Finite Difference Analysis of the Effects of *MHD* on Continuous Dusty Fluid with Volume Faction of Dust Particles

[1]Cole, A. T., [1]Ugwu, U.C., [1]Olayiwola, R. O.
[1]Department of Mathematics, Federal University of Technology, Minna, Nigeria.
Email: cole.temilade@futminna.edu.ng **Mobile:** 07038960556

Abstract
This paper presents a mathematical model of an incompressible, conducting fluid embedded with non-conducting dust particles describing the effects of Magnetohydrodynamics (MHD) on a continuous dusty viscous fluid with volume faction of dust particles incorporating viscosity in the presence of uniform magnetic field. The stretching velocity and wall temperature are assumed to vary accordingly. The dimensionless coupled partial differential equations governing the phenomenon were solved numerically. Solutions were obtained for the velocity of the fluid, dust particles and fluid temperature within the channel. The effects of the various parameters on both the velocity of the fluid and dusty particles as well as the temperature are shown graphically and discussed. It was found that both velocity of the fluid and dust particles increases with the decrease in porous parameter. Increase in volume faction of dust particles increases the velocity of the particles rapidly and increases moderately the velocity of the fluid. Temperature of the fluid also decreases with the increase in Prandtl number.

Keyword: conducting fluid, dusty fluid, dusty particles, FDS, MHD, volume faction.

Introduction
The magnetohydrodynamic (MHD) fluid flow is very important in engineering applications such as aerodynamics equipment and MHD generators. The efficiency of these devices is affected by magnetic, darcy resistant and dust particles. The influence of dust particles on convective flow of dusty viscous fluids has its importance in many application such as wastewater treatment, power plant piping, combustion and petroleum transport. Particularly, the flow and heat transfer of electrically conducting fluids in channels under the effect of a transverse magnetic field occur in magnetohydrodynamic (MHD) accelerators, pumps and generators. This type of flow has uses in nuclear reactors, geothermal systems and filtration, among others. The possible presence of dust particles in combustion MHD generators and their effect on the performance of such devices led to studies of volume fraction of dust particles in non-conducting walls in the presence of uniform transverse magnetic field (Aamin, *et al.* 2018).

The study of convective flow of dusty viscous fluid under the influence of different physical conditions has been carried out by several authors like: Nag and Jana (1979) have studied unsteady couette flows of a dusty gas between two infinite parallel plates, when one plate is kept fixed and the other plate moves in its own plane. The problem has been solved with the help of Laplace Transform technique. It is found that the dust velocity in the case of accelerated start of the plate is less than the fluid velocity, for moderate value of the relaxation time of the dust particles become very fine. It is observed that the magnitude of the shear stress is larger when the plate starts with uniform acceleration than when it is impulsively started to move with uniform velocity. The paper of Nag and Jana (1979) did not examined time dependent plane, transient effects and wave structure of the fluid. Kulshretha and Puri (1981), have investigated the couette flow of a dusty gas due to an oscillatory motion of the plate. The time dependent plate and transient effects have been included.

Singh and Singh (2002) studied the laminar convective flow of an incompressible, conducting, viscous fluid embedded with non-conducting dust particles through a vertical parallel plate channel in the presence of uniform magnetic field and constant pressure gradient taking volume fraction of dust particles into account when one plate of the channel is fixed and the other is oscillating in time and magnitude about a constant non-zero mean .Solutions of the equation governing the flow are obtained for the skin velocity of the fluid and dust particles. The expression for skin friction and heat transfer is also obtained. The effects of various parameters on the velocities, skin friction and heat transfer are discussed. In this study porous parameter is not considered.

Nowadays electromagnetic pumps and their modifications are widely used in metallurgy and materials processing in order to transport and dose (exact batching) melting metal (Ivlev and Baranov, 1993). Saidu, *et al.* (2010) studied laminar convective flow of an incompressible, conducting, viscous fluid embedded in the presence of uniform magnetic field and constant pressure gradient taking volume of particle a dust into account when one plate of the channel is fixed and the other is oscillating in time and magnitude about a constant non-zero mean. It is found that both the velocity of the liquid and dust particles decreases with increase in the porous parameter (ε_3) Dubey and Varshney (2013) worked on the thermal diffusion effect on MHD free convection flow of stratified viscous fluid with heat and mass transfer.

