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Optimal Analysis of Contaminant Invasion in an Unconfined Aquifer System

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ABSTRACT

The analysis of contaminant invasion in an aquifer system, showing the behavior of contaminants for different values of α, b and γ for $0 < t \leq 7$ was formulated. For a uniform source of contamination at $C^u(x) = 1$, it was observed that for different values of $\propto, b$ and γ the level of contamination reduces over the domain.

Keywords: *Contaminant invasion, unconfined aquifer, solute transport, sink fluid, piezometric head*

1. INTRODUCTION

Groundwater offers the most abundant source of water to man. It is the cheapest and the most constant in quality and quantity [1]. It is observed that in many developing countries, groundwater plays a major source of support for domestic needs and irrigation purposes [2]. Water shortages occur quite often in many areas of the world, calling for optimal management of both surface and groundwater resources[3], [4]. Groundwater quality is usually better, since they are naturally more protected. Once polluted, their restoration is more difficult, calling for optimal control of groundwater contamination [5], [6]. The analysis of contaminant invasion into an aquifer system, showing the behavior of contaminants over a time period is of paramount importance in the study of geological behavior of the aquifer system. In [7], a fractured confined porous aquifer is considered. They also came up with a modeled solute equation, that analysed the effect of non-Fickian diffusion into surrounding rocks.

2. MATERIALS AND METHODS

We considered the governing equation for solute transport as given by [7] thus:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x_i}(CV_i) + \frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial C}{\partial x_j}\right) + R_c\,, \quad i,j = 1,2,3 \qquad (1)$$

where,

D_{ij} = Coefficient of hydrodynamic dispersion,

C = Concentration of the solute in the source or sink fluid

R_c = Sources or sinks

$V_i = \frac{K_{ij}}{\theta}\frac{\partial h}{\partial x_i}$ = Seepage velocity

The initial condition (specification of the concentration distribution of solute at initial time $t = 0$), can be written as:

$$C(x) = C^U(x)\,, \quad x \in \Omega \qquad (2)$$

where $C^U(x)$ indicates a known concentration distribution over the domain of interest Ω. $C^U = 1$ for a uniform source of contamination at $x = 0$.

In the model formulation of an unconfined aquifer, three regions are considered; the upper layer (porous layer1; concentration C_{i1}), middle layer (aquifer layer; concentration C_{i2}) and the lower layer (porous layer 2; concentration C_{i3}).

The partial differential equation describing the contaminant transport in the upper layer is given as:

$$\frac{\partial C_{i1}}{\partial \tau} = D_{i1}\frac{\partial}{\partial y}\left[\frac{\partial^\alpha C_{i1}}{\partial y^\alpha}\right] \quad 0 < y < \infty, \quad \tau > 0 \qquad (3)$$

where,

D_{i1} = Effective diffusivity of porous layer
C = contaminant concentration

Similarly, the partial differential equation describing the contaminant transport in the aquifer is given as:

$$\frac{\partial C_{i2}}{\partial \tau} + \beta\frac{\partial^\gamma C_{i2}}{\partial \tau^\gamma} = \bar{D}_2\frac{\partial}{\partial y}\left(\frac{\partial^\lambda C_{i2}}{\partial y^\lambda}\right) + D_2\frac{\partial}{\partial x}\left(\frac{\partial^\lambda C_{i2}}{\partial x^\lambda}\right) - v\frac{\partial C_{i2}}{\partial x}\,, \quad -h < y < 0, 0 < x < \infty, \tau > 0 \qquad (4)$$

where,

$0 < y, \quad \lambda < 1$

D_2 and $\bar{D}_2$ = Effective diffusivities, in the aquifer in the x and y-direction, respectively;
v = Fluid velocity
τ = Time
β = Capacity coefficient

$\frac{\partial^\gamma C_{i2}}{\partial \tau^\gamma}$ is the fraction-in-time derivatives, $\left(\frac{\partial^\lambda C_{i2}}{\partial x^\lambda}\right)$ is the fraction-in-space derivatives with respect to the horizontal

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flow, and $\left(\frac{\partial^{\lambda} C_{i2}}{\partial y^{\lambda}}\right)$ is the fraction-in-space derivatives with respect to the vertical flow.
Thus,

$$L\left[\frac{\partial^{\gamma} C_{i2}}{\partial \tau^{\gamma}}\right] = s^{\gamma-1}\left(sL[C_{i2}] - C_{i2}(x,0)\right) \qquad (5)$$

$$L\left[\frac{\partial^{\lambda} C_{i2}}{\partial x^{\lambda}}\right] = q^{\lambda-1}\left(qL[C_{i2}] - C_{i2}(\tau,0)\right) \qquad (6)$$

