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Effect of Number of Node on the Deflection of a Simply Supported Beam Using Finite Element Analysis

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Abstract: *The effect of the numbers of nodes for a simply supported beam is considered under a pointed load (e.g., vehicle moving on a bridge) using the finite element method and Lagrange polynomial shape function. The deflection problem of the beam was solved analytically and compared with that of the finite element method. The number of nodes considers is 3, 5, 10, 15, 20, and 30. The results of the analytical and finite element method show a very close agreement and have an identical profile. The error became more stable as the numbers of nodes increase, as shown on the graph, that from 30 numbers of nodes the error is very minimal and stable. This error exhibit an exponential function for the deflection of a simply supported beam.*

Keywords: *Beam, simply supported, FEA, deflection, nodes, shape function.*

1.0 INTRODUCTION

A beam is a structural member which resists laterally applied load, [1]. The reaction of a beam to the applied load is a function of the type of the applied load and its end condition. Beams have found applications in various aspects of engineering design and they could fail during service life. The failure of beams can be minimized if detailed behaviour and characteristics of its response to application of load is known [2].

The accuracy of any finite element model depends on two factors i.e., the degree of shape function and the number of mesh size. Also, these two factors determine the computation time required to reach the answer. According to FEA theory, [3] the FE models with a fine mesh (small element size) yields highly accurate results but may take longer computing time. On the other hand, those FE models with coarse mesh (large element size) may lead to less accurate results but smaller computing time, [4]. As the number of mesh increases the error becomes smaller and therefore, when the error became stable, there is no need to increase the size any further, to optimise the computational time. The objective of this paper is to estimate the impact of the number of nodes on the amount of error on the deflection of a simply supported beam numerically. To achieve this objective, a static analysis of the beam deflection and slope was carried out by developing the FEA equation for the beam.

The conclusions drawn from the review of previous researches on the effects of mesh size on the accuracy of the FEA results were mostly carried out using computer software. [5], performed FEA on a cantilever beam of 100 mm and a load of 500 N made from Fe_3O material using Creo 2.0 software. Mesh sizes considered varies between 2 mm – 6 mm with a tetra meshing. The error recorded for deflection was below 0.003%.

Chaphalkar *et al.* [6] performed a modal analysis of cantilever beam Structure Using both Finite Element Analysis and Experimental Analysis technique. transverse vibration of the fixed free beam was investigated, alongside the mode shape frequency. All the frequency values obtained were analyzed with the numerical approach method by using ANSYS finite element package. [6], concluded that the relative error between these two approaches was very minute and the percentage error between the numerical (FEA) approach and the experimental approach are allowed up to 5% to 7%.

Yucheng and Gary [7] presented a systematic study on finding the effects of element size on the accuracy of FEA numerical analysis results, based on the guidelines of choosing the best element size in the finite element modelling approach. Static and buckling analyses were carried out and their results discussed.

Weibing Liu, Mamtimin Geni, and Lei Yu have obtained different FEA accuracy by different element size and type. It is observed that as the curve and surface boundary of the higher-order element can approach boundary accurately, calculation accuracy under hexahedral element is higher than tetrahedral element, and calculation accuracy of model analysis can be improved by increasing the number of nodes.

The usage of a beam in an engineering structure is unavoidable because most engineering structures such as machine and mechanisms, buildings, street and traffic control light, road sign, office furniture, aircraft, sports equipment e.t.c. contains a different kind of beam. For sensitive engineering structures like aircraft, sports equipment e.t.c. exact deformation limits are essential to its performance and safety. This paper seeks to examine the effects of the number of nodal elements on the deflection and slope of a loaded simply supported beam using langrage shape function, [8] .

2.0 METHODOLOGY

2.1 Analytical Method

Consider simply supported beam in Figure 1, carrying a uniformly distributed load (UDL) with a single-pointed load. The reactions at the supported were obtained from the conditions of equilibrium and given by equations 1 and 2.