In this paper we discuss the laminar convective flow of a dusty viscous fluid through a porous medium of non-conducting walls in the presence of uniform transverse magnetic field with volume fraction and considering porous parameter as well as with viscous dissipation term embedded in the Energy equation.

Model Formulation

Extending Saidu *et al* (2010) we neglect the pressure gradient $\frac{\partial p}{\partial x}$ because is already embedded in $g\beta\left(T-T_0\right)$ and incorporating the viscous dissipation term to energy equation, we have thus:

$$N_0 m\frac{\partial u_p}{\partial t}=\gamma\left[\mu\frac{\partial^2 u_p}{\partial y^2}+\rho g\beta\left(T-T_0\right)\right]-KN_0\left(u_p-u_f\right) \quad (1)$$

$$\left(1-\gamma\right)\frac{\partial u_f}{\partial t}=\left(1-\gamma\right)\left[\upsilon\frac{\partial^2 u_f}{\partial y^2}+g\beta\left(T-T_0\right)\right]+\frac{KN_0}{\rho}\left(u_p-u_f\right)+\frac{\sigma\mu_c^2 H_0^2}{\rho}u_f-\frac{\mu}{K_1}u_f \quad (2)$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{C_p}\left[\left(\frac{\partial u_f}{\partial y}\right)^2+\left(\frac{\partial u_p}{\partial y}\right)^2\right] \quad (3)$$

Subject to the initial and boundary conditions

$$\left.\begin{array}{lll} u_f\left(y,0\right)=0, & u_f\left(0,t\right)=0, & u_f\left(h,t\right)=U \\ u_p\left(y,0\right)=0, & u_p\left(0,t\right)=0, & u_p\left(h,t\right)=U \\ ???, 0? = ?_0, & T\left(0,t\right)=T_0, & T\left(h,t\right)=T_1 \end{array}\right\} \quad (4)$$

Where,

υ = kinematic viscosity, u_f = The velocity of the fluid, u_p = Velocity of the dust particles, m = mass of each dust particles, N_0 = number density of dust particles, T = temperature of the fluid, T_0 = initial temperature fluid and wall, β = volumetric coefficient of thermal expansion, C_p = specific heat at constant pressure, γ = volume fraction of dust particles, K = stokes resistance coefficient, H_0 = magnetic field induction, μ_c = magnetic permeability, σ = electric conductivity of the liquid, k = thermal conductivity

K_1 = porous parameter.

Method of Solution

Non-Dimensionalization

Here, we non-dimensionalize equation (1)-(4) in order to reduce the parameter, using the following dimensionless variables:

$$t' = \frac{\upsilon t}{h^2}, \quad \psi = \frac{u_p h}{\upsilon}, \quad y' = \frac{y}{h}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \phi = \frac{u_f h}{\upsilon}, \tag{5}$$

Using (5), and after dropping the prime, (1) – (4) becomes

$$f\frac{\partial \psi}{\partial t} = \gamma \frac{\partial^2 \psi}{\partial y^2} + \gamma Gr\theta - \alpha(\psi - \phi) \tag{6}$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} + Gr\theta + \varepsilon_1(\psi - \phi) + \varepsilon_2 M\phi - \varepsilon_3\phi \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + E_c\left[\left(\frac{\partial \phi}{\partial y}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2\right] \tag{8}$$

with boundary conditions:

$$\left.\begin{aligned} &\phi(y,0)=0; \quad \phi(0,t)=0; \quad \phi(1,t)=b_1 \\ &\psi(y,0)=0; \quad \psi(0,t)=0; \quad \psi(1,t)=b_2 \\ &\theta(y,o)=0; \quad \theta(0,t)=0; \quad \theta(1,t)=1 \end{aligned}\right\} \tag{9}$$

where,

$M = \frac{h^2\sigma\mu_c^2 H_0^2 \upsilon}{\mu}$ (Magnetic Parameter), $\varepsilon_1 = \frac{h^2 K N_0 \upsilon}{\mu(1-\gamma)}$; $\varepsilon_2 = \frac{1}{(1-\gamma)}$; $\varepsilon_3 = \frac{h^2\mu}{K_1\upsilon(1-\gamma)}$ (Porous parameter), $f = \frac{\upsilon N_0 m}{\mu}$ (Mass concentration of dust particles), $Gr = \frac{h^3 g\beta(T_1 - T_0)}{\upsilon^2}$ (Grashof number), $\alpha = \frac{Kh^2}{m\upsilon}$; (Concentration resistance ratio), $\Pr = \frac{\mu c_p}{k}$ (Prandtl number); $E_c = \frac{\upsilon^2}{h^2 C_p(T_1 - T_0)}$ (Eckert number).

