ECONOMIC ORDER QUANTITY MODEL UNDER PARTIAL BACKLOGGING FOR ITEMS EXHIBITING DELAY IN DETERIORATION WITH PRICE, STOCK AND RELIABILITY DEMAND

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ABSTRACT

In this thesis, Mathematical models are formulated using Economic Order Quantity Model Under Partial Backlogging for Items Exhibiting Delay in Deterioration with Price, Stock and Reliability Demand where deterioration does not begin the moment they are stocked until after some period of time. The demand before and after deterioration depends on stock, price and reliability. The major objective is to maximize the profit function in order to determine the best inventory management policy. The result indicated that the profit function is maximized and Numerical examples are given to illustrate the applications of the model.

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CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

Lately, Inventory Management has pulled in genuine consideration from individuals both in academic and enterprises in light of the fact that in this advanced competitive world, improving the utilization of assets is the key issue and principal duty of an association, regardless of whether it is a public sector, private area or government office "business or industry" (Bhattarai, 2015).

For an industrial enterprise to survive and grow, it is highly important that all the pervasive efforts are made to minimize and control the total costs to achieve higher operational efficiency and profitability of an organization. Therefore, modern management has started taking more and more interest in "Inventory Management" as Inventories are highly essential for any enterprise and at the same time it has a direct impact on the financial resource, as it locks up funds. There are lot of possibilities to minimize inventory, both in terms of investment as well as quality, as it is a controllable variable (Bhattarai, 2015).

Inventory comprises of materials, wares, items, etc., which are normally conveyed in stock in order to be consumed or profited by when required. An Economic Order Quantity (EOQ) model which is also referred to as Economic Order Lot-size model is an inventory control model, which decides the ideal quantity to be ordered to satisfy a deterministic demand throughout an arranged time frame to limit cost.

In quite a few articles in the writing on inventory models (concentrating on EOQ), it is accepted that things can be kept for an extensive stretch of time for later use without decay. Notwithstanding, it is an overall information that practically all items on inventory deteriorate over the long haul. Deterioration can be referred to as gloom in quality/number of things kept on inventory for certain reason (Dari and Sani, 2013).

In this thesis, we study some EOQ models of deteriorating items which show delay in deterioration with stock, price, and reliability demand dependent. These are things which do not begin to deteriorate immediately they are kept, until after some period of time. A modification to existing models is introduced of which assume demand to be stock, price and reliability dependent before and after deterioration starts (Sudip and Mahapatra, 2018).

1.1.1 Reasons to Hold Inventory

According to Eiselt and Sandblom (2010), Most businesses hold inventory for many reasons. Among them are:

a) Meeting unexpected demands

The chain of supply and demand really comes into consideration here. Organizations know that consumers expect goods and services exactly when needed. Thus, businesses usually stock up their inventories to meet these unexpected demands. These demands may lead to the overcrowding of inventories.

b) Smoothing seasonal demands

With the coming and going of major events and the changing seasons, most organization have inventories at hand to smoothen the seasonal demands. Holding inventories enable businesses to meet these demands.

c) Taking advantage of price discounts

When a business purchase goods from the manufacturers and suppliers, they usually get price discounts if they buy in bigger bulks. Manufacturers and suppliers give these discounts to attract and maintain regular buyers. Taking advantage of price discounts is helpful at times but not overstocking the inventory should be put into consideration.

d) Hedging against price increase

Businesses usually hold inventory to avoid from the ever-fluctuating market price of inventories. Thus, by having efficient and good inventory system, businesses can control their inventory cost.

e) Economies of scale

Economies of scale can be obtained by purchasing large volumes which allows cost reduction of per unit fixed cost. Also, transportation can get economies of scale through utilization by moving larger volume of products.

1.1.2 Inventory Costs

Inventory is associated with three major costs as follows:

(i) Ordering Cost: This is the cost of placing an order to an outside supplier or releasing a production order to a manufacturer. Ordering Cost covers all costs occurring during the ordering processes of one order regardless of volume or quantity ordered which involves the clerical costs of preparing purchase orders, transportation costs, receiving costs (e.g., unloading and inspection).

(ii) Holding Cost: This is the cost of holding an item in inventory for some given unit of time. Holding costs includes costs of storage space (E.g., warehouse depreciation), Security, Insurance, Forgone interest on working capital tied up in inventory, and Deterioration, theft, spoilage, or obsolescence.

(iii) Shortage Cost: This is the cost incurred when a customer seeks the product and finds the inventory empty. The demand can either go unfulfilled or be satisfied later when the product becomes available. The former case is called a lost sale, and the latter is called a backorder. This cost might be difficult to calculate. Shortage cost might cause losing in sales both current and future since customers might turn to competitors.

Other costs include:

a. Salvage Value: Salvage value of an item is the value of a leftover item when no further inventory is desired. The salvage value represents the disposal value of the item to the firm, perhaps through a discounted sale. The negative of the salvage value is called the salvage cost. If there is a cost associated with the disposal of an item, the salvage cost may be positive.
b. Revenue: Revenue may or may not be included in the model. If both the price and the demand for the product are established by the market and so are outside the control of the company, the revenue from sales (assuming demand is met) is independent of the firm's inventory policy and may be neglected. However, if revenue is not neglected in the model, the loss in revenue must then be included in the shortage cost whenever the firm cannot meet the demand and the sale is lost.
c. Discount Rate: discount rate takes into account the time value of money. When a firm ties up capital in inventory, the firm is prevented from using this money for alternative purposes (Sani, 2014).

1.1.3 A Generalized Inventory Model

The ultimate objective of an inventory model is to answer two questions.

1. How much to order

2. When to order (Dari and Sani, 2013),

The answer to the first question (how much to order) is expressed in terms of what we call the order quantity and the second question (when-to-order) is the inventory level at which a new order should be placed usually expressed in terms of re-order point.

According to Hadley and Whitin (1963), one can summarize the total cost of a general inventory model as a function of its principal components in the following manner:

Total inventory cost = purchasing cost + setup cost (or ordering cost) + holding cost + shortage cost (if shortages are allowed).

1.1.4 Types of Inventory Models

Basically, all inventory models (Economic Order Quantity/Economic Production Quantity) are classified into two categories:

a. Deterministic model and

b. Stochastic model

1.2 Statement of Research Problem

The serious issue in inventory Management is to arrive at ideal inventory levels. Settling on choices about the number of which items are to be kept in the warehouse, when to put in request for the next order, the amounts of goods to be ordered are some of the issues experienced every day. The inability to deal with these issues can altogether increase the total cost of an organization. Holding up an excess of inventory hold down the capital of any organization. Clients then again, lose trust in the organization and look elsewhere if there is no availability of goods. This decreases the profit of the organization and folds up the organization.

The study of balancing the correct levels of inventory can be settled by displaying the inventory framework into a mathematical model. The EOQ model would then be able to be formulated and the outcome analyzed to arrive at the best practices measures in inventory management.

On the other hand, the investigation for finding an EOQ has a few shortcomings. This is the reason, numerous authors needed to make expansions or changes in a few parts of the original EOQ model.

Aside from stock and price, reliability of the item is one acceptable angle that could likewise be added to the EOQ model since it assumes a significant part in the interest of a thing on the lookout. Therefore, in this thesis, stock, price and reliability of an item would be considered as factors that affect the demand rate so as to maximize the total profit per unit time.

1.3 Aim and Objectives

1.3.1 Aim of the Study

The aim of this study is to formulate an economic order quantity model for items which exhibit delayed deterioration with price, stock and reliability demand dependent.

1.3.2 Objectives of the Study

The objectives of the study are:

i. To formulate a model with delayed deterioration under constant deterioration and Price, Stock and Reliability demand consideration in an EOQ model of items.

ii. To investigate the effect of price, stock and reliability on demand of items that exhibit delayed deterioration
iii. To maximize the profit function
iv. To determine the best inventory management policy.

1.4 Scope and Limitation

The scope of the study will be limited to the impact of non-instantaneous deterioration with the demand depending on price, stock and reliability of the product on inventory management of an organization.

The pertinence of this investigation is restricted to items with delay in deterioration in which the demand depends only on price, stock and reliability of the product.

1.5 Methodology

The mathematical model was formulated to derive mathematical relationship for the required models under the stated assumptions after which numerical examples will be used to show the application of the models.

1.6 Justification

The study of inventory management is spread over all fields of research of science, engineering, management and commerce. The inventory control till date, has been considering the various topics of real phenomenon such as supply-demand relation, the natural fact of deterioration of items, settlement of account with in or extra time period etc. The traditional inventory model assumes the demand rate to be constant. But in real life, demand rate is not a fixed quantity and should change in dynamical nature (Sudip and Mahapatra, 2018).

In recent years, most researches in the area of inventory control have been oriented towards the development of more realistic and practical models for decision makers. Recently, deteriorating of items in inventory systems has become an interesting topic due to its practical importance. Keeping in view the rapid change of environment and technology, all such parameters are vital for consideration of the study of inventory control. Deterioration of items due to time, storage, transportation etc. is a natural phenomenon, and it is unavoidable. The study of deterioration is very important due to different deterioration rates of many products (Sudip and Mahapatra, 2018).

In the competitive market situation, the customers are influenced by the marketing policies such as the attractive display of items in the showroom or in big malls. The display of items in large number has a motivational effect on the buyers and attracts the people to buy more, so the demand is influenced by stock status (Madhu *et al*., 2007).

Also, the reliability of an item is being considered of which reliability of an item is the probability that it will adequately perform its specified purpose for a specified period of time under specified environmental conditions. Objective determination of reliability costs will help manufacturers plan operations more effectively since an accurate knowledge of reliability costs allows more accurate profit expectations which may, in turn, lead to some marketing advantages (Cheng, 1991).

