \Big(-\frac{1}{6} M A - \frac{1}{6} A^2 \Big) a \Big) + \Big(\\
& -\frac{1}{6} M A - \frac{1}{6} A^2 \Big) \Big(\frac{1}{6} a^2 + M \Big(-\frac{1}{6} M A - \frac{1}{6} A^2 \Big) \Big) \Big) \eta^9 \\
& - \frac{1}{336} \Big(\frac{1}{2} a \Big(-\frac{1}{2} M A - \frac{1}{2} A^2 \Big) - \frac{1}{8} a (-M A - A^2) \\
& - \frac{1}{4} \Big(-\frac{1}{6} M A - \frac{1}{6} A^2 \Big) a + \frac{1}{2} a \Big(\frac{1}{6} a^2 + M \Big(-\frac{1}{6} M A \\
& - \frac{1}{6} A^2 \Big) \Big) + \Big(-\frac{1}{6} M A - \frac{1}{6} A^2 \Big) \Big(-\frac{1}{2} GrC\, c + \frac{1}{2} M a \\
& + \frac{1}{2} a - \frac{1}{2} GrT\, b \Big) \Big) \eta^8 - \frac{1}{210} \Big(\frac{1}{6} a^2 + M \Big(-\frac{1}{6} M A \\
& - \frac{1}{6} A^2 \Big) + \frac{1}{2} a \Big(-\frac{1}{2} GrC\, c + \frac{1}{2} M a + \frac{1}{2} a - \frac{1}{2} GrT\, b \Big) \\
& + \Big(-\frac{1}{6} M A - \frac{1}{6} A^2 \Big) (-GrC - GrT + 1 + M) \Big) \eta^7 \\
& - \frac{1}{120} \Big(-\frac{1}{2} GrC\, c + \frac{1}{2} M a + \frac{1}{2} a - \frac{1}{2} GrT\, b + \frac{1}{2} a (\\
& -GrC - GrT + 1 + M) \Big) \eta^6 - \frac{1}{60} (-GrC - GrT + 1 \\
& + M) \eta^5 - \frac{1}{720} \Big(\Big(\frac{1}{60} a \Big(-\frac{1}{2} M A - \frac{1}{2} A^2 \Big) - \frac{1}{240} a (\\
& -M A - A^2) - \frac{1}{120} \Big(-\frac{1}{6} M A - \frac{1}{6} A^2 \Big) a \Big) (-M A - A^2) \\
& + \Big(\frac{1}{210} \Big(-\frac{1}{2} M A - \frac{1}{2} A^2 \Big)^2 - \frac{1}{210} \Big(-\frac{1}{6} M A \\
& - \frac{1}{6} A^2 \Big) (-M A - A^2) \Big) a \Big) \eta^{10} - \frac{1}{504} \Big(\Big(\frac{1}{120} a^2 + M \Big(\\
& -\frac{1}{120} M A - \frac{1}{120} A^2 \Big) \Big) (-M A - A^2) + \Big(\frac{1}{60} a \Big(-\frac{1}{2} M A \\
& - \frac{1}{2} A^2 \Big) - \frac{1}{240} a (-M A - A^2) - \frac{1}{120} \Big(-\frac{1}{6} M A \\
& - \frac{1}{6} A^2 \Big) a \Big) a \Big)
\end{aligned}
$$

$$
\begin{aligned}
\theta(\eta) :=\ & 1 + b\,\eta - P\Big(b\Big(\frac{1}{24}\,a\,\eta^4 + \frac{1}{6}\,\eta^3 - \frac{1}{120}\,(M\,A \\
& + A^2)\,\eta^5\Big) + \frac{1}{2}\,Nb\,c\,b\,\eta^2 + \frac{1}{2}\,Nt\,b^2\,\eta^2\Big) - P\Big(-P\Big(\frac{1}{72}\Big(\\
& -\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\Big)\,b\,\Big(-\frac{1}{24}\,M\,A - \frac{1}{24}\,A^2\Big)\,\eta^9 \\
& + \frac{1}{56}\,\Big(\frac{1}{2}\,a\,b\,\Big(-\frac{1}{24}\,M\,A - \frac{1}{24}\,A^2\Big) + \frac{1}{6}\,\Big(-\frac{1}{6}\,M\,A \\
& - \frac{1}{6}\,A^2\Big)\,b\,a\Big)\,\eta^8 + \frac{1}{42}\,\Big(b\,\Big(-\frac{1}{24}\,M\,A - \frac{1}{24}\,A^2\Big) \\