Numerical Solution by Finite Difference Scheme (FDS)

Using explicit finite difference scheme (by using forward finite difference for the order time derivative and central difference for the second order partial derivatives) we transform equation (6) – (9) as thus:

$$\psi_{i,j+1} = \psi_{i,j} + \frac{k\varphi}{fh^2}\left(\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}\right) + \frac{k\varphi}{f} Gr\theta_{i,j} - \frac{k\alpha}{f}\left(\psi_{i,j} - \phi_{i,j}\right) \qquad (10)$$

$$\phi_{i,j+1} = \phi_{i,j} + \frac{k}{h^2}\left(\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}\right) + kGr\theta_{i,j} + k\varepsilon_1\left(\psi_{i,j} - \phi_{i,j}\right) + k\varepsilon_2 M\phi_{i,j} - k\varepsilon_3\phi_{i,j} \qquad (11)$$

$$\theta_{i,j+1} = \theta_{i,j} + \frac{k}{h^2 P_r}\left(\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}\right) + \frac{kE_c}{h^2}\left(\left(\phi_{i+1,j} - \phi_{i,j}\right)^2 + \left(\psi_{i+1,j} - \psi_{i,j}\right)^2\right) \qquad (12)$$

With the initial and boundary conditions as follows

$$\left.\begin{array}{lll} \phi_{i,0} = 0 & \phi_{0,j} = 0 & \phi_{1,j} = b_1 \\ \psi_{i,0} = 0 & \psi_{0,j} = 0 & \psi_{1,j} = b_2 \\ \theta_{i,0} = 0 & \theta_{0,j} = 0 & \theta_{1,j} = 1 \end{array}\right\} \qquad (13)$$

The stability condition is $\varepsilon \leq \frac{1}{2}$. Here we set $h = \frac{1}{10} = 0.1$ and $k = \frac{1}{250} = 0.004$ so that $\varepsilon = \frac{k}{h^2} = \frac{2}{5} < \frac{1}{2}$. A computer program in Pascal codes was written to perform the iterative computations.

Results and Discussions

Analysing the results accruing from the iterations to obtain the various effects of the parameter on the both the velocity of the fluid and that of the particles as well as temperature of the dusty fluid. The results are presented in the tables and graph below.

