APPLICATION OF SCHEFFE'S MODEL IN COMPRESSIVE STRENGTH OPTIMISATION OF CONCRETE USING RICE HUSK ASH AS PARTIAL REPLACEMENT OF CEMENT

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ABSTRACT

This study investigates the economic application of scheffe's model in compressive strength optimisation of rice husk ash-cement concrete. Physical properties of the aggregates such as specific gravity, bulk density and sieve analysis were determined. Specific gravities of fine and coarse aggregates were 2.64 and 2.66 respectively. The bulk densities were 1745kg/m3 and 1660kg/m3 respectively. From the sieve analysis test, the sand belonged to zone 2 and well graded with coefficient of gradation of 1.04. Ninety 150mm x150mm x 150mm cube specimens were produced for the compressive strength test. Model was fitted to data obtained on the compressive strength and mathematical model was developed based on Scheffe's model. The formulated model was tested for adequacy at 95% level of confidence using t-statistic and was found to be adequate. The compressive strength of concrete was observed to decrease with increase in the percentage replacement of cement with rice husk ash. The reduced value of the compressive strength may be due to lower specific gravity value of rice husk ash compared to that of cement. The blending of the two materials caused a reduction in strength value of the end product since specific gravity is strength related. The reduced compressive strength value may also be due to the fact that rice husk ash has less binding properties compared to cement. After 28 days of water curing, the concrete gave an average optimum compressive strength value of 22.84N/mm2 corresponding to a mix proportion of 0.95, 0.05, 1, 2 (cement, rice husk ash, sand, granite) at a water-cement ratio of 0.4.This compressive strength value obtained at 5% replacement is within the recommended value required for plain concrete works, lean concrete, simple foundations, masonry walls and other simple construction works in low-cost housing constructions.

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CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

The high cost of building materials such as cement, has hindered most citizens of Nigeria from affording their own accommodation (Umoh and Ujene, 2013; Aho and Utsev, 2008). In Nigeria and specifically Niger state, there are a lot of natural building and construction materials, namely, laterite, sharp sand, stones, timber and bricks. Despite the availability of these construction materials, most citizens of Nigeria still find it difficult to afford their own shelters (Uwe, 2010). Most of the housing units in Nigeria are constructed using concrete which has Ordinary Portland Cement (OPC) as a basic component material (Anyaogu and Eze, 2013; Onwuka *et al.*, 2013).

Cement is a scarce and costly building and construction material across the globe and the demand for it is growing on daily basis. There is therefore an urgent need to bring down the cost of cement by searching for alternative materials that can partially or fully replace the cement used in concrete and mortars, without affecting its strength, quality and/or other structural characteristics adversely. These materials will help the low income earners to afford their own houses most especially in the developing countries, such as, Nigeria (Coutinho, 2003).

Concrete is the most common and versatile building and construction material in the world. According to Neville and Brooks (1990), concrete is a product of water, cement and aggregates which, when sufficiently hardened, is used to support various structural loads. The strength of concrete depends on the relative proportions of the component materials

(Gambhir, 2004). Invariably, the total cost of concrete production is dependent on the availability and cost of its component materials. To reduce the cost of concrete materials and construction, efforts are being geared towards the use of abundant local building and construction materials either to totally or partially replace cement in concrete production. These materials are called pozzolanas. One of their major advantages in concrete is their slow hydration, which means low rate of heat development, which is very important in concrete production (Otoko and Chinwah, 1991). According to Kovacs (1975), good pozzolanic reaction reduces the porosity of the paste and therefore, improves impermeability of concrete.

The present study on the use of Rice Husk Ash (RHA) as a pozzolan in concrete production is one of such efforts. The use of RHA as partial replacement of cement will reduce the cost of concrete and helps in the provision of low-cost houses for both the rural and urban dwellers. However, the maximum utilization of RHA in concrete production is limited due to lack of knowledge of the structural properties of RHA-cement concrete as well as the production techniques required.

This study is concerned with the application of Scheffe's model in optimizing the compressive strength of concrete using rice husk ash (RHA) as partial replacement of cement. This will involve the formulation of mathematical models and the determination from the formulated mathematical models, the mix proportions that would predict and optimize the compressive strength of RHA-concrete. It is believed that research into the compressive strength of RHA-concrete will enhance its maximum utilization.

1.2 Statement of the Research Problem

Cement is the most important and expensive component of concrete. In Nigeria, the dependence on Portland cement for construction works is a source of concern. This is because cement plants are a significant source of sulphur dioxide, nitrogen dioxide and carbon monoxide and dioxide, which have adverse impacts on human health and the environment. Major projects in Nigeria are sometimes, not completed due to the high cost of production. Although new cement factories are being built to increase the availability of cement, the demand for cement is still high. Consequently, concrete structures and houses needed to accommodate the growing population of Nigeria, are expensive to construct. Low income earners, therefore, find it difficult to afford shelters of their own. Thus, the partial replacement of cement with a pozzolan such as RHA has become a necessity in order to reduce the cost of construction of concrete structures and houses, without adversely affecting the quality and strength. Apata and Alhassan (2012) studied the use of local materials such as rice husk ash, calcined clay, lime and fly ash as partial replacements for cement in concrete. They reported that up to 10% replacement of cement with these local materials can be used for low cost housing construction. Habeeb and Fayyadh (2009) studied the effect of rice husk ash on the average particle size on properties of concrete. They found that low strength values were obtained at early ages, while at age 28 days, concrete containing finer rice husk ash attained higher strength than the concrete sample having coarser rice husk ash particles. Nair *et al*. (2006) in their study on the use of rice husk ash as a replacement material to cement for rural housing, found that rice husk ash could be used to partially replace ordinary portland cement for rural housing.

1.3 Aim and Objectives of the Study

The aim of this study is to apply Scheffe's model in the optimisation of compressive strength of concrete using rice husk ash (RHA) as partial replacement for cement.

The objectives are to:

(i) Determine the physical properties of aggregates.

(ii) Determine the compressive strength of RHA-cement concrete, when RHA is used as partial replacement and the optimum content of RHA in the mix.

(iii) Formulate a mathematical model that will adequately predict the compressive strength of RHA-cement concrete.

1.4 Justification of the Study

This work would be beneficial to the society in the following ways:

(i) The results of this research will form an important part of literature on the use of natural pozzolana as replacement of cement in materials for concrete production.

(ii) The results of the compressive strength from this research will serve as reference materials to both students and practicing civil and structural engineers in design and construction of RHA-cement concrete mixes.

(iii) The models developed will be used as design codes and standards for the determination of design mixes and proportions of RHA-cement concrete.

1.5 Scope of the Study

The scope of this study includes to determine:

i. The physical properties of aggregates used for the laboratory experimentation.

ii. The compressive strength of RHA-cement concrete at zero percent (0%) replacement of cement with RHA. (Control specimen).

iii. The compressive strength of RHA-cement concrete at varying percentages (%) of replacement of cement with RHA.

iv. Formulate a mathematical model based on Scheffe's technique to predict the compressive strength of RHA-cement concretes.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Concrete

Gahlot and Sanjay (2006) defined concrete as a composite material that consists essentially of a binding medium in which is embedded particles or fragments of relatively inert material filler. In Portland cement concrete, the binder is a mixture of Portland cement and water; the filler may be any of the wide variety of natural or artificial aggregates. According to Manasseh (2010), concrete is a mixture of cement, fine and coarse aggregates and water, which are mixed in a particular proportion to get a particular strength. The cement and water react together chemically to produce a paste, which binds the aggregate particles together. The mixture then hardens to form a stone-like material that is good in compression, but weak in tension.

The constituents of concrete are mixture of cement, water, aggregates and sometimes admixtures. Admixtures are sometimes used to alter some of the properties of concrete. The voids in the cement particles are filled as the mixing water hydrates in other to form a void less paste. The paste coats the surface and fills the voids in both fine and coarse aggregates to form a void less mortar or fresh concrete, which hardens with time (Shetty, 2005). The characteristics of concrete depend largely on the types and characteristics of the constituent materials, method used in mixing the constituent materials, proportion of the constituent materials contained in the mix, transporting, placing, finishing, curing of the concrete, presence of entrained air, impurities, admixtures and the age of the concrete (Neville, 2003).

The most commonly used construction material in the world is concrete. It is an integral construction material. Concrete is mainly used in the construction of roads, bridges, highways, tunnels and other civil engineering structures (Neville, 2003). The advantages of concrete as a structural material are: its flexibility in design, ability to be pre-cast or cast in-situ, durability, economy, good compressive strength, availability of raw materials and ability to be cast into any shape (Mehta and Montero, 1997). The strength of hardened concrete are shear strength, compressive strength, static modulus of elasticity, split tensile strength, shear stress, water absorption, flexural strength and shear modulus (Osadebe, 2003). These properties depend largely on the component proportions.

2.1.1 Constituent materials

The primary constituent materials are water, cement, coarse and fine aggregates. The aggregates constitute about 60-80% of the total volume of concrete (Neville, 2003).

2.1.1.1 Cement

(i) Cementing materials

Cement has both cohesive and adhesive properties that give it the ability to bond mineral fragments into a compact whole (Rajput, 2004), it implies that the durability and strength of concrete depends on it. The chemical constituents are as given in Table 2.1 according to Ewa and Ukpata. (2013)

Table 2.1: Chemical Constituent of Cement (Ewa and Ukpata)

Oxide Composition	SiO_2	SO_3	Fe_2O_3	MgO	CaO	Al_2O_3	K_2O	NaO_2
Proportion (%)	20.79	1.48	2.64	1.66	63.34	4.51	1.26	0.18

There are two classification of cement used in the production of concrete. There are hydraulic and non-hydraulic cements. Hydraulic cements are those that set and harden under water as a result of chemical reaction between water and the minerals that make up the cement; a typical example is Portland cement. Non-hydraulic cement, such as, hydraulic lime gives an unstable product when its constituents react with water. Plaster of paris is an example of an unstable product (Neville, 2003).

The type of cement used in concrete production determines the properties of hardened cement (Neville, 2003). Portland cement is the most important cement used for both building and construction works. The most common is ordinary Portland cement.

The four basic compounds present in Portland cement are tricalcium aluminate ($3CaO.Al_2O_3$), tetracalciumaluminoferrite ($4CaO.Al_2O_3.Fe_2O_3$), tricalcium silicate ($3CaO.SiO_2$) and dicalcium silicate ($2CaO.SiO_2$). As the reaction between water and cement takes place, there is a large heat of evolution which leads to the formation of a gel that serves as a binder to the aggregate particles which in effect gives strength and water-tightness to concrete as it hardens. The physical properties of Ordinary Portland Cement (OPC) include setting time, heat of hydration, fineness, compressive strength, soundness and consistency, specific gravity and loss of ignition (Neville, 2003).

