Analytical Solutions to Bloch NMR Flow Equation in Porous System: Future and Emerging Magnetic Resonance Computational Imaging for Medical and Biomedical Engineering

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Abstract: *Background*: In most brain tissue architecture, water diffusion is anisotropic or directional and can be treated as a restricted geometry. This requires the specification of diffusion coefficients as function of space coordinate(s) and such a phenomenon is an indication of non-uniform compartmental conditions observed in porous media. Since the diffusion coefficient becomes directional, the imaging process becomes very complex and the analytical solutions to the NMR diffusion tensor equation with variable coefficients for non-uniform tissue composition can open a new field of research in diffusion tensor imaging.

Metohod: Detailed analytical solutions to the Bloch NMR flow differential equation in cylindrical and spherical restricted geometries needed in the study of general motion of pathological tissues are presented. The system considered was treated as a rigid body which is fundamental to the formulation of the concept of stress in a viscous fluid (blood flow). We show how the analytical solutions to the Bloch NMR flow equation can provide fundamental computational tools that can stimulate interest for future research at the molecular and atomic scales for conceptualization, characterization and development of emerging magnetic resonance imaging theory and applications.

Results and Conclusion: Diffusion tensor imaging is treated theoretically by the analytical solutions to fundamental Bloch NMR equations for Magnetic Resonance Imaging. We derived a differential equation for diffusion in both cylindrical and spherical geometries. The general strategy is to consider the diffusion equation together with a forcing function, which is due to the time dependent magnetic field. First the diffusion equation is considered in terms of the instantaneous change to obtain the spatial and time dependence of magnetization. Then the time dependent magnetization is integrated over time to obtain the steady state solutions for the diffusion tensor equation derived from the Bloch NMR flow equations. It is very interesting to note that the derived experimental parameters are very much related to magnetic resonance imaging parameters where the distribution and nature of axonal injuries can be quantified in terms of refined measurements of tortuosity, permeability, porosity and formation factor.

Keywords: Apparent diffusion coefficient, cylindrical and spherical geometries, diffusion tensor, diffusion NMR equation, permeability, porosity, tortuosity.

INTRODUCTION

The anisotropic nature of water diffusion in white matter has led to another exciting modality that allows investigators to follow different fiber systems in white matter [1-3]. In observing white matter areas, such as the genu or splenium of the posterior corpus callosum, one can appreciate the signal difference caused by anisotropic diffusion. White matter structures appear hypointense relative to the surrounding tissue if the white matter tracts run parallel to the encoding gradient applied, whereas they appear hyperintense when fibers run perpendicular to the encoding gradient.

From the six or more diffusion Weighted (DW) images and the diffusion gradient directions being used, the elements of the diffusion coefficient (D) can be calculated by simple matrix calculus. Unfortunately, neither the diffusion tensor element images nor some grid of ellipsoids is an adequate form that could be presented to the clinician for diagnosis because there is too much information to perceive at once [4].

Traditionally, the Bloch-Torrey equation is used for mapping diffusivity profiles in biological tissues. The Bloch-Torrey equation is usually modified to include a diffusion term with an arbitrary rank Cartesian tensor. This equation is normally solved to give the expression for the generalized Stejskal-Tanner formula quantifying diffusive attenuation in complicated geometries. This makes it possible to calculate the components of higher-rank tensors without using the computationally- difficult spherical harmonic transform. General theoretical relations between the diffusion tensor (DT) components measured by traditional (rank-2) DT imaging (DTI) and 3D distribution of diffusivities, as measured by high angular resolution diffusion imaging (HARDI) methods have been derived [5].

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