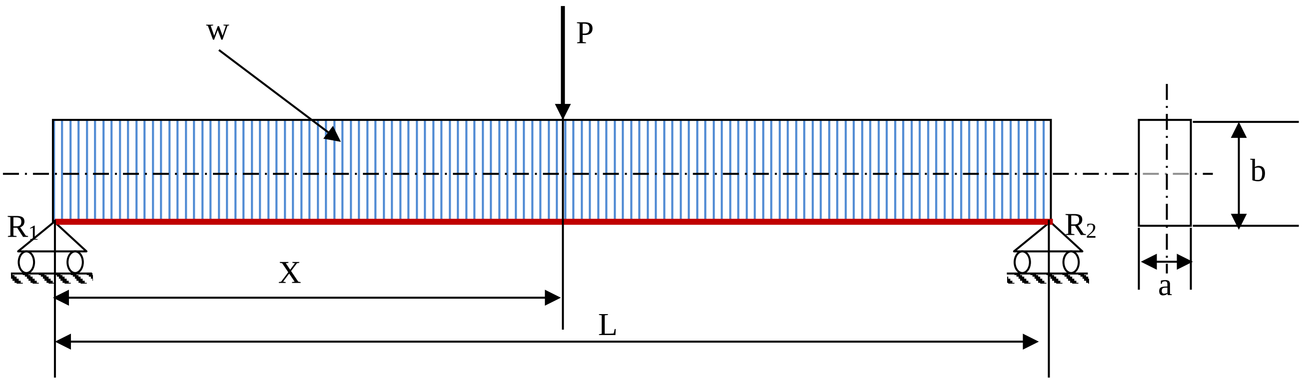


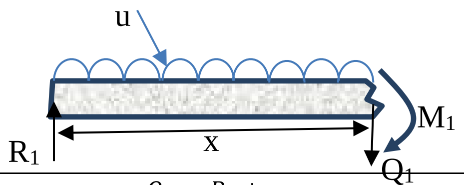
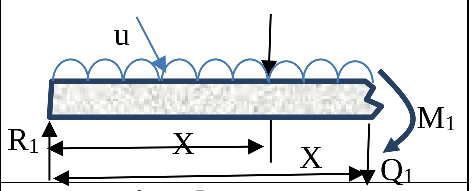
Figure 1: Schematic of a simply supported beam.

$$R_1 = \frac{\left(w.L^2/_2 + p(l - X_1)\right)}{L} \qquad 1$$

$$R_2 = \frac{\left(w.L^2/_2 + pX_1\right)}{L} \qquad 2$$

The shear force and bending moment for the first and second loaded sections are presented in Table 1.

Table 1: Shear force and bending moments of the Beam Loaded Section.

S/No		First Loaded Section	Second Loaded Section
1	**Range**	$0 \le x \le x1$	$x1 \le x \le L$
2	Free Body Diagram		
3	Shear force equation	$Q_1 = R_1 + u.x$	$Q_2 = R_1 + u.x + p$

4	Bending Moment Equation	$M_1 = R_1.x + \frac{u.x^2}{2}$	$M_2 = R_1.x + \frac{u.x^2}{2} + p(L - x_1)$

Substituting the bending moment equation in table 1, into Euler's beam equation (1), yields equation (2);

$EIy'' = M_2$ [9]. 1

$$EIy'' = R_1.x + \frac{w.x^2}{2} + p(L - x_1) \quad 2$$

The deflection and slope of a beam were obtained by integrating equations (1) and (2), one and twice respectively to obtain equations (3) and (4).

$$y' = \theta_1 = \frac{1}{EI}\left(\frac{R_1.x^2}{2} - \frac{w.x^3}{6} - \frac{p(x-x1)^2}{2} + c_1\right) \quad 3$$

$$y = \frac{1}{EI}\left(\frac{R_1.x^3}{6} - \frac{w.x^4}{24} - P\frac{(x-x1)^3}{6} + c_1x + c_2\right) \quad 4$$

And applying the boundary conditions ($x = 0$ *and* $y = 0, x = \frac{l}{2}$, *and* $y' = 0, x = l$ *and* $y = 0.$) to equations (3) and (4) gives the constants of integration, i.e.