(ii) Lime

When limestone is subjected to high temperature, moisture and carbon dioxide is driven off, which results into lime as an end product. Pure limestone melts at a temperature of about 2570˚C. When the molten mineral is cooled, it solidifies into cubic crystals, but when ordinarily prepared, it is non-crystalline, easily powdered and has a specific gravity that ranges from 3.08 to 3.30 (Rajput, 2004).

(iii) Pozzolana

According to the American code, ASTM C618-78 (2005), pozzolanas are siliceous and aluminous materials, which themselves possess little or no cementing property, but in finely divide form and in the presence of water, react with calcium hydroxide at ordinary temperature to form a compound that has cementing properties.

The reaction which occurs during the mixing of pozzolona and OPC reduces the porosity of the paste and improves the impermeability of concrete (Kovacs, 1975). This mixture in optimum proportion also enhances the qualities of concrete and mortar in the fresh and hardened state.

According to Gupta (2013), the merits of pozzolana as partial replacement to cement in the production of concrete as increased resistance to sulfate attack, improved workability, increased resistance to alkali-silica reactivity, higher ultimate strength, reduced bleeding, increased durability and reduced shrinkage. Some typical examples of pozzolana are as follows: rice husk ash, blast furnace slag and fly ash which are further described in the next sections.

(a) Rice husk ash (RHA)

RHA generally referred to as an agricultural by-product of burning husk under controlled temperature of below 800C. The process produces about 25% ash containing 85% to 90% amorphous silica plus about 5% alumina, which makes it highly pozzolanic. A study conducted by (Mehta, 1997) indicated that concrete with RHA required more water for a given consistency due to its absorptive character of the cellular RHA particles. According to Uwe (2010), rice husk ash has more cementitious properties when compared with fly ash and silica fume. Rice husk ash is produced by burning rice husk.

(b) Blast furnace slag

Blast furnace slag is a by-product from the manufacture of iron in the blast furnace. Granulated blast furnace slag is formed by quick cooling of the molten slag in a granulator at high pressure and volume, after which an amorphous and coarse material which has cementitious properties is then formed as the end product (Anyaogu, *et al*, 2013).

(c) Fly ash

It is a by-product of electric generating plants. Fly ash is an artificial pozzolana which is obtained by burning of pulverized coal in electric power plants. Burning of coal at a relatively low temperature tends to maintain the pozzolanic properties as higher temperature turn the glassy particles to crystalline and eventually make them useless as pozzolana (Uwe, 2010). According to Shetty (2005), the amorphous nature of fly ash is due to the fact that it is produced by rapid cooling and solidification of the ash

2.1.1.2 Water

Water is one of the most important constituents of concrete. A paste which holds concrete components together is formed when water is mixed with cement powder. The workability and strength of concrete is greatly affected by the quantity and quality of water used in mixing. Clean water is required for concrete production. Water that contains impurities such as carbonates and bicarbonates of potassium and sodium, calcium chloride, silt, and other impurities in excess should be avoided for concreting because of their numerous effects on the strength properties of concrete.

2.1.1.3 Aggregates

Aggregates are granular materials, such as crushed stone, gravel and sand which mix with cement to produce concrete. They are cheaper when compared to cement. Thus, in this way, any concrete with maximum quantity aggregate is an economical mix. Aggregates gives durability and higher volumetric stability to concrete (Gupta, 2013). Aggregates reduces concrete shrinkage and constitutes about 70-80 percent of the concrete volume (Shetty, 2005). The physical characteristics and chemical compositions of aggregates have many effects on properties of concrete in both its plastic and hardened states (Neville, 2003). Aggregates should have adequate strength and resistance to exposed weather conditions. Aggregates used for concrete production must be clean, durable, well graded and must not have any chemical elements that will adversely affect the concrete strength properties (Deodhar, 2009).

The use, behavior and durability of concrete are determined by surface texture, aggregate strength, specific gravity, water absorption, surface moisture, bulking of fine aggregate, bulk density, shapes and types of deleterious substances present in aggregate (Mehta, 2002).

According to Gahlot and Gehlot (2010), achieving economical production of cement concrete will imply using maximum quantity of aggregate and minimum quantity of cement.

Types of aggregates

According to Singh (2008), aggregates are classified according to grain size, origin and weight/volume. The selection of type, size, quantity and grading of aggregates are done on the basis of workability requirements.

(a) Classification according to size

(i) Fine aggregates

According to Gupta (2013), fine aggregates are aggregates that can pass through sieve of size 4.75mm and are retained on sieve of size 0.06mm. Examples of fine aggregate include fine sands, coarse sands and crushed fines. The maximum size of fine aggregate is 5mm. The function of fine aggregate is to fill all the open spaces in between the coarse aggregates.

(ii) Course aggregates

Aggregates that are retained on the sieve size 4.75mm are course aggregates. Examples of coarse aggregates are gravels, crushed rock, and blast furnace slag. According to Shetty (2005) and Neville (2003), the functions of coarse aggregates are as follows:

(i) They give volumetric stability to concrete as they build high resistance against the deformation due to shrinkage of cement.

(ii) They are cheap fillers for cementing materials.

(iii) They resist the action of applied loads, abrasion, percolation of moisture and weather action.

(b) Classification according to density

According to Neville (2003), the three types of aggregates based on density are as follows:

(i) Normal/standard aggregates

These are usually natural materials, hard crushed rock or natural gravels, which produce concrete of standard strength and have specific gravity values ranging from 2.6 to 2.7.

(ii) Lightweight aggregates

These are artificial and natural materials having lower densities than those of normal aggregates.

(iii) High density aggregates

These are aggregates that are used to make high density concrete where self-weight is considered as the most important design consideration.

(c) Classification according to shape and surface texture

Aggregates are also classified according to shape as elongated, flaky, angular, rounded and according to surface texture as granular, honeycomb, smooth and rough. The aggregate shape and texture influence the properties of aggregates and the strength of concrete.

Aggregates shape and texture affect workability of fresh concrete (Deodhar, 2009). Aggregates having irregular shapes also improve the bond between components of concrete. Well-rounded aggregates need less water and cement for a given workability due to their small surface area.

2.1.2 Properties of concrete

Durability and Strength are most important properties of hardened concrete. The strength properties of hardened concrete are compressive strength, shear strength, flexural strength, split tensile strength, poisson's ratio, static modulus of elasticity, shear modulus, shear stress and water absorption. These properties depend largely, on the component proportions (Osadebe, 2003). According to Gupta (2013), durability of concrete may be defined as the ability of concretes to resist cracking and deterioration due to chemical and atmospheric exposure. The strength and durability of hardened concrete are evaluated based on the 28 days test results.

2.2 Mix Design of Concrete and its Production

In concrete mix design, the fractions of concrete components, such as, cement, water, coarse and fine aggregates are selected to make concrete that has the desired properties at minimum practicable cost (Gupta, 2013).

Also, Gahlot (2009), opined that the main objectives of concrete mix design are to:

(i) Satisfy the properties of hardened concrete, such as, strength and durability.

(ii) Satisfy the requirement of fresh concrete

(iii) Achieve economical design of concrete mixes

In concrete mix design, the components of concrete are measured either by weight or volume.

Furthermore, Simon *et al.* (1997) noted that the general procedures for concrete mixtures proportioning are:

(i) Generating a set of starting trial mix ratios

(ii) Performing one or more trial batches using the initial set of trial mix ratios

(iii) Adjusting the ratios in the subsequent trial batches until the desired properties are achieved.

Examples of typical methods of concrete mix design are the British method, Department of Environment method, International Standards method and the American Concrete Institute method. However, these methods are not commonly used in concrete production and practice as they are both time and energy consuming (Gupta, 2013). They are also not economical as money is wasted in trying to achieve the appropriate mix ratios. Optimisation is therefore a necessity in concrete mixture in order to identify the proper mix ratios that will give the optimal concrete structural properties (Scheffe, 1958).

2.2.1 Optimization techniques

Optimisation of concrete mixture involves using a statistical concept to fit empirical models to the experimental concrete property data (Osadebe, 2003). Optimization is the act of obtaining the best result under any given circumstance. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit, since the effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables (Gen and Cheng, 1997). In concrete mixture optimization, the engineer finds the maximum or minimum value of a multivariable function and the constraints on the decision variables must be satisfied.

The concrete property of interest is a function of the proportions of the component materials, such as, water-cement ratio, cement, fine and coarse aggregates and sometimes, the proportion of the supplementary cementing materials in concrete. Studies have been carried out using optimization techniques to optimize the concrete properties (Osadebe, 2003; Scheffe, 1958).

2.2.2 Scheffe's technique

Scheffe's model is based on simplex lattice design (Scheffe, 1958). According to Becker (1968), a simplex lattice is a structural representation of the lines joining the atoms or components of a mixture. In simplex lattice design; interactions of components occur in the factor space. A factor space is a one-dimensional, two-dimensional, three-dimensional or any other imaginary space, where components of the mixture interact. A factor space consists of three regions, the vertices, borderlines and the inside body space. The (q-1) space is used to define the boundary where interaction of q-components occurs in a mixture (Scheffe, 1958). At any point in the factor space, the total proportion must sum up to unity. It is a constrained optimization model. The simplex lattice design for a 2, 3 and 4-component mixtures are shown in Figures 2.1 to 2.3 respectively.

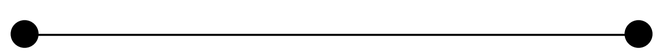

Figure 2.1: Simplex Lattice for a 2-component mixture (Scheffe, 1958)

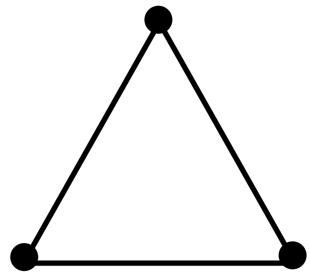

Figure 2.2: Simplex Lattice for a 3-component mixture (Scheffe, 1958)

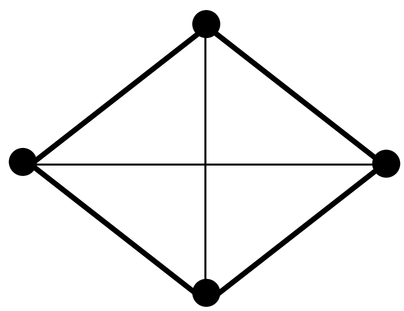

Figure 2.3: Simplex Lattice for a 4-component mixture (Scheffe, 1958**)**

A (q, m) simplex lattice design for q components is made up of points defined by the following coordinate arrangements; the proportions assumed by each component take the m + 1 uniformly equally spaced values in the interval of 0 to 1 given by X_i = 0,1/m, 2/m,…,1(where: q = number of components and m = degree of the polynomial). All possible combinations of the above coordinate settings are called design points. The simplex lattice design consists of all combination of the constituents of the mixture, with each mixture component assuming the above values. According to Scheffe (1958), the experimental region or factor space of interest is defined by the above coordinate values.