Definition of Terms

Backlogging: This means not to meet a certain demand immediately from stock. The customer is assumed to wait until the demand is met eventually after a delay.

Backorder: A client request that can't be filled when introduced, and for which the client is ready to hold on for quite a while.

Backorder cost: The expense of taking care of the backorder (uncommon taking care of, subsequent meet-ups and so on) in addition to whatever loss of goodwill happens because of having to backorder an item.

Inventory Turnover (or stock turn): This is a proportion showing how frequently an organization's inventory is sold and replenished over a period.

Lead time: This is the time between requesting for a replenishment of a product and really receiving the product into inventory. The lead-time can be either deterministic (constant or variable) or probabilistic.

Instantaneous delivery Assuming the lead time of a product is zero, we have an extraordinary instance of instantaneous delivery where there is no requirement for submitting a request ahead of time. In our own study, lead time is taken to be zero; therefore, we have a case of Instantaneous delivery.

Inventory management: is a systematic approach to sourcing, storing, and selling inventory—both raw materials (components) and finished goods (product) with the aim of attaining the correct inventory in a correct place at the correct time in a correct measure in the correct form and the correct cost.

Inventory cycle: This is comprised of the events of detecting a requirement for requesting materials, putting in a request, lead time for getting the material conveyed, accepting the material and utilizing it.

Inventory level: This alludes to the quantity of materials available in inventory that is prepared for use, i.e., the current measure of a product that a business has in stock.

Inventory carrying (holding) cost: This is the expense a business sustains throughout a specific timeframe, to hold and store its inventory.

Order quantity This is the amount of material delivered each time inventory is restocked.

Planned shortages: This is a situation where stock outs are planned.

Set-up cost: This is the expense incurred by setting up a machine or preparing for assembling an order. It includes the design cost, moving of machinery, employee hiring, research and development expenses, and labor cost for cleaning and changing tools or holders.

Time horizon: The period over which the inventory level will be controlled is called the time horizon. It can be finite or infinite depending on the nature of demand.

CHAPTER TWO

2.0 LITERATUE REVIEW

2.1 The Basic Economic Order Quantity (EOQ)

Haris (1915) developed the first inventory model, Economic Order Quantity, which was further generalized by Wilson (1934) who gave a formula to obtain economic order quantity. However, Piasecki (2001) presents an inventory model for calculating optimal order quantity using the Economic Order Quantity (EOQ) method. He indicates that many organizations are not using the EOQ method reason being that poor results were received resulting from wrong data input. He explains that several mistakes made that resulted in the calculation of EOQ in the computer software package are due to the improper understanding of how the data inputs and system setup that control the output by the users. He says that the EOQ is an accounting formula that determines the point at which the combination of order costs and inventory cost are the least.

According to Bowersox (2002), the inventory management needs to be structured in a logical way so that the organization would know when to order and how much to order. Arrangement can be scheduled to happen from month to month, quarterly, half yearly, or yearly. As associations attempt to enhance the stock administration, the EOQ and Re-Order Point (ROP) are necessary instruments that associations can utilize.

Eduina and Orjola (2015) also discussed that EOQ determines the optimal amount of costs that are affected both by the number of inventories held and the number of orders made. Ordering in bulk will increase the number of stocks in the warehouse, while ordering costs will be lowered. At the same time, increasing the number of orders reduces holding costs but increases the ordering costs. EOQ minimizes the amount of these costs. EOQ found a formula that shows the connections between the costs of maintaining and ordering an annual demand for material. The formula is written below:

$$Q = \sqrt{\left((2)*D*S\right)/(H*C)} \tag{2.1}$$

Where;

$Q = \text{the Economic order quantity. This is the variable to be optimized.}$

$D =$
$\text{the annual demand of product in quantity per unit time. This can also be as a rate.}$

$S = \text{the ordering cost of product. This is independent of } Q.$

$C = \text{the Unit cost of a product.}$

$H = \text{Holding cost per unit as a fraction of product cost.}$

This model was based on the basic assumption that demand is constant, and that there is a poor inventory at a fixed rate until it reaches zero. And also, that replenishment is instantaneous.

The research in the field of inventory management comprised diverse types of inventory models dealing with different real-life constraints. Several of these models are variations of the basic EOQ model where the alteration contains the conditions that are encountered in the situation being studied. In spite of these new conditions these models still attempt to determine the optimal order quantity, which is one area where the model developed in this research is separate from other models for the EOQ.

Kisaka (2006) examined the role Economic Order Quantity model in decreasing the cost of raw material inventory at a dairy farm project. He compared the total cost of raw material inventory incurred through the project-employed method with the total cost of raw material inventory which is supposed to be incurred under the EOQ application. Kiasaka found that there was cost saving which could have been observed through employing the EOQ model.

Kumar (2016) discussed that the Economic Order Quantity is a very useful tool for inventory control. EOQ can be applied to finish goods inventories, work-in-progress inventories and raw material inventories. It controls purchase and storage of inventory in a way that maintains an even flow of production and equally avoiding unnecessary investment in inventory.

Silver (1976) extends the classical EOQ model to include supply uncertainty. Two problems are analyzed: One in which the standard deviation of quantity received is independent of quantity ordered, and another in which it is proportional to the quantity ordered. For both cases the optimal order quantity is shown to be a simple modification of the EOQ.

Indresh and Arunkumar (2018) discussed the problem that the current forecasting method brought to a particular firm due to inaccurate forecasting and help recommending alternatives ways to reduce the stock and cost by using more effective prediction EOQ and ROP and this resulted into decrease in holding and ordering cost. And therefore, cause a significant decrease of approximately 10% in the total cost of the company.

2.2 Inventory Models with Instantaneous Deterioration

Kapil (2013) developed an Inventory Model for Deteriorating Items with the effect of inflation. Taking the deterioration rate as time dependent, they discussed two cases when shortages are not allowed and when shortages are allowed with complete backlogging. The problem was formulated analytically results were found to be quite suitable and stable when the solution obtained was also checked for sensitivity analysis.

Muniappan *et al*., (2015) researched on EOQ model for deteriorating items with inflation and time value of money considering time-dependent deteriorating rate and delay payments where shortages are allowed in each cycle and backlogged completely. In their model, the supplier offered the retailer a fixed credit period where the retailer is allowed to buy more items and earn more by selling their products and the retailers are charged the interest on purchasing cost for the delay of payment. The optimal decision variables and order quantities of the products was determined so that the net present value of total system cost over a finite planning horizon is minimized. The sensitivity of the optimal policies with respect to changes in some parameters of the system was analyzed by presenting the numerical results.

An Optimal pricing and replenishment policies for instantaneous deteriorating items with backlogging and trade credit under inflation was developed. The retailer's inventory system was modeled as a profit maximization problem to determine the optimal selling price, optimal order quantity and optimal replenishment time. An algorithm was developed to determine the optimal replenishment policies for the retailer. In the model, numerical examples were presented to illustrate the algorithm provided to obtain optimal profit. The result of the model shows that complete backlogging brings an improvement to the total profit than partially backlogging an item (Sundara and Uthayakumar, 2017).

Jinn-Tsair *et al*., (2016) considered an Inventory lot-size policies for deteriorating items with expiration and advance payments where the optimal cycle time and the cycle fraction of no shortages are the decision variables that minimize the total cost. In their model, each of the decision variables of the total annual relevant cost is shown to be strictly pseudo-convex. Numerical examples are provided to illustrate the model.

Wahyudi and Senator (2008) conducted a research An Inventory Model for Deteriorating Commodity under Stock Dependent Selling Rate. They addressed the decision of the optimal replenishment time for ordering an EOQ to a supplier. The replenishment assumed instantaneous with zero lead time. The model solved the problem of how long is the optimum length of cycle time, then the model determined the EOQ of commodity to be ordered. The problem was solved by first developing a mathematical model and then solving it analytically.

Mishra (2013) developed an inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost where demand rate and holding cost are linear function of time and the backlogging rate is variable and depend on the length of the next replenishment.

The mathematical model was formulated as below:

$$\frac{dI_1(t)}{dt} + \tau_p I_1(t) = -(a+bt); \qquad 0 \le t \le t_1 \qquad (2.2)$$

$$\frac{dI_2(t)}{dt} = \frac{-(a+bt)}{1+\delta(T-t)}; \qquad t_1 \le t \le T \qquad (2.3)$$

With the boundary condition

$I_1(t) = I_2(t) = 0$ at $t = t_1$ and $I_1(t) = IM$ at $t = 0$

And the following Notations and Assumption:

a. A the ordering cost per order.

b. C the purchase cost per unit.

c. h(t) the inventory holding cost per unit per time unit.

d. π_0 the backordered cost per unit short per time unit.

e. π_1 the cost of lost sales per unit.

f. ξ preservation technology (PT) cost for reducing deterioration rate in order to preserve the product, $\xi > 0$.

g. θ the deterioration rate.

h. $m(\xi)$ reduced deterioration rate due to use of preservation technology.

i. τ_p resultant deterioration rate, $\tau_p = \left(q - m(\xi)\right)$.

j. t_1 the time at which the inventory level reaches zero, $t_1 \geq 0$.

k. t_2 the length of period during which shortages are allowed, $t_2 \geq 0$.

l. $T = (t_1 + t_2)$ the length of cycle time.

m. IM the maximum inventory level during $[0, T]$.

n. IB the maximum inventory level during shortage period.

o. $Q = (IM + IB)$ the order quantity during a cycle of length T.

p. $I_1(t)$ the level of positive inventory at time t, $0 < t < t_1$.

q. $I_2(t)$ the level of negative inventory at time t, $t_1 < t < t_1 + t_2$.

r. $TC(t_1, t_2, \xi)$ the total cost per time unit.