$C_2 = 0$, 5

And

$$c_1 = \frac{w.L^3}{24} + \frac{p(L-x1)^2}{6L} - \frac{R_1.L^2}{6} \quad 6$$

2.2 Finite Element Analysis of The Beam

FEA equation of the beam in figure 1, were developed for 3, 5, 10, 15, and 20 numbers of nodes. The equations were based on Hooke's law given by Equation (7),

$[K][x] = [F]$ [10] 7

Equation (7) was applied to all the nodes cases mentioned earlier. But the summary of the analysis of the beam with five nodes is reported. The equation of the beam single element is given by equation (8).

$$\begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \end{bmatrix} = \frac{EI}{L_1{}^3}\begin{bmatrix} 12 & 6L_1 & -12 & 6L_1 \\ 6L_1 & 4L_1{}^2 & -6L_1 & 2L_1{}^2 \\ -12 & -6L_1 & 12 & -6L_1 \\ 6L_1 & 2L_1{}^2 & -6L_1 & 4L_1{}^2 \end{bmatrix}\begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \end{bmatrix} \quad 8$$

The single beam element equation 8, were combined for the five elements shown in Figure 2 to obtain the global matrix equation 9 for the five (5) nodes. The solution of equation 9 gives the slope and deflection profile for the beam.

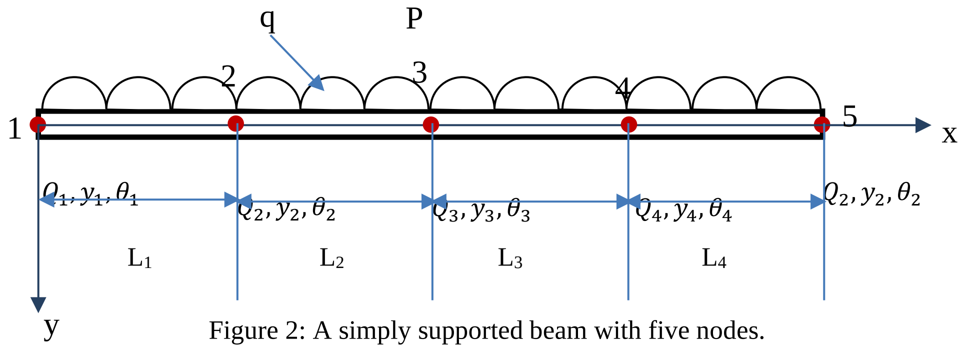


Figure 2: A simply supported beam with five nodes.

$$\frac{EI}{L_1^3}\begin{bmatrix} 12 & 6L_1 & -12 & 6L_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6L_1 & 4L_1^2 & -6L_1 & 2L_1^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -12 & -6L_1 & 12+12 & -6L_1+6L_2 & -12 & 6L_2 & 0 & 0 & 0 & 0 \\ 6L_1 & 2L_1^2 & -6L_1+6L_2 & 4L_2^2+4L_1^2 & -6L_2 & 2L_2^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -12 & -6L_2 & 12+12 & -6L_2+6L_3 & -12 & 6L_3 & 0 & 0 \\ 0 & 0 & 6L_2^2 & 2L_2^2 & -6L_2+6L_3 & 4L_2^2+4L_3^2 & -6L_3 & 2L_3^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -12 & -6L_3 & 12+12 & -6L_3+6L_4 & -12 & 6L_4 \\ 0 & 0 & 0 & 0 & 6L_3 & 2L_3^2 & -6L_3+6L_4 & 4L_3^2 & -6L_4 & 2L_4^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -12 & -6L_4 & 12 & -6L_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6L_4 & 2L_4^2 & -6L_4 & 4L_4^2 \end{bmatrix}\begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \\ y_3 \\ \theta_3 \\ y_4 \\ \theta_4 \\ y_5 \\ \theta_5 \end{bmatrix} = \begin{bmatrix} Q_1 \\ M_1 \\ Q_2 \\ M_2 \\ Q_3 \\ M_3 \\ Q_4 \\ M_4 \\ Q_5 \\ M_5 \end{bmatrix} + \begin{bmatrix} \frac{qL_1}{2} \\ -\frac{qL_1^2}{12} \\ \frac{qL_2}{2} \\ -\frac{qL_2^2}{12} \\ \frac{qL_3}{2} \\ -\frac{qL_3^2}{12} \\ \frac{qL_4}{2} \\ -\frac{qL_4^2}{12} \\ \frac{qL_5}{2} \\ -\frac{qL_5^2}{12} \end{bmatrix} \quad 9$$

