# AN EVALUATION OF LEAST SQUARE MODELS FOR TIDAL HARMONIC ANALYSIS AND PREDICTION IN APAPA DOCKYARD, LAGOS, NIGERIA

BY

WATHANAFA, Abubakar Marka
M.Tech/SET/2017/6701

DEPARTMENT OF SURVEYING AND GEOINFORMATICS,
FEDERAL UNIVERSITY OF TECHNOLOGY,
MINNA

AUGUST, 2021

**ABSTRACT**

The harmonic method of tidal analysis has been considered as more accurate than the non-harmonic methods. This is because the precise knowledge of the astronomic tidal constituents of a river channel enhances a comprehensive analysis of the tidal behavior in relation to the astronomic constituents which are the predominant factors responsible for tidal patterns and variation. Over the years, the Ordinary Least Squares technique (OLS) has been used for tidal harmonic analysis. However, due to issues of performance speed, computational effectiveness and operational limitations base on confidence interval, the use of the Iteratively Re-weighted Least Squares (IRLS) is becoming increasingly prominent. This study presents a comparative analysis of both methods of tidal harmonic analysis using 12months sea level observational data taking at 10minutes interval at Dockyard Tide Gauge (Lagos state). A computational experiment based on some sort of hierarchical data input scenario has been conducted. The 12months data has been spited into four different data types being; (i) short term data without gaps (5months), (ii) short term data with gaps (4% missing data), (iii) multi-months data without gaps (12months) and (iv) multi-months data with gaps (5% missing data). The harmonic constants for each tidal constituent in all four observational scenarios were computed using both methods and the corresponding confidence interval and SNR determined for each method using the UT-tidal analysis hydrographic tool in MATLAB as designed by Codiga (2012). Tidal values at specific times based on the determined constituent values were thereafter predicted. It was discovered that there is no significant difference in tidal prediction between the results obtained from the use of both the OLS and the IRLS method for analyzing data not exceeding five months at 95% confidence interval with the OLS method performing better than the IRLS for short term data period. However, in the multi- months data period, the IRLS method performed better that the OLS method in both scenarios (complete data and data set with omissions). The study concludes that while long term data might be better for river tidal characteristics determination with values of 1.457m and 1.6495m for the Mean Low Water Springs (MLWS) and MHWS respectively, short term data is best for tidal prediction.

**TABLE OF CONTENTS**

## LIST OF TABLES

## LIST OF FIGURES

**Figures** **pages**

## LIST OF PLATE

**Plate** **Page**

# CHAPTER ONE

# 1.0 INTRODUCTION

## 1.1 Background to Study

Tides are the periodic movements of the sea waters i.e. periodic rises and falls in water level due to changes in the attractive forces of the Moon and Sun upon the rotating Earth (Badejo, 2017). The rise and fall of tide is accompanied by horizontal movement of the water called tidal current. It is necessary to distinguish clearly between tide and tidal current, although the relation between them is complex and variable. While tide is the vertical rise and fall of the water, tidal current is the horizontal flow. Tides are caused by the balancing effects of gravitational pull of the sun and moon on the rotation of the Earth. Generally, tides are classified according to the tidal pattern they exhibit. Locations in the ocean where there are two high then two low tides daily are classified as having semidiurnal tides while locations that have only one high tide followed by one low tide daily have diurnal tide (Reddy, 2001). Although few cases of mixed tidal pattern exist, most water bodies have semidiurnal tides. The times and amplitude of the tides at the coast are influenced by the alignment of the Sun and Moon, by the pattern of tides in the deep ocean and by the shape of the coastline and near-shore bathymetry (Mellor, 1996).

The principal tidal forces are generated by the Moon and Sun (Consoli, 2013). The Moon is the main tide-generating body. Due to its greater distance, the Sun‟s effect is only 46 percent of the Moon's. Conventionally, observed tides differ considerably from the tides predicted by equilibrium theory since size, depth, and configuration of the basin or waterway, friction, landmasses, inertia of water masses, Coriolis acceleration, and other factors are neglected in this theory. Nevertheless, equilibrium theory is sufficient to describe the magnitude and distribution of the main tide-generating forces across the surface of the Earth.

The gravitational effect of the Moon on the surface of the Earth is the same when it is directly overhead as when it is directly underfoot. Because the direction of the Earth's rotation is in consonance with the Moon's orbit around the Earth, the period of the Moon is slightly above day (about 24 hours and 50minutes). In the course of this daily movement, the Moon passed overhead and underfoot once, hence the period of strongest tidal forcing is 12hours and 25 minutes. It is important to note that the position of the Moon in relation to the Earth (overhead or underfoot) does not necessarily determine the high tides although, the period of the forcing determines the time between high tides.

Asides the moon, the gravitational attraction of the Sun on the Earth also allows for some secondary tidal effects. Whenever the Earth, Moon and Sun are aligned, the tidal effects of the Sun and Moon (loosely described as the M and S tidal constituents) reinforce themselves thereby causing higher high waters and lower low waters. This near-alignment occurs at the full moon and new moon; with the recurring extreme tides termed as spring tides while those with the smallest range are called the neap tides. The neap tides occur around the first and last quarter moons.

Tides or rather water level information are useful for several purposes. Tidal information is very useful for navigation, harbour and near shore engineering constructions, Long term climate studies, flood management, hydrographic survey, oil exploration and exploitation activities(Badejo, 2017).

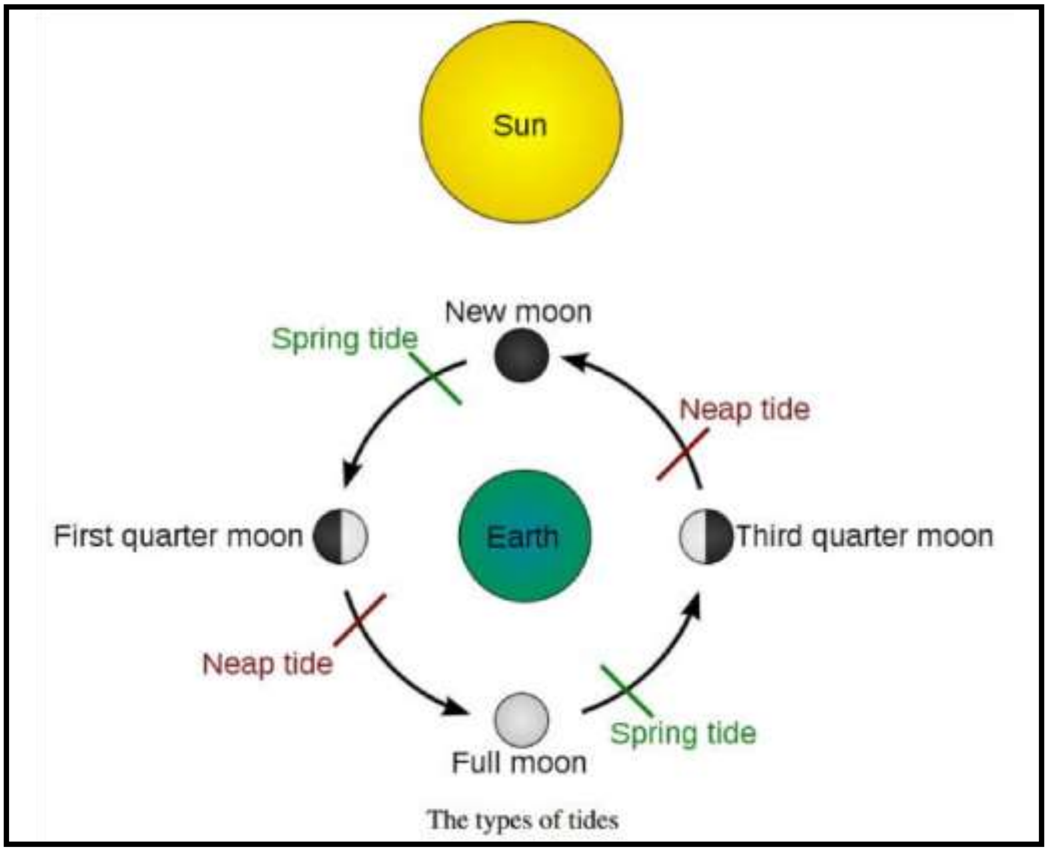


Figure 1.1: Range variation for different types of tides (Parker, 2007)

Tidal analysis and prediction is very useful for planning, management and decision making in Ports and Harbour operations. Amongst others, tidal analysis and prediction is useful for:

i. flood forecasting and monitoring
ii. Storm surge warning
iii. Study of Tsunami effects
iv. Prediction of upwelling and fishing operations
v. Study of horizontal and vertical crustal movements with the aid of GPS and Gravimeter for determining the secular sea level changes.

vi. Study of Precise Inter-island drift.

vii. Study of the effect of rise in sea level caused by global green House effect and other phenomenon.

viii. The hourly tidal data are used on a global basis for applications such as altimeter calibration and assimilation of sea level data numerical models.

ix. Local behaviour of sea (Chart datum, Mean Sea Level, Lowest low & highest High tide etc.) are required for major construction activities along the coast, e.g., Harbour development (Jian, 2003).

x. Efficient utilization of energy generation potential of hydro-dams (Jian, 2003)

All over the world, studies of tidal variation, analysis and prediction is an important aspect of hydrographic surveys given its established importance to navigational security as well as national economy. In Nigeria, issues of safety on our territorial waters (inclusive of the inland water ways) are strictly the jurisdiction of the Nigerian Navy Force (NNF). While the NNF is charged with security issues, compliance of safety rules especially in dredging and water channelization is entrusted in the Nigerian Inland Waterways Agency (NIWA). The NNF achieves this aim through the Nigerian Navy Hydrographic Office (NNHO).

"The Nigerian Navy Hydrographic Office (NNHO) is the coordinating center for all national hydrographic matters in Nigeria. Its primary responsibility is to produce charts, coordinate all the hydrographic surveys that are carried out on the Nigerian waters and prepare annual tables of tidal prediction across the Nigerian water ways. In the discharge of its duties, several organs both within and outside the NNF contribute either in whole or part. Some of these agencies include the National Emergence Management Agency (NEMA), Office of the Surveyor General of the Federation (OSGOF) etc.

Generally, two basic kinds of tidal analysis methods exist being the harmonic and nonharmonic methods. Of these two, the use of non-harmonic tidal prediction method is mostly utilized for two reasons. The first reason is to allow for optimal usage of the space in the Tide Tables. Daily predictions of the time and heights of all tides were included for all tide stations, then there would be too many pages. On the other hand, recording the differences in tidal time and height at a station takes only one line in the conventional tide table thus a total of about 60 stations per page. Therefore, by calculating tidal time and height difference between stations via the non-harmonic comparison method, several stations could be accommodated for in the tide tables. Secondly, in thepast, most stations had only few data thereby making it nearly impossible to carry out a reliable harmonic analysis.

Nevertheless, the greatest prediction accuracy and also the greatest understanding of the tidal dynamics of any water body can only be obtained from harmonic analysis (Parker,

2007). It is against this backdrop that this study presents a harmonic analysis of part of the Lagos Lagoon using data obtained from an automatic tide guage at Dockyard Naval cantonment (near East mole), Apapa, Lagos.

## 1.2 Statement of Research Problem

Mathematically, the ordinary least squares (OLS) method exhibits a poor performance in the presence of outliers. A reliable alternative to the OLS is given by the robust regression technique by iteratively updating the weights. This yields the iteratively re-weighted least squares (IRLS). The IRLS has been extensively used in statistics, geodetic, geophysics and harmonic analysis literatures (Huber and Ronchetti, 2009; Codiga, 2011).

Codiga (2011) presented the use of iteratively reweighted least squares (IRLS) technique for analysis and prediction of tides. The development was motivated by the need to carry out tidal analysis on a long span sequence of irregularly spaced data. Since the datasets are irregularly spaced, determination of observational weights as in the case of the OLS (or weighted least squares (WLS) as the case may be) becomes a herculean task. He therefore proposed that an iteratively re-weighted system would efficiently continue to impose weights on each observation in the sequence until convergence is achieved. Although, the author claims this technique is very efficient for analysis of multi staged data covering more than one-year, there are only few documented research on the performance of the model for short time series tidal observations.

Furthermore, there are limited documented scientific investigation to validate the comparative strength of IRLS over the OLS in short term and long term tidal analysis. This paper therefore presents a comparative evaluation of the computational reliability of the IRLS and the OLS techniques of tidal analysis and prediction with a view to identifying circumstances that might warrant the need for IRLS in either short term or long span data using five months and twelve months data respectively.

### 1.3 Aim and Objectives

The aim of this study is to perform harmonic tidal analysis of the tides at Apapa Dockyard Tide guage station (TGS), Lagos, Nigeria using OLS and IRLS models with a view to identify the preferred model for tidal prediction given different observational scenarios.

The objectives of the study are to:

i. Determine the harmonic constituents for Apapa Dockyard TGS using OLS and IRLS techniques. ii. Examine the performance of the OLS and IRLS techniques under various data scenarios.

iii. Examine the confidence interval and error estimates of the determined constituents.

iv. Predict tides at specified times given for the different data scenarios based on the determined harmonic constituents.

## 1.4 Research Questions

1. How can the OLS and IRLS models be implemented for determination of tidal harmonic constituents?
2. Under what data scenario can the OLS and IRLS techniques perform to optimum capacity for constituent determination?
3. What relationship exists between the computed constituents' confidence interval for harmonic constituents and the Signal to Noise ratio (SNR)?
4. What kind of data scenario is best for tidal prediction?

## 1.5 Justification of the Study

Tidal prediction and analysis of water bodies is an essential task for safe navigation and flood hazard mitigation. Despite numerous studies across the world, local efforts on tidal prediction in Nigeria is limited to the national tide tables prepared by the Nigerian Navy Hydrographic Office (NNHO). These national tide tables because they were generated by non-harmonic tidal analysis do not provide the amplitude, phase and phase lag of the astronomic tidal constituents that make up the analyzed tides. Although the tables have served well for navigational purposes, they are grossly deficient for advanced scientific applications in hydrography such as (i) flood forecasting in the maritime domain(ii) atmospheric studies and (iii) oceanic tidal loading determination in Nigeria‟s maritime domain.

Another drawback to non-harmonic tidal analysis is that, it ignores the modulation of the perihelion which is effectively constant over historical time of about 18.6 year time series (Pawlowicz, 2002). This modulation value is required to resolve all the listed frequencies (because there is a minimum of one distinct wavelength for each constituent which differs from other constituents).In order to handle this, it is assumed that the phase/amplitudes of response sinusoids (having similar frequencies) are equal to those of the equilibrium response (Kowalick and Luick, 2019).