The mixes for the points x_1, x_2, x_3 is obtained from or the factor space is obtained from

$N = \frac{(q+m-1}{m(q-1)!}$as follows: For example, for m =2 and q = 3, the number of experiment is:

$$(x_1\ x_2\ x_3) = (1\ 0\ 0), (0\ 1\ 0), (0\ 01), (\tfrac{1}{2},\tfrac{1}{2},0),(\tfrac{1}{2},0,\tfrac{1}{2}),(\ 0, \tfrac{1}{2},\tfrac{1}{2}) = 6 \qquad (2.1)$$

2.3 Adequacy Test of the Model

The predicted and experimental values are tested for adequacy using the following hypotheses.

Null hypothesis Ho: There is no significant difference between the experimental and predicted values at 5% level of significance.

Alternative hypothesis H_1: There is a significant difference between the experimental and predicted values at 5% level of significance.

The Null hypothesis (H_0)" is the hypothesis that is being tested for validity. Alternative hypothesis (H_1) is accepted when the Null hypothesis has been rejected" (Nwachukwu, 2005). The Null hypothesis (H_0) being true means that the difference between the predictive and experimental values is very small.

2.4 Literature Works without Optimization

Apata and Alhassan (2012) studied the use of local materials, such as, rice husk ash, calcined clay, lime and fly ash as partial replacements for cement in concrete. They reported that up to 10% replacement of cement with these local materials can be used for low cost housing construction.

Habeeb and Fayyadh (2009) studied the effect of rice husk ash average particle size on properties of concrete. They found that low strength values were obtained at early ages, while at age 28 days, concrete containing finer rice husk ash attained higher strength than the concrete sample having coarser rice husk ash particle.

The research work of Wada *et al*. (2000) was on the strength properties of concrete containing highly reactive rice husk ash. They stated that rice husk ash mortar and concrete gave higher compressive strength than the control mortar and concrete. They reported that silica fume and rice husk ash gave higher strength values than ordinary portland cement concrete at 28 days of curing and above at 15% replacement. Aho and Utsev (2008) investigated the compressive strength of hollow sandcrete blocks made with rice husk ash as partial replacement for cement. They concluded that blocks made with 30% rice husk ash replacement of cement met the requirements specified by BS 2028 (1968). Mehta and Pirtz (2000) studied the use of rice hush ash to reduce temperature in high strength mass concrete and found that rice husk ash is very capable of bringing down the temperature of mass concrete compared to normal cement concrete.

Donster (2009) who studied the use of silica fume in in-situ concrete reported that the use of silica fume in in-situ concrete during construction increased the cohesiveness of fresh concrete.

Makarand *et al.* (2014) in their research found that rick husk ash improved the structural properties of concrete. Sakr (2006) studied the effects of silica fume and rice husk ash on the properties of heavy weight concrete. He found that 30-40% replacements of cement with rice husk ash was adequate for structural concrete. Rukzon *et al.* (2006) investigated the effect of grinding on the chemical and physical properties of rice husk ash and the effects of rice husk ash fineness on mortar properties and found that pozzolanas that have finer particles produced higher pozzolanic reaction. Ettu *et al*. (2013a) studied the variation of ordinary portland cement-rice husk ash-sawdust ash-composites strength under prolonged curing. They concluded that the compressive strength values obtained using the pozzolanas can be improved by grinding them to finer particles.

Cordeiro *et al.* (2009) studied concrete containing Brazilian rice husk ash and rice straw ash. They reported that grinding rice husk ash to finer particles increased the pozzolanic reaction of rice husk ash. Also, they concluded that combining rice husk ash with lime produced a weak cementitious material, which could be used for stabilization of laterite and improve the bearing strength of the material. Fadzil *et al.* (2008) carried out research on the properties of tenary blended cement concrete containing rice husk ash and fly ash as partial replacement materials. They reported that the compressive strength of concrete with the tenary blended cement, produced low strength at early ages compared to normal cement concrete, but higher than ordinary blended cementitious concrete with fly ash.

Cisse and Laquerbe (2000) studied the mechanical properties of sandcrete blocks with rice husk ash as partial replacement. They reported that sandcrete blocks produced using coarse rice husk ash as partial replacement, had greater mechanical resistance than the control sandcrete blocks. Rukzon and Chindaprasirt (2006) studied the compressive strength of fly ash- rice husk ash blended cement. They reported that the compressive strength values at ages of 28 and 90 days of the ordinary blended cement mortar with 10% and 20% rice husk ash, were a little higher than the control values, but less than the strength values obtained using fly ash. They concluded that 30% of rice husk ash- fly ash was suitable for structural concrete.

Rajput *et al*. (2013) studied the effect of rice husk ash as pozzolana on strength of mortar. They concluded that 10% replacement of cement with rice husk ash gave the optimum value of compressive strength at the age of 28 days.

Ismail and Waliudin (1996) carried out research on the effect of rice husk ash on high strength concrete. They concluded that optimum replacement of cement by rice husk ash for structural concrete was 10-20%. Ramezanianpour *et al*. (2010) investigated the mechanical properties and durability of concrete containing rice husk ash as supplementary cementing material. They reported that burning rice husks at temperatures below 700^0C produced rice husk ash with high pozzolanic activity. Nair *et al.* (2006) carried out research on the use of rice husk ash as a replacement material to cement for rural housing. They found that rice husk ash could be used to partially replace ordinary portland cement for rural housing. Agbede and Obam (2008) studied the strength properties of ordinary portland cement-rice husk ash blended sandcrete blocks. They found that good quality sandcrete blocks were obtained with 17.5% percentage replacement of ordinary portland cement. Malhotra and Mehta (2004) reported that ground rice husk ash with finer particle size improved concrete properties.

Ganesan *et al.* (2008) evaluated the effect of replacement of ordinary portland cement with rice husk ash on concrete. They found that the compressive strength of rice husk ash blended cement concrete increased with curing period but decreased as the percentage of rice husk ash increased in concrete. Dabai *et al*. (2009) studied the effect of rice husk ash as a cement admixture and concluded that rice husk ash can be used as cement replacement material at 10% and 20% replacements respectively.

Piyush and Vandana (2014) studied concrete properties using rice husk ash and marble powder. They found that when 21% of cement is replaced by 16% rice husk ash and 5% marble powder, compressive and flexural strength values were more than the values for normal concrete. Kartinia and Mahmud (2008) in their study on improvement of mechanical properties of rice husk ash concrete with super plasticizer, found that concrete containing up to 30% rice husk ash can attain strength of $30N/mm^2$ at 28 days. DaoVan and PhamDuy (2008) studied several key properties of high strength concrete using Rice Husk Ash as cement replacement material in concrete production. They concluded that 10% replacement of cement with Rice Husk Ash was acceptable for structural concrete.

Ramezanianpour and Khani (2009) investigated the effect of rice husk ash on mechanical properties and durability of sustainable concrete. They found that rice husk ash as a natural pozzolan improved the durability of concrete. Zemke and Woods (2009) used rice husk ash as partial replacement of cement in concrete. They found that up to 30% replacement was acceptable for structural concrete. Harunur and Keramat (2010) studied the durability of rice husk ash blended cement mortar and found that up to 20% replacement percentage was adequate for structural concrete. Abdhilash and Arbind (2011) studied the use of rice husk ash as partial replacement of cement in concrete production and found that only about 10% replacement was suitable for structural concrete. Kartini (2011) studied the use of rice husk ash in concrete production. He found that rice husk ash is a pozzolanic material that has the potential for use as partial replacement to cement. Malleswara and Patnaikuni (2012) studied the performance of rice husk ash concrete exposed to sea water. They found that only about 7.5% replacement showed better compressive strength values. Maurice and Godwin (2012) studied the compressive strength of rice husk ash blended cement concrete.

They concluded that about 5-10% replacement was adequate for structural concrete. Marthong (2012) studied the effect of rice husk ash properties. They concluded that only 20% replacement was the optimum value for structural concrete. Umoh and Ujene (2013) studied the use of rice husk ash blended cement in lateritic brick and concrete production and found that a mix ratio of 1:1:2 containing 30% rice husk ash and 50% rice husk ash in the mix ratio of 1:2:3 are suitable for masonry and insulating concrete respectively.

Umoh (1997) carried out research on the effect of rice husk ash on strength of low workability concrete and found that optimal percentage replacements of 30-40% of cement with rice husk ash produced a concrete of compressive strength in the range of 15-25N/mm^2. Chik *et al.* (2011b) studied the properties of ordinary portland cement-rice husk ash blended cement concrete blocks. They concluded that 15% percentage replacement was the optimal percentage for good performance of mansory blocks. Nagrale *et al.* (2012) investigated the effect of rice husk ash on properties of ordinary portland cement blended concrete and found that concrete strength increased with increase in rice husk ash percentage replacements in the range of 15-25%. Ramasamy (2012) investigated the compressive strength and durability properties of rice husk ash blended cement concrete.

He found that the compressive strength increased after 90 days of curing and the optimum percentage replacement was 7%. Karim *et al*. (2012) reviewed the effect of rice husk ash on properties of normal concrete and found that up to about 20-30% replacement was adequate for structural concrete. Rodriguez de Sensale (2006) studied the strength development of concrete with rice husk ash. He found that the compressive strength of rice husk concrete improved with increase in rice husk ash content. Zhang and Malhotra (2004) studied the properties of high performance concrete using rice husk ash as supplementary cementing material. They found that rice husk ash blended cement concrete produced higher compressive strength values at various ages up to 730 days compared with the control values. Abalaka and Okoli (2013) studied the strength development and durability properties of concrete containing pre-soaked rice husk ash. They reported that 20% replacement of cement with rice husk ash gave higher value of compressive strength for both water cured and uncured concrete cubes. Rashid *et al*. (2010) carried out research on the effect of incorporating rice hush ash on the strength and durability of concrete. They found that 20% replacement of cement with rice husk ash gave better values of compressive strength and porosity over plain mortar.

Chik *et al.* (2011a) in their study on the effect of rice husk ash on the performance of concrete block found that 10 -15% replacement of cement with rice husk ash were the suitable values for structural concrete. Sinulingga *et al.* (2014) studied the effect of three different types of rice husk ash as admixture for ordinary portland cement and found that 10-15% replacement of cement with rice husk ash were the optimum values for structural concrete. Hashem *et al.* (2013) investigated the acid resistance of portland cement pastes using rice husk ash and cement kiln dust as additives and found that the presence of rice husk ash and cement kiln dust in cement pastes improved the resistance of the mortar to sulphuric acid attack. Narrayan (2005) reported that 5-10% replacement of cement with rice hush ash can improve the workability and impermeability of concrete mixes.