Similarly, the unconstraint equations of the beam with 3, 10, 15 and 20 nodes were derived and the solution obtained. The shape function of an elastic beam was derived from the Lagrange polynomial ([11]) equation (10).

$$L = \prod_{\substack{m=1 \\ m\neq k}}^{n} \frac{(x-x_m)}{(x_k-x_m)} \quad 10$$

Equation 10 was applied to each element of the beam to obtain the shape functions. The derived shapes from equation (10 are presented by equations (11-15).

$$N1 = 0.0417x^4 - 0.4167x^3 + 1.4583x^2 - 2.0833x + 1.0000 \quad 11$$

$$N2 = -0.1667x^4 - 1.5x^3 + 4.33x^2 - 4.0x \quad 12$$

$$N3 = 0.25x^4 - 2x^3 + 4.7x^2 - 3.0x \quad 13$$

$$N4 = -0.1667x^4 + 1.667x^3 + 2.33x^2 + 1.33x \quad 14$$

$$N5 = 0.0417x^4 - 0.25x^3 + 0.4583x^2 - 0.25x \quad 15$$

Equations 11-15 were assembled into equation (16), known as the entire shape function of the beam in Figure 2.

$$N = [N1\ N2\ N3\ N4\ N5\ ] \quad 16$$

This process of shape function development was applied to beams with 3, 10, 15 and 20 nodes too.

3.0 RESULTS AND DISCUSSIONS

The beam data used to evaluate the two methods i.e. analytical and FEA are shown in table 2. And the results are shown in table 3 and figure 3 - 4.

Table 2: Numerical Values for computation

S/N	Quality	Value
1	Beam length	4 m
2	load position X1	2 m
3	UDL	25 kN/m^2
4	Pointed Load	10 KN
5	Beam width, a	0.5 m
6	beam breadth, b	a/4 m
E	Young modulus of statics	29.5 kN/m

$$y(x) = [N1\ N2\ N3\ N4\ N5]\begin{bmatrix} 0 \\ 420.185 \\ 387.21 \\ 316.18 \\ 563.02 \\ -0.925 \\ 387.21 \\ -317.66 \\ 0 \\ -422.04 \end{bmatrix}$$

$$y(x) = [N1\ N2\ N3\ N4\ N5\ N6\ N7\ N8\ N9\ N10]\begin{bmatrix} 0 \\ 223.5 \\ 400.08 \\ 537.76 \\ 585.55 \\ 537.76 \\ 400.08 \\ 223.5 \\ 0 \end{bmatrix}$$

Table 3: deflection results for 3,5,10,15 and 20 nodes

x	**Analytical Result**	**No. of Nodes 3**	**No. of Nodes 5**	**No. of Nodes 10**	**Nos. of Nodes 15**	**Nos. Nodes 20**
0	0	0	0	0	0	0
0.5	225.48	222.46	200.31	223.50	226.60	230.11
1	421.19	381.36	387.21	400.08	408.89	418.09
1.5	558.53	476.69	516.83	537.76	542.29	555.01
2	610.17	508.47	562.79	585.55	592.45	603.33
2.5	558.53	476.69	516.20	537.76	542.29	555.01
3	421.19	381.36	385.66	400.08	408.89	418.09
3.5	225.48	222.46	197.26	223.50	226.60	230.11
4	0	0	0	0	0	0