Furthermore, the non-harmonic analysis does not allow for an easy determination of the characteristics of the resulting phase/amplitude in a deterministic way. Generally, the fit would include elements of the confidence interval for the deterministic part (Pawlowicz, 2002).

For these reasons, the harmonic analysis and prediction approach is a preferred method of tidal analysis and prediction. Nevertheless, several harmonic analysis techniques have continued to evolve in response to growing computational demands for tidal analysis and prediction. Two major harmonic analysis techniques are the Ordinary Least Squares (OLS) and the Iteratively Reweighted Least Squares (IRLS). This study therefore presents harmonic analysis of the tides at an Automatic Tide Guage station located at Dockyard, Apapa using both the ordinary least squares (OLS) and the Iteratively Reweighted Least

Squares (IRLS) technique with a view to evaluating the computational effectiveness, performance speed and operational limitations of both models based on the confidence interval generated from each method.

Apapa Dockyard station was chosen for this study because of the availability of data. Efforts to access data from other tide stations proved abortive as such, the study utilized only the Apapa station data which was available to the researcher at the time of the study. The outcomes of this study would be very relevant and important to the Nigerian Navy, The inland waterways authority and hydrographers generally.

### 1.6 Study Area

The Lagos lagoon has a length of more than 50 km and width of between 3 to 13 km wide. It is separated from the Atlantic Ocean by a long sand spit of about 5 km wide with swampy margins on the lagoon side. Lagos state has a surface area of approximately 6,354.7sq km. With the exception of the Commodore channel, the lagoon is fairly shallow and is usually plied by ferries, boats, barges and daughter-vessels. The Lagos Lagoon averages 2-4 m deep, but is 10 m deep in the entrance at the Commodore channel (Okusipe, 2008). Lagos Lagoon discharges its water content into the Atlantic Ocean through the Lagos harbour. The Lagos harbour or Commodore Channel is 0.5 km to 1 km wide and 10 km long. The Lagos port is located at Apapa in a broad western branch off the main channel of the harbour. The Lagos Lagoon is tidal, water from the Atlantic Ocean moves into the lagoon during high tides and recedes during low tides. The Lagos lagoon is affected by a powerful long shoredrift. It is fed by several rivers, the most important of which are the Ogun, Ona/Ibu, Oshun, Shasha and Oni.

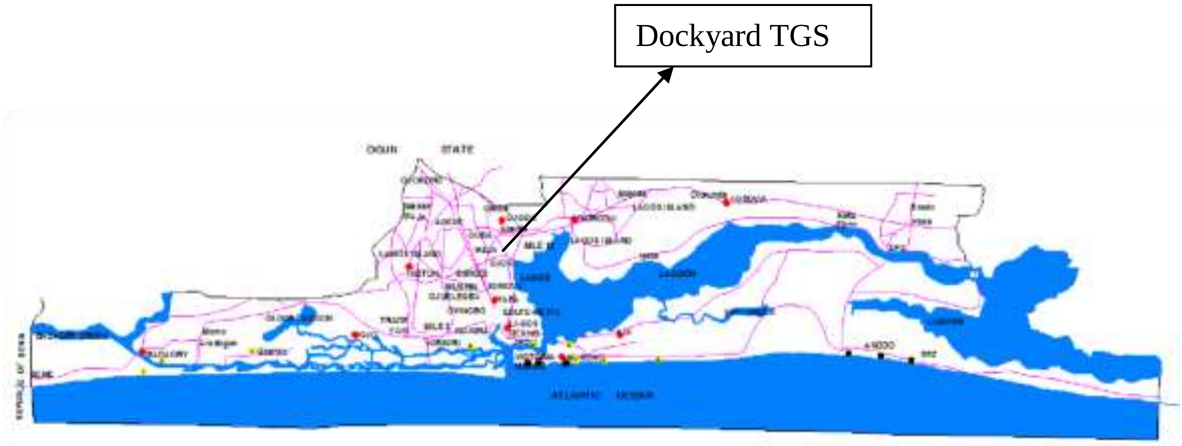



Figure 1.1: Study Area (Uduma-Olugu and Oduwaye, 2014)

# CHAPTER TWO

# 2.0 LITERATURE REVIEW

## 2.1 Historical Background of Tidal Analysis

Tidal prediction is an age-long task that had been attributed to the hydrographer in the marine environment. Till date, tidal prediction remains one of the oldest and most accurate requirements for safe navigation on the ocean and open seas (Hicks, 2006). In fact, there were already tide tables for the Tsientang River in China's early as 1056 and the Thames River (at the London Bridge) early in the 1200's (Hicks, 2006). Tide prediction began as early as there was discovery of the relationship that exists between tide and the continuously changing location (position and phase) of the moon. This relationship was already established and well known even to the early hydrographers/mariners. The concept of tidal prediction began with the invention of the rule-of-thumb technique for tide prediction. These techniques were often treated as treasured family secrets and handed down generational lines (Parker, 2007).

In 1687, Isaac Newton published "the Principia" a text in which he explained the principle of tides by using the theory of gravitational attraction of masses. Based on the theory of universal gravitation, Newton accounted for the tide-generating forces as natural responses to the Moon and Sun's attractions. Further supported by other theories such as the equilibrium theory of Pierre-Simon Laplace, Newton affirmed that the occurrence of tides would still be observed peradventure a non-inertial ocean were evenly covering the whole Earth (Lisitzin, 1974). Up until now, studies on tide-generating forcesare still relevant in tidal theory and should be considered as being influenced by bathymetry, Earth's rotation, and other factors (Wahr, 1995).

Notable works on tidal theory and analysis are listed in table 2.1

Table 2.1(a): Past efforts in tidal analysis (Pugh, 2004)

| S/No | Contributor | Contribution | Year |
|---|---|---|---|
| 1 | William Thomson | Re-wrote the Laplace equation to describe the Kelvin waves | 1750 |
| 2 | Daniel Bernoulli | Tidal table for France using the Moon's tidal interval | 1752 |
| 3 | John Lubbock | Produced the tide tables for England with nonharmonic method | 1832 |
| 4 | William Thomson | Use of Fourier technique for tidal analysis and prediction | 1860 |
| 5 | George Darwin | Developed the classical tidal harmonics theory using the Lunar theory | 1870's |
| 6 | Arthur Thomas Doodson | Developed the Tide generating potentials | 1921 |

Doodsonelaborated on the formal treatment of the slowest astronomical periodicities and identified 388 different tidal frequencies of which 62 constituents are sufficient to be used in marine tidal prediction (Parker, 2007).

The following are among the major tidal constituents contributing to the astronomical tide:

M2 - Principal lunar semidiurnal constituent (speed: 28.984 degrees per mean solar hour)

S2 – Principal solar semidiurnal constituent (speed: 30.000 degrees per mean solar hour);

N2 - Larger Lunar elliptic semidiurnal constituent (speed: 28.440 degrees per mean so larhour)

K1 - Luni-solar declinational diurnal constituent (speed: 15.041 degrees per mean solar hour)

O1 - Lunar declinational diurnal constituent (speed: 13.943 degrees per mean solar hour)

M4 - First overtide of M2 constituent (speed: 2 x M2 speed)

M6 - Second overtide of M2 constituent (speed: 3 x M2 speed)

S4 - First overtide of S2 constituent (speed: 2 x S2 speed)

MS4 - a compound tide of M2 and S2 (speed: M2 + S2 speed).

A method for harmonic analysis and prediction independently from the work of Thomson and Darwin for the Coast and Geodetic Survey. Ferrel's innovation was based on the previous works by Laplace in the area of tidal theory. Later on, Rollin Harris (1897-1907) made further improvements to the theory of tidal prediction and analysis (Parker, 2007). At the time Doodson was working on Tidal generating potentials in Britain, Paul Schureman was also working on the use of astronomical equations and Fourier techniques for harmonic analysis and prediction. Schureman discovered that tidal energy is mostly found in these diurnal and diurnal bands, hence, he analyzed water level measurements by harmonic method. He also determined the energy level is at each tidal frequencies in different locations.

One major contribution of Schureman to tidal analysis was identifying the tidal frequency responsible for the tidal energy using the *tidal harmonic constituent.* The tidal harmonic constituents were represented by the amplitude and phase lag (jointly termed as

*Harmonic constants)*. The amplitude is defined as the maximum height of a tide while the epoch is the time the amplitude occurs. The duo of the amplitude and epoch are useful tidal constituents for predicting the tide at other times (Cartwright, 1999).

Before the advent of computers, Thomson developed a method automate tide predictions using the harmonic tidal constituents by inventing a mechanical analog tide predicting machine. This machine consists of several gears and pulleys systems. Another tide prediction machine was designed by Rollin Harris in 1912 in the workshops of the U.S. Coast and Geodetic Survey. This machine used 37 tidal constituents and predicted tide elevations and / or tidal currents. The major limitation of these tide predicting machines was that it involved so much of manual energy before it could analyze water level data and calculate the tidal harmonic constants that would be represented by the wheels on the machines. Given the kind of manual efforts required to operate these machines, their use became too time consuming and labor intensive. Consequently, it took several weeks before annual data could be harmonically analyzed (Pawlowicz *et al.,* 1992).

The tide predicting machines served as the means of tidal prediction for most countries until they were replaced with the use of computers in the mid-1960's. Up until now, tidal analyses and tidal predictions are done using high-speed computers. Similarly, the methods of obtaining water level have improved over the years. The previous method of using tide staffs has been replaced with automatic tide gauges that rely on acoustic methods. Other techniques that are recently employed for obtaining water level data include satellite altimetry, GNSS receiver units attached with buoys, laser systems, and radar systems (Badejo *et al.*, 2012).

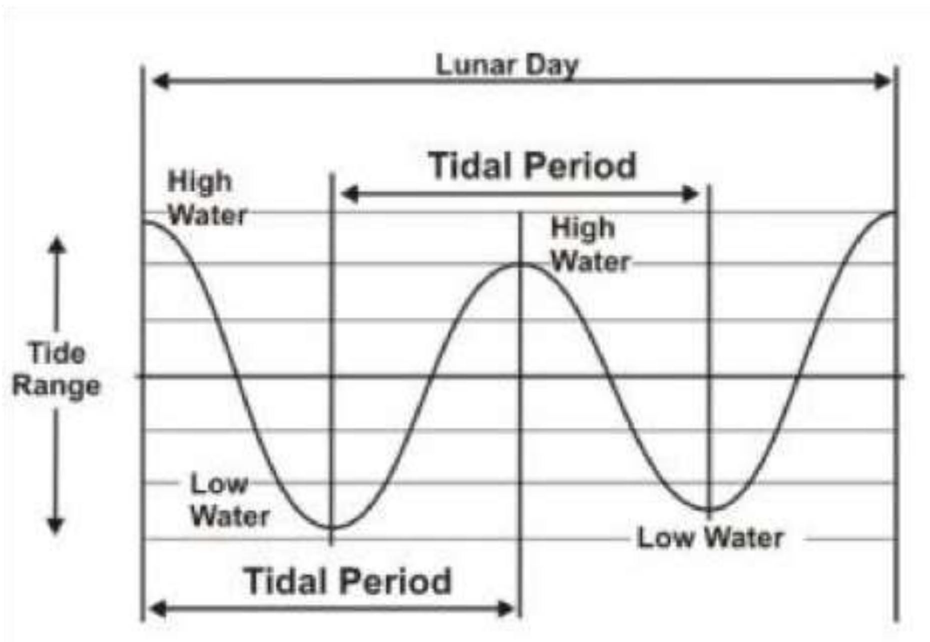
The first published tide prediction tables in the U.S appeared in *The American Almanac* in 1830 and were produced by the U.S. Coast Survey. These predictions consists MHW predictions for Boston, New York, and Charleston. By 1887, low water predictions were included in the American Almanac. Finally in 1867 the U.S. Tide Tables produced by the Tide division of the Coast Survey were published. It was the first official tide table to be produced by a government agency.

Doodson (1921) introduced some technical refinements to tidal harmonic analysis as is used in practice. Further improvements thereafter are associated with the elimination of the effects of minor constituents, station inference, precise specification of astronomical inputs, treatment of unevenly spaced data, treatment of vector data, and development of numerically efficient software e.g. Godin, 1972; Foreman, 1977; Pawlowicz, 2002 and Codiga, 2011). The calculations for tidal predictions via harmonic constituents are laborious, and the advent of the new numerically efficient programs developed in recent times have completely phased off the need for tide-predicting machines.

In Nigeria, tidal analysis and prediction is left to the jurisdiction of the office of the Hydrographer of the Navy. Also, the Office of the Surveyor General of the Federation (OSGOF), and several oil exploration companies (e.g Mobil, Shell, AGIP, e.t.c) within the coastal environments are actively involved in tidal analysis and prediction within their environment. Due to lack of appropriate synergy between the Navy and other relevant marine related authorities and institutions, the publications on the national tide tables is limited to non-harmonic methods.

## 2.2 Definition of Terminologies and Concepts Associated with Tidal Analysis

This section presents a definition of basic tide related terminologies. *Tides* are the periodic motion of the waters of the sea caused by the changing gravitational effects of the moon and the sun as they change position relative to the rotating Earth (Ojinaka, 2007). Although, the *astronomical* forces are responsible for tides, the behavior of the tides in oceans and connected bays depends on the *hydrodynamics*, that is, by the physics of the water movement. Proper understanding of tidal analysis and prediction requires good knowledge of astronomic-tidal forcing and the hydrodynamics of the oceans, bays and rivers. The vertical




movement of the water surface is usually referred to as the *tide*, while the accompanying horizontal motion is referred to as the *tidal current.* In its simplest form, the graphical plot of changing tidal height looks like a sine wave, with the maximum height reached by the water surface (i.e amplitude) called *high water,* and the lowest height (i.e trough) is called *low water*(Figure 2.1).

Figure 2.1: Tidal curve showing basic components of the tide (Parker, 2007)

In Figure 2.1, the sine curve oscillates about the *mean sea level* (MSL). The difference in height between high and low water is called *tide range*. Tidal *period* is defined as the time difference between successive high or low water. Typically, this is usually about 12.42 hours for most water bodies but can be more in some few cases. *Tidal frequency* is the inverse or reciprocal of tidal period. It is either expressed in cycles per day (cpd) or cycles per lunar day. The lunar day being 0.84hours longer than the solar day (Parker, 2007).