Assumptions

i. The demand rate is time dependent that is if 'a' is fix fraction of demand and 'b' is that fraction of demand which is vary with time then demand function is $f(\mathrm{t}) = \mathrm{a} + \mathrm{bt}$, where $\mathrm{a} > 0$, $\mathrm{b} > 0$.

ii. Preservation technology is used for controlling the deterioration rate.

iii. Holding cost is linear function of time $h(t) = \alpha + \beta t$, $\alpha \geq 0$, $\beta \geq 0$.

iv. Shortages are allowed and partially backlogged.

v. The lead time is zero.

vi. The replenishment rate is infinite.

vii. The planning horizon is finite.

viii. The deterioration rate is constant.

ix. During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for next replenishment. $B(t) = \frac{1}{1+\delta(T-t)}\delta$. So that the backlogging rate for negative inventory is, B (t) is backlogging parameter and $(T-t)$ is waiting time $(t_1 < t < T)$.

In the model, shortages are allowed to partially backlogged and the total cost was optimized by obtaining an analytical solution.

2.3 Inventory Models with Non-Instantaneous Deterioration

An investigated optimal replenishment policy for non – instantaneous deteriorating items with imprecise demand rate was determined. Modified graded mean integration representation method was used to defuzzified the objective function in fuzzy sense (Kamlesh *et al*., 2015).

Their model was formulated as:

$$\frac{dI_1(t)}{dt} = -D, \qquad 0 \le t \le t_d \tag{2.4}$$

$$\frac{dI_2(t)}{dt} = -\theta I_2(t) - D, \qquad t_d \le t \le T \tag{2.5}$$

With the following notations and assumptions:

Notation

K: The ordering cost per order.

t_d : The length of deterioration free time.

D : Demand rate units per unit time which is imprecise in nature and characterized by triangular fuzzy number $D_1 = (D - \delta_1, D, D + \delta_2)$, where $\delta_1, \delta_2 \ge 0$.

C_d : The deterioration cost per unit.

h : Unit holding cost per unit time.

Q : The order quantity.

T : Length of replenishment cycle $(t_d \le T)$.

θ: The deterioration rate of the on-hand inventory over $[t_d, T]$.

$I_1(t)$: The inventory level at time t $(0 \le t \le t_d)$ in which the product has no deterioration.

$I_2(t)$: The inventory level at time t $(t_d \le t \le T)$ in which the product has deterioration.

$TC(T)$: The total cost per unit time of inventory system.

Assumptions

a. The inventory system involves single non-instantaneous deteriorating item.

b. The on-hand inventory deteriorates with constant rate θ, where $0 \prec \theta$.

c. there is no replacement or repair of deteriorated units during the period under consideration.

d. shortages are not allowed to avoid the lost sales.

e. replenishment rate is infinite and lead time is zero.

f. the system operates for an infinite planning horizon.

Numerical example was used to establish the convexity of defuzzified objective function.

Farughi *et al*., (2014) discussed the Pricing and inventory control policy for non-instantaneous deteriorating items with time- and price-dependent demand and partial backlogging. They maximized the total profit per unit time by determining the optimal price, the replenishment time, and economic order quantity. They proposed an algorithm to the modeled problem, proved that the problem statement is concave function and that the optimal solution is global.

An Inventory Model for Non–Instantaneous Deteriorating Items under Conditions of Permissible Delay in Payments for n-Cycles was studied. An inventory model was developed to find the minimum relevant inventory cost per unit time and shortages were allowed to be backordered. A numerical example was presented to illustrate the model of which the sensitivity analysis was also study. (Maragatham and Lakshimidevi, 2013).

Shalu (2014) developed An Inventory Model for Non – Instantaneous Deteriorating Products with Price and Time Dependent Demand where Shortages are allowed and completely backlogged. The effectiveness of the model was demonstrated by considering numerical examples.

The following mathematical Models was formulated:

$$\frac{dQ_1}{dt} = -d(p)e^{-\theta t}, \qquad 0 \le t \le t_d \tag{2.6}$$

$$\frac{dQ_2}{dt} + \lambda Q_2 = -d(p)e^{-\theta t}, \qquad t_d \le t \le t_1 \tag{2.7}$$

$$\frac{dQ_3}{dt} = -d(p)e^{-\theta t}, \qquad t_1 \le t \le T \tag{2.8}$$

With the following assumptions and Notations:

P Selling price per unit

θ A constant governing the decreasing rate of demand

$D(t,p)$ The demand rate which is a decreasing function of time and selling price p where demand rate is given by $D(t,p) = d(p)e^{-\theta t}$ where $d(p) = a - p$ is function of selling price

Q The Replenishment quantity

t_d The length of time during which the product has no deterioration

t_1 The time at which the inventory level becomes zero

c Purchasing cost per unit item

T The length of replenishment cycle

c1	Shortage cost per unit per unit time
Q1	The inventory level at time t where $t\varepsilon\left(0,t_d\right)$
Q2	The inventory level at time t where t ∈ (t_d, t_1)
Q3	The inventory level at time t where t ∈ (t_d, T)
TP	Total profit
λ	Deteriorating rate and $0 < \lambda < 1$
H	Holding cost (excluding interest charges) per unit per unit time
I	The inventory size after clearing the backlogs.
S	Shortage of inventory
A	Ordering cost per cycle

a. Demand is a function of time as well as selling price.
b. Replenishment is instantaneous with known constant leading time.
c. The time horizon of the inventory is infinite.
d. Shortages are allowed and completely backlogged.
e. Holding cost is constant.
f. There is no replacement or repair of decayed units during the period of consideration.
g. A single product is considered.

With this, it was observed that in case of increasing demand rate and increasing rate of deterioration, it is profitable for retailer to decrease the cycle time.

In the model Periodic Review Inventory Policy for Non-Instantaneous Deteriorating Items with Time Dependent Deterioration Rate, no shortages and different demand rates was considered for pre- and post- deterioration periods. The model minimized the total cost per unit length of an inventory cycle by determining the optimal reorder interval and the optimal order quantity. The model was illustrated by presenting the numerical examples and the sensitivity analysis was carried out. (Anwesha and Manisha, 2015).

Gusti *et al*., (2016) studied an Integrated single-vendor multi-buyer production-inventory policy for food products incorporating quality degradation where the kinetic model is applied to represent the quality degradation of the raw material at the vendor who is the manufacturer, and the shelf-life based pricing is adopted to characterize the value degradation of finished goods at the buyers who are the retailers. The joint profit of the entire system is maximized by establishing a mathematical model. The numerical test validated that the model yields a better profit when compared with the benchmark model.

2.4 Inventory Models of Deteriorating Items with Constant and Varying Demand Rate

Mishra *et al*., (2013) developed an inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging. They provided analytical solution by minimizing the total inventory cost. They illustrated the result with numerical examples. They concluded that the model can be applied to optimize the total inventory cost for the business enterprises where both the deterioration rate and the holding cost are time dependent.

Their model can be expressed as follows:

$$\frac{dI_1(t)}{dt} = -D(t) - \theta(t) I_1(t), \qquad 0 \le t \le t_1$$

$$(2.9) \frac{dI_2(t)}{dt} = -\beta D(t) \qquad t_1 \le t \le T$$

(2.10)

Where the assumptions and notations are as follows:

i. Deterioration rate is time proportional.

ii. $\theta(t) = \theta t$, where θ is the rate of deterioration; $0 \prec \theta \prec 1$.

iii. Demand rate is time dependent and linear, i.e., $D(t) = a + bt$; $\mathrm{a, b} \succ 0$ and are constant.

iv. Shortage is allowed and partially backlogged.

v. C_2 is the shortage cost per unit per unit time.

vi. β is the backlogging rate; $0 \le \beta \le 1$.

vii. During time t1, the inventory is depleted due to the deterioration and demand of item. At time t_1, the inventory becomes zero and shortage starts occurring.

viii. There is no repair or replenishment of deteriorating item during the period under consideration.

ix. Replenishment is instantaneous; lead time is zero. x.

x. T is the length of the cycle.

xi. The order quantity of 1 cycle is q.

xii. Holding cost h(t) per unit time is time dependent and is assumed $h(t) = h + \alpha t$, where $\alpha > 0; h > 0$.

xiii. C is the unit cost of an item.

xiv. IB is the maximum inventory level during the shortage period.

xv. I_0 is the maximum inventory level during $(0, T)$.

xvi. S is the lost sale cost per unit.

They obtained a result that indicated the validity and stability of the model.

Hemmati *et al*., (2017) discussed the Inventory of complementary products with stock-dependent demand under vendor-managed inventory with consignment policy. They considered a vendor-managed inventory with consignment stock policy. They jointly determined the number of shipments and replenishment lot sizes as decision variables in order to maximize the total profit. While conducting the numerical study, it was shown that the quantity of transfers and demand of both products increases as the complementation rate increases.

An Inventory Model with Three Rates of Production Rate under Stock and Time Dependent Demand for Time Varying Deterioration Rate with Shortages was considered. A two parameter Weibull distribution used to represent the deterioration rate. The optimal total cost and the optimal time schedule of the plan for the proposed model was determined and the developed model was illustrated with some numerical examples. The increase of stock level of a product was observed to have a positive impact on the demand of the product. It was also observed that consumer satisfaction and earning of potential profit can be acquired if production rate is being varied. (Sharmaila and Uthayakumar, 2016).