2.4.2 Works with optimization

Obam (2009) formulated a mathematical model for the optimisation of modulus of rice husk ash blended cement concrete using Osadebe's regression theory. Anyaogu and Ezeh (2013) used Scheffe's theory to optimise the compressive strength of fly ash blended cement concrete. Onwuka *et al.* (2013) used Scheffe's theory to optimise the compressive strength of sawdust ash blended cement concrete. Obam (1998) formulated a mathematical model for the optimisation of compressive strength of palm kernel shell aggregate concrete using Scheffe's theory. Onwuka et al. (2013b) predicted the crushing strength of sawdust ash-clay fired brick using Scheffe's optimisation theory. Mama and Osadebe (2011), Osadebe and Nwankonobi (2007) formulated an optimisation model based on Scheffe's theory for laterized concrete mix proportioning in building construction. Osadebe (2003) formulated a generalized model for prediction of compressive strength of concrete as a multi-variate function of the properties of its constituent materials using Osadebe's theory.

Ezeh *et al.* (2010) developed a mathematical model for the optimisation of compressive strength of cement-sawdust ash sandcrete block using Scheffe's regression theory.

Osadebe and Ibearugbulem (2009) used Osadebe's alternative model to optimize the compressive strength of periwinkle shell-granite concrete. Obam (2006) investigated the accuracy of the predicted results using Scheffe's second and third degree optimisation regression polynomials. Obam and Osadebe (2006) formulated a mathematical model for the optimisation of rice husk ash blended concrete using Osadebe's regression theory.

Ndububa and Osadebe (2007) developed a mathematical model for the optimisation of flexural strength of fibre-cement mixture using Scheffe's simplex lattice theory. Osadebe and Nwankonobi (2007) formulated a mathematical model for the optimisation of flexural strength of palmnutfibre reinforced cement based composites using Scheffe's theory.

Obam (2007) found that rice husk ash concrete generally produce low compressive strength. Orie and Osadebe (2009) found that mound soil cement blended concrete can be used in construction but the mound soil content should not exceed 7% by weight of the cement for optimal flexural strength performance. Ibearugbulam *et al*. (2013) in their research on a new regression model for optimising concrete mixes found that the new regression model can be used in concrete mix design with advantages over the existing Scheffe's simplex and Osadebe's regression models.

Gahlot and Gehlot (2009) identified four procedures for proportioning concrete ingredients. These are Mix Design Methods according to the Indian Standard Recommended Guidelines, The ACI Mix Design Method, The USBR Mix Design Practice and The British Design Method. However, these procedures are based on empirical relationships, charts and graphs developed from extensive experimental investigations. Using the four mix design procedures would produce uneconomical concrete; they are also time and energy consuming. With increasing cost of the concrete constituent materials, the prediction of concrete mixture proportions becomes a necessity. Also, the increased number of concrete constituent materials makes prediction of concrete mixture proportions a necessity. The selection of proportions of concrete constituent materials to produce concrete of desired characteristics at a minimum practicable cost is called concrete mix design (Neville, 1996).

The Scheffe's method has not been used to optimize the mix constitutes of concrete to determine the compressive strength.

CHAPTER THREE

3.0 MATERIALS AND METHODS

3.1 Materials

The materials utilized in this study are Ordinary Portland Cement (OPC), sharp river sand, granite, rice husk ash and water.

Cement: The cement used as the binding agent was OPC brand of Dangote cement 42.5R with properties conforming to BS 12 :(1996).

Water: clean potable water devoid of acid and organic substances obtained from the University tap.

Coarse aggregate: Crushed granite of igneous rock with size range of 10 – 20mm conforming to BS 882: (1983).

Fine aggregate: Locally available river sand, passing through 4.75mm BS sieve conforming to grading of BS 882: (1983).

RHA: A supplementary cementitious material was obtained in sufficient quantities from Badeggi in Katcha Local Government Area of Niger State

3.1.1Batching and mixing of materials

The mixing and batching process of the materials was done by weighing cement and the percentage of RHA to cement. RHA varied from 5- 25% (at an interval of 5% that is 5,10,15,20 and 25%) replacement of OPC in concrete cubes.

3.2 Mix Ratios for Scheffe's Model

The response function to be predicted or optimized is given by Equation (2.34) and it is a function of the component factors X_1, X_2 , X_3 , X_4 and X_5 . These factors are subject to the constraint given by Equation (2.2) and also subject to the constraint that none of the component materials will have a negative value. The response is compressive strength. The property is dependent on the component proportions. The component materials are water, cement, rice husk ash, sand and granites. For each response, there are thirty (30) runs of experiment. The first fifteen (15) runs were obtained using Equation (2.3) to formulate the model and are called the trial mixes. These values are given in Table 3.4. Additional fifteen (15) mix ratios were also generated for each response and were used as check points to validate the model for the response. The mix ratios shown on Tables 3.4 and 3.5 are based on Scheffe's (5, 2) factor space for points 1 to 5.

3.3 Concrete Mix Design

The concrete mix system containing cement, water, RHA, granite and sand was designed based on Scheffe's statistical experimental method. Detailed calculations were carried out in order to obtain the different mix ratios for the design. From the obtained mix ratios, an Excel Spreadsheet was used in the calculation of the required quantities of materials that was used in this work.

3.4 Laboratory Tests

The laboratory works involved all the tests that were performed in the laboratory to determine the compressive strengths of the concrete produced in the laboratory.

(a) Physical property tests

The fine aggregate sample was tested to determine the following physical properties. These properties include sieve analysis, specific gravity, and bulk density.

Sieve analysis of fine aggregate (Sand 500g)

$$\text{Fineness Modulus} = \text{Total cummulative \% retained up to } 150\mu m \text{ sieve } /100 \quad (3.1)$$

$$= 340.32 /100 = 3.4$$

Coefficient of uniformity, Cu

$$Cu = D60/D10 \quad (3.2)$$

Coefficient of curvature, Cc

$$Cc = (D30)2 / (D10xD60) \quad (3.3)$$

where:

D60 =Size of sieve at 60% passing = 0.61mm, D10=Size of sieve at 10% passing = 0.260mm

D30 = Size of sieve at 30% passing = 0.40mm

Sieve analysis of coarse aggregate (Granite 1000g)

$$\text{Fineness Modulus} = \text{Total cummulative \% retained up to } 150\mu m \text{ sieve } /100 \quad (3.4)$$

$$= 424.06 /100 = 4.2$$

Sample preparation

Step 1: The rice husk ash burned sample was carried out at the step B laboratory of the Federal University of Technology Minna. The resulting rice husk ash was left to cool at room temperature. After which the sample was sieved to obtain finer particle size.

Step 2: The river sand was sieved so as to remove all deleterious content. The needed size of the sample was collected by passing it through a 4.75mm sieve size. The crushed granite with almost 20mm in size was obtained, and then the sieve analysis, bulk density and specific gravity tests were carried out.

Step 3: The constituent materials were then accurately measured.

Step 4: Concrete was mixed by varying the proportions of cement from 5 to 25% of RHA. Using a 150mm x 150mm x 150 mm steel cube mould, 3 cubes were cast from each of the 30 mix ratios based on Scheffe's model, making a total of 90 cubes. The freshly mixed concrete was filled into the mould in three compacted layers. The concrete was allowed to harden for 24 hours. The mould was removed and the cubes were water-cured for 28 days in the curing tank by total immersion.

At the end of the 28 days the cubes were crushed in the Universal Crushing machine to determine the compressive strength of each mix. The results and average of each test points were obtained as;

$$\text{Compressive Strength} = \frac{\text{compressive load of cube at failure (N)}}{\text{cross sectional area of mould (mm}^2\text{)}} \tag{3.5}$$

3.5 Method for the Optimization of Concrete

The objectives of this research would be achieved by applying Scheffe's mathematical model using simplex lattice design (Scheffe's, 1958).

3.5.1 Formulation of simplex lattice design

Simplex is a factor space or a polygon. The simplest simplex is a straight line; others could be a plane, tetrahedron or any other imaginary solids whose dimension is above three. RHA-OPC concrete is a five component mixture consisting of water, OPC, RHA, sand and crushed granite rock. This was analyzed using a five dimensional simplex lattice (pentahedron).

Scheffe (1958) introduced polynomial regression to model the response, called (q,n) polynomial which have to be of low degree (n)

where q = number of component

n = degree of polynomial

$$\text{if } n=1:\ f(x) \sum_{i=1}^{q} \beta_i x_i \tag{3.6}$$

$$\text{if } n=2: f(x) \sum_{i=1}^{q} \beta_i x_i + \sum_{1 \le i \le j \le q}^{q} \beta_{ij} x_i x_j \tag{3.7}$$

$$\text{if } n=3:\ f(x) \sum_{i=1}^{q} \beta_i x_i + \sum_{1\le i\le j\le q}^{q} \beta_{ij} x_i x_j + \sum_{1\le i\le j\le k\le q}^{q} (\beta_{iij} x_i^2 x_j + \beta_{iij} x_i x_j x_k) \tag{3.8}$$

He also showed that the number of coefficient k in a (q,n) lattice is given by $C_q^n + n - 1$; where

$$k = \frac{q(q+1)\dots(q+n-1)}{n!} \qquad (3.9)$$

Hence, for our five criteria or parameter case

$$k = \frac{5(5+1)}{2!} = 15$$

For the five component mixture with two degree of reaction, the number of coefficient is 15. The diagrammatic expression of the criteria in Scheffe's proposition is shown in Figure 3.1

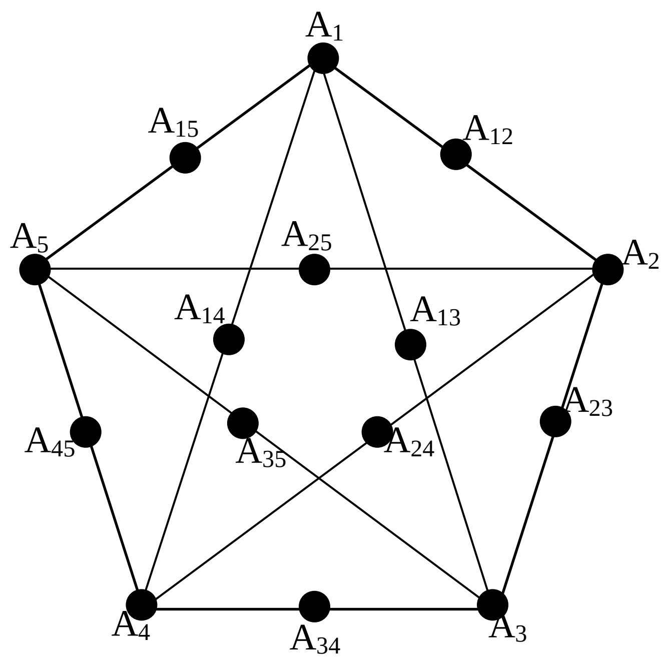

Figure 3.1: A (5,2) Pentahedron simplex lattice, representing Five-Component Concrete Mix

3.5.2 Five component factor space

The first five pseudo mix ratios are located at the vertices of the pentahedron simplex: that is, $A_1[1:0:0:0:0]$, $A_2[0:1:0:0:0]$, $A_3[0:0:1:0:0]$, $A_4[0:0:0:1:0]$ and $A_5[0:0:0:0:1]$.