Usually, the 2 daily high and low waters are not of the same height. The higher of the 2 daily high waters is called the *higher high water* (HHW), while the lower of the 2 daily high waters is called the *Lower High Water* (LHW). Consequently, the low water also has the higher of the pair referred as the *higher low water* (HLW) while its pair is referred to as the *lower low water* (LLW). The height difference between two successive high waters (that is between HHW and LHW) is called the *high water diurnal inequality*. Likewise the height difference between two successive low water is called *low water diurnal inequality*. *Mixed* *tides* occur whenever the value of the diurnal inequality is significant. This significance may become so high, that there might eventually be only one high water and one low water per day thus resulting in a diurnal tide. Therefore, depending on the number and nature of the tidal highs and lows, tides are classified as either semi diurnal, diurnal or mixed tides (Ojinaka, 2007).

The tidal range is also not constant but changes continually throughout the month in response to the phase of the Moon. Towards new or full Moon, the tidal range and by extension the diurnal inequalities are larger. Tides during this period and exhibiting this characteristics are called *spring tides*(earlier shown in Figure 1.1). Conversely, during the first quarter and third quarter, the tidal range is small and such tides are usually referred to as *neap tides*. Springs tides occur when theEarth, moon, and Sunare all on the same line, such that their tidal effect on the Earth is cumulative. On the contrary, neap tides occur when the Moon and the Sun are at opposite locations, such that, the pull of the Sun on the one side is balanced by the Moon on the other side and vice versa (Smith *et al,* 1997).

Although, tidal prediction depends largely on the knowledge of astronomical forcing, other tide related variables such as tidal range, timing, types and current depend on the hydrodynamics of the tide. Again, since the river hydrodynamics are greatly influenced by river morphometry, then a strong relationship exists between tides and morphometry especially in shallower water. The variations in tidal range on a water body is shown in *corange charts*, while variation in the phase lag is shown in *co-tidal charts*(Parker, 2007).

Water levels are usually measured at tide gauge stations by erecting a vertically mounted tide staff such that it‟s zero mark defines a vertical reference surface from where height measurements are referred. This reference level is called a *vertical datum*. A vertical datum is either a *tidal datum*, *Orthometric datum* or *ellipsoidal datum*. A tidal datum is connected to tidal dependent surfaces, an Orthometric datum is a gravity dependent, while an ellipsoidal datum depends on the space-based computation of positions on a 3D Earth model. Ellipsoidal heights are geometric and easily realized using the GNSS receiver units but are often not practically acceptable for hydrography-related operations.

A *tide datum* at a particular location is generally defined as the mean height at a given stage of the tide. In order to minimize the significance in tide variations, a tidal datum such as mean high water (MHW) is defined. The MHW is the average of all the high water readings over a 19-year period. Nineteen years is selected because it provides an average value for the 18.6 years lunar nodal cycle. By taking average over such a long period, the effects of most meteorological variations on water level are eliminated (Ojinaka, 2007).

Tidal datums provide the vertical reference for bathymetric and other shoreline operations especially in the production of nautical and navigational charts. There are several tidal datums such as mean lower low water (MLLW), mean low water (MLW), mean sea level

(MSL),mean tide level (MTL), mean high water (MHW), mean higher high water (MHHW), etc.. Tidal datums are referenced to the land through high precision differential leveling operations on a vertical reference frame. Such reference frame is realized through the establishment of *benchmarks.* The value of a tidal datum can change over long times due to Earth movements (earthquakes, landslides e.t.c) or Sea Level Rise. Figure 2.2 presents the various tidal datums and their relationship with the ocean (Jenkins, 2014).

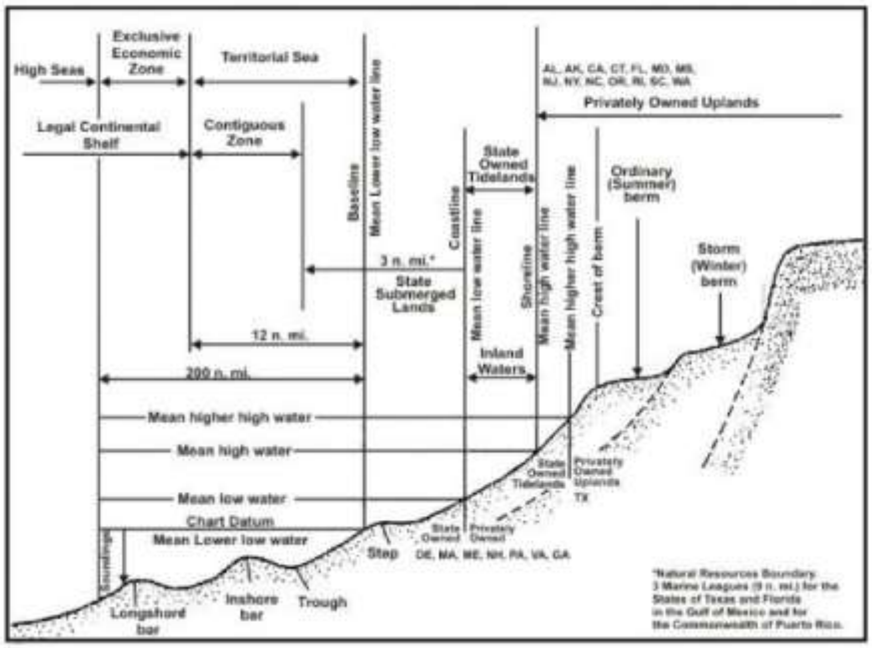



Figure 2.2: Tidal datum‟s in relation to the ocean shore-line (Parker *et al*, 2003)

## 2.3 Tidal Forcing

According to Newton's Law of gravitational attraction, the pull of gravity decreases linearly with mass and inversely according to the square of the distance between them. Therefore, the moon being the closest celestial body to the Earth exerts greater influence on the Earth than the Sun; hence has the greater contributory influence to the formation of tides (Davis *et al.*, 1996). The Earth is a rigid body, so every material point in it executes an identical orbit, and is therefore subject to the same centrifugal force, as illustrated in figure 2.3, but

Gravitational force will continuously change with varying distance between the points and the Moon (Rahman *et al.*, 2017).Again, because gravitational force is greater than centrifugal force on the hemisphere closest to the Moon and vice versa beyond the hemisphere, the opposite hemispheres have net forces in opposite directions, causing the ocean to bulge on both sides. The action of these forces as the Earth spins results in two daily tides (Pugh, 1987).

Supposing that the centrifugal force is due to the rotation of the Earth on its axis thus constant for every latitude, then there would be an imbalance with lunar attraction which is longitude dependent. Moreover, the centrifugal force due to the Earth‟s spin permanently deforms the Earth‟s surface into a spheroid (as opposed to the spherical shape that would ensue from self-gravitation only).

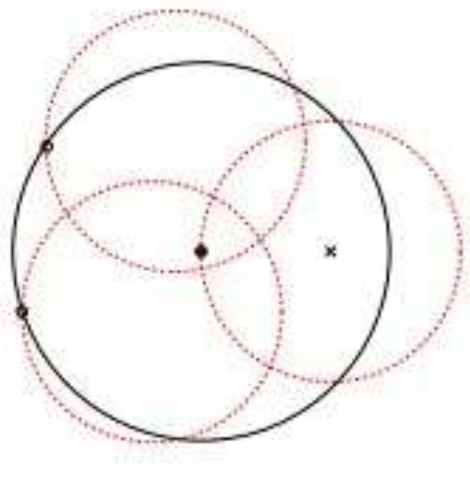


Figure 2.3: The center of the Earth, shown as a filled dot, rotates about the center of mass of the Earth-Moon system, indicated by a „x‟ mark. Dashed circles show the orbital movement of the points shown by a „o‟ mark on Earth‟s surface and the center of the Earth (Cartwright, 1999).

This implies that centrifugal force is compensated by the Earth‟s gravitational field and as such, tides are also influenced by other celestial bodies asides from the Sun and Moon. We can calculate the surface elevation that would result from this forcing. The assumption is that the elevation would be such that net terrestrial gravitational force would exactly compensate for the lunar force at the point (Doodson, 1921).

## 2.4 Theoretical Framework (Concept of Tidal Analysis)

Tides are caused by the Luni-solar pull. This is due to the fact that the impact of a celestial body (the moon or the sun) on the tides of the Earth obeys the Newton's law of gravitation. (Dean, 1966;Godin, 1988, Forrester, 1988).Consequently, equation 2.1 as given by Badejo and Akintoye (2017) is general adopted for tidal prediction.

$$h_t = S_0 + \sum_{t=1}^{n}(H_i Cos[\omega_i t + a_i]) \quad (2.1)$$

Where:

$h_t$ = predicted height of tide

$S_0$ = Height of mean sea level above datum used $H_i$

= Amplitude of tidal constituent I

$\omega_i$ = Angular frequency of the tidal constituent I

t = Time n = Number of harmonic constituents

$a_i$ = Phase of the harmonic constituent

Due to the fact that the orbit of the moon is not constant but rotates slowly with a period of 18.61 years; the amplitude (H) and phase (a) of each tidal are also not constant but change slowly as a result of the rotation of the moon"s orbit. In order to accommodate for the effect of orbital rotation on the amplitude and phase of the tidal harmonic constituents, a nodal factor (f) and an astronomical argument (v + u) are introduced to modify equation 2.1 to become 2.2 as shown (Codiga, 2011):

$$h_t = S_0 + \sum_{t=1}^{n}(f_i H_i Cos[\omega_i t + (v_i + u_i + a_i)]) \quad (2.2)$$

Where:

$v_i$ = phase angle for constituent I at time zero

$u_i$ = nodal angle for constituent I at time zero

$f_i$ = nodal factor for constituent I

Equation 2.1 is used to predict the tidal heights, while some further processing is done to extract the parameters relating to the high and low waters. Usually, the important harmonic parameters in tidal prediction and analysis are the angular speed, amplitude and phase lag. The angular speed defines the frequency of the harmonic constituent and is given in terms of degrees per solar hour.

## 2.5 Developments in Tidal Harmonic Analysis

Since its inception, several developments have arisen in the study and harmonic analysis of tides. Conventionally, the ordinary least squares (OLS) approach is the utilized method for harmonic tidal analysis. However, with improvements in computing technology and advances in computer systems, many versions of harmonic analysis software / program have been developed (Munk and Cartwright, 1966, Godin, 1972). This OLS approach fortidal analysis and prediction have been the basis for most tidal analysis and predictions programs such as Tidana, Tide pack and Versertile Tides (Foreman, 2004).

Tianhang and Vanicek, (1988) used sequential least squares adjustment for tidal analysis and prediction. For seven tidal constituents with fifteen unknowns, sixty six percent of central processing unit time was saved by using the sequential least squares adjustment over standard and conventional least squares adjustment. Later, the Kalman filtering method was used in determining the parameters of the tide level model (Yen *et al*, 1996)). The Kalman filtering method was used to directly estimate the harmonic parameters. The method was however limited to determining the main constituent tides before tidal prediction.

Tsai and Lee (1999) applied the back-propagation neural network for tide prediction given data for both diurnal and semi-diurnal tides. The method was further applied by Shu (2003) also for tidal prediction. Again, Lee (2003) applied the back-propagation method with short-term data from tidal level data at Taichung Harbour in Taiwan. Comparisons withthe OLS method indicate that the backpropagation neural network mode can also efficiently predict tides given long-term tide data.

Codiga (2011) presented the use of iteratively reweighted least squares (IRLS) technique for analysis and prediction of tides. The development was motivated by the need to carry out tidal analysis on a long span sequence of irregularly spaced data. Since the datasets are irregularly spaced, determination of observational weights as in the case of the OLS (or weighted least squares (WLS) as the case may be) becomes a herculean task. He therefore proposed that an iteratively re-weighted system would efficiently continue to impose weights on each observation in the sequence until convergence is achieved. Although, this technique has been utilized for analysis of multi staged data covering more than one-year, there has been no documented research on the performance of the model for short time series tidal observations.

## 2.6 Least Square Models for Tidal Harmonic Analysis and Prediction

Based on the conventional tidal analysis equation given in equation (2.2), tidal predictions are made based on pre-determined amplitude, phase lags of the contributory harmonic constituents. As earlier identified in section 2.4, several solution approaches have been developed for solving equation 2.2; among which the least squares approach are mostly utilized. Amongst the least squares approaches, the OLS is most common and will in this study be compared with the IRLS technique. Presented in the sub-sections that follow are the model equations for the OLS and the IRLS solution approach respectively as given by Codiga (2011).

**2.6.1 Ordinary Least Squares (OLS)**

Equation (2.2) can be expanded into equation (2.3) as follows:

$h_t = S_0 + \sum_{t=1}^{n}\big(f_i H_i Cos\ \omega_i t + (v_i + u_{i_i})\big] \cos a_i + {}^{n}{}_{t=1} f_i H_i Cos\ \omega_i t +$

$_i + a_i)] \sin$

$(v_i + u\ a_i$

(2.3)

Let $A_i = H_i \cos \alpha_i$ and $B_i = H_i \sin \alpha_i$

Then, the tidal harmonic prediction model becomes equation (2.4):

$$h_t = S_0 + \sum_{t=1}^{n}\big(A_i f_i\ Cos[\omega_i t + (v_i + u_{i_i})]\big) + \sum_{t=1}^{n}\big(B_i f_i\ sin\ [\omega_i t + (v_i + u_{i_i})]\big)$$

(2.4)

Matrix A will be created as:

| $S_0$ | $A_1$ | $B_1$ | $A_2$ | $B_2$ .. |
|---|---|---|---|---|
| 1 | $f_1 \cos[w_1 t_1 + (v_1 + u_1)]$ | $f_1 \sin[w_1 t_1 + (v_1 + u_1)]$ | $f_2 \cos[w_2 t_1 + (v_1 + u_1)]$ | $f_2 \sin[w_2 t_1 + (v_1 + u_1)]$. |
| 1 | $f_1 \cos[w_1 t_2 + (v_1 + u_1)]$ | $f_1 \sin[w_1 t_2 + (v_1 + u_1)]$ | $f_2 \cos[w_2 t_2 + (v_1 + u_1)]$ | $f_2 \sin[w_2 t_2 + (v_1 + u_1)]$ |

.

(2.5)

Adopting the OLS formulation and taking Matrix A as shown in equation 2.5, the determination of the tidal harmonic parameters is obtained by equation (2.6):

$$X = (A^T P A) L \qquad (2.6)$$

Where:

$X$ = Parameters

$A$ = Design Matrix

$P$ = Weight Matrix

$L$ = Matrix of observables

### 2.6.2 Iteratively Re-Weighted Least Squares (IRLS)

However, if the IRLS is to be adopted, the weight matrix changes from being a static matrix, hence, the formulation changes to equation (2.7)

$$X = (A^T P^k A)^k L \qquad (2.7)$$

Where:

$P^k$ = Weights with subsequent re-weights

K = i-th iteration

Because minimizing the weighted squared error in an approximation can often be done analytically (or with an infinite number of numerical calculations), it is the base of many iterative approaches including the iteratively re-weighted least squares solution (Burrus, 2012).