Madhu *et al*., (2007) investigated an Economic Production Quantity Models with Shortage, Price and Stock-Dependent Demand for Deteriorating Items. They provided an expression for various optimal indices and cost analysis, carried out sensitivity analysis which are consistent with tic incentives by taking numerical illustration and then employed Newton's method for optimizing the cost.

A study on an order level EOQ model for deteriorating items in a single warehouse system with price depended demand and shortages was carried out. They minimized the total cost by developing the optimum replacement policy and decision rule. The results are being illustrated with the help of numerical example and discussed the sensitivity of the solution with the changes of the values of the parameters associated with the model (Duari, and Tripti, 2014).

An Inventory Model for Deteriorating Items with Permissible Delay in Payment and Inflation Under Price Dependent Demand where shortages are being allowed and backlogged was studied. It was assumed that the inventory manager is being permitted by the supplier to settle his accounts within a given specified time period. The model was illustrated by citing Numerical examples and the sensitivity of the model to change in model parameters was analyzed. The total profit over the planning horizon was maximized by determining the optimum ordering policy (Manisha and Hare, 2012).

Chih-Te *et al.*, (2009) developed a Retailer's Optimal Pricing and Ordering Policies for Non-Instantaneous Deteriorating Items with Price-Dependent Demand and Partial Backlogging where the backlogging rate is variable and dependent on the waiting time for the next replenishment. They maximized the total profit per unit time by determining the optimal selling price, the length of time in which there is no inventory shortage and the replenishment cycle time simultaneously and found the optimal solution by developing an algorithm. They illustrated theoretical results by providing numerical examples and also carried out the sensitivity analysis.

Lakshmana (2015) considered An EOQ Model for Deteriorating Items with Selling Price Dependent Demand and Time-Varying Holding Cost under Partial Backlogging and maximized the profit function by solving the model analytically.

He formulated the following model:

$$\frac{d}{dt}I(\mathrm{t}) + \lambda(s) = -a(t)I(t), \qquad 0 \le t \le t_1 \qquad (2.11)$$

$$\frac{d}{dt}I(t) = -\beta\lambda(s), \qquad t_1 \le t \le T \qquad (2.12)$$

With the following Notations and assumptions:

a. $a(t) = \eta t$, where η is the rate of deterioration; $0 < \eta < 1$

b. demand rate is selling price dependent i.e., $\lambda(s) = a - bs$; $\mathrm{a, b} > 0$ and are a constant

c. Shortage is allowed and partially backlogged.

d. C1 is the holding cost per unit per unit time.

e. C2 is the shortage cost per unit per unit time.

f. A is ordering cost.

g. β is the backlogging rate, $0 \le \beta \le 1$.

h. During time t_1, the inventory is depleted due to the deterioration and demand of item. At time t_1, the inventory becomes zero and shortage starts occurring.

i. There is no repair or replenishment of deteriorating item during the period under consideration.

j. Replenishment is instantaneous.

k. Lead time is zero.

l. T is the length of the cycle.

m. Ordering quantity in one cycle is Q.

n. C is the unit cost of an item.

o. I_0 is the maximum inventory level during $(0,T)$.

p. $I(t)$ is inventory level at time t.

q. IB is the maximum inventory level during the shortage period.

r. S is the lost sale cost per unit.

He presented the result of the model by numerical illustrations which indicated the validity and stability of the model.

The inventory model of flexible demand for demand for price, stock and reliability with deterioration under inflation incorporating delay in payment was studied where shortage was allowed under inflation. Situation of the credit period less than or greater than the cycle time for settling the account is considered. Numerical example was given and sensitivity analysis was carried out to analyze the effect of the parameters on the optimal solution. (Sudip and Mahapatra, 2018).

Sunjay *et al.*, (2014) carried out a model on Optimum Ordering Interval for Deteriorating Items with Selling Price Dependent Demand and Random Deterioration. They considered a special form of two parameter Weibull function by Covert and Philip as deterioration rate and the result was obtained by choosing appropriate values of the parameter.

Nita *et al.*, (2011) studied Optimal ordering and pricing policy for price sensitive stock–dependent demand under progressive payment scheme and developed an Algorithm to search for the optimal decision policy. The model was supported with a numerical example and sensitivity was also carried out to investigate critical parameters.

Hameed (2008) studied an Optimal Policy for a Deteriorating Inventory Model with Finite Replenishment Rate and with Price dependent Demand Rate and Cycle Length dependent Price. In the model, the optimal sequence of the cycles was found and different optimal selling prices, optimal order quantities and optimal maximum inventories for the cycles with unequal lengths were obtained by using a dynamic programming-based solution algorithm. A numerical example was used to demonstrate the accuracy of the solution procedure.

A study was carried out on the Effect of Reliability on Varying Demand and Holding Cost on Inventory System incorporating Probabilistic Deterioration where shortages are allowed and partially backlogged at a fixed rate. The study considered the demand rate to be high initially and then reduces in the later stage. the dependency of the holding cost of the inventory system on the reliability of the item makes the study more realistic. The numerical examples were given and the sensitivity analysis was studied so as to examine the effect of changes on the optimal total inventory cost. (Sudip and Mahapatra, 2020).

Tripathi *et al*., (2017) conducted a research on Inventory Models for Stock-Dependent demand and Time varying holding cost under different trade credits where the second order approximations are used for exponential terms. Mathematical 9.0 software was used to obtain the optimal solutions and the model was illustrated by numerical examples and sensitivity analysis.

2.5 Inventory Models with Imperfect Quality

Mehmood and Wahab (2010) studied The Economic order quantity model for items with imperfect quality with learning in inspection which considered situations of lost sales and backorders. Their work considered the case where the buyer's inspection process undergoes learning while screening for defective items in a lot. Their results indicated the total transfer of learning remains better for both the lost sales and the backorders set-up. They also tested for the sensitivity analysis of the model and the result indicated that annual profit increases with the learning exponent in screening.

A study was carried out on Economic Order Quantity Model for Deteriorating items with Imperfect Quality where the inspection rate is assumed to be more than the demand.

The mathematical model was formulated as below:

$$\frac{dI(t)}{dt}+\theta I(t)=-D, \qquad 0\le t\le t_1 \qquad (2.13)$$

$$\frac{dI(t)}{dt}+\theta I(t)=-D, \qquad t_1\le t\le T \qquad (2.14)$$

With the following Notations and Assumptions:

a. A constant fraction $\theta(0\le\theta\le 1)$ of the on-hand inventory deteriorates per unit time.

b. The demand rate is known, constant, and continuous.

c. Shortages are not allowed.

d. The lead-time is known and constant.

e. The replenishment is instantaneous.

f. The screening process and demand proceeds simultaneously, but the screening rate (λ) is greater than demand rate (D), $\lambda > D$.

g. The defective items exist in lot size (Q). Also assume that percentage defective random variable (α) is uniformly distributed with its pdf as f(α), $E(\alpha) = \int_a^b \alpha f(\alpha) d\alpha$, $0 < a < b < 1$.

where;

i. c the purchasing cost per unit
ii. h the holding cost per order
iii. A the ordering cost per order
iv. p the selling price per unit
v. s the salvage value per defective unit, $s < c$
vi. β the screening cost per unit

It was assumed that the retailer was enabled to fulfill the demand, out of the products which are found to be of perfect quality, along with the screening process and to validate the model, a hypothetical example was considered. (Chandra and Mandeep, 2011).

2.6 Other Inventory Models

Zohreh *et al*., (2014) researched on an Economic Order Quantity Model with Completely Backordering and Nondecreasing Demand under Two-Level Trade Credit. They investigated on how both the seller and the buyer apply trade credit as a strategic tool to stimulate customer's demand. They maximized the profit of the retailer's model and used genetic algorithm to obtain the replenishment decisions optimally. They demonstrated with numerical examples the profitability of the developed two-level supply chain with backorder.

In the research work, Inventory Management Models and their effects on uncertain demand, the effects of demand uncertainty on inventory management and evaluation of the difference on uncertain demand subject to demand controls are determined and the models are used. The Economic Order Quantity, The Activity-Based Costing, and Just-in-time are being studied. The research was conducted through the use of quantitative research methods because of the descriptive nature of the research (Ndivhuwo and Charles, 2016).

Maragatham and Gnanvel (2017) considered A Purchasing Inventory Model for Breakable items with Permissible Delay in Payments and Price Discount when shortages are allowed and backlogged.

They formulated the below Mathematical model:

When $t_1 > t_r$,

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t), \qquad 0 \le t \le t_r \qquad (2.15)$$

With the condition $t = 0$ and $I(t) = 0$.

When $t_1 < t_r$,

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t), \qquad 0 \le t \le t_1 \qquad (2.16)$$

With the condition $t = 0$ and $I(t) = 0$.

When $t_1 = t_r$,

$$\frac{dI(t)}{dt} + \theta I(t) = -D(t), \qquad t_1 \le t \le t_2 \qquad (2.17)$$

With the condition $t = 0$ and $I(t) = 0$.

With the notations and assumptions:

a. A - The ordering cost per order.

b. P- The purchasing rate per unit time per year.

c. Q – The initial inventory level.

d. D(t) – The demand rate at any time $t \geq 0$, $D(t) = a + bt$, $a, b > 0$ where a, b are positive constants.

e. T- The length of replenishment cycle.

f. r- The price discount.

g. h- The holding cost per unit per unit time.

h. θ- The rate of damageable items.

i. I_θ - Interest which can be earned per year.

j. I_p- Interest payable per year.

k. C_1- The shortage cost for backlogged items per unit per year.

l. t_1- The time at which the inventory level reaches zero.

m. t_r- The replacement time and trade credit period.

n. m_0-The mark up of selling price for damaged items.

o. B(Q)-The number of damaged units per unit of time at time t and is a function of current inventory level Q.

p. TC - The minimum total cost per unit time.

i. The supplier offers price discount to his retailer at $t_1 < t_r$.

ii. The breakable items are replaced when end of the trade credit period.

iii. The time horizon is infinite.

iv. The lead time is zero.

v. Shortages are allowed and backlogged.

vi. Selling price for damaged items Sd is a multiple of purchasing cost.

vii. $S_d = m_0 * P \quad 0 \le m_0 \le 1$.