Ten other pseudo mix ratios located at the mid points of the lines joining the vertices are: $A_{12}[0.5:0.5:0:0:0]$, $A_{13}[0.5:0:0.5:0:0]$, $A_{14}[0.5:0:0:0.5:0]$, $A_{15}[0.5:0:0:0:0.5]$,

$A_{23}[0{:}\,0.5{:}\,0.5{:}\,0{:}\,0]$, $A_{24}[0{:}\,0.5{:}\,0{:}\,0.5{:}\,0]$, $A_{25}[0{:}\,0.5{:}\,0{:}\,0{:}\,0.5]$, $A_{34}[1{:}\,0{:}\,0.5{:}\,0.5{:}\,0]$, $A_{35}[0{:}\,0{:}\,0.5{:}\,0{:}\,0.5]$ and $A_{45}[0{:}\,0{:}\,0{:}\,0.5{:}\,0.5]$.

Now according to Scheffe (1958), for a five component of RHA-OPC concrete, the proportion x_i of the i^{th} component of the mixture must satisfy the following constraints:

$x_i \geq 0$

$(i=1,2,3,4,5)$ (3.10)

and the sum of all proportions of the constituents of the five-component of RHA cement concrete must be equal to unity, therefore

$$\sum_{i=1}^{q} x_i = 1 \qquad (3.11)$$

for the five-component RHA cement concrete,

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1 \qquad (3.12)$$

The equation of response Y, for the five pseudo mix ratios is given as:

$$Y = b_0 + \sum b_i x_i + \sum b_i x_i x_j + \dots\dots\dots \qquad (3.13)$$

where $0 \leq i \leq j \leq 5$

expanding Equation (3.12) by substituting the values of ‘i’ and ‘j’, we obtain

$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{22}X_2^2$

$$+b_{23}X_2X_3+b_{24}X_2X_4+b_{25}X_2X_5+b_{33}X_3{}^2+b_{34}X_3X_4+b_{35}X_3X_5+b_{44}X_4{}^2+b_{45}X_4X_5+b_{55}X_5{}^2 \quad (3.14)$$

Where b is a constant coefficient

Multiplying equation (3.5) by b_0

$$b_0x_1 + b_0x_2 + b_0x_3 + b_0x_4 + b_0x_5 = b_0 \quad (3.15)$$

Multiplying Equation (3.11) successively by $x_1, x_2,\ x_3,\ x_4$ and x_5, and rearranging the products

$$x_1^2 = x_1 - x_1x_2 - x_1x_3 - x_1x_4 - x_1x_5$$

$$x_2^2 = x_2 - x_1x_2 - x_2x_3 - x_2x_4 - x_2x_5$$

$$x_3^2 = x_3 - x_1x_3 - x_2x_3 - x_3x_4 - x_3x_5 \quad (3.16)$$

$$x_4^2 = x_4 - x_1x_4 - x_2x_4 - x_3x_4 - x_4x_5$$

$$x_5^2 = x_5 - x_1x_2 - x_2x_5 - x_3x_5 - x_4x_5$$

Substituting Equations (3.15) and (3.16) into Equation (3.14)

$$Y=(b_0+b_1+b_{11})X_1 + (b_0+b_2+b_{22})X_2 + (b_0+b_3+b_{33})X_3 + (b_0+b_4+b_{44})X_4 + (b_0+b_5+b_{55})X_5 + (b_{12} - b_{11} - b_{22})X_1X_2 + (b_{13} - b_{11} - b_{33})X_1X_3 + (b_{14} - b_{11} - b_{44})X_1X_4 + (b_{15} - b_{11} - b_{55})X_1X_5 + (b_{23} - b_{22} - b_{33})X_2X_3 + (b_{24} - b_{22} - b_{44})X_2X_4 + (b_{25} - b_{22} - b_{55})X_2X_5 + (b_{34} - b_{33} - b_{44})X_3X_4 + (b_{35} - b_{33} - b_{55})X_3X_5 + (b_{45} - b_{44} - b_{55})X_4X_5 \quad (3.17)$$

The reduced second degree polynomial for a quinary system is as follows:

$$Y = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 \quad (3.18)$$

Denoting:

$$\beta_i = b_0 + b_i + b_{ii} \quad \text{and} \quad \beta_{ij} = b_{ii} - b_{jj} \quad (3.19)$$

The solution of equation (3.14) as given by Scheffe (1958) for the coefficient of the polynomial is:

$$\beta_i = Y_i \quad \text{and} \quad \beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j \quad (3.20)$$

where$\beta_i = \beta_1, \beta_2, ..., \beta_5$

$\beta_{ij} = \beta_{12}, \beta_{13} ..., \beta_{45}$

Equation (3.14) is the response function for optimization of RHA-cement concrete of five components. The terms n_i and n_{ij} are the responses (compressive strength) at the points i and ij

3.6 Actual and Pseudo Mix Ratios

Scheffe (1958) in his mixture design expressed the relationship between the actual mix ratios and the pseudo mix ratios with the following equation

$$[Z] = [A]\,[X] \quad (3.21)$$

where $Z =$ Actual mix ratio

$X =$ Pseudo mix ratio

A = Constant; a five by five matrix, and would be obtained from the first five mix ratios

The first five actual mix ratio, water: cement: RHA: sand: granite with 5 to 25% replacement of OPC with RHA, generated from the basic mix designed are:

$Z_1[0.45:0.95:0.05:1:2]$, $Z_2[0.6:0.9:0.1:2:3]$, $Z_3[0.55:0.85:0.15:1.5:3]$,

$Z_4[0.5:0.8:0.2:2:2]$,

and $Z_5[0.65:0.75:0.25:2:4]$. The corresponding pseudo mix ratios are of an identity matrix thus:

$X_1[1:0:0:0:0]$, $X_2[0:1:0:0:0]$, $X_3[0:0:1:0:0]$, $X_4[0:0:0:1:0]$, and $X_5[0:0:0:0:1]$.

Where X_1=Proportion of water/cement ratio

X_2= Proportion of OPC

X_3= Proportion of RHA

X_4= Proportion of sand

X_5= Proportion of granite

Table 3.1 shows the initial set of mix ratios for five components RHA – OPC concrete

Table 3.1: Initial set of Mix Ratios (Actual) for five component of RHA – OPC concrete

S/N	Principal Coordinates	Water/Cement ratio	Cement	RHA	Sand	Granite
1	N_1	0.40	0.95	0.05	1	2
2	N_2	0.45	0.90	0.10	1.5	3

3	N_3	0.50	0.85	0.15	2	2
4	N_4	0.55	0.80	0.20	2	4
5	N_s	0.60	0.75	0.25	3	6

From the actual mix ratios, a Z-matrix is formed whose transpose will give the A-matrix. This would be employed to determine the actual mix ratios for all the experimental and control points. These matrices are indicated below.

$$Z = \begin{bmatrix} 0.40 & 0.95 & 0.05 & 1 & 2 \\ 0.45 & 0.90 & 0.10 & 1.5 & 3 \\ 0.50 & 0.85 & 0.15 & 2 & 2 \\ 0.55 & 0.80 & 0.20 & 2 & 4 \\ 0.60 & 0.75 & 0.25 & 3 & 6 \end{bmatrix}$$

$$Z^T = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} = A$$

Thus

$$\text{Matrix A} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix}$$

Substituting matrix A into equation (3.17), yields

For A_{12}:

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \left[\begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix}\right] \begin{Bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.425 \\ 0.925 \\ 0.075 \\ 1.25 \\ 2.5 \end{bmatrix}$$

For A_{13}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \left[\begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix}\right] \begin{Bmatrix} 0.5 \\ 0 \\ 0.5 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.45 \\ 0.90 \\ 0.10 \\ 1.50 \\ 2 \end{bmatrix}$$

For A_{14}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0 \\ 0 \\ 0.5 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.475 \\ 0.875 \\ 0.125 \\ 1.5 \\ 3 \end{bmatrix}$$

For A_{15}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0.5 \\ 0 \\ 0 \\ 0 \\ 0.5 \end{Bmatrix} = \begin{bmatrix} 0.50 \\ 0.85 \\ 0.15 \\ 2 \\ 4 \end{bmatrix}$$

For A_{23}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.5 \\ 0.5 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.475 \\ 0.875 \\ 0.125 \\ 1.75 \\ 2.5 \end{bmatrix}$$

For A_{24}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.5 \\ 0 \\ 0.5 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.50 \\ 0.85 \\ 0.15 \\ 1.75 \\ 3.5 \end{bmatrix}$$

For A_{25}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.5 \\ 0 \\ 0 \\ 0.5 \end{Bmatrix} = \begin{bmatrix} 0.525 \\ 0.825 \\ 0.175 \\ 2.25 \\ 4.5 \end{bmatrix}$$

For A_{34}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.5 \\ 0.5 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.525 \\ 0.825 \\ 0.175 \\ 2 \\ 3 \end{bmatrix}$$

For A_3

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.5 \\ 0 \\ 0.5 \end{Bmatrix} = \begin{bmatrix} 0.55 \\ 0.80 \\ 0.20 \\ 2.5 \\ 4 \end{bmatrix}$$

For A_{45}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \\ 0.5 \end{Bmatrix} = \begin{bmatrix} 0.575 \\ 0.775 \\ 0.225 \\ 2.5 \\ 5 \end{bmatrix}$$

The obtained actual values for the experimental points with their corresponding pseudo mix ratios are shown in Table 3.2.