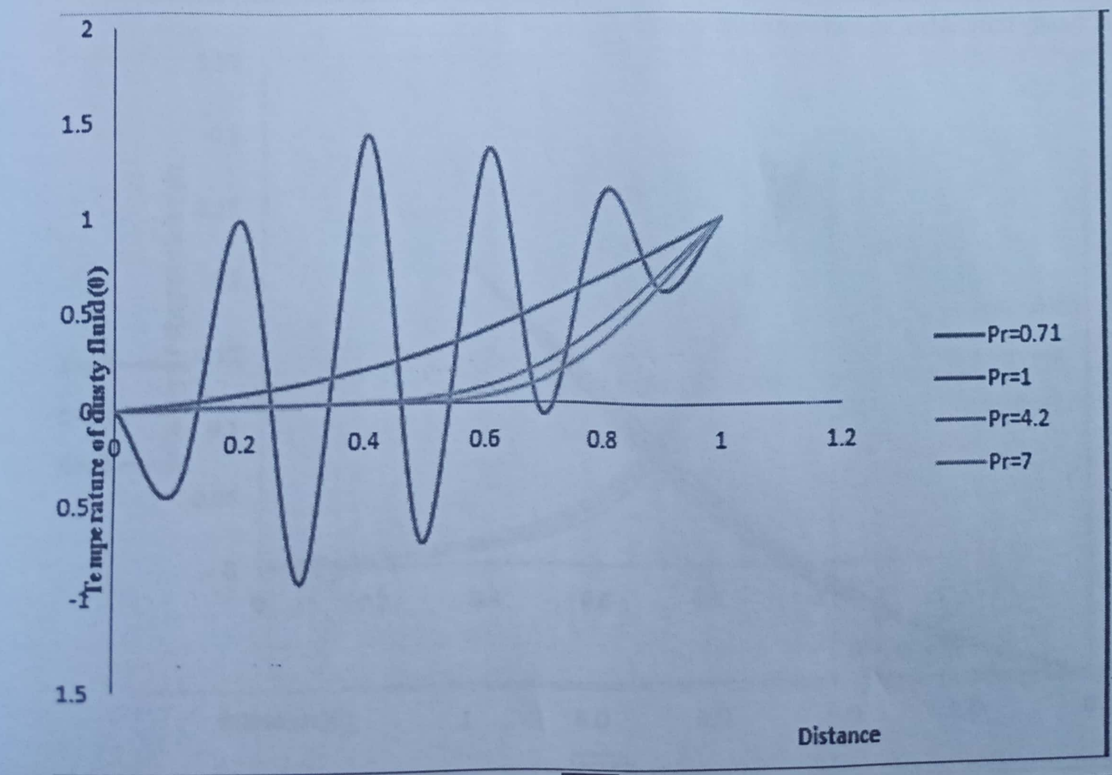

Figure 1: Effects of Pr on temperature profiles

Figure 1 shows the Temperature of dusty fluid θ (y, t) with various values of Prandtl number (Pr) when t =0.1, Gr=5, f=0.5, M=1. It is observed that at Pr<1, temperature is seen to be oscillating. Also, there is decrease in temperature with increase in Prandtl number at steady time. This reveals that the effect of increasing the Prandtl number is to decrease the temperature distribution in the flow region. It is evident that large values of Pr results in thinning of thermal boundary layers. It is the ratio of viscous force to thermal force. The prandtl number is large when thermal conductivity is less than one and viscosity is large; and it is small when viscosity is less than one and thermal conductivity is large.