In this model, the total inventory cost was minimized and the optimal time interval was discussed. They considered replacement provision or price discounting by the supplier by the supplier for damageable items. Numerical examples were used to illustrate the results.

Hong *et al*., (2017) studied a case study of Inventory Management in a Manufacturing Company in China. They used a case study to identify major factors that affect inventory management practices, considered efficient and effective inventory management approaches, and observed the influence of supplier cooperation on supply chain improvement. The inventory management in the improvement of supply chain management was optimized.

Makoena and Olufemi (2019) investigated on Economy order quantity model for growing items with incremental discounts. In their model, they developed a lot sizing model for growing items if the supplier of the items offers incremental quantity discounts. They derived a mathematical model to determine the optimal inventory policy so as to minimize the total inventory and solved the model which was illustrated by numerical examples. They observed that increment in quantity discounts resulted in reduced purchasing costs and that ordering very large quantities has downsides as well.

They concluded that procurement and inventory managers when making purchasing decisions can use the model owing to the importance of growing items in the food supply chains.

Following Lakshmana (2015), in an EOQ Model for Deteriorating Items with Selling Price Dependent Demand and Time-Varying Holding Cost under Partial Backlogging, we observed the following limitations:

1. He did not consider delayed deterioration
2. He only considered selling price dependent demand without considering stock and reliability dependent demand.

In order to fill the gap of Lakshmana (2015), we considered the following

1. Delayed deterioration
2. Selling price dependent demand with stock and reliability dependent demand.

These distinguishes our research from Lakshmana. (2015).

CHAPTER THREE

3.0 MATERIALS AND METHODS

3.1 Model Definition

In this chapter, an EOQ model for deteriorating inventory that exhibits delay in deterioration in which the demand depends on price, stock and reliability wherein shortages are allowed and partially backlogged was formulated.

3.2 Assumptions and Notation

The assumptions and Notation to the Economic order quantity for items that exhibit delay in deterioration with price, stock and reliability demand under partial backlogging is being given below:

Notation

Q The order quantity in one cycle

T The replenishment cycle time

C The unit cost of the item

B The ordering cost per order

h_1 The inventory holding cost per unit item

h_2 The shortage cost per unit per time

h_3 The unit cost of lost sales

θ The rate of deterioration

s The purchasing cost per unit

p Selling price per unit, where $p > s$

T_1 The length of time in which the product exhibits no deterioration

T_2 The Length of time in which the stock level vanishes

$N_1(t)$ The inventory level at time $t\varepsilon[0, T_1]$

$N_2(t)$ The inventory level at time $t\varepsilon[T_1, T_2]$, where $T_2 > T_1$

$N_3(t)$ The inventory level at time $t\varepsilon[T_2, T]$

N_0 The maximum inventory level

S The maximum amount of demand backlogged

β backlogging rate, $0 \le \beta \le 1$

$D(N(t);r,p)$ demand rate where $N(t)$ is inventory level at time t, r, is the reliability, p is price of the stock

Assumptions:

a) Replenishment rate is infinite
b) Lead time is zero
c) The deterioration rate is constant on the on-hand inventory per unit time and there is no repair of the deterioration item within the cycle.
d) Demand rate is $D(N(t);r,p) = k(p)\left[xpr^6 + \alpha N(t)\right]$ where $k(p) = \gamma e^{\left(\frac{p\delta}{r}\right)}$ is the price factor where $x, y, \delta > 0$ are the parameter. α is the stock dependent consumption rate parameter $0 \le \alpha \le 1$; and r is the reliability. (Sudip and Mahapatra, 2018).
e) Shortages are allowed and partially backlogged

3.3 Model Formulation

The differential equation of the proposed inventory system can be written in the form of mathematical model under consideration based on the above assumptions is as follows:

$$\frac{dN_1(t)}{dt} = -k(p)\left[xpr^y + \alpha N_1(t)\right] \qquad 0 \le t \le T_1 \qquad (3.1)$$

$$\frac{dN_2(t)}{dt}+\theta N_2(t)=-k(p)\left[xpr^y+\alpha N_2(t)\right] \qquad T_1 \le t \le T_2$$

$$(3.2)\frac{dN_3(t)}{dt}=-k(p)xpr^y \qquad T_2 \le t \le T$$

(3.3)

Where:

Equation (3.1) depends only on demand and the differential equation that describes the situation before deterioration sets in, equation (3.2) describe the situation in which after deterioration sets in, depletion of inventory will depend on both demand and deterioration and equation (3.3) describes the situation in which the inventory level is on zero when shortages has set in.

3.4 Solution to the Formulated Model

Solving (3.1) using integrating factor, we have

$$\frac{dN_1(t)}{dt}+\alpha k(p)N_1(t)=-k(p)xpr^y \tag{3.4}$$

$$N_1(t).e^{\alpha k(p)t}=-k(p)xpr^y.\int e^{\alpha k(p)t}dt+E \tag{3.5}$$

$$N_1(t)=Ee^{-\alpha k(p)t}-\frac{k(p)xpr^y}{\alpha k(p)} \tag{3.6}$$

Applying the boundary condition ($N_1(t)=N_0$ at $t=0$) in (3.6)

$$E=N_0-\frac{xpr^y}{\alpha} \tag{3.7}$$

Substituting (3.7) into (3.6) we have

$$N_1(t) = \left(N_0 + \frac{xpr^y}{\alpha} \right) e^{-\alpha k(p)t} - \frac{xpr^y}{\alpha} \tag{3.8}$$

Re-writing (3.2), we have,

$$\frac{dN_2(t)}{dt} + (\theta + \alpha k(p)) N_2(t) = -k(p) xpr^y \tag{3.9}$$

Solving (3.9) using integrating factor, we have

$$\frac{dN_2(t)}{dt} + \sigma N_2(t) = -k(p) xpr^y \tag{3.10}$$

Where $\sigma = \theta + \alpha k(p)$

$$I.Fe^{\int \sigma dt} = e^{\sigma t} \tag{3.11}$$

$$N_2(t) e^{\sigma t} = -k(p) xpr^y \int e^{\sigma t} dt + F \tag{3.12}$$

$$N_2(t) e^{\sigma t} = -\frac{k(p) xpr^y}{\sigma} . e^{\sigma t} + F \tag{3.13}$$

$$N_2(t) = Fe^{-\sigma t} - \frac{k(p) xpr^y}{\sigma} \qquad \text{where F is a constant} \tag{3.14}$$

Applying the boundary condition ($N_2(t) = 0$ at $t = T_2$) in (3.14)

$$0 = Fe^{-\sigma T_2} - \frac{k(p) xpr^y}{\sigma} \tag{3.15}$$

$$F = \frac{k(p) xpr^y}{\sigma} . e^{\sigma T_2} \tag{3.16}$$

Substituting (3.16) in (3.14) we have,

$$N_2(t)=\left(\frac{k(p)xpr^y}{\sigma}.e^{\sigma T_2}\right)e^{-\sigma t}-\frac{k(p)xpr^y}{\sigma}$$

(3.17)

$$N_2(t)=\frac{k(p)xpr^y}{\sigma}.e^{\sigma(T_2-t)}-\frac{k(p)xpr^y}{\sigma} \tag{3.18}$$

$$N_2(t)=\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-t)}-1\right) \tag{3.19}$$

Now, considering continuity of $N_1(t)$ and $N_2(t)$ at point $t=T_1$, that is, $N_1(t)=N_2(t)$, the maximum inventory level for each cycle can be obtained as follows.

Therefore, from (3.8) and (3.19), we have that

$$\left(N_0+\frac{xpr^y}{\alpha}\right)e^{-\alpha k(p)T_1}-\frac{xpr^y}{\alpha}=\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right) \tag{3.20}$$

$$N_0e^{-\alpha k(p)T_1}=\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)+\frac{xpr^y}{\alpha}-\frac{xpr^y}{\alpha}e^{-\alpha k(p)T_1} \tag{3.21}$$

$$N_0=\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}\right)-\frac{xpr^y}{\alpha}e^{-\alpha k(p)T_1}\left(e^{\alpha k(p)T_1}\right) \tag{3.22}$$

$$N_0=\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}\right)-\frac{xpr^y}{\alpha} \tag{3.23}$$

$$N_0=\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right) \tag{3.24}$$

Now, substituting the value of N_0 in (3.24) into (3.8) we have,

$$N_1(t)=\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)e^{-\alpha k(p)t}-\frac{xpr^y}{\alpha}$$
$$0\le t\le T_1 \qquad (3.25)$$

During the shortage time interval $[T_2, T]$ the demand at time t is partially backlogged and the model equation is given in (3.3).