Table 3.2: Mixture Proportions for Actual and Pseudo Components for formulation of the Model

s/n	Response	Actual Components					Pseudo Components				
		Water	Cement	RHA	Sand	Granite	Water	Cement	RHA	Sand	Granite
		Z_1	Z_2	Z_3	Z_4	Z_5	X_1	X_2	X_3	X_4	X_5
1	Y_1	0.40	0.95	0.05	1	2	1	0	0	0	0
2	Y_2	0.45	0.9	0.1	1.5	3	0	1	0	0	0
3	Y_3	0.50	0.85	0.15	2	2	0	*0*	1	0	0
4	Y_4	0.55	0.8	0.2	2	4	0	0	0	1	0
5	Y_5	0.60	0.75	0.25	3	6	0	0	0	0	1
6	Y_{12}	0.425	0.925	0.075	1.25	2.5	0.5	0.5	0	0	0
7	Y_{13}	0.45	0.90	0.10	1.5	2	0.5	0	0.5	0	0
8	Y_{14}	0.475	0.875	0.125	1.5	3	0.5	0	0	0.5	0
9	Y_{15}	0.50	0.85	0.15	2	4	0.5	0	0	0	0.5
10	Y_{23}	0.475	0.875	0.125	1.75	2.5	0	0.5	0.5	0	0
11	Y_{24}	0.50	0.85	0.15	1.75	3.5	0	0.5	0	0.5	0
12	Y_{25}	0.525	0.825	0.175	2.25	4.5	0	0.5	0	0	0.5
13	Y_{34}	0.525	0.825	0.175	2	3	0	0	0.5	0.5	0
14	Y_{35}	0.55	0.8	0.20	2.5	4	0	0	0.5	0	0.5
15	Y_{45}	0.575	0.775	0.225	2.5	5	0	0	0	0.5	0.5

To validate the model, extra fifteen points (control points) C_1, C_2, C_3, C_4, C_5, C_{12}, C_{13}, C_{14}, C_{15}, C_{23}, C_{24}, C_{25}, C_{34}, C_{35}, C_{45} of observations were determined and used in the ANOVA test. These control points served as control mix ratios of the RHA-cement concrete mixes in this study. The pseudo mixes used in validating the model were chosen arbitrarily from the lines joining the vertices with the consideration that each must sum up to unity. These values are:

$X_1[0.4{:}0{:}0.4{:}0{:}0.2], X_2[0{:}0.6{:}0{:}0.4{:}0], X_3[0.8{:}0{:}0.2{:}0{:}0],$

$X_4[0{:}0.4{:}0{:}0.6{:}0], X_5[0.6{:}0{:}0{:}0{:}0.4], X_{12}[0{:}0{:}0.8{:}0.2{:}0],$

$X_{13}[0.6{:}0.2{:}0{:}0{:}0.2], X_{14}[0{:}0.4{:}0{:}0.4{:}0.2], X_{15}[0.2{:}0{:}0{:}0{:}0.8], X_{23}[0{:}0{:}0.2{:}0.8{:}0],$

$X_{24}[0.2{:}0{:}0.6{:}0{:}0.2], X_{25}[0{:}0.2{:}0.4{:}0.2{:}0.2], X_{34}[0{:}0.8{:}0.2{:}0{:}0],$

$X_{35}[0{:}0.2{:}0{:}0.2{:}0.6]$ and $X_{45}[0.2{:}0.2{:}0.2{:}0.2{:}0.2]$.

Substituting these values into Equation (3.17) gives:

Control point for A_1

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0.4 \\ 0 \\ 0.4 \\ 0 \\ 0.2 \end{Bmatrix} = \begin{bmatrix} 0.48 \\ 0.87 \\ 0.13 \\ 1.8 \\ 2.8 \end{bmatrix}$$

Control point for A_2

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.6 \\ 0 \\ 0.4 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.49 \\ 0.86 \\ 0.14 \\ 1.7 \\ 3.4 \end{bmatrix}$$

Control point for A_3

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0.8 \\ 0 \\ 0.2 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.42 \\ 0.93 \\ 0.07 \\ 1.2 \\ 2 \end{bmatrix}$$

Control point for A_4

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.4 \\ 0 \\ 0.6 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.51 \\ 0.84 \\ 0.16 \\ 1.8 \\ 3.6 \end{bmatrix}$$

Control point for A_5

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0.6 \\ 0 \\ 0 \\ 0 \\ 0.4 \end{Bmatrix} = \begin{bmatrix} 0.48 \\ 0.87 \\ 0.13 \\ 1.8 \\ 3.6 \end{bmatrix}$$

Control point for A_{12}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.8 \\ 0.2 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.51 \\ 0.84 \\ 0.16 \\ 2 \\ 2.4 \end{bmatrix}$$

Control point for A_{13}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0.6 \\ 0.2 \\ 0 \\ 0 \\ 0.2 \end{Bmatrix} = \begin{bmatrix} 0.45 \\ 0.9 \\ 0.1 \\ 1.5 \\ 3 \end{bmatrix}$$

Control point for A_{14}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.4 \\ 0 \\ 0.4 \\ 0.2 \end{Bmatrix} = \begin{bmatrix} 0.52 \\ 0.83 \\ 0.17 \\ 2 \\ 4 \end{bmatrix}$$

Control point for A_{15}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0.2 \\ 0 \\ 0 \\ 0 \\ 0.8 \end{Bmatrix} = \begin{bmatrix} 0.57 \\ 0.78 \\ 0.22 \\ 2.5 \\ 5.4 \end{bmatrix}$$

Control point for A_{23}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.2 \\ 0.8 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.54 \\ 0.81 \\ 0.19 \\ 2 \\ 3.6 \end{bmatrix}$$

Control point for A_{24}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0.2 \\ 0 \\ 0.6 \\ 0 \\ 0.2 \end{Bmatrix} = \begin{bmatrix} 0.5 \\ 0.85 \\ 0.15 \\ 2 \\ 2.8 \end{bmatrix}$$

Control point for A_{25}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.2 \\ 0.4 \\ 0.2 \\ 0.2 \end{Bmatrix} = \begin{bmatrix} 0.52 \\ 0.83 \\ 0.17 \\ 2.1 \\ 3.4 \end{bmatrix}$$

Control point for A_{34}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.8 \\ 0.2 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 0.46 \\ 0.89 \\ 0.11 \\ 1.6 \\ 2.8 \end{bmatrix}$$

Control point for A_{35}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.2 \\ 0 \\ 0.2 \\ 0.6 \end{Bmatrix} = \begin{bmatrix} 0.56 \\ 0.79 \\ 0.21 \\ 2.5 \\ 5 \end{bmatrix}$$

Control point for A_{45}

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{bmatrix} 0.40 & 0.45 & 0.50 & 0.55 & 0.60 \\ 0.95 & 0.90 & 0.85 & 0.80 & 0.75 \\ 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\ 1 & 1.5 & 2 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \end{bmatrix} \begin{Bmatrix} 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \end{Bmatrix} = \begin{bmatrix} 0.5 \\ 0.85 \\ 0.15 \\ 1.9 \\ 3.4 \end{bmatrix}$$

The obtained actual values for the control points with their corresponding pseudo mix ratio are shown in Table 3.3.

Table 3.3: Mixture Proportions for the Actual and Pseudo Components of Control Observation Points

S/N	Points	Actual Components					Pseudo Components				
		Water Z_1	Cement Z_2	RHA Z_3	Sand Z_4	Granite Z_5	Water X_1	Cement X_2	RHA X_3	Sand X_4	Granite X_5
1	C_1	0.48	0.87	0.13	1.8	2.8	0.4	0	0.4	0	0.2
2	C_2	0.49	0.86	0.14	1.7	3.4	0	0.6	0	0.4	0
3	C_3	0.42	0.93	0.07	1.2	2	0.8	*0*	0.2	0	0
4	C_4	0.51	0.84	0.16	1.8	3.6	0	0.4	0	0.6	0
5	C_5	0.48	0.87	0.13	1.8	3.6	0.6	0	0	0	0.4
6	C_{12}	0.51	0.84	0.16	2	2.4	0	0	0.8	0.2	0
7	C_{13}	0.45	0.9	0.1	1.5	3	0.6	0.8	0	0	0.2
8	C_{14}	0.52	0.83	0.17	2	4	0	0	0	0.4	0.2
9	C_{15}	0.57	0.78	0.22	2.7	5.4	0.2	0	0	0	0.8
10	C_{23}	0.54	0.81	0.19	2	3.6	0	0	0.2	0.8	0
11	C_{24}	0.5	0.85	0.15	2	2.8	0.2	0.2	0.6	0	0.2
12	C_{25}	0.52	0.83	0.17	2.1	3.4	0	0.8	0.4	0.2	0.2
13	C_{34}	0.46	0.89	0.11	1.6	2.8	0	0.2	0.2	0	0
14	C_{35}	0.56	0.79	0.21	2.5	5	0	0.2	0	0.2	0.6
15	C_{45}	0.5	0.85	0.15	1.9	3.4	0.2	0	0.2	0.2	0.2

3.7 Preparation of Concrete Specimen and Structural Properties Tests

The quantities of the concrete component materials obtained from mix design were weighed and kept at a particular location. Thereafter, the mixing of measured quantities of concrete ingredients took place. Mixing was achieved manually with a shovel. The mixing process continued until a uniform mix was obtained. The well-mixed concrete was finally compacted into concrete moulds and then vibrated by shaking to achieve full compaction and also to remove air voids. The surface of the compacted concrete was then troweled to smoothen the concrete surface. The concrete was demoulded after 24 hours of casting and then transferred to a curing tank and allowed to cure for 28 days before testing.

3.8 Statistical Student's T-test for the Model

From Table 4.8, the value of total variation was calculated:

$\sum D = 8.48$

$N = 15$

$D = (\sum Di)/N = 8.48/15 = 0.57$

$S^2 = \sum(YE - YP)^2/(N-1) = 2.49$

$S = \sqrt{2.49} = 1.25$

Actual value of total variation in t-test

$t = (D*N \text{^}0.5)/S = 1.76$

Allowable total variation in t-test:

Degree of freedom = N- 1 = 15

5% significant for two-tailed test 1= 0.975

Allowable total variation in t-test = $t_{(0.975,\ 14)}$ = 2.14 (obtainable from statistical t-distribution table).

CHAPTER FOUR

4.0 RESULTS AND DISCUSSION

4.1 Results

4.1.1 Physical properties of aggregates

The results of specific gravity, bulk density, and sieve analysis test carried out on the coarse and fine aggregates are presented in Tables 4.1 to 4.6 respectively

(a) Particle size distribution

From the particle size distribution of fine aggregate, given in Table 4.1, the grading curve of the fine aggregate shown in Figure 4.1 was obtained by plotting the percentage passing against sieve sizes.