## 2.7 Review of Related Works

Several developments have arisen in the study and harmonic analysis of tides. Conventionally, the ordinary least squares (OLS) approach is the utilized method for harmonic tidal analysis. However, with improvements in computing technology and advances in computer systems, many versions of harmonic analysis have been developed.

Codiga (2011) presented the use of iteratively reweighted least squares (IRLS) technique for analysis and prediction of tides. The development was motivated by the need to carry out tidal analysis on a long span sequence of irregularly spaced data. Since the datasets are irregularly spaced, determination of observational weights as in the case of the OLS (or weighted least squares (WLS) as the case may be) becomes a herculean task. He therefore proposed that an iteratively re-weighted system would efficiently continue to impose weights on each observation in the sequence until convergence is achieved. Although, this technique has been utilized for analysis of multi staged data covering more than one-year, there has been no documented research on the performance of the model for short time series tidal observations.

## 2.8 Identified Research Gaps

So far, the OLS technique has remained the most preferred technique for tidal harmonic analysis given its relative simplicity and the presence of a large number of existing software programs that utilize the method. Although, the method is well suited for short term data not exceeding one-year, results from such computational approach become less accurate for records longer than one year. Furthermore, the OLS approach requires that a regularly distributed data sampling. These data requirements for optimum performance of the OLS approach are however sometimes difficult to achieve especially in developing nations where tidal data collection is often not considered a necessity. On the other hand, the IRLS approach has the ability to effectively analyze tidal data covering more than one year and also can be used for irregularly spaced dataset.

In this study, the performance of both models shall be evaluated with a view to determining most suitable option between them for tidal analysis given the following conditions:

(i) Five months equally spaced dataset of tidal observation (short period of regularly spaced tidal data analysis)

(ii) Five months irregularly spaced dataset of tidal observations (short period irregularly spaced tidal data analysis)

(iii) Twelve months equally spaced dataset of tidal observation (long span sequence of regularly spaced tidal data analysis)

(iv) Twelve months irregularly spaced dataset of tidal observations (long span sequence of irregularly spaced tidal data analysis)

# CHAPTER THREE

## 3.0 MATERIALS AND METHODS

### 3.1 Materials

Materials in this study refer to the various data sets and data types used for the research. Since this study is an empirical assessment of the performance of two harmonic tidal analysis models; the data sets used are mainly secondary data. Nevertheless, the reliability of the data remains uncompromised as we see in further discussions as presented below.

#### 3.1.1 Sea level (tidal values)

Twelve months continuous observations of sea level readings taken with an automatic tide gauge at Dockyard Apapa Lagos were used for this study. The observations were taken at 10minutes interval and recorded digitally into the accompanying workstation. The twelve months data spanning from 1$^{st}$ November, 2018 till 31$^{st}$ October, 2019 comprises of a total of 52,560 tidal observations (sea level readings). Table 3.1 shows some features of the observed sea level readings.

Table 3.1: Description of sea level data used

| S/No | Parameter | Value |
|---|---|---|
| **1** | Number of observations | 52,560 |
| **2** | Observational interval | 10minutes |
| **3** | Duration | 12months (1 year) |
| **4** | Data gaps | No |
| **5** | Tidal regime | Semi diurnal |

A sample of the sea level data (tidal observation readings) taken is provided in table 3.2 below. The rest of the data is presented in appendix "A". The data was obtained from the Nigerian Navy Hydrographer of the Office (NNHO).

Table 3.2: Sample readings from the Dockyard Automatic tide gauge station

| Date | Time | Depth |
|---|---|---|
| **1/11/2018** | 12:50:00 PM | 1.985 |
| **1/11/2018** | 1:00:00 PM | 2.01 |
| **1/11/2018** | 1:10:00 PM | 2.029 |
| **1/11/2018** | 1:20:00 PM | 2.047 |
| **1/11/2018** | 1:30:00 PM | 2.1 |
| **1/11/2018** | 1:40:00 PM | 2.126 |
| **1/11/2018** | 1:50:00 PM | 2.177 |
| **1/11/2018** | 2:00:00 PM | 2.173 |
| **1/11/2018** | 2:10:00 PM | 2.223 |
| **1/11/2018** | 2:20:00 PM | 2.274 |
| **1/11/2018** | 2:30:00 PM | 2.323 |
| **1/11/2018** | 2:40:00 PM | 2.344 |
| **1/11/2018** | 2:50:00 PM | 2.377 |
| **1/11/2018** | 3:00:00 PM | 2.404 |

Source: The Nigerian Navy Hydrographer Office (NNHO, 2019)

### 3.1.2 Automatic tide gauge (ATG)

A tide gauge (also known as mareograph, marigraphor sea-level recorder is a device for measuring the change in sea level relative to a vertical datum. At an automatic tide gauge (ATG) station, sensors continuously record the height of the water level with respect to a height reference surface close to the geoid. Water enters the device by the bottom pipe (far end of the tube, see Plate I), and electronic sensors measure its height and send the data to a computer (Ojinaka, 2007). Since the measurements are taken and recorded digitally, it is usually possible to observe at accurately regular interval for a long period of time.

A tide gauge, which is one component of a modern water level monitoring station, is fitted with sensors that continuously record the height of the surrounding water level. This data is critical for many coastal activities, including safe navigation, sound engineering, and habitat restoration and preservation (NOAA, 2020).

Plate I: An Automatic Tide Gauge (NOAA, 2019)

### 3.1.3 Data quality

Since the tide gauge is an automatic gauge station, the accuracy of the readings is 2mm. Also, the precise coordinates of the gauge station was collected and used for computation of nodal/satellite correction in the determination of ellipsoidal parameters. The specifications of the ATG used for this study is as summarized in Table 3.3 below.

Table 3.3: Properties of the Dockyard ATG

| S/No | Parameter | Value |
|---|---|---|
| **1** | Filmware version | 0741705B3 |
| **2** | Battery Level | 5.3 |
| **3** | Tide Master Serial No | 50454 |
| **4** | Station ID | 2 (Apapa Dockyard) |
| **5** | Calibration Date | 29/06/2015 |
| **6** | Mode | B3 |
| **7** | Pressure Units | M |
| **8** | Output format | Tide-master |
| **9** | User Pressure calculated: | |
| | Gain | 0.951 |
| | Offset | -0.1018 |
| **10** | Vale Pressure calculated: | |
| | P0 | 0 |
| | P1 | 0.0032 |
| | P2 | -1.7573 |

### 3.1.4 Tidal characteristics of the Lagos Lagoon

Based on the observed sea level data, the Lagos Lagoon as observed from the Dockyard tide gauge station has a semi-diurnal tidal regime. As seen in the overlay plot shown in Figure 3.1, it is observed that the tidal station is semi-diurnal having two daily high and low waters respectively. This is expected as the Lagos Lagoon is semi-diurnal (Badejo *et al.*, 2012). This further confirms that the tidal readings obtained from the ATG are correct and reliable.

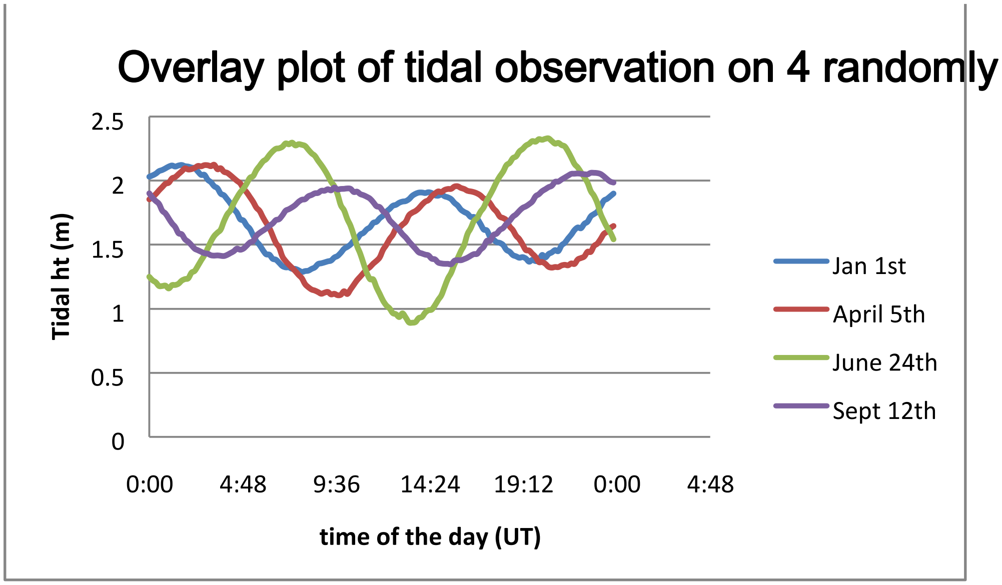



Figure 3.1: Overlay plot of tidal observations on 4 randomly selected days in 2019.

## 3.2 Methods

Standard geodetic methods as stipulated in previous hydrographic texts and acceptable by known hydrographic standards were adopted in this study. A hierarchical solution approach is implemented in this study in order to attain the study objectives. The solution approach utilized is described in a four stepped procedural approached written below and further illustrated in Figure 3.2.

Step 01: Determination of harmonic constituents‟ parameters (amplitude, phase and frequency) using OLS and IRLS models.

Step 02: Determine the confidence interval and error estimates for the analyzed constituents (amplitude, phase and frequency) determined in objective 1 based on the determined Signal to Noise Ratio (SNR) using the Covariance matric approach.

Step 03: Predict 24hours tides for one day (1month away from the observed data) based on the determined constituent values.

Step 04: Statistical assessment of the performance of the OLS and IRLS harmonic prediction model for the predicted tides and determination of tidal characteristics of a water body.

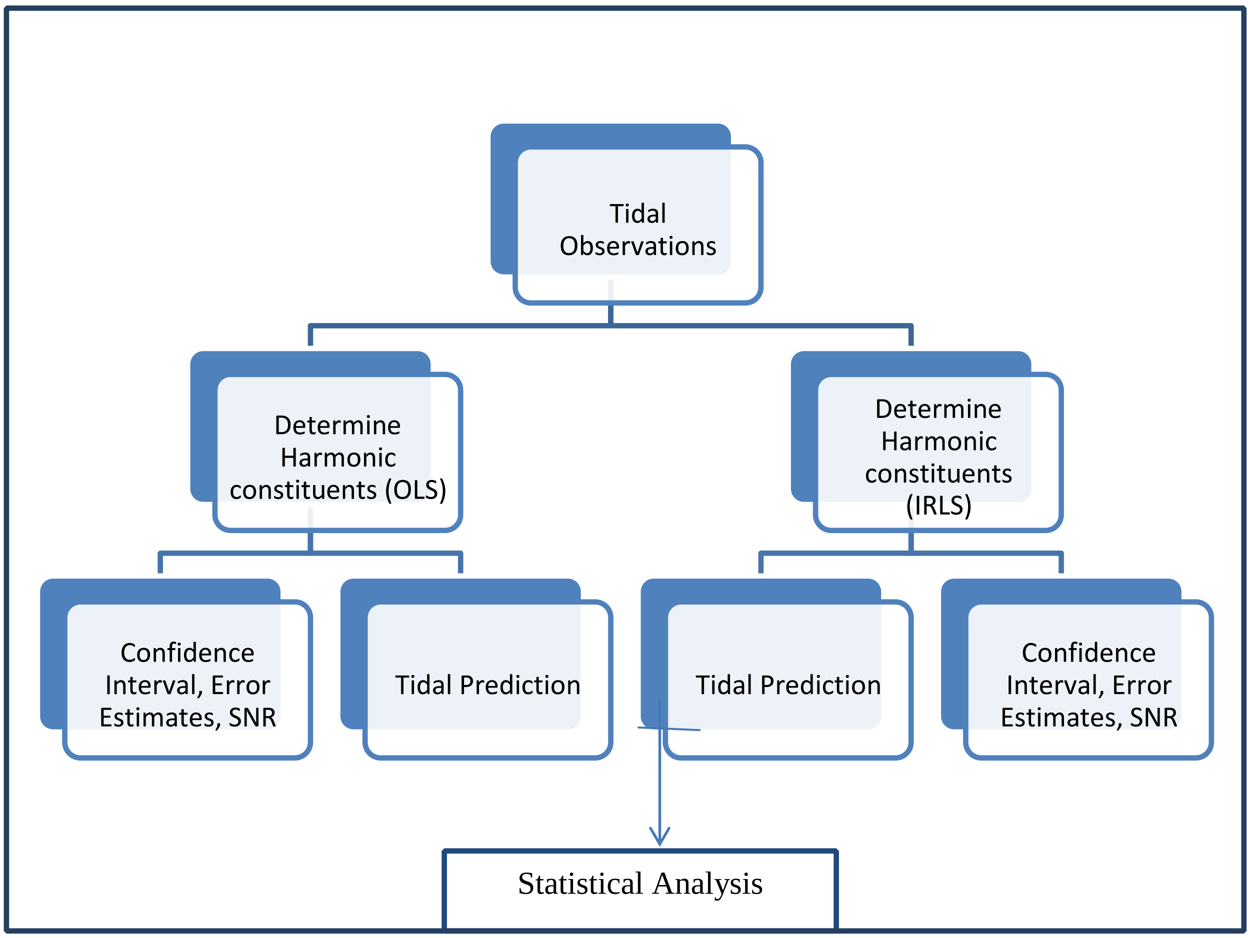



Figure 3.2: Hierarchical diagram showing work flow

As earlier specified, this study shall focus on four major kinds of data set being;

(i) Short period of regularly spaced tidal data (five months equally spaced dataset of tidal observation)

(ii) Short period irregularly spaced tidal data analysis (five months irregularly spaced tidal observations i.e white noise induced by presence of observational gaps)

(iii) Long span long sequence of regularly spaced tidal data analysis (Twelve months equally spaced dataset of tidal observation)

(iv) Long span sequence of irregularly spaced tidal data analysis (Twelve months irregularly spaced dataset of tidal observations i.e white noise induced by presence of observational gaps).

In order to achieve the desired result, Table 3.4 presents the data breaks and prediction dates used for the various stages of data analysis.

