**ANALYSIS OF SOME FACTORS THAT AFFECT ACCURACY IN LONG WAVELENGTH GEOID DETERMINATION USING GRAFLAB**

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**ABSTRACT**

In recent times, the advent of computer based tools for long wavelength geoid computation has popularized the ease of long wavelength geoid and gravity anomaly computation. Evaluating these gravity field functionals however require a proper understanding of the effects of certain computational preferences in the overall geoid computation process as was applied in the study area of Minna. Four computational factors were identified for analysis in this study using the grafLab program. The factors considered are: the Associated Legendre Function (ALF), choice of maximum degree of expansion of harmonic model, choice of geo-potential model (EGM96 and EGM2008) and choice of ellipsoidal surface (WGS 84 and GRS 80). The results obtained confirm the numerical stability of the ALF’s even at ultra high degrees within the study area (Minna metropolis of Niger state). However, the degree of expansion and ellipsoid of computation greatly affect the result of the computed long wavelength geoid. A geoid undulation difference ranging between 0.93m – 0.94m was observed between the WGS84 and GRS80 ellipsoid while haphazard fluctuation in values was observed between different degrees of expansion. Long wavelength geoid maps of the study area was produced from the maximum degree expansion of each harmonic model used.

**Keywords**: *Associated Legendre function (ALF), co-ordinate systems, ellipsoidal surfaces, geo-potential models, gravity field functionals and spherical harmonics expansion.*

1. **Introduction**

The ease with which 3D point positioning is achieved with GNSS receivers is gradually leading to the replacing of the conventional spirit leveling height determination technique with GNSS/Leveling (Blewitt, 1997; Fotopulous, 2003; Odumosu et al, 2016). However, the GNSS/leveling technique provides its users with ellipsoidal heights rather than orthometric heights therefore requiring that an accurate regional geoid model exists from which the orthometric equivalent of the ellipsoidal heights could be determined (Aleem,2013).

Geoid modeling has long been an area of research in geodesy in which several methods of its determination evolved over time. Of these methods, the remove-restore compute (RRC) method is common practice. The RRC method takes advantage of the fact that the geoid is composed basically of 3 wavelengths (long, medium and short), hence the method removes the effect of the long wavelength reference field then smooth the resulting residual gravity before later restoring the effects of the long wavelength and topography (Nsombo, 1996).

The long wavelength geoid computation is achieved through spherical harmonic analysis and approximations of the earth gravity field as given by Heskainem and Moritz (1969) in (1).

N(r,) =

where:

N(r,) = Geoid undulation as a function of r,

r, = spherical co-ordinates (radial, azimuth and polar)

a = semi major axis

and = fully normalized harmonic coefficients or stokes coefficients

= Associated Legendre polynomial

= degree and order

With advancements in satellite geodesy and subsequent space gravity missions (GRACE, CHAMP and GOCE), Global Gravity Models (GGM) have been developed from where fully normalized harmonic co-efficients can be obtained. Therefore from (1), once the normalized harmonic co-efficients have been obtained, the task of long wavelength geoid modeling is a computational problem. The requisite computational considerations in the execution of (1) will then be:

1. Computing the associated Legendre Polynomial
2. Choice of maximum degree of expansion of the harmonic model
3. Choice of computational surface (reference ellipsoid)
4. Choice of geo-potential model from where the harmonic coefficients used for the computation are extracted

The grafLAB program has been used in this study to identify the effect of these stated factors on the achieved accuracy in long wavelength geoid modeling.

1. Associated Legendre Functions (ALF)

Computation of the fully normalized associated Legendre functions is given by Hofmann-Wellenhof and Moritz (2005) as (2) below

(2)

1 for m = 0

2 for m 0

where:

m and n = degree and order

t =

= fully normalized associated Legendre function

Either the recursive formulae and explicit formulae in (3) and (4) respectively is then used for computing the associated legendre functions (ALF) prior to input in the fully normalized ALF evaluation as given by (Torge, 2001; hofmann-wellenhof and Moritz, 2005)

= (3)

= (4)

However due to the numerical instability of the recursive and explicit formulae at high and ultra high degrees of expansion of the spherical harmonic especially at high latitudes, modified recursive formulae and the use of the Clenshaw’s summation been developed to ensure numerically stable results at ultra high degree of expansion of the spherical harmonics (Holmes and Featherstone, 2002). These modified approaches include;

1. The modified Forward Column Method (MFCM)
2. The mordified Forward Row Method (MFRM)

The standard forward column (an example of the conventional recursive-based computation technique) and the modified forward column (an example of the modified recursion-based technique) have been evaluated in this study in an attempt to ascertain the computational suitability of both techniques for long wavelength geoid computation in the equatorial region taking Minna metropolis of Niger state in Nigeria as case study.

1. Global Geo-potential Models (GGM)

The solution to the spherical harmonic expansion of the earth’s gravity field corresponds to a spectral decomposition of the gravity field in which the coefficients of the series expansion provide the amplitudes of the respective spectral parts (Torge, 2001). Once the spherical harmonic coefficients and of a global gravity field model are given, the quantities of the various gravity functional can be computed in its geographical distribution (Lambeck, 1990; Kaula, 1996). Presently, many GGM’s have been developed some of which include the EGM96 developed from synthesizing orbital tracking from numerous satellite missions (Roman et al, 2010), EGM2008 from a combination of terrestrial gravity data and GRACE data (Roman et al, 2010), GGM05S from GRACE (Tapley et al, 2013), GGM05G from combination of GRACE and GOCE (Bettadpur et al, 2015) e.t.c.