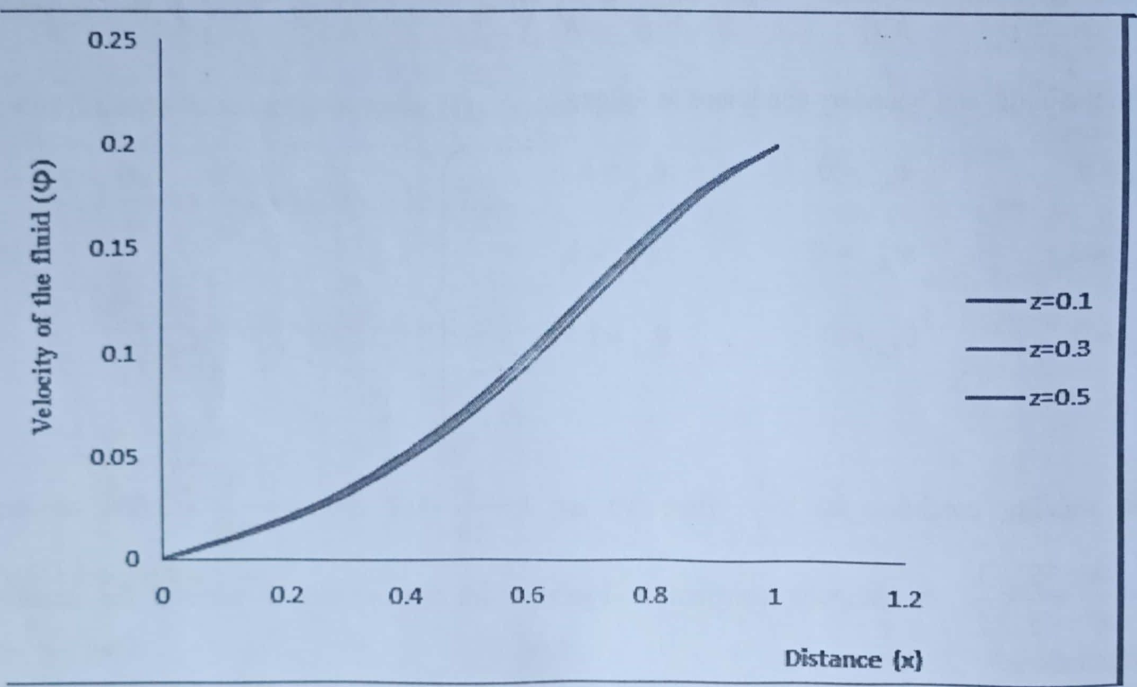

Figure 2: Effects of γ on velocity profiles

Figure 2 shows the effect of various values of volume faction of dust particles (γ) on velocity of the fluid when t=0.1, Pr=1, f=0.5, Ec=1. It is observed that increase in the volume faction of dust particles leads to small increase in the velocity of the fluid at steady time. Also, an increase in the volume fraction of dust particle, increases the velocity profiles for both the fluid and dust phase. This behaviour is observed as the volume occupied by the dust particle per unit volume of the fluid is higher than the dust concentration that is fluid particles moves faster than dust particles

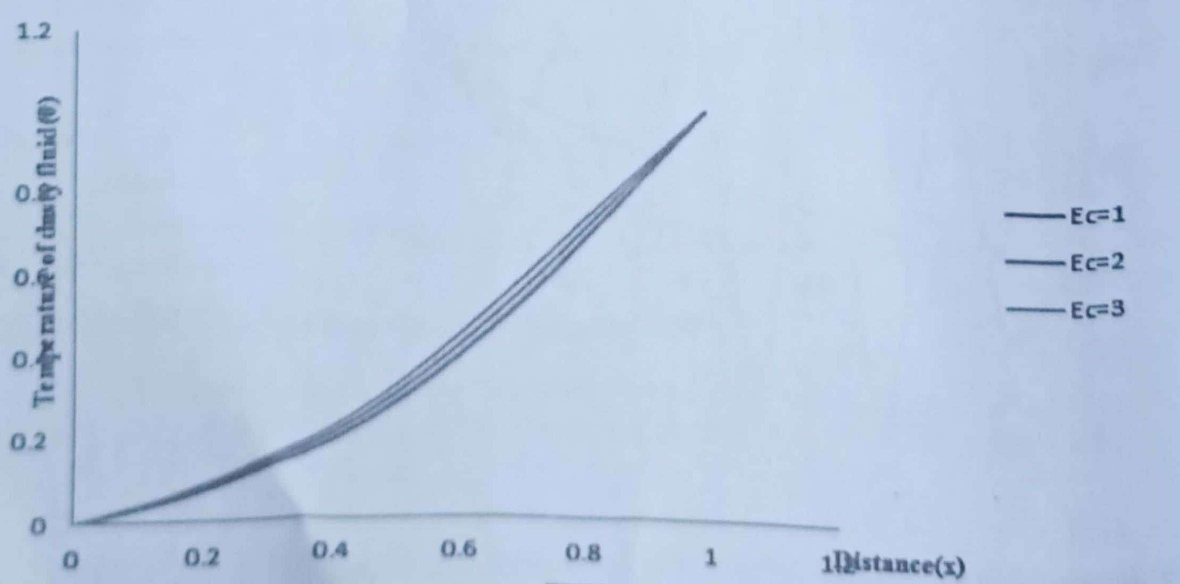

Figure 3: Effects of Ec on temperature profiles

Figure 3 shows the effect of various values of Eckert number (Ec) on the temperature(θ) when t= 0.1, Gr=5, f=0.5, M=1. It is observed the temperature increases with an increase in Eckert number at a steady time. Effects of Eckert number on the temperature profiles is as a result of the viscous dissipation. Viscous dissipation can change the temperature distribution by playing a role like an energy source, which leads to affects heat transfer rates. Thus, temperature of both the fluid and dust increases with increase in the values of Ec. This is because heat energy is stored in the liquid due to frictional heating and it's true in both cases.