Table 3.4: Data period used for study

| **S/No** | **Analysis** | **Dates of tidal observation** | **Prediction date** | **Missing days** |
|---|---|---|---|---|
| **1** | Short period (no observational gaps) | 1st November 2018 31st March, 2019 | 7th May, 2019 | Nill |
| **2** | Short period (with observational gaps) | 1st November 2018 31st March, 2019 | 7th May, 2019 | 6days |
| **3** | Long span (no observational gaps) | 1st November 2018 31st October, 2019 | 7th Dec. „019 | Nill |
| **4** | Long span (with observational gaps) | 1st November 2018 31st October, 2019 | 7th Dec. „019 | 17days |

### 3.2.1 Determination of harmonic constituents

The algorithm used here for making phase and amplitude estimates is based on algorithms and MATLAB codes described Codiga (2011) as modified after Godin (1972), Foreman(1977), and Foreman and Neufeld (1991). In a series of arranged data, where the data is contained in a vector and the observation times are regularly spaced at an interval, let M be an odd number. The time axis is defined such that the origin (or central time) is time $t_{(M+1)/2}$. Missing observations for the OLS and IRLS can be handled by using a missing data marker in the input vector (by MATLAB convention this is NaN, the IEEE arithmetic representation for Not-a-Number). This regular interval restriction does not arise from the least-squares fit itself but rather from the automated constituent-selection algorithm and is also a requirement when spectra are estimated in one of the confidence interval algorithms.

The time series is passed to the analysis program "ut-analysis" tide along with a variety of (mostly optional) parameters. The tidal response is modeled as given in equation 2.1 while the constituents are determined as given by equation 2.2.

#### 3.2.1.1 Implementation for the OLS

In order to implement equation 2.1 and 2.2 in the OLS, all tide heights provided are used as the set of observation matrix (L). Then, observation equations are generated adopting a unit weight matrix as the weight of the observations. The observation equations are such as is given in equations 2.3 and 2.4. Since, amplitude, phase, and all the constituents are unknowns, the observation equation is first solved simultaneously using exact number of equations as unknowns. This is to determine the apriori-initial estimate for each of the unknowns. These estimated values are then used to compute the observation equations in order to fill up the design matrix given in equation 2.5.

Given the design matrix, matrix of observations and a unit matrix for weight, least squares solution for the parameters (tidal phase, tidal amplitude and astronomic arguments) can be obtained using equation 2.6.

### 3.2.1.2 Implementation for the IRLS

Implementing the IRLS is similar to that of the OLS, except that the estimates obtained from the OLS are again taken as a second iteration to recompute the design matrix. Also, the diagonal values of the variance -covariance matrix of the OLS solution, are now used to re-weight the observations, giving rise to a new weight matrix.

Given the new design matrix and weight matrix, the parameters are again solved and the solution becomes that of the IRLS.

### 3.2.2 Determination of confidence intervals and error estimates

A conversion from errors in the cos/sine amplitudes toerrors in standard parameters (amplitude and phase) can be done using a linearized analysis. Consider a constituent k:

Let $k = F(A_k, B_k)$be a non-linear function of these parameters, either the amplitude or the Greenwich phase. Then if $A_k,\ B_k$ are independent random variables, we can find a linearized estimate of the standard error of x in terms of the standard errors of the

sinusoid amplitudes is given by Rahman *et al*., (2017)

$$\sigma_\gamma{}^2 = (\frac{\partial F}{\partial A_k})^2 \sigma^2{}_A + (\frac{\partial F}{\partial B_k})^2 \sigma^2{}_B \qquad (3.1)$$

Where the partial derivatives can be derived exactly (buttediously).

Alternatively the non-linear mapping can be handle directly using a „parametric bootstrap"" (Efron and Tibshirani, 1993). In this situation the residual variance estimates are used by the code to simulate a number of realizations or replications of the analysis by taking the estimates of the sinusoid amplitudes and adding Gaussian noise with the appropriate variance to them. All of these realizations are then converted non-linearly to standard parameters using Eqs. 3.1and an estimate of the standard error computed from this replicate data set directly. Once a standard error is determined, 95% confidence intervals can be estimated using standard techniques. Alternatively, in this study, a signal-to-noise power ratio (SNR) was computed based on the square of the ratio of amplitude to amplitude error. Simulations performed in tsynth (Codiga, 2011) in which the variability of analyses carried out on a fixed data set with different noise realization are compared with estimated confidence intervals show that the linear procedure appears to be adequate for real time series (e.g. tidal height), as long as the SNR > 10; and is probably not bad for SNR as low as 2 or 3. The nonlinear procedure gives similar results to the linearized procedure at high SNR, and is more accurate at low SNR.

### 3.2.3 Predict 24hours tides for one day

Tidal prediction was done with the "ut-analysis" MATLAB package by the implementation of equation 2.6. However, the OLS and IRLS solution approaches are implemented using equations 2.6 and 2.7 respectively as earlier discussed in section 2.6.

### 3.2.4 Statistical assessments and determination of tidal characteristics

The standard Analysis of Variance (ANOVA) Test was used to test for the presence of any significant difference in the means of predicted results by the OLS and IRLS models. Thereafter the tidal characteristics for the water body were determined by harmonic calculations based on formulae given by Stephenson (2017) as follows:

MLWS (Mean Low Water Springs) = Z - (M2 + S2) (3.2)

MLWN (Mean Low Water Neaps) = Z –(M2 - S2) (3.3)

MSL (Mean Sea Level) = Z (3.4)

MHWN (Mean High Water Neaps) = Z + (M2 - S2) (3.5)

MHWS (Mean High Water Springs) = Z + (M2 + S2) (3.6)

The value of the MSL (Z) was determined using the formula (3.7) given by Popoola (1985), Data from 1982 – 1985 was obtained directly from Popoola (1985), while data from 2018 – 2019 was gotten directly from the observational data provided. The data gap was ignored and eliminated when utilizing equations 3.2 – 3.6.

$$MSL = \frac{\sum_{1}^{N} T_{obs}}{N} \quad (3.7)$$

Where:

$T_{obs}$ = Tidal observations

N = number of tidal observation

# CHAPTER FOUR

# 4.0 RESULTS AND DISCUSSIONS

## 4.1 Result Presentation and Discussion

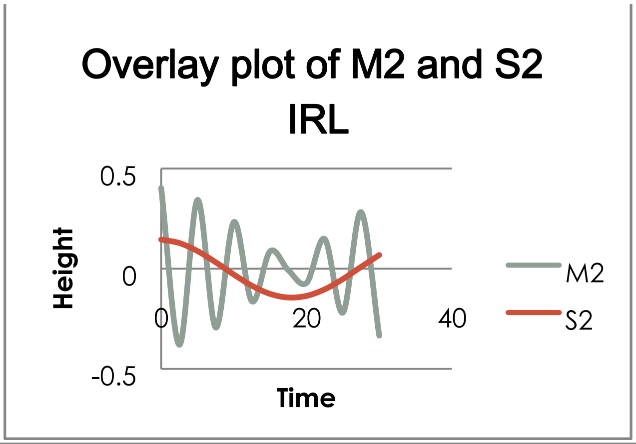
The results obtained in this study are presented in two phases. The phases are divided along the line of the length of the observation i.e the six month period observation and the twelve months period observation. Each phase is further discussed in two sections being the observational series without data gap and the series with data gaps. The ut-analysis tidal program written by Codiga (2011) was implemented for the analysis and the results obtained are as presented in sections 4.2 - 4.4.

## 4.2 Short Period without Observational Gaps

Table 4.1 shows the determined constituents‟ parameters as well as their confidence interval, SNR and PE. Based on the SNR and the PE, thirteen constituents are most significant being M2, S2, K1, N2, O2, MU2, M4, MS4, MN4, L2, MSF, MK3 and J1. As expected the M2 constituent (being the principal lunar semidiurnal constituent which relates to the moon) and the S2 (being the principal solar semidiurnal constituent) have the greatest impact with 76.5%, 9.69% then 76.47% and 9.79% when analyzed by the IRLS and OLS respectively. Graphical plot of the M2 and S2 constituents is shown in Figure 4.1.

Table 4.1: Determined Parameters of tidal harmonic constituents by OLS and IRLS technique (short observation period without gaps)

| | Iteratively Re-weighted Least Squares (IRLS) | | | | | | Ordinary Least S | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Const.** | **Amp** | **Phase** | **Amp_ci** | **Phase_ci** | **PE** | **SNR** | **Amp** | **Phase** | **Amp_ci** | **P** |
| **M2** | 0.405 | 107.0 | 0.000851 | 0.120 | 76.55% | 870000 | 0.405 | 107 | 0.00107 | |
| **S2** | 0.144 | 176.0 | 0.000872 | 0.336 | 9.69% | 110000 | 0.145 | 176 | 0.00103 | |
| **K1** | 0.129 | 23.6 | 0.000874 | 0.442 | 7.70% | 83000 | 0.129 | 23.4 | 0.00101 | |
| **N2** | 0.102 | 89.0 | 0.001000 | 0.473 | 4.87% | 40000 | 0.102 | 88.7 | 0.00101 | |
| **O1** | 0.027 | 311.0 | 0.001010 | 1.920 | 0.35% | 2800 | 0.0264 | 311 | 0.001 | |
| **MU2** | 0.026 | 47.8 | 0.000840 | 1.850 | 0.31% | 3600 | 0.0255 | 45 | 0.000905 | |
| **M4** | 0.022 | 343.0 | 0.000770 | 2.170 | 0.22% | 3100 | 0.0218 | 343 | 0.000918 | |
| **MS4** | 0.013 | 75.7 | 0.000768 | 3.800 | 0.08% | 1200 | 0.0133 | 75.5 | 0.000978 | |
| **MN4** | 0.011 | 295.0 | 0.000815 | 4.180 | 0.05% | 640 | 0.0098 | 298 | 0.00114 | |
| **L2** | 0.009 | 142.0 | 0.000807 | 5.090 | 0.04% | 470 | 0.00966 | 147 | 0.001 | |
| **MSF** | 0.007 | 185.0 | 0.000837 | 7.010 | 0.03% | 300 | 0.00841 | 179 | 0.00104 | |
| **MK3** | 0.006 | 272.0 | 0.000914 | 8.690 | 0.02% | 170 | 0.00656 | 274 | 0.000925 | |
| **J1** | 0.006 | 39.9 | 0.001120 | 9.980 | 0.02% | 98 | 0.00511 | 54 | 0.00113 | |
| **2Q1** | 0.005 | 57.6 | 0.000905 | 12.000 | 0.01% | 110 | 0.00475 | 49.4 | 0.0014 | |
| **MM** | 0.005 | 59.4 | 0.000931 | 10.500 | 0.01% | 100 | 0.00457 | 48.1 | 0.00111 | |
| **SK3** | 0.004 | 44.6 | 0.000936 | 12.400 | 0.01% | 82 | 0.00419 | 172 | 0.0013 | |
| **ETA2** | 0.004 | 194.0 | 0.001050 | 12.200 | 0.01% | 62 | 0.00417 | 41 | 0.00105 | |
| **EPS2** | 0.003 | 0.8 | 0.000842 | 30.200 | 0.01% | 65 | 0.00387 | 124 | 0.00121 | |
| **OO1** | 0.003 | 158.0 | 0.001030 | 21.400 | 0.00% | 37 | 0.00335 | 19.4 | 0.00105 | |
| **M6** | 0.003 | 94.6 | 0.000884 | 15.300 | 0.00% | 41 | 0.0027 | 98.4 | 0.00091 | |
| **Q1** | 0.002 | 47.3 | 0.001060 | 21.200 | 0.00% | 20 | 0.00236 | 11.9 | 0.00128 | |
| **UPS1** | 0.002 | 111.0 | 0.001260 | 27.000 | 0.00% | 14 | 0.00235 | 1.58 | 0.000918 | |
| **ALP1** | 0.002 | 356.0 | 0.000987 | 47.100 | 0.00% | 21 | 0.00226 | 169 | 0.000942 | |
| **2MN6** | 0.002 | 62.9 | 0.000722 | 23.500 | 0.00% | 33 | 0.00222 | 12 | 0.00114 | |
| **3MK7** | 0.002 | 270.0 | 0.000806 | 23.100 | 0.00% | 26 | 0.00216 | 62.9 | 0.000947 | |
| **M3** | 0.002 | 151.0 | 0.000847 | 22.800 | 0.00% | 23 | 0.00183 | 38.6 | 0.000921 | |
| **2MS6** | 0.002 | 193.0 | 0.000735 | 25.100 | 0.00% | 22 | 0.00182 | 275 | 0.00101 | |



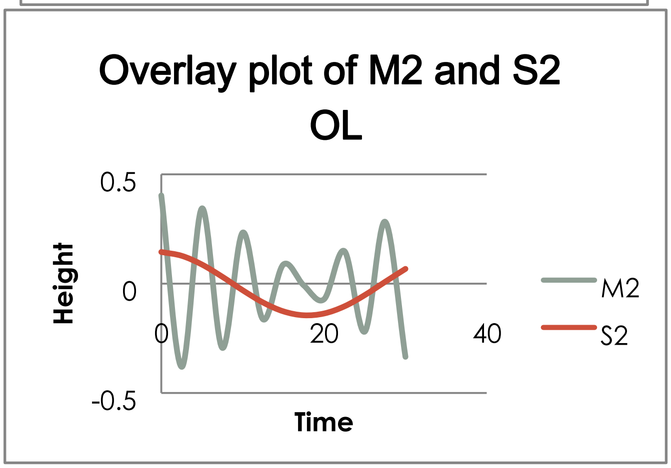




Figure 4.1: M2 and S2 constituents determined by IRLS (Left)

and OLS (Right)

Visual inspection of Figure 4.1 reveals that there is no significant difference between the determined parameters using the IRLS and the OLS technique.Table 4.2 presents the statistics of the predicted tidal values at the Dockyard station using both techniques. The predicted values were compared with tidal observations taken at the station on the day of the prediction ($7^{th}$ May, 2019). Similarly, Figure 4.2 presents a graphical plot of the overlay of the actual observation and the predicted observations using both methods.

