**CROSSOVER ANALYSIS OF BGI GRAVITY DATASETS ACROSS NIGERIA**

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**ABSTRACT**

The global gravity database of the Bureau Gravimetric International (BGI) has been used as a reliable source of gravity data for scientific inquiry and gravity field modelling by many countries, especially in regions with sparse terrestrial gravity data. However, considering the heterogeneity of the sources of the data in the BGI database, this study investigated the observational consistency of the BGI datasets over Nigeria. To that end, 4511 gravity data points covering Nigeria were collected from the BGI database and used in this study. The 4511 points were results of campaigns conducted by 15 different organizations at different times between 1938 and 1983. Taking the dataset obtained from the International Gravity Standardization Network 1971 (IGSN-71) as the control points, Crossover adjustment has been performed on all the existing datasets across the country for detection of observational inconsistencies in the datasets. Analysis of the Crossover adjustment at intersection points with the IGSN-71 datasets indicate high consistency of the dataset from British Antarctic survey, with values of -1.03 mgals, 2 mgals and 0.8 mgals in absolute gravity, free Air (FA) anomaly, and bouguer anomaly, respectively.

**Keywords**: *BGI, gravity database, gravity field, least squares*

**1.0 Introduction**

Determining an accurate national gravimetric geoid model upon which reliable conversion of ellipsoidal to orthometric heights will be based is dependent on the accuracy of the point gravity measurements that serve as input in the overall geoid modelling process.

Terrestrial gravity observations with the use of either the relative or absolute gravimeter are usually a time-consuming exercise resulting in multiple observation campaigns especially when very large area is to be covered. As a means of accuracy check, each observational campaign is tied to known gravity control points (Osazuwa and Ajakaiye, 1987). However, due to instrumental errors, navigational errors, and other error sources, significant inconsistencies often exist between the different observational campaigns (Wessel and Watts, 1988; Denker and Roland, 2005).While appropriately structured stochastic and mathematical models could be used to minimize instrumental errors (systematic errors) within each cruise (Hwang, Wang and Lee, 2002), individual random errors within cruises accumulate to result in inconsistencies in measured gravity values. Such errors as well as other observational inconsistencies result in obvious differences in the parametric values of same observation point between cruises.

Analysis and subsequent adjustment of these parametric differences at similar observation points known as crossover error analysis (COA) or crossover adjustment has become a well known method for minimizing observational inconsistencies in multiple gravity campaigns (Wenzel, 1992; Denker and Roland, 2005). The crossover differences are used for determining the error parameter per observational campaign using the ordinary least squares (OLS) approach.

Performing the crossover analysis requires that a standard base measurement is adopted upon which the bias per observational regime will thereafter be computed. In this research, the data points which form part of the International Gravity Standardization Network 1971 (IGSN-71) that are located within the study area were adopted as base values due to the accuracy of their measurements and subsequent adjustments to which the points were subjected (Morelli et al, 1972). Besides, since these points are consistent with global standards in terms of observational datum, there reliability is further justified as a stable base for this analysis.

In countries like Nigeria which have a very sparse gravity point distribution, archived gravity data from recognised gravity data repositories mostly the BGI. However, the accuracy of these archived dataset need to be ascertained prior to their use for geoid modelling and other geodetic or geophysical studies. It is for this reason that this research is conducted using the Crossover analysis method to determine the observational consistency of the BGI dataset over Nigeria.

**2.0 Crossover analysis**

Standard crossover error estimation (COE) procedure considering observational bias between tracks / campaigns was utilized in this study. The COE model is as given in the following equations

$l\_{i}+ v\_{i}+ b\_{i}= g\_{x}$ (1)

$l\_{j}+ v\_{j}+ b\_{j}= g\_{x}$ (2)

where $l\_{i}, l\_{j}$ are gravity observations of tracks i and j at the crossover point;

$v\_{i}, v\_{j}$ are corresponding observation residuals of tracks i and j;

$b\_{i}, b\_{j}$ are bias parameters for tracks i and j.