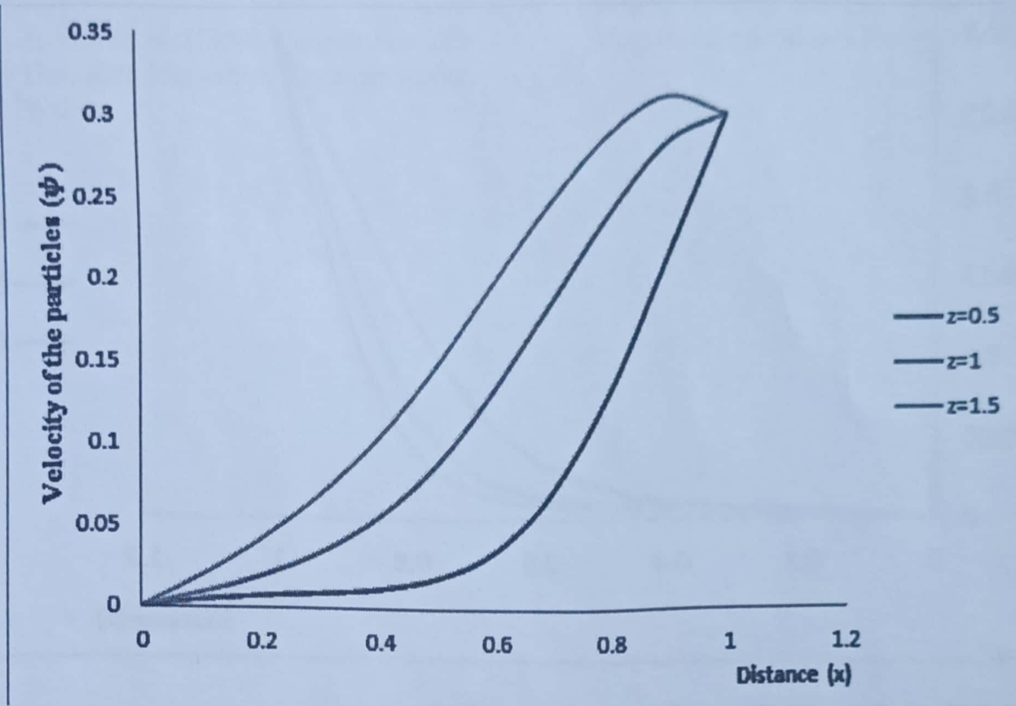

Figure 4: Effects of on velocity profiles

Figure 4 shows the effect of various values of volume faction of dust particles ( ) on the velocity of the fluid **when t = 0.1, Gr=5, Ec=1, f=0.5**. It is observed that increase in the volume faction of dust particles increases the velocity of the particles at steady time.

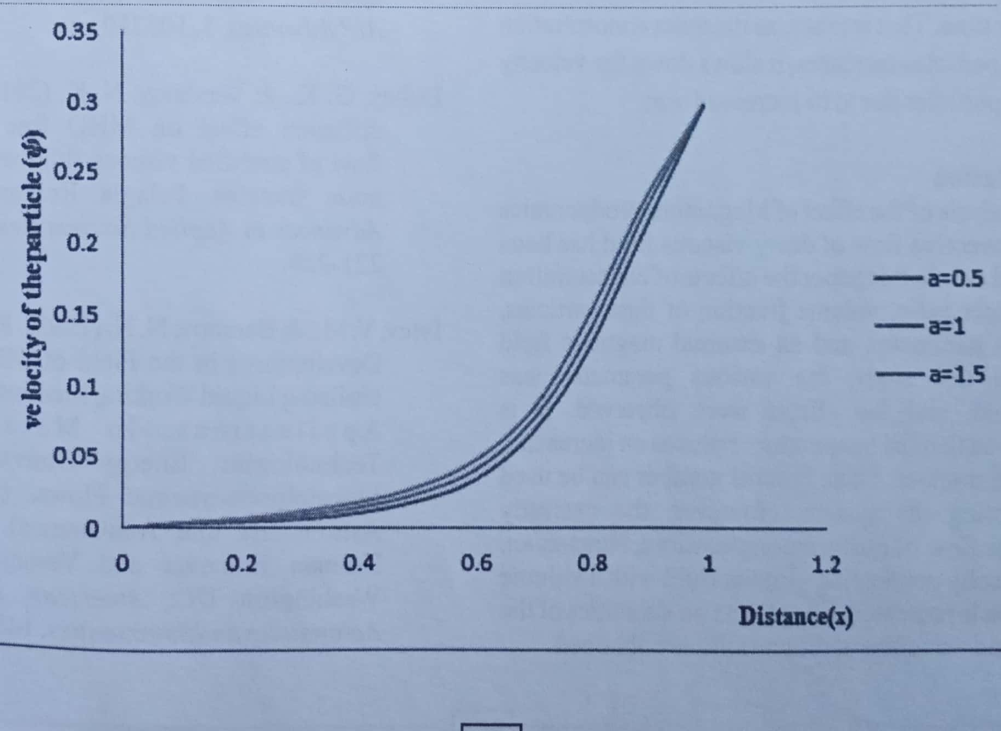

Figure 5: Effects of *a* on velocity profiles

Figure 5 shows the effect of Velocity of the particles ψ (y,t) with various values of Concentration ratio ($\propto$) when t = 0.1, Gr=5, Ec=1, f=0.5 it is observed that the velocity of the particles increases with increase in the Concentration ratio at steady time