Table 4.2: Statistics of prediction results

| Parameter | IRLS | OLS |
|---|---|---|
| Max | -0.0011 | 0.0019 |
| Min | -0.2643 | -0.2562 |
| RMSE | 0.0213 | 0.0204 |

From the statistics obtained in table 4.2 and the overlay plot in Figure 3, negligible difference is observed in the prediction by OLS and IRLS using short term data (3 months). In fact, the Root Mean Square Error (RMSE) of predictions suggests that the OLS is better suited for short term tidal analysis than the IRLS method. A probably reason for this is due to over-fitting of the data with the observations (Yen *et al*, 1996)

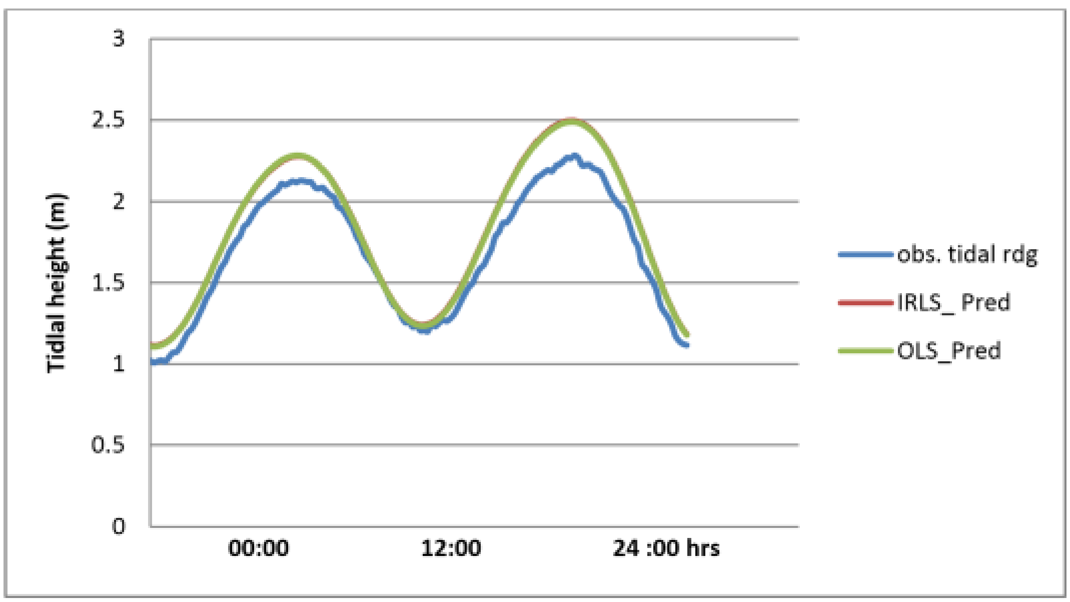



Figure 4.2: Overlay plot of observed and predicted tidal observations

Table 4.3 presents an analysis of variance (ANOVA) test conducted to test for the level of significance of differences in the predicted tidal values using the IRLS and the OLS techniques.

Table 4.3: ANOVA test for short period with regularly spaced tidal data

**SUMMARY**

| Groups | Count | Sum | Average | Variance |
|---|---|---|---|---|
| IRLS_ Pred | 144 | 263.05 | 1.83 | 0.189 |
| OLS_Pred | 144 | 262.58 | 1.82 | 0.189 |

ANO VA

| Source of Variation | SS | df | MS | F | P-value | F crit |
|---|---|---|---|---|---|---|
| Between Groups | 0.00077 | 1 | 0.00076 | 0.0041 | 0.9493 | 3.874 |
| Within Groups | 54.082 | 286 | 0.1891 | | | |
| Total | 54.083 | 287 | | | | |

As seen in table 4.3, it is observed that the F-crit (3.87) exceeds the F-table (0.0041), hence, we accept the null hypothesis that there is no significant difference between the predicted values using OLS and IRLS. This is further buttressed by the value of the variance being the same for the prediction results obtained from both the OLS and IRLS methods.

Based on the determined constituents, the tidal characteristics for the Apapa tide station was computed using equations 3.2 - 3.6 given in section 3.2.4. Table 4.4 presents the results of the harmonic tidal characteristics of the Apapa Dockyard Station as computed.

Table 4.4: characteristics of Dockyard computed with short observational data without gaps

| S/No | Tidal Characteristic | Value (m) |
|---|---|---|
| **1** | Mean High Water Springs | 1.004 |
| **2** | Mean Low Water Springs | 2.102 |
| **3** | Mean High Water Neaps | 1.814 |
| **4** | Mean Low Water Neaps | 1.292 |
| **5** | Mean Sea Level | 1.553 |

### 4.3 Short Period with Observational Gaps

Again Table 4.5 shows some of the significant harmonic constituents as well as their confidence interval, SNR and PE. In this case of data with observational gaps, only the first nine parameters maintained their level of most significance in the same order as in the case without missing data. Here the thirteen most significant parameters are M2, S2, K1, N2, O2, MU2, M4, MS4, MN4, L2, MSF, MK3 and 2Q1. Again as expected, the M2 constituent (being the principal lunar semidiurnal constituent which relates to the moon) and the S2 (being the principal solar semidiurnal constituent) have the greatest impact.

Table 4.5 Determined Parameters of tidal harmonic constituents by OLS and IRLS technique (short observation with gaps)

| | Iteratively Re-weighted Least Squares (IRLS) | | | | | | Ordinary Least Squares (OLS) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Const. | Amp | Phase | Amp_ci | Phase_ci | PE | SNR | Amp | Phase | Amp_ci | Phase_ci | PE | SNR |
| M2 | 0.405 | 107 | 0.000966 | 0.129 | 76.71% | 680000 | 0.405 | 107 | 0.000979 | 0.137 | 76.60% | 660000 |
| S2 | 0.144 | 176 | 0.000928 | 0.357 | 9.67% | 92000 | 0.145 | 176 | 0.001 | 0.387 | 9.75% | 80000 |
| K1 | 0.128 | 23.7 | 0.000906 | 0.434 | 7.61% | 76000 | 0.128 | 23.5 | 0.000999 | 0.511 | 7.64% | 63000 |
| N2 | 0.102 | 89 | 0.00091 | 0.457 | 4.85% | 48000 | 0.102 | 88.7 | 0.00115 | 0.541 | 4.87% | 31000 |
| O1 | 0.0273 | 311 | 0.000909 | 1.59 | 0.35% | 3500 | 0.0262 | 310 | 0.00116 | 2.28 | 0.32% | 2000 |
| MU2 | 0.0258 | 48 | 0.000772 | 1.96 | 0.31% | 4300 | 0.0256 | 45.2 | 0.000948 | 2.02 | 0.30% | 2800 |
| M4 | 0.0219 | 343 | 0.000822 | 2.27 | 0.22% | 2700 | 0.0219 | 342 | 0.000875 | 2.49 | 0.22% | 2400 |
| MS4 | 0.0133 | 75.3 | 0.000882 | 4.04 | 0.08% | 880 | 0.0133 | 75 | 0.000883 | 4.33 | 0.08% | 870 |
| MN4 | 0.0106 | 294 | 0.00103 | 4.48 | 0.05% | 400 | 0.0098 | 297 | 0.000956 | 5.3 | 0.04% | 400 |
| L2 | 0.00867 | 139 | 0.00091 | 4.86 | 0.04% | 350 | 0.0095 | 144 | 0.000981 | 5.34 | 0.04% | 360 |
| MSF | 0.00563 | 177 | 0.000931 | 10.1 | 0.01% | 140 | 0.0069 | 173 | 0.000965 | 9.36 | 0.02% | 200 |
| MK3 | 0.00599 | 270 | 0.000866 | 8.91 | 0.02% | 180 | 0.0063 | 272 | 0.00105 | 9.68 | 0.02% | 140 |
| 2Q1 | 0.00461 | 59.3 | 0.00103 | 13.6 | 0.01% | 78 | 0.0051 | 55.8 | 0.00105 | 13.2 | 0.01% | 90 |
| J1 | 0.00588 | 33.1 | 0.00123 | 10.1 | 0.02% | 88 | 0.00467 | 44.5 | 0.00128 | 12.9 | 0.01% | 51 |
| SK3 | 0.0043 | 44.2 | 0.000932 | 12.4 | 0.01% | 82 | 0.00427 | 40.1 | 0.00111 | 15.1 | 0.01% | 56 |
| ETA2 | 0.0043 | 193 | 0.00116 | 12.7 | 0.01% | 53 | 0.00414 | 171 | 0.00121 | 14.4 | 0.01% | 45 |
| OO1 | 0.00267 | 155 | 0.00101 | 23.7 | 0.00% | 27 | 0.00362 | 121 | 0.0014 | 18.7 | 0.01% | 26 |

The M2 and S2 components had percentage contribution of 76.71%, 9.67% then 76.6% and 9.75% respectively. Although, the difference in level of significance is negligible, the data gaps introduced some noise in the computed parameters.This seemingly insignificant difference in computed harmonic constituents contributed quite significantly to the value of the predicted tides. This is substantiated by an increase of 0.002m and 0.001m in the RMSE of predictions obtained from the IRLS and OLS methods respectively (Table 4.6).

Table 4.6: Statistics of predictions (short observation period with observational gaps)

| **Parameter** | **IRLS** | **OLS** |
|---|---|---|
| Max | -0.0044 | -0.0017 |
| Min | -0.2672 | -0.2589 |
| RMSE | 0.0220 | 0.0211 |

Again, although an insignificant difference is noticed, the RMSE obtained from the predictions based on OLS determined constituents is better than that of the IRLS determined constituents. This suggests that the OLS technique is suitable for tidal harmonics analysis and prediction so long as the dataset available for analysis does not exceed five months period even if there are observational gaps.

It should be noticed that six day were omitted out of 151 days in the performed analysis. This corresponds to an omission of about 840 observations out of 21,744 observations. All data were originally observed at 10minutes interval. This omission makes for about 4% of the total number of observations. Although, further analysis was not performed to ascertain the response of both models as the number of omissions increase, it is expected that the

IRLS could perform better than the OLS if the level of omission had exceeded 10% of required

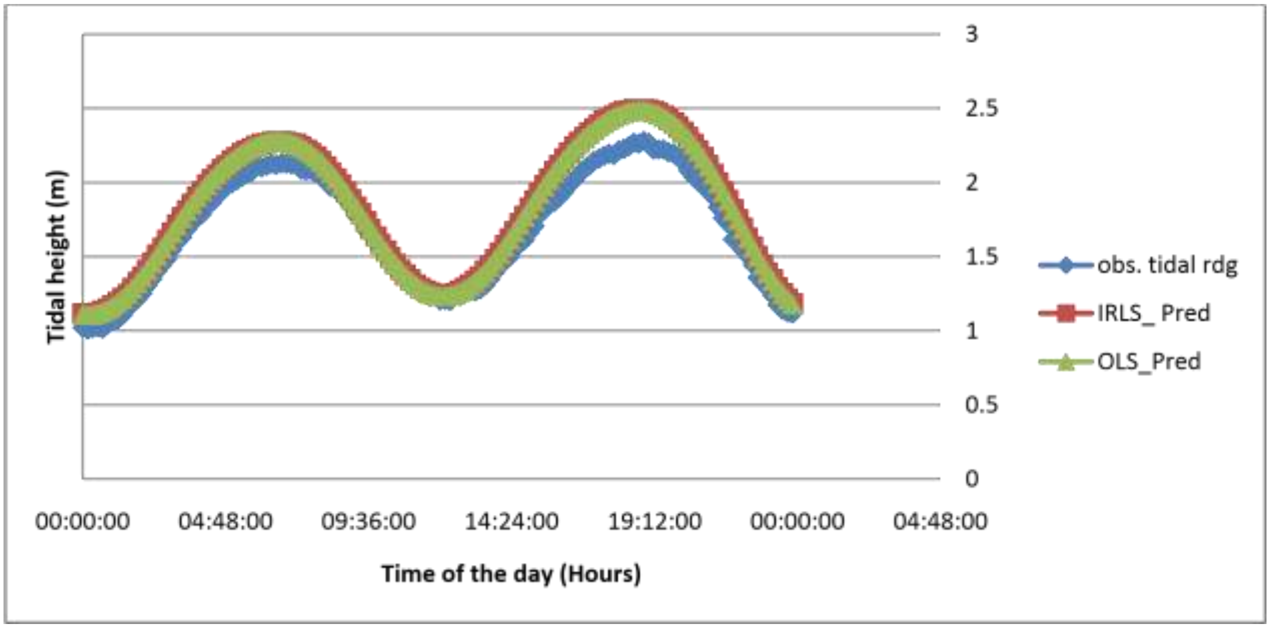



Figure 4.3: Overlay plot of observed and predicted tidal observations (short observation period with gaps)

observations given the observational interval. Furthermore, the study also discovered that an omission of over 4 consecutive days in the data entry was unable to resolve at all. This is because; the inconsistency in data interval became too large to be approximated by the „missing data"" making the input vector of the MATLAB program.

A graphical plot of the predicted and observed tides on the prediction date is shown in Figure 4.3, while an ANOVA test to confirm the statistical insignificance of the differences at 95% confidence interval is shown in Table 4.7. The prediction is based on the predictive equation given in equation 2.2.

As seen in table 4.7, it is observed that the F-crit (3.87) exceeds the F-table (0.0041), hence, we again accept the null hypothesis that there is no significant difference between the predicted values using OLS and IRLS at 95% confidence interval. The p-value of 0.95

indicates a 95% certainty/confidence of the ANOVA test about the determined relationship between the OLS and IRLS results.

able 4.7: Anova Single Factor Test

SUMMARY

| *Groups* | *Count* | *Sum* | *Average* | *Variance* |
|---|---|---|---|---|
| IRLS_ Pred | 144 | 263.049908 | 1.82673547 | 0.18896739 |
| OLS_Pred | 144 | 262.580063 | 1.82347266 | 0.18922879 |

ANOVA

| *Source of Variation* | *SS* | *df* | *MS* | *F* | *P-value* | *F crit* |
|---|---|---|---|---|---|---|
| Between Groups | 0.00076651 | 1 | 0.00076651 | 0.00405349 | 0.94927982 | 3.87417834 |
| Within Groups | 54.082053 | 286 | 0.18909809 | | | |
| Total | 54.0828195 | 287 | | | | |

Based on the determined harmonic constituents, the tidal characteristics at Dockyard tide gauge station in Apapa, Lagos state are as given in Table 4.8. It is noticed that the obtained tidal characteristics are the same as those determined in table 4.4 (short observational period without data gaps). This observation further suggests that an omission of about 4% in data observation would not necessarily compromise the accuracy of harmonic tidal analysis and predictions in cases where short observation period is used for such analysis.

Table 4.8 characteristics of Dockyard computed with short period tidal observation withgaps

| S/No | Tidal Characteristic | Value (m) |
|---|---|---|
| **1** | Mean High Water Springs | 1.004 |
| **2** | Mean Low Water Springs | 2.102 |
| **3** | Mean High Water Neaps | 1.814 |
| **4** | Mean Low Water Neaps | 1.292 |
| **5** | Mean Sea Level | 1.553 |

## 4.4 Long Span Observation Period without Observational Gaps

In the case of long span observation, fourteen months observationwas analyzed. A total of 36 constituents were estimated with the M2, L2, K1 and S2 being the four most significant constituents. The percentage energy for each of the four most significant constituents in ascending order are 42.26%, 24.03%, 8.73% and 3.95% respectively for the IRLS estimation and 42.39%, 28.28%, 8.73% and 5.62% respectively for the OLS estimation. The significance of the M2 and S2 constituents are expected being the principal semidiurnal constituents since the Lagos Lagoon is known to be a semi-diurnal water body.