$l\_{j}+ v\_{j}+ b\_{j}= g\_{x}$ is the final gravity value

Combining 1 and 2 the crossover differences can be obtained in the form of an observation equation as given in 3 (Denker and Roland, 2005):

$(l\_{i}- l\_{j}) + (v\_{i}- v\_{j}) = d\_{ij}+ v\_{ij}= b\_{j}- b\_{i}.$ (3)

$d\_{ij}$ = $(l\_{i}- l\_{j})$

Since the standard least squares observation equation requires that

$L^{b}+ V=L^{a}$ (4)

where $L^{b}$ = unadjusted observations

$V$ = observational residuals

$L^{a}$ = adjusted observations

Therefore 3 can be easily solved using the conventional least squares approach. Having identified cross over points, appropriate observation and subsequently design and normal matrix were formed and executed accordingly. From equation 3, it should be noted that $b$ is a 2-element vector comprising of the track bias and translation parameters (Marsh et al, 1982; Rapp, 1983).

As is expected in most least squares problems, the accuracy of the adjustment process depends on the spread of the control points, therefore the crossover adjustment can only be practically relevant when all other points to be corrected by the derived parameters are within the limits of the crossover points (Yen et al, 1995).

**3.0 Data used and data treatment procedure**

The BGI database comprises of datasets collected by several institutions and agencies over several years. The data used, year of observation and number of points collected are as presented in table 1. The data points used in this study were a total of 4511 points.

As presented in the table 1, the BGI data covering the study area is a collection of data observed from (15) sources between 1938 and 1984. Consequent upon this time lag and difference in observing agencies, observational inconsistencies resulting in discrepancies in parametric values of cross over points is expected.

Table 1: Gravity data from various sources

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **S/N** | **Data Source** | **Number of points** | **Accuracy (mgals)** | **Date of Observation** |
| 1 | IGSN-71 (BGI) | 1 | 0.040 | 1-11-1961 |
| 2 | Euro/African Secondary Calibration Line survey (BGI/IGSN-71) | 4 | 0.032 | 1-11-1965 |
| 3 | British Antarctic Survey (BGI) | 63 | Not available | 1-11-1975 |
| 4 | Academy of Science, France (BGI) | 11 | Not available | 1-11-1938 |
| 5 | Princeton University, USA (BGI) | 220 | Not available | 1-1-1969 |
| 6 | University of Leeds (BGI) | 789 | Not available | 1-11-1984 |
| 7 | Geological survey of Nigeria (BGI) | 987 | Not available | 1-11-1961 |
| 8 | Shell Exploration Company (BGI) | 69 | Not available | 1-1-1965 |
| 9 | University of Ibadan (BGI) | 192 | Not available | 1-1-1978 |
| 10 | Ahmadu Bello University, Zaria (BGI) | 151 | Not available | 1-1-1978 |
| 11 | University of Ibadan (BGI) | 303 | Not available | 1-1-1978 |
| 12 | University of Calabar and Leeds (BGI) | 1074 | Not available | 1-1-1984 |
| 13 | Cratchley, C. R (1960) BGI | 460 | Not available | 1-1-1960 |
| 14 | Anonymous observer (BGI) | 69 | Not available | Unknown |
| 15 | Garcia, G (IGC / BGI) | 117 | Not available | 1-11-1967 |

A plot of the spatial distribution of all these data points across the country is presented in figure 1



Figure 1: spatial spread of all BGI data points across the country

The crossover error analysis (COA) technique was employed in-order to determine the internal consistency of the BGI datasets as earlier described in section 2.0 and equations 2 and 3. Results obtained after the COA is as presented in table 3 of the results section.

In selecting crossover points, the following factors were considered to ensure accuracy in the obtained results.

1. how to minimize error due to interpolation technique adopted for gravity prediction at crossover point
2. changes in gravity with respect to latitude (i.e., the spatial dependence of gravity on difference in latitude)
3. the spatial autocorrelation of gravity signals with increasing distance apart.