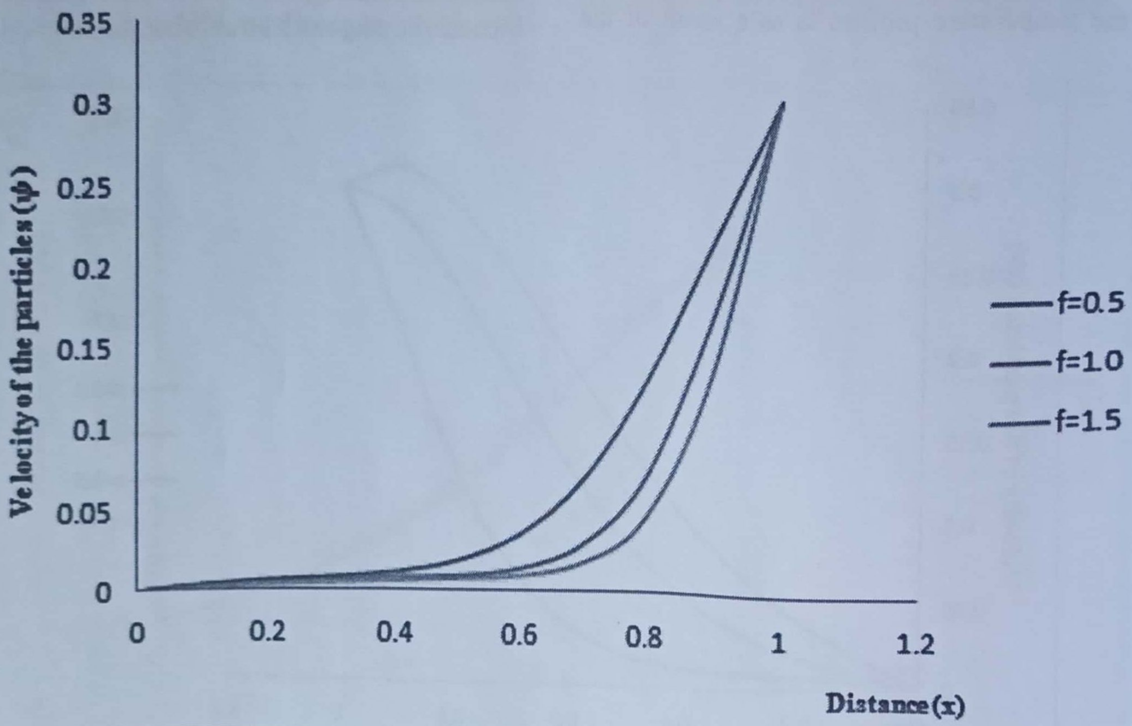

Figure 6: Effects of *f* on velocity profiles

Figure 6 shows the effect of various values of Mass concentration (*f*) on the velocity of the particle **when t=0.1, Gr=5, Ec=1, φ=0.1**. It is observed that the velocity of the particle decreases with increase in the Mass concentration of dust particles at a steady time. That is to say, as the mass concentration of the particles increases, it slows down the velocity of the particles due to its increased size.

Conclusion

An analysis of the effect of Magnetohydrodynamics on convective flow of dusty viscous fluid has been carried out. In this paper the effects of concentration resistance ratio, volume fraction of dust particles, porous parameter, and an external magnetic field on. In this study, the various parameter was analysed, and the effects were observed. It is observed that the temperature reduces on increasing Prandtl number. Thus, Prandtl number can be used in cooling the system. However, the unsteady laminar flow of dusty, incompressible, Newtonian, electrically conducting viscous fluid with a volume fraction is presented. The effects on velocities of the fluid and velocities of the particles are obtained.

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