Now,

$$\bar{D}_2\left(\frac{\partial^{\lambda} C_{i2}}{\partial y^{\lambda}}\right) = D_1\left(\frac{\partial^{\alpha} C_{i1}}{\partial y^{\alpha}}\right) \text{ at } y = 0 \qquad (7)$$

$$\frac{\partial^{\lambda} C_{i2}}{\partial y^{\lambda}} = 0 \text{ at } y = -h \qquad (8)$$

$$C = \frac{1}{h}\int_{-h}^{0} C_{i2}\, dy \qquad (9)$$

The Advection–Dispersion equation for the Concentration of Contaminants in the Aquifer is formulated in the following form:

$$\frac{\partial C}{\partial \tau} + \beta\frac{\partial^{\gamma} C}{\partial \tau^{\gamma}} = \frac{D_1}{h}\frac{\partial^{\alpha} C_{i1}}{\partial y^{\alpha}}\bigg|_{y=0} + D_2\frac{\partial}{\partial x}\left(\frac{\partial^{\lambda} C}{\partial x^{\lambda}}\right) - v\frac{\partial C}{\partial x}, \quad 0 < x < \infty, \quad \tau > 0 \qquad (10)$$

$$\varphi(t,x) = 1 - \left(\frac{-1+e^{-t}}{t} - \frac{-1+e^{xb\gamma\cos(\pi\gamma)+x\pi\cos\beta^2}}{x(b\gamma\cos(\pi\gamma)+\pi\cos\beta^2)}\right)\left(\frac{\sin x b \sin\pi\gamma}{\gamma+1} - \frac{x\sin\pi\beta}{\beta+1}\right) \qquad (11)$$

Applying Duhamel's Theorem [7], and for a uniform source of contamination at $x = 0$, we have:

$$C(t,x) = \frac{\partial}{\partial t}\int_0^t C_0(t-\tau)\varphi(\tau - x, x)d\tau \qquad (12)$$

$$C = 1 - \left[\left(\frac{-1+e^{-t}}{t}\right) - \left(\frac{xb\gamma + xt^{\beta}\left(\frac{1}{2}\gamma\right) + \frac{1}{2}}{1-\gamma}\right) + \left(\frac{xt^{\gamma}\left(\frac{1}{2}\beta\right) + \frac{1}{2}}{1+\beta}\right)\right] \qquad (13)$$

$$C = \frac{-\Gamma(-\beta-1)\Gamma(-\gamma+1) + xt^{\beta}\Gamma(-\gamma+1) + x}{\Gamma(-\beta-1)\Gamma(-\gamma+1)} + \frac{bxt^{\gamma}\Gamma(-\beta-1)}{\Gamma(-\beta-1)\Gamma(-\gamma+1)} \qquad (14)$$

where,

$$\beta = \frac{\alpha}{\alpha+1} \qquad (15)$$

$$\Gamma(\beta) = \int_0^1 t^{\beta-1}e^{-t}dt \qquad (16)$$

$$\Gamma(\beta+1) = \int_0^1 t^{\beta}e^{-t}dt \qquad (17)$$

α = Diffusive transport into the confining layer
γ = Diffusive transport within the layer
b = Size of the aquifer

Equation (14) describes the behaviour of contaminants invasion under a uniform source of contamination $C^u = 1$ for different values of values of α, b and γ for $0 < t \leq 7$.

3. RESULTS AND DISCUSSION

On substituting the values $\alpha = 1.00, b = 0.5$ and $\gamma = 0.5$ for $0 < t \leq 7$ into equation (14), as illustrated in Table 1 we obtain the corresponding aquifer concentration distribution graph given by Figure 1.

Table 1 Concentration distribution values for $\alpha = 1.00, b = 0.5$, and $\gamma = 0.5$

α	b	γ	t
1.00	0.50	0.50	$0 < t \leq 7$

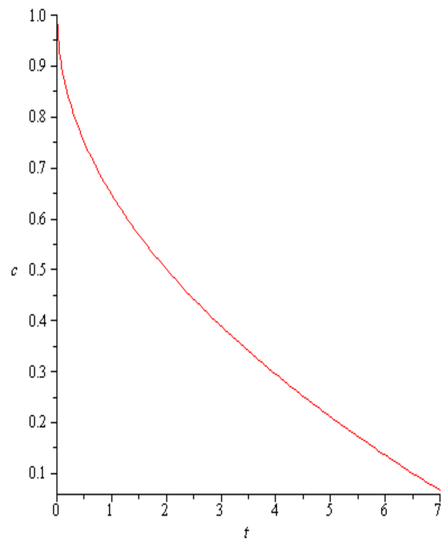

Figure : Aquifer concentration distribution for Values of $\alpha = 1.00, b = 0.5$, and $\gamma = 0.5$

Figure 1 reveals that for a uniform source of contamination, $C^u = 1$, for values of $\alpha = 1.00, b = 0.5$, and $\gamma = 0.5$ the level of contamination is at its peak.

On substituting the values $\alpha = 0.50, b = 0.5$, and $\gamma = 0.5$ for $0 < t \leq 7$ into equation (14), as illustrated in Table 2

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we obtain the corresponding concentration distribution graph given by Figure 2.