Based on these factors, crossover point selection was done considering a spatial separation threshold of less than or equal to 2km as the maximum acceptable distance difference for points to be selected as crossover points.

From the collected datasets, six sets were not analysed for reasons mentioned in table 2.

Table 2: Datasets not analyzed

|  |  |
| --- | --- |
| **Dataset**  | **Reason** |
| Ahmadu Bello University, Zaria | No CO Points |
| Observations by Garcia | No CO Points |
| University of Calabar | No standard points for CO |
| Cratchley | No absolute gravity data |
| Garcia | Outside national boundary |
| Anonymous observer | Outside national boundary |

CO = crossover

**5.0 Results and discussions**

The result of the cross over analysis is as summarized in table 3. Two classes of inconsistent datasets were identified. The first class of inconsistent datasets are those with parametric differences exceeding 25 mgals in absolute gravity across all the observed natural crossover points. Such points were therefore considered as unacceptable datasets (Figure 2).

Another class of inconsistent datasets (though not eliminated) were data sets with varied level of crossover error i.e. while acceptable level of error was achieved at some points, unacceptable values greater than 25mgal were obtained at other points. Although COA was applied to such datasets, results obtained showed poor accuracy level.

Points with poor spatial spread of crossover points were however not adjusted by the standard crossover adjustment technique. Rather the mean of identified crossover errors was adopted.

After the crossover analysis and adjustment, a total of 2634 gravity points were found to be consistent. These datasets could therefore be considered as being a internally consistent with themselves making then suitable for geoid modelling and other associated national gravity-related tasks.

Table 3: Results of crossover analysis and adjustment (heights (Ht) are in metres).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Standard dataset | **Analyzed dataset** | **Max observed diff (mgals) before adjustment (Abs Grav/FA/Bou/Ht)** | **Min observed diff before adjustment** | **Adopted corrn. factor (mgals) (Abs grav/FA/Bou)** | **Max observed diff (mgals)after adjustment (Abs Grav)** | **Min observed diff (mgals) after adjustment (Abs Grav)** | **Std dev of unit weight after COA**  | **Remarks** |
| **IGSN Data** | British Antarctic Survey (BGI) | 1.03 / 2 / 0.8 / 9.6 | Not applicable | Not required | Not Applicable | Not Applicable | Not readjusted | Consistent |
| **IGSN Data & Source 2** | French Academy of Science | -15.57 / -20.00 / -19.00 | -8.53 / -36.50 / -23.40 | -12.81 (Mean) | 4.28 | 1.63 | 0.00002 | Not Consistent. COA was not performed because of the poor spatial spread of the control data compared to the profile data available. This is especially necessary as the predictions from COA became expectedly poorer with increasing distance from the CO point. The mean of observed CO differences was thus adopted for data correction |
| **IGSN Data** | Princeton University, USA  | -4.8 / -0.9 / -2.4 | Not applicable | Not required | Not applicable | Not applicable | Not readjusted | Consistent |
| **Source 4** | University of Leeds | 10.1 / -0.2 / 3.5 | -3.6 / 0.5 / -1 | COA  | 5.23  | -8.47  | 0.000004 | COA performed |
| **IGSN Data** | University of Ibadan (BGI) | 1.07 / 0.2 / 0.2 | Not applicable | Not required | Not applicable | Not applicable | Not readjusted | Consistent |
| **Source 6** | Shell Exploration Company | 32.8 / -2.8 / 9.9 | -75 / -15 / -14 | Outlier | Not Applicable | Not Applicable | Not readjusted | Outlier |
| **Source 2** | University of Ibadan (BGI) set 2 | 2.12 / 4.3 / 3.6 | -0.28 / -7.3 / -4.7 | 1.02 (Mean) | 1.1 | -1.3 | 0.000009 | Consistent |
| **Source 8** | Geological Surveys of Nigeria  | 8.15 / -4.7 / -0.1 | -14.53 / 3.9 / -2.8 | COA  | 2.995 | -6.142 | 22.5077 | Inconsistent. Large value of standard deviation further reflects the inconsistence in gravity values within the crossover area. |



Figure 2: Crossover analysis between University of Ibadan dataset and Shell data set.