However, the L2 and K1 correspond to the Luni-solar declinational semi-diurnal and diurnal constituents. The significance of the K1 constituent on a long term is surprising and suggests the need for a longer period of observation in-order to ascertain this variation. Nevertheless, such blend may lead to the assumption that the Lagos Lagoon occasionally exhibits a mixed tidal behavior. Table 4.9 presents an extract of the harmonic constituents derived from both the OLS and IRLS techniques.

Predictions based on the determined constituents also yielded similar levels of accuracy with those obtained from the short period of observation. However the IRLS technique produced better accuracy than the OLS technique in this case (see Table 4.10). It is also observed that the residuals obtained in both cases are quite high (Figure 4.4). This is expected as tidal prediction is often done using short range of data and not with long span data. However, further investigation is required on this.

Table 4.9: Extract of parameters of tidal harmonic constituents by OLS and IRLS technique (long span observation without gaps)

| | Iteratively Re-weighted Least Squares (IRLS) | | | | | | Ordinary Least Squares (OLS) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Constituent** | **Amp** | **Phase** | **Amp_ci** | **Phase_ci** | **PE** | **SNR** | **Amp** | **Phase** | **Amp_ci** | **Phase_ci** | **PE** | **SNR** |
| **M2** | 0.0739 | 288 | 0.00598 | 4.28 | 42.26% | 680 | 0.0689 | 288 | 0.00572 | 4.57 | 42.39% | 530 |
| **K1** | 0.0557 | 208 | 0.00543 | 6.43 | 24.03% | 510 | 0.0562 | 208 | 0.00576 | 6.73 | 28.28% | 310 |
| **L2** | 0.0336 | 252 | 0.00494 | 9.25 | 8.73% | 170 | 0.0298 | 252 | 0.00549 | 10 | 7.94% | 100 |
| **S2** | 0.0226 | 194 | 0.00613 | 14.3 | 3.95% | 45 | 0.0251 | 194 | 0.00613 | 13.2 | 5.62% | 76 |
| **ETA2** | 0.0225 | 289 | 0.00648 | 18.8 | 3.93% | 41 | 0.0168 | 289 | 0.00631 | 24.1 | 2.51% | 24 |
| **O1** | 0.0202 | 159 | 0.00615 | 15.5 | 3.17% | 49 | 0.0187 | 159 | 0.00619 | 18 | 3.12% | 32 |
| **N2** | 0.019 | 244 | 0.00575 | 15.4 | 2.80% | 61 | 0.0167 | 244 | 0.00638 | 18.9 | 2.48% | 27 |
| **EPS2** | 0.0179 | 214 | 0.00516 | 17.8 | 2.49% | 46 | 0.0116 | 214 | 0.00491 | 29.8 | 1.21% | 13 |
| **MM** | 0.0145 | 67.7 | 0.00555 | 21.7 | 1.63% | 29 | 0.0151 | 67.7 | 0.00546 | 24.5 | 2.05% | 32 |
| **SN4** | 0.0121 | 33.6 | 0.00508 | 29.3 | 1.14% | 17 | 0.00475 | 33.6 | 0.00573 | 70.9 | 0.20% | 5.2 |
| **MU2** | 0.0113 | 131 | 0.00537 | 25.8 | 0.99% | 14 | 0.0106 | 131 | 0.00533 | 31.7 | 1.00% | 12 |
| **MSF** | 0.0111 | 209 | 0.00545 | 30 | 0.96% | 22 | 0.0109 | 209 | 0.00536 | 29.5 | 1.06% | 12 |
| **Q1** | 0.0101 | 192 | 0.00743 | 37.9 | 0.79% | 12 | 0.00705 | 192 | 0.00669 | 56.3 | 0.44% | 4.5 |
| **M4** | 0.01 | 56.5 | 0.00444 | 30.8 | 0.78% | 17 | 0.00658 | 56.5 | 0.00546 | 45.9 | 0.39% | 6.9 |
| **J1** | 0.00708 | 321 | 0.00668 | 73.2 | 0.39% | 4 | 0.00777 | 321 | 0.00597 | 51.9 | 0.54% | 4.3 |

Table 4.10: Statistics of predictions (long span observation period without observational gaps)

| **Parameter** | **IRLS** | **OLS** |
|---|---|---|
| **Max** | 0.4742 | 0.4781 |
| **Min** | -0.9471 | -0.9431 |
| **RMSE** | 0.2118 | 0.2236 |

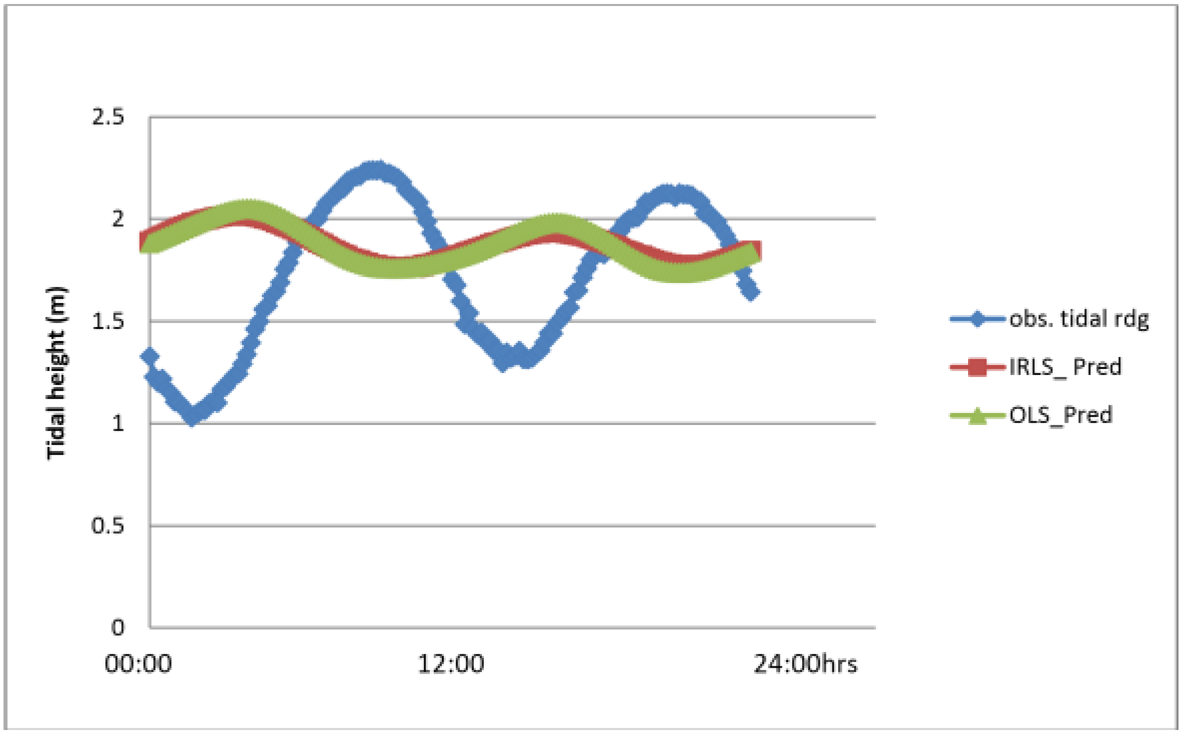



Figure 4.4: Overlay plot of observed and predicted tidal observations (long span observation period without gap)

ANOVA Test shown in Table 4.11 indicates that there is no significant difference between the predicted values by IRLS and OLS. It is observed that the F-crit (3.87) exceeds the Ftable (0.2922), hence, we again accept the null hypothesis that there is no significant difference between the predicted values using OLS and IRLS at 95% confidence interval. However, the p-value of 0.59 indicates a 59% certainty/confidence level of the ANOVA test about the determined relationship between the OLS and IRLS results.

Table 4.11: ANOVA Single Factor Test

| **SUMMARY** | | | | | | |
|---|---|---|---|---|---|---|
| ***Groups*** | *Count* | *Sum* | *Average* | *Variance* | | |
| **IRLS_ Pred** | 144 | 269.523419 | 1.87169041 | 0.00657592 | | |
| **OLS_Pred** | 144 | 270.34994 | 1.87743014 | 0.00966003 | | |
| **ANOVA** | | | | | | |
| ***Source of Variation*** | *SS* | *df* | *MS* | *F* | *P-value* | *F crit* |
| **Between Groups** | 0.002372 | 1 | 0.002372 | 0.29219153 | 0.58924027 | 3.87417834 |
| **Within Groups** | 2.32174058 | 286 | 0.00811797 | | | |
| **Total** | 2.32411258 | 287 | | | | |

Although, no significance was observed even with long span data, the better performance of the IRLS technique over the OLS technique indicates that the IRLS is better suited for muti-year data than the OLS. However, because the data used was only fourteen months data it is not possible to state if the OLS data would be suitable for analyzing data taken over a longer period of time. Efforts by the researcher to get longer period of data from the Nigerian Navy was unsuccessful. Table 4.12 presents the tidal characteristics computed at the Dockyard tide gauge based on the long span harmonic computation.

Table 4.12: Tidal characteristics of Dockyard computed with long span tidal observation without gaps

| S/No | Tidal Characteristic | Value (m) |
|---|---|---|
| **1** | Mean High Water Springs | 1.4565 |
| **2** | Mean Low Water Springs | 1.6495 |
| **3** | Mean High Water Neaps | 1.6043 |
| **4** | Mean Low Water Neaps | 1.5017 |
| **5** | Mean Sea Level | 1.553 |

### 4.5 Long Span Observation Period with Observational Gaps

Similar trend as observed with the long span data without gaps was seen with the long span data with observation gaps. Extract of the determined parameters using the IRLS and OLS methods for this case is presented in Table 4.13 while statistics of predictions from both methods is presented in Table 4.14.

As is the case of long span data without observational gaps, the M2, K1, L2 and S2 constituents were the leading four constituents. We observe that the 17days missing data resulted in as much as about 10cm difference in computed amplitude of the leading M2 constituent. This implies that the absence of 2448 out of 52,460 observations (about 5%) could result in very significant difference in the computed parameters of the constituents. While such percentage of omission had insignificant effect on the computed parameters, subsequent tidal prediction and also river tidal characteristics determination in the short period observation; it is noticed that this omission had significant effect on the computed tidal characteristics of the river computed by such constituents as shown in Table 4.15.

These results indicate that the use of long term tidal observations is good for tidal characteristics determination but not optimal for tidal prediction. Furthermore, the results obtained show that the IRLS model is a better option for long term tidal analysis where such is to be used for river tidal characteristics determination.

Table 4.13: Extract of parameters of tidal harmonic constituents by OLS and IRLS technique (long span observation with gaps)

| | Iteratively Re-weighted Least Squares (IRLS) | | | | | | Ordinary Least Squares (OLS) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Constituent** | **Amp** | **Phase** | **Amp_ci** | **Phase_ci** | **PE** | **SNR** | **Amp** | **Phase** | **Amp_ci** | **Phase_ci** | **PE** | **SNR** |
| **M2** | 0.0679 | 289 | 0.00607 | 4.74 | 38.00% | 480 | 0.0628 | 288 | 0.00566 | 5.03 | 38.32% | 470 |
| **K1** | 0.0564 | 209 | 0.00553 | 6.38 | 26.23% | 400 | 0.0569 | 207 | 0.00592 | 6.78 | 31.47% | 350 |
| **L2** | 0.0372 | 244 | 0.00514 | 8.08 | 11.38% | 200 | 0.0338 | 256 | 0.00547 | 9.02 | 11.10% | 150 |
| **S2** | 0.0232 | 288 | 0.00649 | 18.5 | 4.41% | 49 | 0.0239 | 207 | 0.00599 | 12.2 | 5.58% | 61 |
| **N2** | 0.0228 | 212 | 0.00623 | 14.9 | 4.29% | 52 | 0.0166 | 238 | 0.00648 | 19.6 | 2.67% | 25 |
| **ETA2** | 0.0193 | 246 | 0.00569 | 15.6 | 3.06% | 44 | 0.0166 | 290 | 0.00633 | 24.3 | 2.67% | 26 |
| **O1** | 0.018 | 170 | 0.00585 | 18.1 | 2.66% | 36 | 0.0163 | 165 | 0.00589 | 22 | 2.58% | 29 |
| **MM** | 0.0158 | 200 | 0.0059 | 19.3 | 2.07% | 28 | 0.0125 | 78.9 | 0.00537 | 33.2 | 1.53% | 21 |
| **EPS2** | 0.0127 | 79.2 | 0.00547 | 24.6 | 1.32% | 21 | 0.0107 | 199 | 0.00522 | 31.2 | 1.11% | 16 |
| **MU2** | 0.012 | 56.1 | 0.00509 | 29.8 | 1.18% | 21 | 0.00882 | 138 | 0.00552 | 37 | 0.76% | 9.8 |
| **MSF** | 0.0104 | 177 | 0.00533 | 28.6 | 0.88% | 15 | 0.00659 | 192 | 0.00478 | 53.7 | 0.42% | 7.3 |
| **M4** | 0.00941 | 54.4 | 0.00433 | 33.3 | 0.73% | 18 | 0.00634 | 56.3 | 0.00542 | 47.7 | 0.39% | 5.3 |
| **Q1** | 0.00887 | 342 | 0.00632 | 50.2 | 0.65% | 8 | 0.00574 | 212 | 0.00713 | 70.3 | 0.32% | 2.5 |

Table 4.14: Statistics of predictions

| Parameter | IRLS | OLS |
|---|---|---|
| **Max** | 0.4549 | 0.4508 |
| **Min** | -0.9411 | -0.9320 |
| **RMSE** | 0.2062 | 0.2178 |

Table 4.15: Tidal characteristics of Dockyard computed with long span period tidal observation with gaps

| S/No | Tidal Characteristic | Value (m) |
|---|---|---|
| **1** | Mean High Water Springs | 1.4619 |
| **2** | Mean Low Water Springs | 1.6441 |
| **3** | Mean High Water Neaps | 1.5977 |
| **4** | Mean Low Water Neaps | 1.5083 |
| **5** | Mean Sea Level | 1.553 |

Although, there are no officially published tidal characteristics for the Dockyard tide gauge which could be used to check the results of the computed tidal characteristic obtained from the study, it is suggested that the determined characteristics using the long term data without gaps is the most reliable. This assumption is based on established theories of tidal characteristics determination as given by Stephenson (2017).

Comparison of the determined tidal characteristics of the Dockyard tide gauge from all the data used is presented in table 4.16.