Table 4.1: Sieve analysis of fine aggregate (Sand 500g)

Sieve size (mm)	Weight of empty sieves (g)	Weight of sieves + sample (g)	Weight of sample retained (g)	Percentage retained (%)	Cumulative percentage retained (%)	Percentage passing (%)
5.00	476.0	477.9	1.9	0.38	0.38	99.62
3.35	468.3	476.4	8.1	1.62	2.0	98.0
2.36	426.7	461.4	24.7	4.94	6.94	93.06
2.00	418.0	432.7	14.7	2.94	9.88	90.12
1.18	385.7	441.8	56.1	11.22	21.1	78.9
0.85	353.4	400.8	47.4	9.48	30.58	69.42
0.6	468.2	520.1	51.99	10.4	40.98	59.02
0.425	435.4	526.6	91.22	18.24	59.22	40.78
0.3	312.0	404.3	92.3	18.46	77.68	22.32
0.15	420.7	490.1	69.4	13.88	91.56	8.44
0.075	378.1	416.5	38.4	7.68	99.24	0.76
Pan	298.6	302.4	3.8	0.76	100	0.00

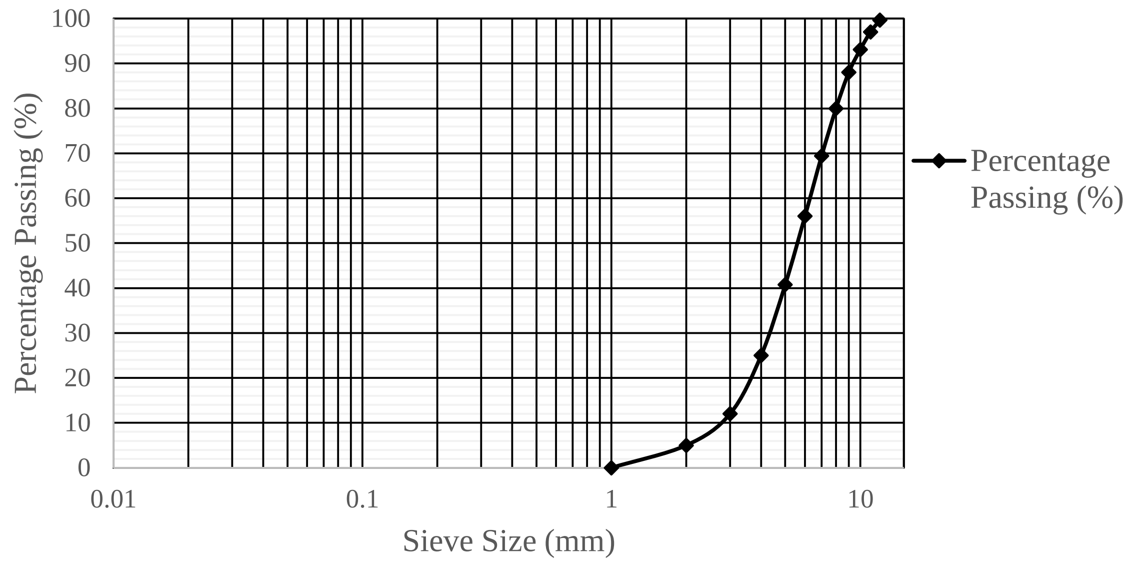

Figure 4.1: Particle Size Distribution Curve for Fine Aggregate

From the grading curve, the coefficient of uniformity, Cu and coefficient of curvature, Cc were calculated using Equations (3.2) and (3.3) respectively.

After substitution, the value of Cu is 2.17. This value is less than 4 for well-graded sand. And, the value of Cc is 1.04. This value falls within the recommended range of values (1-3) for a well graded aggregate.

(b) Results of sieve analysis of coarse aggregate (granite)

The results of Sieve Analysis of coarse Aggregate carried out are presented in Table 4.2

Table 4.2: Sieve analysis of coarse aggregate (Granite 1000g)

Sieve size (mm)	Weight of empty sieves (g)	Weight of sieves + sample (g)	Weight of sample retained (g)	Percentage retained (%)	Cumulative percentage retained (%)	Percentage passing (%)
28.0	1550.0	1550.0	0.0	0.0	0.0	100
20.0	1470.0	1512.0	42.0	4.2	4.2	95.8
14.0	1410.0	1678.7	268.7	26.87	31.07	68.93
10.0	1370.0	1950.0	580.0	58.0	89.07	10.93
6.5	1350.0	1457.0	107.0	10.7	99.77	0.23
5.0	1480.0	1481.9	1.8	0.18	99.95	0.05
3.35	468.3	468.8	0.5	0.05	100.0	0.0
Pan	830.0	830.0	0.0	0.0	0.0	

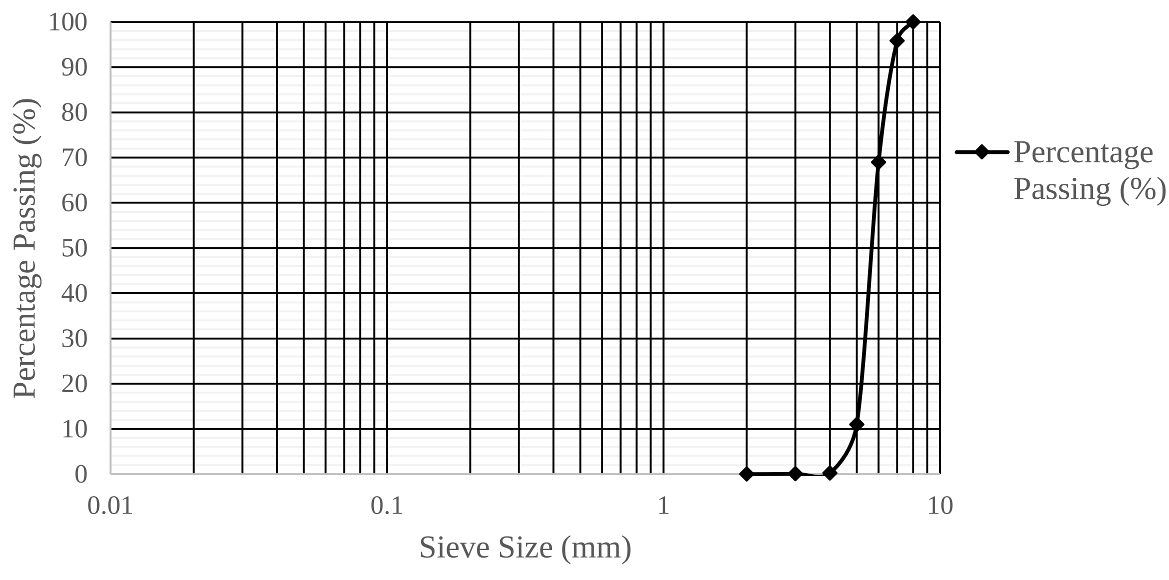

Figure 4.2: Particle Size Distribution Curve for Coarse Aggregate

(c) Results of specific gravity of aggregate (sand)

The results of specific gravity of sand carried out are presented in Table 4.3

Table 4.3: Specific gravity of aggregate (Sand)

Trials	1	2	3
Weight of Cylinder, W_1 (g)	120	120	120
Weight of Cylinder + Dry Sample, W_2 (g)	162.4	172.6	167.5
Weight of Cylinder + Sample + Water, W_3 (g)	453.8	461.1	457.2
Weight of Cylinder + Water, W_4 (g)	427.7	428.1	427.6
Weight of Dry Sample, $W_2 – W_1$ (g)	42.4	52.6	47.5
Weight of Water, $(W_4 – W_1) – (W_3 – W_2)$ (g)	16.3	19.6	17.9
$Gs = \frac{(W_2 - W_1)}{(W_4 - W_1) - (W_3 - W_2)}$	2.6	2.68	2.65
Average Specific Gravity		2.64	

(d) Results of specific gravity of aggregate (granite)

The results of specific gravity of granite carried out are presented in Table 4.4

Table 4.4: Specific gravity of aggregate (Granite)

Trials	1	2	3
Weight of Cylinder, W_1 (g)	116.5	116.5	116.5
Weight of Cylinder + Dry Sample, W_2 (g)	180	176.3	169.4
Weight of Cylinder + Sample + Water, W_3 (g)	473.3	471.9	467.1
Weight of Cylinder + Water, W_4 (g)	433.7	434.6	434.2
Weight of Dry Sample, $W_2 – W_1$ (g)	63.5	59.8	52.9
Weight of Water, $(W_4 – W_1) – (W_3 – W_2)$ (g)	23.9	22.5	20
$Gs = \frac{(W_2 - W_1)}{(W_4 - W_1) - (W_3 - W_2)}$	2.66	2.66	2.65
Average Specific Gravity		2.66	

(e) Results of bulk density of fine aggregate (sand)

The results of Bulk Density of Fine Aggregate carried out are presented in Table 4.5

Table 4.5: Bulk Density of Fine Aggregate (Sand)

	Uncompacted			Compacted		
Trials	1	2	3	1	2	3
Weight of mould, W_1 (Kg)	1.08	1.08	1.08	1.08	1.08	1.08
Weight of mould + Sample, W_2 (kg)	3.87	3.91	3.98	4.03	4.05	4.06
Volume of mould, V (m^3)	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017
Weight of Sample, $W_2 – W_1$ (kg)	2.79	2.83	2.9	2.95	2.97	2.98
Bulk density = $\frac{W_2 - W_1}{V}$ (kg/m^3)	1641.8	1664.71	1705.88	1735.29	1747.06	1752.94
Average bulk density (kg/m^3)		1670			1745	

(f) Results of bulk density of coarse aggregate (granite)

The results of Bulk Density of Coarse Aggregate carried out are presented in Table 4.6

Table 4.6: Bulk Density of Coarse Aggregate (Granite)

	Uncompacted			Compacted		
Trials	1	2	3	1	2	3
Weight of mould, W_1 (Kg)	1.08	1.08	1.08	1.08	1.08	1.08
Weight of mould + Sample, W_2 (kg)	3.71	3.78	3.40	3.91	3.92	3.88
Volume of mould, V (m^3)	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017
Weight of Sample, $W_2 – W_1$ (kg)	2.63	2.7	2.32	2.83	2.84	2.8
Bulk density = $\frac{W_2 - W_1}{V}$ (kg/m^3)	1547.06	1588.24	1364.70	1664.71	1670.59	1647.06
Average bulk density (kg/m^3)		1500			1660	

4.1.2 Results of compressive strength of RHA- OPC concrete

Based on Tables 3.2 and 3.3, the values of compressive strength of concrete at 0% replacements of cement with rice husk ash are presented on Table 4.7A & B. The values of the compressive strength were obtained using Equation (2.1)

Table 4.7A: Compressive strength of the cubes obtained from the experiment and model

Points	**Compressive Strength(N/mm^2)**			**Mean Experiment Compressive Strength(N/mm^2)**	**Predicted Compressive Strength of concrete cubes (N/mm^2)**
	Replicate 1	Replicate 2	Replicate 3		
Y_1	27.20	26.50	25.47	26.39	26.39
Y_2	22.27	21.75	23.60	22.54	22.54
Y_3	21.37	22.13	20.25	21.25	21.25
Y_4	20.69	21.09	21.43	21.07	21.07
Y_5	20.57	20.17	21.93	20.89	20.89
Y_{12}	24.98	25.25	25.07	25.10	25.10
Y_{13}	24.00	25.42	23.33	24.25	24.25
Y_{14}	21.06	21.28	21.14	21.16	21.16
Y_{15}	22.72	22.11	21.44	22.09	22.09
Y_{23}	21.09	21.00	20.94	21.01	21.01
Y_{24}	20.35	22.25	23.46	22.02	22.02
Y_{25}	20.78	19.29	21.04	20.37	20.37
Y_{34}	21.87	19.47	19.14	20.16	20.16
Y_{35}	20.30	22.12	21.27	21.23	21.23
Y_{45}	20.05	20.55	20.45	20.35	20.35

Table 4.7B: Compressive strength of the cubes obtained from the experiment and model

Points	Compressive Strength(N/mm^2)			Mean Experiment Compressive Strength(N/mm^2)	Predicted Compressive Strength of concrete cubes (N/mm^2)
	Replicate 1	Replicate 2	Replicate 3		
C_1	23.15	24.09	23.11	23.45	22.84
C_2	22.12	22.21	22.42	22.25	22.16
C_3	24.50	23.20	22.32	23.34	25.36
C_4	23.82	21.6	21.15	22.19	21.86
C_5	23.15	20.92	20.43	21.50	22.85
C_{12}	24.65	24.30	23.11	24.02	20.57
C_{13}	34.67	37.23	37.63	36.51	38.56
C_{14}	20.98	19.91	24.18	21.69	21.13
C_{15}	21.21	22.09	22.55	21.95	21.09
C_{23}	20.97	22.56	22.23	21.92	20.47
C_{24}	29.10	28.09	27.32	28.17	26.10
C_{25}	26.54	27.82	25.86	26.74	25.40
C_{34}	22.09	20.74	23.47	22.10	20.88
C_{35}	20.38	22.19	22.23	21.60	20.34
C_{45}	18.15	18.85	16.13	17.71	17.05

The mix ratios from Tables 3.2 and 3.3 were used to obtain the compressive strength at various replacement quantities of Ordinary Portland Cement (OPC) with Rice Husk Ash (RHA). The results of the compressive strength are presented on Table 4.7A & B. The values were obtained using Equations (2.1). The compressive strength test results shown on numbers 1-15 were obtained from the first fifteen trial mix ratios given on Table 3.2. These values were used to formulate the models. The test results in the Table 4.7B were obtained based on the control mix ratios given on Table 3.3. These values are the control points used to validate the formulated model.