The dataset from Shell was removed from the datasets used in this research on the basis of the identified inconsistencies with the base data (University of Ibadan dataset). All four crossover points utilized, revealed crossover errors ranging from -49.2 mgals – 65.9 mgals in absolute gravity value.

Another interesting point to note from table 3 is that very high error in absolute gravity value does not necessarily correspond to same magnitude of error in the resulting anomaly. This is because the gravity anomaly is also a function of the adopted height value. It is for this reason that the COA was done based on absolute gravity values rather than on the anomaly values.

**6.0 Conclusion**

Crossover analysis of the BGI datasets across the country has been performed and observational inconsistencies identified in the datasets. Datasets from 6 sources have been excluded from the crossover analysis some due to lack of crossover points for analysis and others due to their location being outside the country’s boundary. The remaining 8 data sources analysed reflect varied level of consistency or otherwise with the IGSN data which has been adopted as the base data for this study. Data from the British Antarctic survey was found to be most consistent with the IGSN data with error values of 1.03 mgals, 2 mgals and 0.8mgals in absolute gravity, free air and bouguer anomaly respectively.

This study therefore has identified a total of 2634 observationally consistent gravity points from within the BGI datasets which could be subjected to further internal consistency checks prior utilisation for national gravity field modelling.

Other internal consistency checks recommended for further study to which these 2634 data points could be subjected to includes:

1. Analysis of the closure of loops
2. Lowpass filtering of gravity signals for detection and removal of outliers via gravity smoothing

**List of abbreviations**

CO= Crossover

COA= Crossover analysis

BGI= Bureau Gravimetric International

**REFERENCES**

### Denker H and M. Roland (2005). C[ompilation and evaluation of a consistent marine gravity data set surrounding Europe](http://link.springer.com/chapter/10.1007/3-540-27432-4_42).A window on the Future of Geodesy, Springer.

Esan, O. (2000). Spectral analysis of gravity field data and error in view of sub-decimeter geoid determination in Canada. MSc dissertation, Department of Geomatics Engineering, University of Calgary.

Hwang C., Wang C. G. and Lee, L. H. (2002).Adjustment of relative gravity measurements using weighted and datum-free constraints. *Journal of Computers & Geosciences* Vol 28 pp 1005–1015.

Marsh, J. G., CheneyR. E., Martin, T. V. and McCarthy, J. J. (1982).Computation of a precise mean sea surface in the eastern North Pacific using satellite altimetry, *EOS Trans. AGU,* Vol 63,178-179.

Morelli, C, Gantar, C., Honkasalo, T., McConnel, R., Szabo, B., Tanner, J., Uotila, U. and Wallen, C. (1972). The International gravity standardization network 1971. Report of the working group on international gravity standards of the IAG to the IUGG XV general assembly, Moscow.

Osazuwa, I. B and Ajakaiye D. E (1987): Gravity control network at airports in Nigeria. *Journal of Geodynamics*, 7, 303 – 317.

Rapp, R.H (1983). The determination of geoid undulations and gravity anomalies from Seasat altimeter data. *Journal of Geophysical research, 88*, 1552-1562.

Wessel, P. and Watts,A. B. (1988).On the accuracy of marine gravity measurements.*Journal of Geophysical Research*, 93(B1):393–413.

Yen, H. Y, W. T. Liang, B. Y. Kuo, Y. H. Yeh, C. S. Liu, D. Reed, N. Lundberg, F. C. Su and H. S. Chung (1995). A regional gravity map for the subduction-collision zone near Taiwan.*TAO, Vol 6. No. 2*, 233 – 250.