Therefore, integrating (3.3), we have

$$\int dN_3(t)=-\int k(p)xpr^y dt \qquad (3.26)$$

$$N_3(t)=-k(p)xpr^y t+G \qquad (3.27)$$

Applying the boundary condition $N_3(T_2)=0$ we have,

$$0=-k(p)xpr^y t+G \qquad (3.28)$$

$$G=k(p)xpr^y T_2 \qquad (3.29)$$

$$N_3(t)=-k(p)xpr^y t+k(p)xpr^y T_2 \qquad (3.30)$$

$$N_3(t)=-k(p)xpr^y(t-T_2) \qquad (3.31)$$

Putting $t=T$ in (3.31) we have the maximum amount of demand backlogged per cycle to be

$$S=-N_3(t)=k(p)xpr^y(T-T_2) \qquad (3.32)$$

We obtain the order quantity by adding N_0+S i.e., by adding (3.24) and (3.32)

Therefore,

$$Q = N_0 + S = \frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+k(p)xpr^y\left(T-T_2\right) \quad (3.33)$$

The cost of holding inventory in stock is computed for until it is sold or used, which is inventory carrying cost H.C, and is given by:

$$HC = h\int_0^{T_1} N_1(t)\,\mathrm{dt} + h\int_T^{T_2} N_2(t)dt \quad (3.34)$$

By substituting (3.19), (3.25) into (3.34) we have,

$$HC = h\int_0^{T_1}\left(\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)e^{-\alpha k(p)\mathrm{t}}-\frac{xpr^y}{\alpha}\right)dt + h\int_{T_1}^{T_2}\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-t)}-1\right)\right)dt \quad (3.35)$$

$$HC = h\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\int_0^{T_1} e^{-\alpha k(p)\mathrm{t}}dt -$$
$$h\frac{xpr^y}{\alpha}\int_0^{T_1} 1dt + h\frac{k(p)xpr^y}{\sigma}\int_{T_1}^{T_2} e^{\sigma(T_2-t)}dt - h\frac{k(p)xpr^y}{\sigma}\int_{T_1}^{T_2} 1dt$$

(3.36)

$$HC = h\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left[-\frac{1}{\alpha k(p)}e^{-k(p)t}\right]_0^{T_1}$$
$$-h\frac{xpr^y}{\alpha}[t]_0^{T_1} + h\frac{k(p)xpr^y}{\sigma}e^{\sigma T_2}\int_{T_1}^{T_2} e^{-\sigma t}dt - h\frac{k(p)xpr^y}{\sigma}\int_{T_1}^{T_2} dt \quad (3.37)$$

$$HC = h\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left(-\frac{1}{\alpha k(p)}e^{-k(p)T_1}+\frac{1}{\alpha k(p)}\right)$$
$$-h\frac{xpr^y}{\alpha}T_1 + h\frac{k(p)xpr^y}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)} - h\frac{k(p)xpr^y}{\sigma}(T_2-T_1) \quad (3.38)$$

$$HC = \frac{h}{\alpha k(p)}\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left(1-e^{-k(p)T_1}\right)-$$
$$h\frac{xpr^y}{\alpha}T_1 + h\frac{k(p)xpr^y}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)} - h\frac{k(p)xpr^y}{\sigma}(T_2-T_1) \quad (3.39)$$

Shortage due to stock out is accumulated in the system during the interval [T_2, T]. The optimum level of shortage is present at $t = T$; therefore, the total shortage cost during this time period is as follows:

$$SC = h_2 \int_{T_2}^{T} \left(-N_3(t)\right) dt \tag{3.40}$$

$$SC = h_2 \int_{T_2}^{T} \left(k(p) xpr^y (T - T_2)\right) dt \tag{3.41}$$

$$SC = h_2 \left(k(p) xpr^y (T - T_2)\right) [t]_{T_2}^{T} \tag{3.42}$$

$$SC = h_2 \left(k(p) xpr^y (T - T_2)\right)(T - T_2) \tag{3.43}$$

$$SC = h_2 k(p) xpr^y (T - T_2)^2 \tag{3.44}$$

Due to stock out during (T_2, T), shortage is accumulated, but not all customers would be willing to wait for the next lot size to come.

Therefore, this results in some loss of sale which accounts to loss in profit.

Hence, lost sale cost is calculated as follows:

$$LSC = h_3 \int_{T_2}^{T} (1 - \beta) k(p) xpr^y dt \tag{3.45}$$

$$LSC = h_3 (1 - \beta) k(p) xpr^y [t]_{T_2}^{T} \tag{3.46}$$

$$LSC = h_3 (1 - \beta) k(p) xpr^y (T - T_2) \tag{3.47}$$

The purchase cost denoted by PC is as follows

$$PC = sQ$$

$$= s\left(\frac{k(p)xpr^{y}}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^{y}}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+k(p)xpr^{y}\left(T-T_2\right)\right) \quad (3.48)$$

The total cost is the sum of ordering cost, purchase cost, inventory holding cost, shortage cost and lost sales.

$$K\left(T,T_2,s\right)=OC+PC+HC+SC+LSC \quad (3.49)$$

$$\begin{aligned}
&= B+s\left(\frac{k(p)xpr^{y}}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^{y}}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+k(p)xpr^{y}\left(T-T_2\right)\right)+\\
&\frac{h}{\alpha k(p)}\left(\frac{k(p)xpr^{y}}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^{y}}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^{y}}{\alpha}\right)\left(1-e^{-k(p)T_1}\right)- \quad (3.50)\\
&h\frac{xpr^{y}}{\alpha}T_1+h\frac{k(p)xpr^{y}}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)}-h\frac{k(p)xpr^{y}}{\sigma}\left(T_2-T_1\right)\\
&+h_2k(p)xpr^{y}\left(T-T_2\right)^2+h_3\left(1-\beta\right)k(p)xpr^{y}\left(T-T_2\right)
\end{aligned}$$

The sales Revenue (denoted by SR) is given as

$$SR = Selling\ price * Total\ demand\ over\ the\ cycle$$

$$SR = p\left(\int_0^{T_2}\left(k(p)xpr^{y}+\alpha N_0\right)dt-N_3\left(t\right)\right)$$

$$SR=p\left(\int_0^{T_2}\left(k(p)xpr^{y}+\alpha\left(\frac{k(p)xpr^{y}}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^{y}}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)\right)\right)dt+k(p)xpr^{y}\left(t-T_2\right)\right)$$

(3.51)

$$SR=p\left(\left(k(p)xpr^{y}+\alpha\left(\frac{k(p)xpr^{y}}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^{y}}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2+k(p)xpr^{y}\left(T-T_2\right)\right)$$

(3.52)

$$SR=p\left(\left(k(p)xpr^{y}+\frac{\alpha}{\sigma}\left(k(p)xpr^{y}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)\right)+\left(xpr^{y}\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2+k(p)xpr^{y}\left(T-T_2\right)\right)$$

(3.53)

Let P (T, T_2, s) be the profit rate function, since the profit rate function is the total sales revenue per unit minus Total cost per unit.

$$P(T,T_2,s)=SR-K(T,T_2,s) \tag{3.54}$$

$$P(T,T_2,s)=p\left(\left(k(p)xpr^y+\frac{\alpha}{\sigma}\left(k(p)xpr^y\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)\right)+\left(xpr^y\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2+k(p)xpr^y(T-T_2)\right)-$$
$$\left(\begin{array}{l} B+s\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+k(p)xpr^y(T-T_2)\right)+\\ \frac{h}{\alpha k(p)}\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left(1-e^{-k(p)T_1}\right)\\ -h\frac{xpr^y}{\alpha}T_1+h\frac{k(p)xpr^y}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)}-h\frac{k(p)xpr^y}{\sigma}(T_2-T_1)\\ +h_2k(p)xpr^y(T-T_2)^2+h_3(1-\beta)k(p)xpr^y(T-T_2) \end{array}\right)$$

(3.55)

Total profit per unit time is

$$TP(T_2,T,s)=\frac{1}{T}\left(\begin{array}{l} p\left(\left(k(p)xpr^y+\frac{\alpha}{\sigma}\left(k(p)xpr^y\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)\right)+\left(xpr^y\left(e^{\alpha k(p)T_1}-1\right)\right)\right)T_2+k(p)xpr^y(T-T_2)\right)-\\ \left(\begin{array}{l} B+s\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+k(p)xpr^y(T-T_2)\right)+\\ \frac{h}{\alpha k(p)}\left(\frac{k(p)xpr^y}{\sigma}\left(e^{\sigma(T_2-T_1)}-1\right)\left(e^{\alpha k(p)T_1}\right)+\frac{xpr^y}{\alpha}\left(e^{\alpha k(p)T_1}-1\right)+\frac{xpr^y}{\alpha}\right)\left(1-e^{-k(p)T_1}\right)\\ -h\frac{xpr^y}{\alpha}T_1+h\frac{k(p)xpr^y}{\sigma^2}e^{\sigma T_2}e^{-\sigma(T_2-T_1)}-h\frac{k(p)xpr^y}{\sigma}(T_2-T_1)\\ +h_2k(p)xpr^y(T-T_2)^2+h_3(1-\beta)k(p)xpr^y(T-T_2) \end{array}\right) \end{array}\right)$$

(3.56)

Differentiating (3.56) with respect to T_2, T, and s, we have

$$\frac{\partial TP(T_2,T,s)}{\partial T_2},\ \frac{\partial TP(T_2,T,s)}{\partial T},\ \frac{\partial TP(T_2,T,s)}{\partial p}$$

Representation of Variable Terms

$A = K(p)xpr^{y}$, $\quad A_1 = \dfrac{\alpha}{\sigma}$, $\quad A_2 = xpr^{y}$,

$A_3 = B + s$, $\quad A_4 = \dfrac{K(p)xpr^{y}}{\sigma}$,

$A_5 = \dfrac{xpr^{y}}{\alpha}$, $\quad A_6 = \dfrac{h}{\alpha k(p)}$,

$A_7 = \dfrac{h}{\alpha} xpr^{y}$, $\quad A_8 = \dfrac{hK(p)xpr^{y}}{\sigma^2}$

$A_9 = \dfrac{hK(p)xpr^{y}}{\sigma}$, $\quad A_{10} = h_2 K(p)xpr^{y}$ and

$A_{11} = h_3(1-\beta)K(p)xpr^{y}$.