The effects of different percentage replacements of OPC with RHA on the compressive strength of RHA-OPC concrete are shown in Figures 4.3.

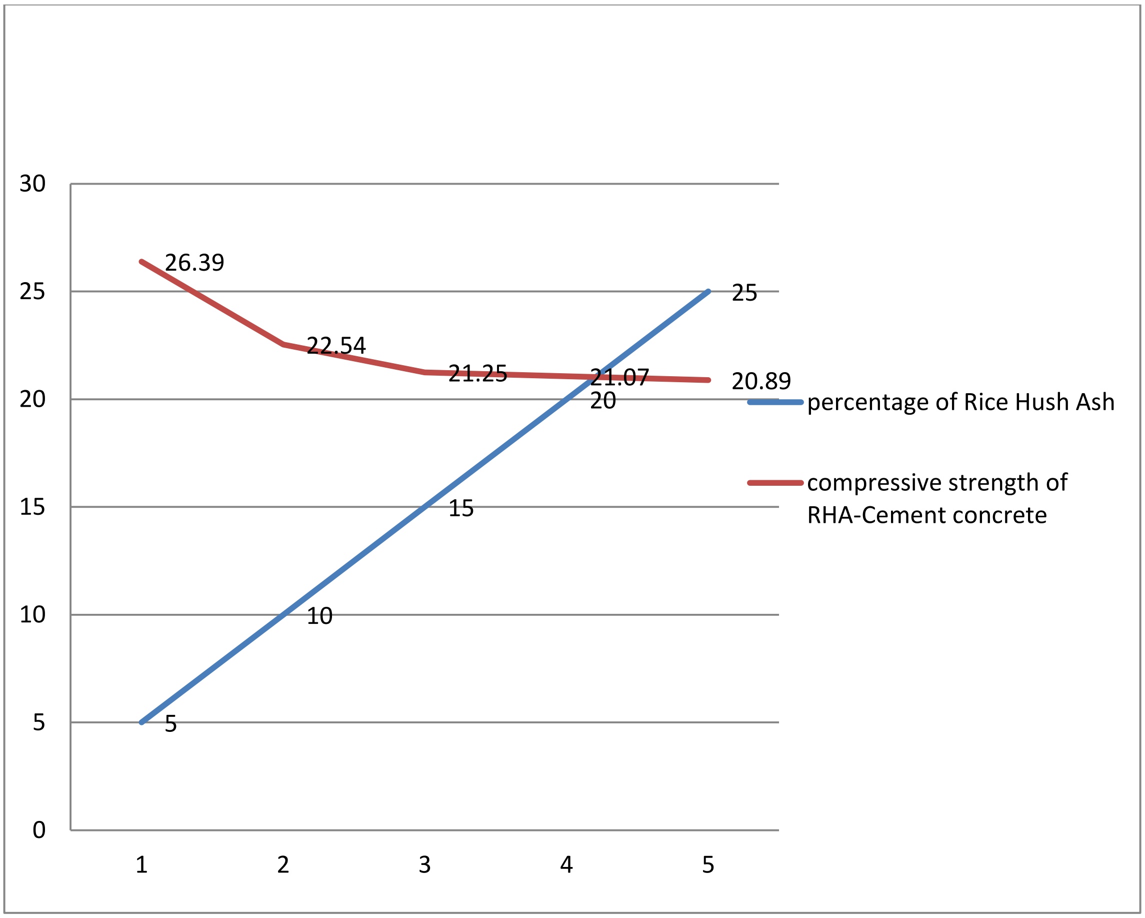

Figure 4.3: Relationship between compressive strength and rice hush ash content at 28

Table 4.8: Statistical student's t-test for the model

	Two – Tailed t-Test				
Point	Y_E	Y_P	$D = Y_E - Y_P$	$M_D - D$	$(M_D - D)^2$
C_1	23.45	22.84	0.61	-0.04	0.002
C_2	22.25	22.16	0.09	-0.09	0.01
C_3	23.34	25.36	-2.02	2.02	4.08
C_4	22.19	21.86	0.33	-0.33	0.11
C_5	21.5	22.85	-1.35	1.35	1.82
C_{12}	24.02	20.57	3.45	-3.45	11.90
C_{13}	36.51	38.56	-2.05	2.05	4.20
C_{14}	21.69	21.13	0.56	-0.56	0.31
C_{15}	21.95	21.09	0.86	-0.86	0.74
C_{23}	21.92	20.47	1.45	-1.45	2.10
C_{24}	28.17	26.1	2.07	-2.07	4.28
C_{25}	26.74	25.4	1.34	-1.34	1.80
C_{34}	22.1	20.88	1.22	-1.22	1.49
C_{35}	21.6	20.34	1.26	-1.26	1.59
C_{45}	17.71	17.05	0.66	-0.66	0.44
			$\sum D = 8.48$		$\sum(M_D - D)^2 = 34.87$

From Table 4.8, the calculated t is 1.76. The value obtained from standard statistical t-distribution table (2.14) is greater than the calculated t-value (1.76).

This means that difference between the two set of cubes compressive strength is less than allowable difference. Hence null hypothesis is accepted and alternate hypothesis is rejected. Therefore, the regression equation is adequate for the prediction of compressive strength of rice husk ash-cement concrete.

4.2 Discussion

(a) Constituent materials of concrete

The effect of partial replacement of cement with rice husk ash on the compressive strength of concrete is as shown in Figure 4.3. It can be observed from Figure 4.3 that compressive strength decreases with increases in the percentage replacement of OPC with rice husk ash.

The reduced value of the compressive strength may be due to lower specific gravity value of rice husk ash compared to that of OPC. The blending of the two materials caused a reduction in strength value of the end product since specific gravity is strength related. The reduced compressive strength value may also be due to the fact that rice husk ash has less binding properties compared to OPC. After 28 days of water curing, the concrete gave an average optimum compressive strength value of 26.39 N/mm^2 corresponding to a mix proportion of 0.95, 0.05, 1, 2 (cement, rice husk ash, sand, granite) at a water-cement ratio of 0.4.

This compressive strength value obtained at 5% replacement is within the recommended value required for plain concrete works, lean concrete, simple foundations, masonry walls and other simple construction works in low-cost housing construction (Deodhar, 2009).

The specific gravity value of sand used is 2.64. This value agrees with Shirley (1975), who reported that the specific gravities of normal density aggregates lie between 2.5 and 3.0. The results of the sieve analysis are compared with the grading limits for fine aggregates as recommended by BSI (1983) and observed that the sand used belongs to zone 2. The sand that belongs to zone 2 is not too coarse and not too fine, thus, making it suitable for concrete works. The value of coefficient of uniformity (Cu) is 2.17, which is less than 4 for well-graded sand. The value of coefficient of gradation (Cc) is 1.04. This value falls between 1 and 3 showing that the fine aggregate is well graded (BS 1377, 1975).

(b) Analysis of prediction models

The compressive strength test results are presented in Table 4.7A & B and test of adequacy of the formulated model is presented in Table 4.8 respectively. A model based on Scheffe's technique was formulated for the compressive strength. The compressive strength model was tested for adequacy using Student's t-test. The null hypothesis is accepted for the compressive strength model based on the Student's t-test indicating that the model is adequate. The optimal value of the compressive strength, which can be predicted by Scheffe's model is 22.84 N/mm^2. The optimal value of compressive strength being 22.84N/mm^2 is above the minimum value of 15 N/mm^2 recommended for construction of non-load bearing walls and mass concrete foundations as earlier suggested by Deodhar, (2009) on similar concrete predictions.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

From this study, the following conclusion can be made:

The physical properties of the aggregates were determined. The results obtained from the tests indicated that the aggregates are suitable for concrete works.

The optimal compressive strength of RHA-OPC concrete obtained was 22.84N/mm^2 using Scheffe's model. This value was obtained at 5% replacement and it shows that RHA-OPC concrete is acceptable for plain concrete works, lean concrete, simple foundations and other simple construction works in low-cost housing construction. Also the compressive strength of RHA-OPC concrete decreases with increase in percentage replacement of cement with rice husk ash.

The Scheffe's based model for the prediction and optimization of compressive strength of rice husk ash-cement concrete determined is given by the expression:

$$Y = 26.39X_1 + 22.54X_2 + 21.25X_3 + 21.07X_4 + 20.89X_5 + 2.54X_1X_2 + 1.72X_1X_3 - 10.28X_1X_4 - 5.6X_1X_5 - 3.54X_2X_3 + 0.86X_2X_4 - 5.38X_2X_5 - 4X_3X_4 + 0.64X_3X_5 - 2.5X_4X_5$$

This model can predict the value of the optimal compressive strength when the mix ratios are specified and can also generate the mix ratios for any specified compressive strength. The formulated model was tested for adequacy at 5% level of significance and was found to be adequate. The model developed is reliable, easy to use and saves time

5.2 Recommendations

Based on the results obtained from this study, the following recommendations are made:

(i) The mathematical model developed in this study can be used as design codes and standards for optimal design of RHA-OPC concrete mix ratios.

(ii) The model developed in this study can be used for optimisation of any compressive strength of concrete made from cement, rice husk ash, sand, granite and water.

(iii) Research work should be carried out to ascertain the workability of the concrete that correspond with the predicted mix ratios.

(iv) Further research can be carried out to ascertain the accuracy of the formulated model using higher order polynomials.

5.3 Contribution to Knowledge

The study established that the optimal compressive strength of RHA-OPC concrete using Scheffe’s model as 22.84 N/mm^2 at 5% replacement of OPC with RHA and which is acceptable for plain concrete works, simple foundations and other construction works. Also the expression for the Scheffe’s model used for the prediction was suggested in the work.

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