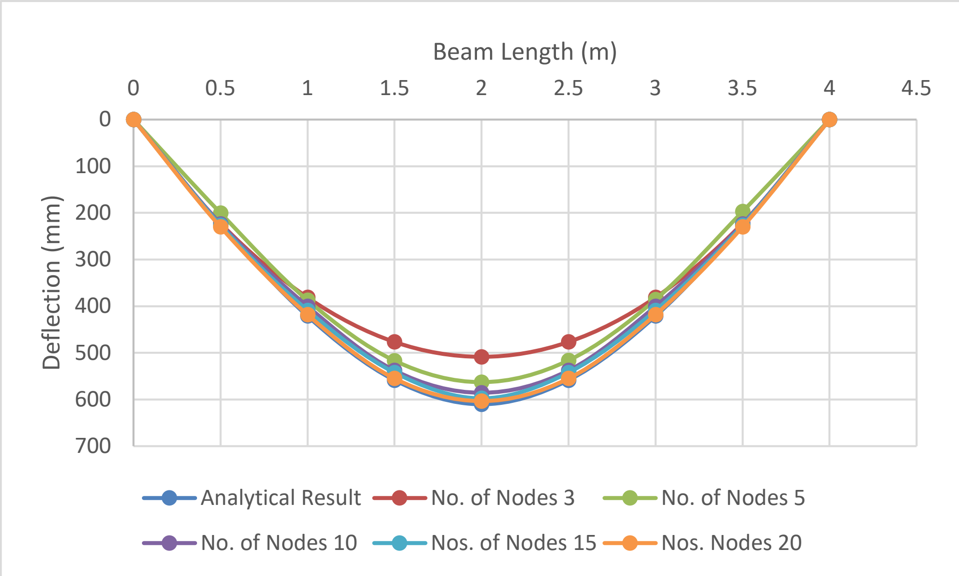


Figure 3: Comparison and results of analytical method and that of the FEA methods

Figure 3, shows the deflection of the beam as the numbers of the nodes are increased. Also, it shows the deflection of the beam from the analytical computation. And as can be seen from Figure 3, the deflection error decreases with the increased numbers of beam elements (i.e., nodes). The error of the FEA method computation as compared with the analytical method is presented in Figure 4. From Figure 4 the error drops sharply when the number of the nodes are few (i.e. 3 and 5) and the error becomes minimal with the higher number of nodes (i.e. 10, 15 and 30). The result is in close agreement with that of [12] and [7].

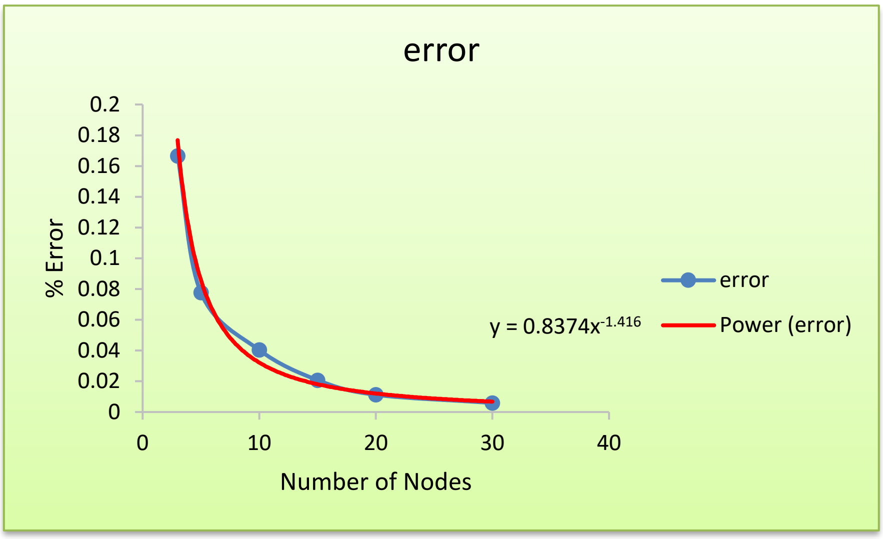


Figure 4: Effects of the number of nodes of beam deflection.

4.0 CONCLUSIONS

Based on the results of analytical and finite element analysis of a simply supported beam with pointed load the following conclusions are made:

1. As the numbers of the FEA nodes on the beam increase, the deflection get close to that of the analytical method.
2. The error became more stable as the numbers of nodes increase because the graph shows that from 20 numbers of nodes the error is very minimal and stable. This error exhibit an exponential function for the deflection of a simply supported beam.

3. This is consistent with the theory of the FEM, as increasing the number of elements reduces the error, which in turn improves the accuracy of the solution.

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