$$\frac{\partial TP(\mathrm{T}_2,\mathrm{T},\mathrm{s})}{\partial T_2} = \frac{1}{T}\left(\begin{array}{l} P\Big(A + A_1\left(e^{\sigma(T_2-T_1)}\right) + A_2\left(e^{\alpha k(p)T_1} - 1\right)\Big) + P\Big(0 + A_1\left(A\left(\sigma e^{\sigma(T_2-T_1)}\right)\right) + A_2(0)\Big)T_2 \\ +(-A) - A_3\left(A_4\left(\sigma e^{\sigma(T_2-T_1)}\right)e^{\alpha k(p)T_1}\right) + A_5(0) - A + A_6\left(A_4\left(\sigma e^{\sigma(T_2-T_1)}\right)\left(e^{\alpha k(p)T_1}\right) + A_5(0) + 0\right)\left(1 - e^{-\alpha k(p)T_1}\right) \\ -A_7(0) + A_8(0) - A_9 + A_{10}(-2T + 2T_2) + A_{11}(-1) \end{array}\right) \tag{3.57}$$

That is, (3.57) becomes

$$\frac{\partial TP(\mathrm{T}_2,\mathrm{T},\mathrm{s})}{\partial T_2} = \frac{1}{T}\left(\begin{array}{l} P\Big(A + A_1\left(\left(e^{\sigma(T_2-T_1)}\right)\right) + \left(A_2\left(e^{\alpha k(p)T_1} - 1\right)\right)\Big) + P\Big(A_1\left(A\left(\sigma e^{\sigma(T_2-T_1)}\right)\right)\Big)T_2 \\ -A - A_3\left(A_4\left(\sigma e^{\sigma(T_2-T_1)}\right)e^{\alpha k(p)T_1}\right) - A + A_6\left(A_4\left(\sigma e^{\sigma(T_2-T_1)}\right)\left(e^{\alpha k(p)T_1}\right)\right)\left(1 - e^{-k(p)T_1}\right) \\ -A_9 + A_{10}(2T_2 - 2T) - A_{11} \end{array}\right) \tag{3.58}$$

$$\frac{\partial TP(\mathrm{T}_2,\mathrm{T},\mathrm{s})}{\partial T} = -\frac{1}{T^2}\begin{pmatrix} P\Big(A + A_1\Big(A\Big(e^{\sigma(T_2-T_1)}-1\Big)\Big)+\Big(A_2\Big(e^{\alpha k(p)T_1}-1\Big)\Big)\Big)T_2 + A(T-T_2) \\ -A_3\Big(A_4\Big(e^{\sigma(T_2-T_1)}-1\Big)\Big(e^{\alpha k(p)T_1}\Big)+A_5\Big(e^{\alpha k(p)T_1}-1\Big)+A(T-T_2)\Big) \\ +A_6\Big(A_4\Big(e^{\sigma(T_2-T_1)}-1\Big)\Big(e^{\alpha k(p)T_1}\Big)+A_5\Big(e^{\alpha k(p)T_1}-1\Big)+A_5\Big)\Big(1-e^{-\alpha k(p)T_1}\Big) \\ -A_7T_1 + A_8e^{\sigma T_1} - A_9(\mathrm{T}_2-\mathrm{T}_1) + \mathrm{A}_{10}(T-T_2)^2 + A_{11}(T-T_2) \end{pmatrix}$$

$$+\frac{1}{T}\big(P(0) + A - A_3(A) + 0 + 0 + 2A_{10}(T-T_2) + A_{11}\big) \tag{3.59}$$

This Implies that;

$$\frac{\partial TP(\mathrm{T}_2,\mathrm{T},\mathrm{s})}{\partial T} = -\frac{1}{T^2}\begin{pmatrix} P\Big(A + A_1\Big(A\Big(e^{\sigma(T_2-T_1)}-1\Big)\Big)+\Big(A_2\Big(e^{\alpha k(p)T_1}-1\Big)\Big)\Big)T_2 + A(T-T_2) \\ -A_3\Big(A_4\Big(e^{\sigma(T_2-T_1)}-1\Big)\Big(e^{\alpha k(p)T_1}\Big)+A_5\Big(e^{\alpha k(p)T_1}-1\Big)+A(T-T_2)\Big) \\ +A_6\Big(A_4\Big(e^{\sigma(T_2-T_1)}-1\Big)\Big(e^{\alpha k(p)T_1}\Big)+A_5\Big(e^{\alpha k(p)T_1}-1\Big)+A_5\Big)\Big(1-e^{-\alpha k(p)T_1}\Big) \\ -A_7T_1 + A_8e^{\sigma T_1} - A_9(\mathrm{T}_2-\mathrm{T}_1) + \mathrm{A}_{10}(T-T_2)^2 + A_{11}(T-T_2) \end{pmatrix}$$
$$+\frac{1}{T}\big(A - A_3A + 2A_{10}(T-T_2) + A_{11}\big) \tag{3.60}$$

and similarly,

$$\frac{\partial TP(\mathrm{T}_2,\mathrm{T},\mathrm{s})}{\partial p} = \frac{1}{T}\Big(T_2\Big(A + A_1\Big(A\Big(e^{\sigma(T_2-T_1)}-1\Big)\Big)+\Big(A_2\Big(e^{\alpha k(p)T_1}-1\Big)\Big)\Big)\Big) \tag{3.61}$$

To maximize the profit function $P(\mathrm{T}_2,\mathrm{T},\mathrm{s})$ per unit time, the optimal values of T_2, T and p was obtained by solving the following equations:

$$\frac{\partial TP(\mathrm{T}_2,\mathrm{T},\mathrm{s})}{\partial T_2} = 0, \qquad \frac{\partial TP(\mathrm{T}_2,\mathrm{T},\mathrm{s})}{\partial T} = 0, \qquad \frac{\partial TP(\mathrm{T}_2,\mathrm{T},\mathrm{s})}{\partial p} = 0, \tag{3.62}$$

The condition for maximization of $P(\mathrm{T}_2, \mathrm{T}, \mathrm{s})$ is

$$M = \begin{bmatrix} \frac{\partial^2 TP(\mathrm{T}_2,\mathrm{T},\mathrm{p})}{\partial T_2^{\,2}} & \frac{\partial^2 TP(\mathrm{T}_2,\mathrm{T},\mathrm{p})}{\partial T_2, \partial T} & \frac{\partial^2 TP(\mathrm{T}_2,\mathrm{T},\mathrm{p})}{\partial T_2, \partial s} \\ \frac{\partial^2 TP(\mathrm{T}_2,\mathrm{T},\mathrm{p})}{\partial T_2, \partial T} & \frac{\partial^2 TP(\mathrm{T}_2,\mathrm{T},\mathrm{p})}{\partial T^2} & \frac{\partial^2 TP(\mathrm{T}_2,\mathrm{T},\mathrm{p})}{\partial T, \partial s} \\ \frac{\partial^2 TP(\mathrm{T}_2,\mathrm{T},\mathrm{p})}{\partial T_2, \partial s} & \frac{\partial^2 TP(\mathrm{T}_2,\mathrm{T},\mathrm{p})}{\partial T, \partial s} & \frac{\partial^2 TP(\mathrm{T}_2,\mathrm{T},\mathrm{p})}{\partial s^2} \end{bmatrix} < 0 \quad (3.63)$$

The value of T_2^*, T^* *and* s^* was used to the optimal value, equation (3.56) provides the maximum profit function per unit time PT^* of the inventory system.

The model was solved and the analysis of the Result is presented in chapter four.

CHAPTER FOUR

4.0 RESULTS AND DISCUSSION

4.1 Analysis of Results

In this chapter, we presented the data used for 'Economic Order Quantity for Items that Exhibit Delay in Deterioration with Price, Stock and Reliability Demand Under Partial Backlogging'. Computational results are performed using Excel office 2016 and Maple2015, Intel(R) Core (TM) i5-3340M CPU @ 2.70GHz 8.00 GB memory.

4.2 Numerical Examples

To exhibit delayed deterioration with constant deterioration and Price, Stock and Reliability demand consideration in an EOQ model of items, to investigate the effect of price, stock and reliability on demand of items that exhibit delayed deterioration and to maximize the profit function in order to determine the best inventory management policy, numerical examples were given for illustrations.

In a Large store the demand rate does not only depends upon the amount of the stock but also depends upon the reliability as well as the price of the item so that demand rate is D(N(t); r, p) where γ = 100, δ = 1.4, x= 100, p = 6, r = 1, y = 3, α = 0.15 β = 0.6. Let us consider that the item deteriorates at constant rate 0.1 part of the total inventory. Let the shortages cost be N2per unit item and N250 to order the total inventory. Let the cost of each item be N3, selling price is N20; to hold the item it requires N0.6 per unit and the reliability of the item is 1. Now we have to maximize the profit per unit item per unit time for the above situations of inventory system. We consider the following information as input parameters for the proposed inventory model, we have p = 20, θ = 0.1, B = 250 per order, h= 0.6unit, s= 3 per item, per year, T =1, T_1 = 0.5, T_2 = 1.2.

4.3 Discussion of Results

Table 4.1 shows the effect of stock dependent consumption rate parameter against the demand.