Table 2: Concentration distribution values for $\alpha = 0.50, b = 0.5$, and $\gamma = 0.5$

α	b	γ	t
0.50	0.50	0.50	$0 < t \leq 7$

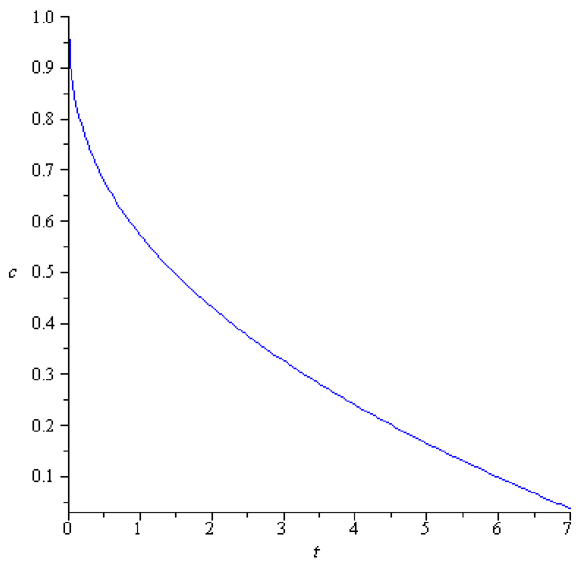

Figure 2 :Aquifer Concentration Distribution for Values of $\alpha = 0.50, b = 0.5$, and $\gamma = 0.5$

Figure 2 shows that a reduction of the value of α by 0.05 leads to a reduction in the level of concentration.

On substituting the values $\alpha = 0.20, b = 0.5$, and $\gamma = 0.5$ for $0 < t \leq 7$ into equation (14), as illustrated in Table 3 we obtain the corresponding aquifer concentration distribution graph given by Figure 3.

Table 3 :Concentration distribution values for $\alpha = 0.20, b = 0.5$, and $\gamma = 0.5$

α	b	γ	t
0.20	0.50	0.50	$0 < t \leq 7$

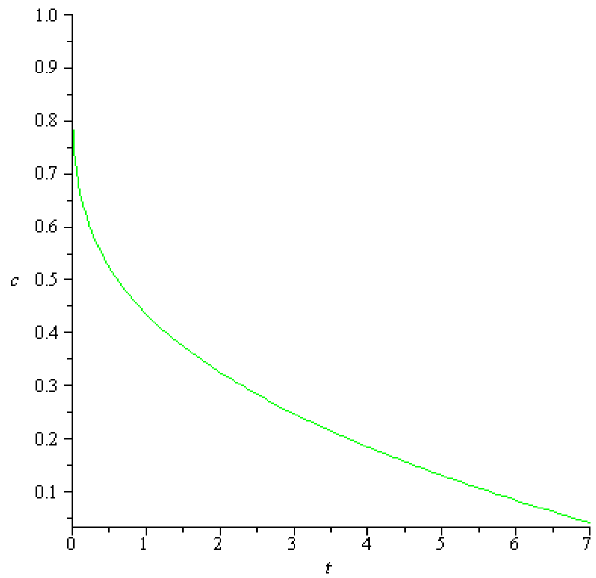

Figure 3: Aquifer concentration distribution for values of $\alpha = 0.20, b = 0.5$, and $\gamma = 0.5$

From Figure 3, it is observed that a further reduction of the value of α to 0.20, results in further reduction in the level of concentration.

For the purpose of comparison, Figure 4 combines the graphs of Figures 1 to Figure 3.

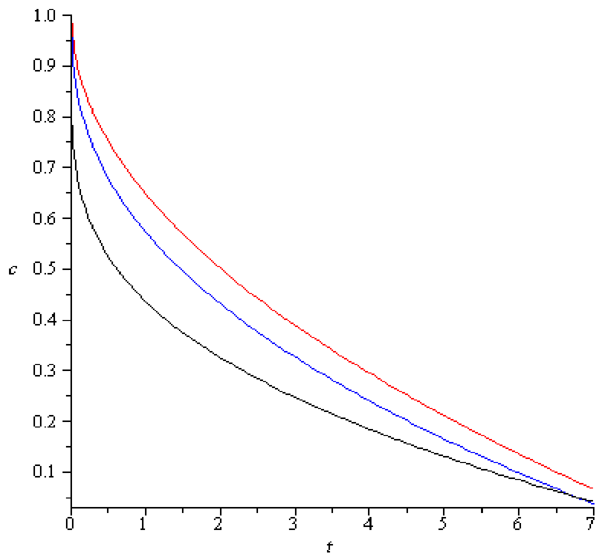

Figure 4: Aquifer concentration distribution for different Values of α, b and γ.

4. CONCLUSION

From the results of Figures 1, 2 and 3, it is conclusive that for a uniform source of contamination, the level of contamination reduces over time depending on the values of α, b and γ.

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