Table 4.16: Overview of determined harmonic tidal characteristics of the Lagos Lagoon at Dockyard tide gauge.

| Parameter | Short period (m) _ no gap | Short period (m) _ gap | Long span_ gap (m) | no Long span_ gaps (m) |
|---|---|---|---|---|
| **MLWS** | 1.004 | 1.004 | 1.4565 | 1.4619 |
| **MHWS** | 2.102 | 2.102 | 1.6495 | 1.6441 |
| **MHWN** | 1.814 | 1.814 | 1.6043 | 1.5977 |
| **MLWN** | 1.292 | 1.292 | 1.5017 | 1.5083 |

## 4.6 Pair-wise Comparison of Results

Table 4.17 presents a summary of the computed confidence interval and the Signal to Noise ratio (SNR) for the M2 (principal constituent) in each of the models used. It is obvious from the Table that the SNR increases with increase in the confidence interval of determination of the constituent and vice versa. This conforms with the findings of Oho and Suzuki (2012) and thus provides a quick means to determining the accuracy of a tide prediction model. Based on the obtained results presented in sections 4.1 - 4.3, Table 4.18 shows a pair-wise comparison of the OLS and IRLS models under various data scenarios.

Table 4.17: Confidence intervals and SNR derived for M2 constituent for each observational scenario.

| | **OLS** | | | | **IRLS** | | | |
|---|---|---|---|---|---|---|---|---|
| **Parameters** | **SS_No gap** | **SS_ gap** | **LS_No gap** | **LS_ gap** | **SS_No gap** | **SS_ gap** | **LS_No gap** | **LS_ gap** |
| Conf Int. | 76.47 | 76.6 | 42.39 | 37.32 | 76.55 | 76.71 | 42.26 | 38 |
| SNR | 5500 | 6600 | 5300 | 4700 | 8700 | 6800 | 6800 | 4800 |

The table ratings used in Table 4.18 are based on a 4-point likert scale as follows:

(i) Very good (SNR > 6000)

(ii) Good (5500 < SNR < 6000)

(iii) Fair (5000 < SNR < 5500)

(iv) Not good (SNR < 5000)

Table 4.18: Comparison of the OLS and IRLS methods.

| | | **OLS** | | **IRLS** | |
|---|---|---|---|---|---|
| **S/No** | **Parameter** | **Regularly spaced** | **Irregular Spaced** | **Regularly spa ced** | **Irregular Spaced** |
| 1 | Computational effectiveness | ***SS***: Very Good<br>***LS***: Fair | ***SS***: Good<br>***LS***: Not good | ***SS***: Very Good<br>***LS***: Good | ***SS***: Good<br>***LS***: Not good |
| 2 | Performance speed | ***SS***: 75secs<br>***LS***: 148secs | ***SS***: 102secs<br>***LS***: 176secs | ***SS***: 101secs<br>***LS***: 164secs | ***SS***: 132secs<br>***LS***: 192secs |
| 3 | Operational limitation | ***SS***: Very appropriate<br>***LS***: Not Appropriate | ***SS***: Appropriate<br>***LS***: Not Appropriate | ***SS***: Very appropriate<br>***LS***: Not Appropriate | ***SS***: Very appropriate<br>***LS***: Not Appropriate |

SS = Short Span data

LS = Long Span data

On the overall, the study showed that the OLS model is more suitable and accurate for tidal prediction given short term observational data (both when there are gaps and when there are no gaps) having outperformed the IRLS technique in this study with a RMSE of 0.0204m as against the IRLS with RMSE of 0.0213m in the case of complete data without gaps and RMSE of 0.0211m as against the IRLS with RMSE of 0.0220m in the case of data having observational gaps. However, in the case of tidal prediction using long span data, the IRLS had better RMSE in both scenarios (complete data set and missing data) with RMSE of 0.2118m and 0.2062m as against OLS model with RMSE of 0.2236m and

0.2178m respectively. This shows that the IRLS model is better for long span data tidal analysis (Codiga, 2011). Nevertheless, given the RMSE and goodness of fit (R), is is seen that the short term data is best for tidal prediction. This shows that tidal prediction is better done using short term observation (Badejo *et al*, 2012). Nevertheless, the study discovered that the long term tidal observation data might be more useful for determination of the tidal characteristics of the water body like the MHWS, MLWS, MHWN and the MLWN.

Although, there are no existing data that could be used to validate this, this assumption is based on existing literature which presupposes a period of 19years data for determination of such river characteristics (Ojinaka, 2007). This is justified by the significant difference in values obtained when analysis is performed using the four different observational scenarios implemented in this study.

# CHAPTER FIVE

## 5.0 CONCLUSION AND RECOMMENDATIONS

### 5.1 Conclusion

The OLS and IRLS methods have been used in this study to determine the tidal harmonic constituents for the Apapa Dockyard tide station and the results obtained have been presented in 4respectively. Based on the obtained results, the research objectives have been met and the following conclusions can be drawn.

The study by Codiga (2011) has been further validated and the developed MATLAB codes have been implemented (domesticated) in the determination of harmonic constituents and parameters (amplitude, phase and frequency) for different tidal observational scenarios using the UT-Tidal analysis hydrographic tool in MATLAB. From the results obtained, a total of 36 constituents were determined by both the IRLS and OLS models when the short term observation was analyzed while 38 constituents were determined from both models when the long span data was analyzed. This shows that a short observation period (with total data of about 21,744 observations) is sufficient to extract all relevant harmonic constituents required for tidal prediction and analysis.

Furthermore, the study showed that the confidence intervals and error estimates for the analyzed constituents (amplitude, phase and frequency) can be determined by the Signal to Noise Ratio (SNR). From the study, it is seen that the most significant constituents had the largest percentage energy (PE) and SNR. The study discovered that for short term data, the

M2 and S2 constituents are most significant as expected for a semi-diurnal water body. However, for the long term data (long span), the L1 and K1 constituents also appear to have some significance following the M1 constituent. This pattern of constituent significance explains that on a long term, most water bodies occasionally exhibit mixed tides (Badejo *et al*, 2012).

The study also confirmed the suitability of the use of the SNR values for estimating the accuracy of the determined constituents as large SNR signify large confidence intervals in the determination of the harmonic constituent as against lower SNR for low confidence interval of determination.

The study identified that while long term data might be better for river tidal characteristics determination with values of 1.457m and 1.6495m for the Mean Low Water Springs (MLWS) and MHWS respectively, short term data is best for tidal prediction. Hence, the study confirmed that short term data is better suited for tidal prediction than long term data (Badejo *et al*, 2012). This justifies the need for the establishment of several tide gauges at different locations along the waterways. It is only when there are recent, frequent and regular tidal data that accurate prediction can be done for ship navigational safety on the water ways.

It can therefore be concluded that there is no statistically significant difference between the OLS and IRLS models for tidal prediction when short term tidal observations are used. It is also concluded that there exists a statistically significant difference in results of tidal prediction when using short term and long span data.

## 5.2 Recommendations from the Study

Consequent upon the outcomes of this study, the following recommendations are proffered:

1. Tidal prediction along navigational routes should be based on short term observations not exceeding five months.
2. The Nigerian Navy Hydrographic office should make its data available for a longer period of time (say 10years) to facilitate further research in this area.

3. The outcomes of this study should be applied by the oil and gas sector and other stake holders in the maritime industry for tidal prediction towards ensuring navigation safety

4. The study recommends that automatic tide gauge should be set up at relevant tide stations along all major Nigerian waterways (such as Escravos, Bonny and Iddo) and be manage through collaborations between the NNHO and the NIWA.

### 5.3 Recommendations for Further Study

Based on the results obtained from the study, it is recommended that further analysis should be performed using a real long span data covering about five years for East Mole, Apapa, Calabar and Bonny tide gauges. The results obtained at these stations can then be compared with known national values on the performance of this method for river tidal characteristics determination.

## REFERENCES


Badejo, O. T. and Akintoye, S. O (2017). High and Low water prediction at Lagos Harbour, Nigeria. *Nigerian Journal of Technology (NIJOTECH)*. 36(3), 944–952. http://dx.doi.org/10.4314/njt.v36i3.39

Cartwright, D.E., 1999. *Tides – A Scientific History*, Cambridge University Press, Cambridge, UK, 292 pages

Codiga D. L. (2011). Unified Tidal Analysis and Prediction Using Untied Mat lab Functions. Report NO; URI/GSO technical report 2011-01 *University of Rhodes Island.* DOI: 10. 13/40/ RG.2.1.3761

Consoli, S., Recupero, D. R. and Zavarella, V. (2013). *A survey on Tidal analysis and forecasting*, US, 239-256.

Davis, M. A. Jones J. E. and Xing J. A. (1996). A Review of Recent Development in Tidal Hydrodynamic Modelling. *Part I and II. J. of Hydraulic Engineering* ASCE 123, 278 – 302.

Dean, R.G., (1966). Tides and Harmonic Analysis. In: *Estuary and Coastline Hydrodynamics*, (Ed.) Arthur T. Ippen, McGraw-Hill Book Company, New York, 197-230.

Doodson, A.T., (1921). The harmonic development of the tide-generating potential. Proceedings of the Royal Society of America, 100: 305-329.

Efron, B. and Tibshirani, R. J. (1993).An Introduction to the Bootstrap. Monogram of Statistics and Applied Probability, New York. Chapman and Hall.

Foreman, M. G. G. (2004).*Manual for Tidal Currents Analysis and Prediction.* Pacific Marine Science Report 78-6, Inst. of Ocean Sciences, Patricia Bay, Victoria, B.C., 70 pages. (Online revised version of 1978 original)

Foreman, M.G.G., and E.T. Neufeld (1991).Harmonic analysis of long time series. *International Hydrographic Review,* 68(1): 85-108.

Forrester. Warren D., (1983) *Canadian Tide Manual*, Dept. of Fisheries and Oceans, Ottawa, CA, 137 pages.

Godin, Gabriel, (1988). *Tides.* CICESE, Ensenda, Mexico, 290 pages

Hicks S. D. (2006). Understanding Tides, Center for Operational Oceanographic Products and Services. National Ocean Service NOAA 66 pages.

Huber, P.J. and Ronchetti, E.M. (2009) Robust Statistics. Wiley Series in Probability and Statistics. 2nd Edition, John Wiley & Sons, Inc., Hoboken.

Jian, A. S. (2003). Prediction and Analysis of Tides and Tidal currents. *International Hydrographic Review*, 24 – 29.

Jenkins, A. P. (2014). Oblique internal tide generation. An MSc thesis submitted to the graduate division of the University of Hawai. 55.

Kowalick, T. and Luick, W. (2019). Modern Theory and Practice of Tide Analysis and Tidal Power, Austidies Consulting Australia. 216 pages.

Lee, T. (2003). Back-propagation neural network for long-term tidal predictions. MSc thesis of the Department of Construction Technology, Leader University, Tainan Taiwan, 709.

Lisitzin, E. (1974). Sea LEVEL Changes. Limnology and oceanography, Elsevier science publishers Amsterdam oxford. 286 pages

Mellor, G. L (1996).*Introduction to physical oceanography*. Springer, U. S. A. ISBN 1563962101

Mellor, G., Sirpa H., Tal E., and Richard P. (2002). A generalization of a sigma coordinate ocean model and an inter-comparison of model vertical grids. In: *Ocean Forecasting: Conceptual Basics and Applications*, N. Pinardi and J. Woods (Eds.), Springer, Berlin, pp 55-72.

Munk, W. H. and Cartwright, D. E. (1966).Tidal spectroscope and prediction. *Philosophy and Transactions of the Royal Society of America*, 259, 533-581.

NOAA, (2021). Changing Technology for Tide Measurements the Way it was Retrieved Online (a) www. Celebrating 200 years. NOAA. GOV/Foundation/Tides.

Oho, E. and Suzsik, K. (2012). Highly Accurate SNR Measurement Using the Covariance of two SEM images with the identical view. Scanning 34, (2010), 43 – 50.

Ojinaka, O. J. (2007). Principles of *Hydrographic Surveying_ from Sextant to Satellite*. First Edition. International Standard book No 978-8143-10-5

Okusipe, O. M (2008). Lagos Lagoon Crystal Profile: Information Database for Planning Theory. PhD Thesis of the Department of URP UNILAG.

Parker, B. B. (2007): Tidal analysis and prediction. NOAA Special Publication NOS COOPS 3, U.S. Department of Commerce.378. (Available online at http://tidesandcurrents.noaa.gov.)

Pawlowicz, R., B. Beardsley, and S. Lentz, 2002: Classical tidal harmonic analysis including error estimates in MATLAB using T_TIDE. *Computer and Geosciences*, 28, 929–937.

Pawlowicz, R., Bob B., and Steven Lenz, 2002. Classical tidal harmonic analysis including error estimates in MATLAB using T_TIDE, *Computers & Geosciences*, 28:929-937

Popoola, B. M. (1985). A study of Annual sea level variation in Nigerian waters. A BSc project report of the Department of Surveying, University of Lagos, Nigeria.

Pugh, d. t. (2004). Changing Sea Levels. Effect of Tides, Weather and Climate. Cambridge University Press. 265 pages

Pugh, D.T., (1987). *Tides, Surges, and Mean Sea Level*. John Wiley & Sons, New York, 472 pages

Rahman, M. M. Paul G. C. and Houque, A. (2017). A study on Tidal Potential and Tide Generating force. *Ganit Journal of the Bagladesh Mathematics Society*. 37, 29 – 37

Reddy, M. P. M. and Affholder, M. (2001).*Descriptive physical oceanography: State of the Art*. Taylors and Francis, England.

Shu, J. J (2003). Prediction and Anal sis of Tides and Tidal currents. *International Hydrographic Review*. 4 (2), 43 - 56.

Smith, A. J. Ambrosius, and A. C., Wakker, K. F. Woodworth P. L. and Vassie J.M. (1997). Comparism between the Harmonic and Response Methods of Tidal Analysis Using TOPEX/POSEIDON *Altimetry Journal of Geodesy*. 71.695 – 703

Smith, R.S. (1979). The Lagos Consulate: University of California Press. 143 pages

Tianhang, H and Vanicek, P. (1988). Towards a Real-time Tidal Analysis and Prediction, Department of Surveying Engineering, University of New Brunswick, Fredericton, N.B. Canada.

Tsai, C. P., Lee, T. L. (1999). Back-propagation neural network in tidal-level forecasting. *Ocean Engineering*. AS E 12 (4), 195–202.

Uduma – Olugu, N. and Okuwaye, L. (2010): the Regeneration of Lagos Lagoon Water Fronts for Recreation and Tourism. Proceedings of the Real Corps International Conference. 18-20 may 2010. Vienina A ties for everyone. 757 – 762.

Wahr, J. (1995). *Geodesy and gravity*. Colorado, Samizdat press Ltd.

Yen, P. H., Jan, C. D., Lee, Y. P., Lee, H. F. (1996).Application of Kalman filter to short term tide level prediction. *Oceanographic Engineering*, 122 (5), 226–231.