Table 4.1: Stock Dependent Consumption Parameter Rate Against Demand Rate

∝	R	P	K(p)	D
0.15	1	20	74.59123	96125655
0.2	1	20	74.59123	3.77E+09
0.25	1	20	74.59123	6.67E+10s
0.3	1	20	74.59123	5.75E+12
0.35	1	20	74.59123	2.25E+14
0.4	1	20	74.59123	8.77E+15
0.45	1	20	74.59123	3.43E+17
0.5	1	20	74.59123	1.34E+19
0.55	1	20	74.59123	5.22E+20
0.6	1	20	74.59123	2.04E+22

∝= stock dependent consumption rate parameter (0.15 – 0.6), r = is the reliability (1), p = selling price (20), k(p) =The price factor (74,59123), D= demand rate (96125655 – 2.04E+22).

Table 4.1 shows that as the stock dependent consumption rate parameter increases, the price factor k(p) is constant which increases the demand for goods this is as a result of the attraction brought by products display on shelf, its popularity and variety to the customers, because when there is low stock in the shop, goods are most times treated as though they are not fresh even though on the other hand a customer can think that a large amount of stock means the item is of less demand because the other customers are not buying but when the stock is well optimized, the demand keep increasing and thereby increasing the total profit.

Table 4.2 shows the effect of price factor on the demand.

Table 4.2: Table of Price Factor Against Demand Rate

∝	R	P	K(p)	D

0.15	1	20	61.07014	96125655
0.15	1	20.5	74.59123	2.69E+09
0.15	1	21	91.10594	9.63E+10
0.15	1	21.5	111.277	5.93E+12
0.15	1	22	135.9141	7.64E+14
0.15	1	22.5	166.0058	2.52E+17
0.15	1	23	202.76	2.69E+20
0.15	1	23.5	247.6516	1.21E+24
0.15	1	24	302.4824	3.22E+28
0.15	1	24.5	369.4528	7.52E+33

$\propto$ = stock dependent consumption rate parameter (0.15), r = is the reliability (1), p = selling price (20 – 24.5), k(p) =The price factor (61.07014 – 369.4528), D= demand rate (96125655 – 7.52E+33).

Table 4.2 shows that as the selling price increases, the price factor k(p) increases, of which the demand for goods also increases. The price of a product in the market has both negative and positive aspects. Some customers can buy a product because it is cheap and has a low price while some other customers believes that the product with the higher price is of more quality than the one with a lower price therefore, increasing the demand for the product with the higher price which also increases the total profit.

Table 4.3 shows the effect of reliability of goods on the demand.

Table 4.3: Reliability Against Demand Rate

∝	R	P	K(p)	D
0.15	1	20	61.07014	96125655
0.15	1.2	20	69.78062	96304691
0.15	1.4	20	76.75315	96509914
0.15	1.6	20	82.43606	1.03E+08
0.15	1.8	20	87.14545	1.12E+08
0.15	2	20	91.10594	1.22E+08
0.15	2.2	20	94.47986	1.34E+08
0.15	2.4	20	97.3867	1.46E+08
0.15	2.6	20	99.91609	1.6E+08
0.15	2.8	20	102.1364	1.75E+08

∝ = stock dependent consumption rate parameter (0.15), r = is the reliability (1- 2.8), p = selling price (20), k(p) =The price factor (61.07014 – 102.1364), D= demand rate (96125655 – 7.52E+08).

Table 4.3 shows that as the reliability of goods increases, even though the price factor k(p) increases, but the demand for goods also increases. As it has been discussed above, even though some customers so far, the product is cheap, they do not really care if the product is of good quality or not, they go for it. But we still have more demand from a product that is more reliable even if the price factor of the demand increases. This shows that most customers prefer the product to be reliable and expensive than to be cheap and not reliable. Thus, the reliability of a particular product increases the demand for the product which in returns, increase the total profit.

The analysis of the table 4.4 is given from the equation (3.56). The table 4.4 shows the effect of deterioration of goods against the Total Profit in thousands of Naira

Table 4.4: Deterioration Rate Against the Total Profit per Time

θ	β	T_1	T_2	T	TP
0.1	0.6	0.5	1.2	3	131
0.15	0.6	0.5	1.2	3	126
0.2	0.6	0.5	1.2	3	122
0.25	0.6	0.5	1.2	3	118
0.3	0.6	0.5	1.2	3	114
0.35	0.6	0.5	1.2	3	111
0.4	0.6	0.5	1.2	3	107
0.45	0.6	0.5	1.2	3	104
0.5	0.6	0.5	1.2	3	100
0.55	0.6	0.5	1.2	3	097

θ = Deterioration rate (0.1- 0.55), β=backordering rate (0.6), T_1 =The length of time in which the product exhibits no deterioration (0.5), T_2 =The Length of time in which the stock level vanishes (1.2), T= The replenishment cycle time (3), and TP = the total profit per unit time.

Table 4.4 shows that as the deterioration of a product increases at a given time, the Total profit of the product decreases as a result of the reduction in the quantity and quality of the product that can be sold at a given time. As a product deteriorates, the willingness for a customer to pay for the product start dropping once the product is getting to its expiration date, or when the product is no more in good shape due to spoilage, or when the product decreases in its usefulness or in its obsolescence state. During this period, if necessary, actions are not taken for example, discounting the price or quantity of the deteriorated product, the demand rate would drastically be reduced and therefore, causing a decrease in the total profit due to deteriorated product. On the other hand, too much of price discounts or quantity discounts on a product to generate enough sales can bring a decrease to the total profit.

The analysis of the table 4.5 is given from the equation (3.56). Table 4.5 shows the effect of replenishment cycle time against the Total Profit in thousands of Naira

Table 4.5 Replenishment Cycle Time Against the Total Profit per Time

θ	β	T_1	T_2	T	TP
0.1	0.6	0	1.2	3	264
0.1	0.6	0	1.2	3.2	247
0.1	0.6	0	1.2	3.4	232
0.1	0.6	0	1.2	3.6	219
0.1	0.6	0	1.2	3.8	207
0.1	0.6	0	1.2	4.0	196
0.1	0.6	0	1.2	4.2	186
0.1	0.6	0	1.2	4.4	177
0.1	0.6	0	1.2	4.6	169
0.1	0.6	0	1.2	4.8	161

θ = Deterioration rate (0.1), β=backordering rate (0.6), T_1 =The length of time in which the product exhibits no deterioration (0), T_2 =The Length of time in which the stock level vanishes (1.2), T= The replenishment cycle time (3 – 4.8), and TP = the total profit per unit time.

Table 4.5 shows that as the replenishment cycle time increases, the Total profit of the goods decreases especially when instantaneous deterioration of the product is being considered. As a result of this, the quality of the product is worsened when the replenishment cycle time is increased. When the replenishment cycle time of a product is being increased, it gives room for a product to deteriorate more in such that sometimes, the product might no longer even meet the changing demand of a customer when the product stays long in the store, this may result in decrease in demand and therefore cause a decrease in the total profit.

The analysis of table 4.6 is given from the equation (3.56). Table 4.6 shows the effect of backlogging rate on the Total Profit in thousands of Naira

Table 4.6: Backlogging Rate Against the Total Profit per Time

θ	β	T_1	T_2	T	TP
0.1	0.6	0.5	1.2	3	14867
0.1	0.7	0.5	1.2	3.2	14873
0.1	0.8	0.5	1.2	3.4	14880
0.1	0.9	0.5	1.2	3.6	14886
0.1	1.0	0.5	1.2	3.8	14892
0.1	1.1	0.5	1.2	4.0	14899
0.1	1.2	0.5	1.2	4.2	14906
0.1	1.3	0.5	1.2	4.4	14912
0.1	1.4	0.5	1.2	4.6	14919
0.1	1.5	0.5	1.2	4.8	14925

θ = Deterioration rate (0.1), β=backordering rate (0.6 – 1.5), T_1 =The length of time in which the product exhibits no deterioration (0.5), T_2 =The Length of time in which the stock level vanishes (1.2), T= The replenishment cycle time (3 – 4.8), and TP = the total profit per unit time.

Table 4.6 shows that as the backordering rate of a product increases, the Total profit of the product increases also, this is because backordering allows customers to continue purchasing items that are not readily available and so one can keep accumulating sales even when products are not physically available for delivery, this reduces unnecessary inventory cost such as holding cost or cost due to the deterioration of the product, this process therefore enhances the total profit of an organization.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In conclusion, the model equations using Economic Order Quantity for Item that Exhibits Delay in Deterioration in relation to Price, Stock and Reliability Demand Under Partial Backlogging are solved and the solutions exhibit delayed deterioration. The Price, Stock and Reliability of the item vary proportionately with the demand.

As a result, the research work shows that when there is increase in stock and improvement on the reliability of a product, the demand for the product would still increase notwithstanding the increase in price of the product.

This would bring a positive effect on the total profit thereby indicating the best inventory management policy to be adopted for maximum profit.

5.2 Recommendations

The following are our recommendations:

1. Further studies should be carried out on Economic Order Quantity for Items that Exhibit Delay in Deterioration as a function of Price, Stock and Reliability Demand comparing when shortages are allowed and when shortages are not allowed.
2. Also, a research on an Economic production Quantity for items that exhibit delay in deterioration in relation to price, stock, and imperfect quality demand should be carried out.

5.3 Contribution to the Knowledge

The following are our contributions to knowledge:

1. The Study helps an organization in handling product deterioration *vis-à-vis* profit maximization
2. It provides information to organizations on how to prioritize production or ordering of products that are reliable over cheap and unreliable products in order to ensure customer satisfaction as obtained in the analysis of the solution which revealed that when reliability, $r = 1$, demand rate, $D = 96125655$ and when reliability, $r = 2.8$, demand rate, $D = 1.75E + 08$.

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