& + \frac{1}{12}\,a^2\,b + \frac{1}{2}\,\Big(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\Big)\,b\Big)\,\eta^7 + \frac{1}{30}\,\Big(\frac{5}{12}\,b\,a \\
& + \Big(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\Big)\,\big(Nt\,b^2 + Nb\,c\,b\big)\Big)\,\eta^6 + \frac{1}{20}\,\Big(\frac{1}{2}\,b \\
& + \frac{1}{2}\,a\,\big(Nt\,b^2 + Nb\,c\,b\big)\Big)\,\eta^5 + \frac{1}{12}\,\big(Nt\,b^2 + Nb\,c\,b\big)\,\eta^4\Big) \\
& - Nb\,c\,P\,\Big(b\,\Big(\frac{1}{120}\,a\,\eta^5 + \frac{1}{24}\,\eta^4 - \frac{1}{720}\,(M\,A + A^2)\,\eta^6\Big) \\
& + \frac{1}{6}\,Nb\,c\,b\,\eta^3 + \frac{1}{6}\,Nt\,b^2\,\eta^3\Big) - 2\,Nt\,b\,P\,\Big(b\,\Big(\frac{1}{120}\,a\,\eta^5 \\
& + \frac{1}{24}\,\eta^4 - \frac{1}{720}\,(M\,A + A^2)\,\eta^6\Big) + \frac{1}{6}\,Nb\,c\,b\,\eta^3 \\
& + \frac{1}{6}\,Nt\,b^2\,\eta^3\Big) + b\,\Big(-\frac{1}{15120}\,\Big(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\Big)\,(-M\,A \\
& - A^2)\,\eta^9 - \frac{1}{6720}\,\Big(\frac{1}{2}\,a\,(-M\,A - A^2) + \Big(-\frac{1}{6}\,M\,A \\
& - \frac{1}{6}\,A^2\Big)\,a\Big)\,\eta^8 - \frac{1}{2520}\,\Big(-M\,A - A^2 + \frac{1}{2}\,a^2\Big)\,\eta^7 \\
& + \frac{1}{720}\,a\,\eta^6 + \frac{1}{120}\,\eta^5 + \frac{1}{15120}\,\Big(-\frac{1}{2}\,M\,A - \frac{1}{2}\,A^2\Big)^2\,\eta^9 \\
& + \frac{1}{3360}\,a\,\Big(-\frac{1}{2}\,M\,A - \frac{1}{2}\,A^2\Big)\,\eta^8 + \frac{1}{2520}\,(-M\,A - A^2 \\
& + a^2)\,\eta^7 + M\,\Big(\frac{1}{720}\,a\,\eta^6 + \frac{1}{120}\,\eta^5 - \frac{1}{5040}\,(M\,A \\
& + A^2)\,\eta^7\Big) - GrT\,\Big(\frac{1}{120}\,\eta^5 + \frac{1}{720}\,b\,\eta^6\Big) - GrC\,\Big(\frac{1}{120}\,\eta^5 \\
& + \frac{1}{720}\,c\,\eta^6\Big)\Big) - Nb\,Ln\,c\,b\,\Big(\frac{1}{120}\,a\,\eta^5 + \frac{1}{24}\,\eta^4 \\
& - \frac{1}{720}\,\big(M\,A + A^2\big)\,\eta^6\Big)\Big)
\end{aligned}
$$

$$\phi(\eta) := 1 + c\,\eta - Ln\,c\left(\frac{1}{24}\,a\,\eta^4 + \frac{1}{6}\,\eta^3 - \frac{1}{120}\,(M\,A + A^2)\,\eta^5\right) - Ln\left(-Ln\,c\left(\frac{1}{72}\left(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\right)\left(-\frac{1}{24}\,M\,A - \frac{1}{24}\,A^2\right)\eta^9 + \frac{1}{56}\left(\frac{1}{2}\,a\left(-\frac{1}{24}\,M\,A - \frac{1}{24}\,A^2\right) + \frac{1}{6}\left(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\right)a\right)\eta^8 + \frac{1}{42}\left(-\frac{1}{8}\,M\,A - \frac{1}{8}\,A^2 + \frac{1}{12}\,a^2\right)\eta^7 + \frac{1}{72}\,a\,\eta^6 + \frac{1}{40}\,\eta^5\right) + c\left(-\frac{1}{15120}\left(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\right)(-M\,A - A^2)\,\eta^9 - \frac{1}{6720}\left(\frac{1}{2}\,a\,(-M\,A - A^2) + \left(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\right)a\right)\eta^8 - \frac{1}{2520}\left(-M\,A - A^2 + \frac{1}{2}\,a^2\right)\eta^7 + \frac{1}{720}\,a\,\eta^6 + \frac{1}{120}\,\eta^5 + \frac{1}{15120}\left(-\frac{1}{2}\,M\,A - \frac{1}{2}\,A^2\right)^2\eta^9 + \frac{1}{3360}\,a\left(-\frac{1}{2}\,M\,A - \frac{1}{2}\,A^2\right)\eta^8 + \frac{1}{2520}\,(-M\,A - A^2 + a^2)\,\eta^7 + M\left(\frac{1}{720}\,a\,\eta^6 + \frac{1}{120}\,\eta^5 - \frac{1}{5040}\,(M\,A + A^2)\,\eta^7\right) - GrT\left(\frac{1}{120}\,\eta^5 + \frac{1}{720}\,b\,\eta^6\right) - GrC\left(\frac{1}{120}\,\eta^5 + \frac{1}{720}\,c\,\eta^6\right)\right)\right) + \frac{1}{Nb}\left(Nt\,P\left(b\left(\frac{1}{24}\,a\,\eta^4 + \frac{1}{6}\,\eta^3 - \frac{1}{120}\,(M\,A + A^2)\,\eta^5\right) + \frac{1}{2}\,Nb\,c\,b\,\eta^2 + \frac{1}{2}\,Nt\,b^2\,\eta^2\right)\right)$$

$$S(\eta) := 1 + d1\,\eta - Le\,d1\left(\frac{1}{24}\,a\,\eta^4 + \frac{1}{6}\,\eta^3 - \frac{1}{120}\,(M\,A + A^2)\,\eta^5\right) - Le\left(-Le\,d1\left(\frac{1}{72}\left(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\right)\left(-\frac{1}{24}\,M\,A - \frac{1}{24}\,A^2\right)\eta^9 + \frac{1}{56}\left(\frac{1}{2}\,a\left(-\frac{1}{24}\,M\,A - \frac{1}{24}\,A^2\right) + \frac{1}{6}\left(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\right)a\right)\eta^8 + \frac{1}{42}\left(-\frac{1}{8}\,M\,A - \frac{1}{8}\,A^2 + \frac{1}{12}\,a^2\right)\eta^7 + \frac{1}{72}\,a\,\eta^6 + \frac{1}{40}\,\eta^5\right) + d1\left(-\frac{1}{15120}\left(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\right)(-M\,A - A^2)\,\eta^9 - \frac{1}{6720}\left(\frac{1}{2}\,a\,(-M\,A - A^2) + \left(-\frac{1}{6}\,M\,A - \frac{1}{6}\,A^2\right)a\right)\eta^8 - \frac{1}{2520}\left(-M\,A - A^2 + \frac{1}{2}\,a^2\right)\eta^7 + \frac{1}{720}\,a\,\eta^6 + \frac{1}{120}\,\eta^5 + \frac{1}{15120}\left(-\frac{1}{2}\,M\,A - \frac{1}{2}\,A^2\right)^2\eta^9 + \frac{1}{3360}\,a\left(-\frac{1}{2}\,M\,A - \frac{1}{2}\,A^2\right)\eta^8 + \frac{1}{2520}\,(-M\,A - A^2 + a^2)\,\eta^7 + M\left(\frac{1}{720}\,a\,\eta^6 + \frac{1}{120}\,\eta^5 - \frac{1}{5040}\,(M\,A + A^2)\,\eta^7\right) - GrT\left(\frac{1}{120}\,\eta^5 + \frac{1}{720}\,b\,\eta^6\right) - GrC\left(\frac{1}{120}\,\eta^5 + \frac{1}{720}\,c\,\eta^6\right)\right)\right)$$