Each geo-potential model has a maximum degrees of expansion that corresponds to the degree and order of its stokes coefficients. Therefore given the stokes coefficients, gravity field functionals can be computed to user required degrees of harmonic expansion provided the maximum degree of the geopotential model in use is not exceeded. By implication, the EGM96 have spatial feature resolution of about 100km while the EGM2008 have a resolution of about 10km in most region because they are complete to degrees and order 360 and 2190 respectively (Roman et al, 2010).

1. Reference Ellipsoidal Surfaces

All GGM’s are geocentric in nature because they have global coverage. The degree 1 spherical harmonic coefficients () are related to the geocentre coordinates and are taken as zero if the coordinate systems’ origin coincides with the geocentre. Similarly, the degree 2 coefficients are connected to the time dependent mean rotational pole position. Therefore, the degree 1 and 2 terms of the spherical harmonic coefficients as well as the adopted value for the semi-major axis represent the reference ellipsoidal parameters of choice.

In practice, the WGS84 and GRS84 ellipsoidal surfaces are the most frequently used surfaces upon which gravity field functional especially the long wavelength geoid are computed. For this reason, care must be taken when a regional model is to be implemented towards ensuring that coordinates are specified in the appropriate nationally adopted global ellipsoidal surface and if coordinates in a local surface are to be used, appropriate transformation parameters must be specified and included in the model before such geoid models are computed.

1. Study Area

The study area (Figure 1) for this research is Minna metropolis of Niger state in Nigeria.

It lies between Latitude 09°24N to 09°44N and Longitude 06°25E to 06°45E and is part of North-central Nigeria. The area experiences distinct dry and wet seasons with annual rainfall varying from 1,100mm to 1,600mm. Temperature 23°c, wind NE, at 16 km/h, 28% humidity. The state is located in the low latitudes and therefore could effectively serve as a basis for testing the performance of long wavelength geoid computational characteristics in the low latitudes and equatorial region in general.

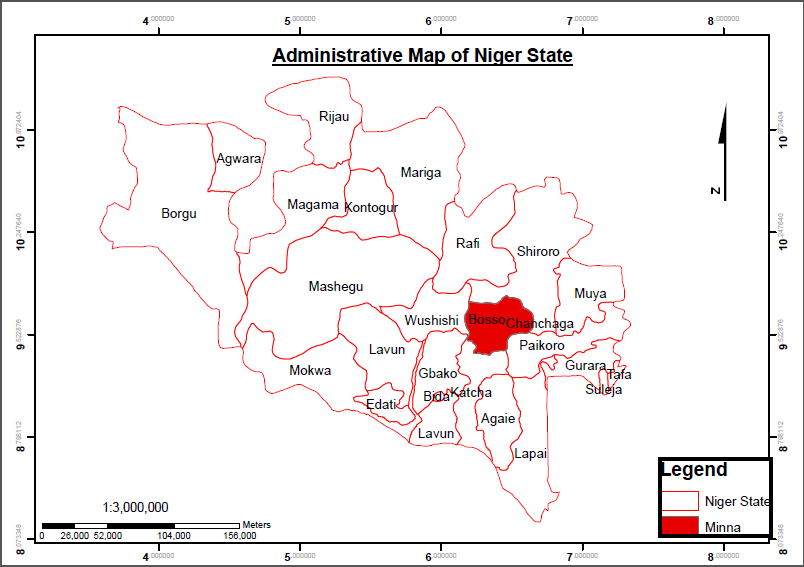


Figure 1: Administrative map of Niger state with Minna highlighted in red (study area)

1. Materials and Methods

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ALF | | | | Reference Ellipsoid | | | | Choice of GGM | | Degree of Expansion | |
| FCR | | Modified FCR | | WGS84 | | GRS80 | |
| EGM96 | EGM08 | EGM96 | EGM08 | EGM96 | EGM08 | EGM96 | EGM08 | EGM96 | EGM08 | EGM96 | EGM08 |
| L | L | L | L | L | L | L | L | L | L | L | L |
|  | H |  | H |  | H |  | H |  | H |  | H |
|  | UH |  | UH |  | UH |  | UH |  | UH |  | UH |

The Stokes coefficients for EGM96 and EGM2008 were downloaded from the NGA website at “http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm96” and <http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008> respectively. Equations 1 – 4 were then implemented at 1km grid interval across the study are using the grafLab computational package (Bucha and Janak, 2013). To effectively identify the result of the computational preferences under study, the computation was performed as summarized in table 1. The degrees of expansion of the spherical harmonic model were classified into three. Degrees 180 and 360 were adopted as low (L) degrees, degrees 720 and 1800 were adopted as high (H) degrees while degree 2190 was adopted as the ultra-high (UH) degree.

Table 1: computational scheme

L = Low degrees of harmonic expansion

H = High degrees of harmonic expansion

UH = Ultra high degrees of harmonic expansion

FCR = Forward Column Recursion

1. Results and Discussion of results

The FCR and modified FCR were tested at the low, high and ultra-high degrees within the study area and using both the EGM96 and EGM2008 models. Results obtained are as summarized in table 2. Similarly, the effect of choice of reference ellipsoid on geoid computation result was tested as shown in table 3.

Table 2: Summary of results of test on computational efficiency based on preferred choice of ALF

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | | ALF | | | | Diff (FCR - MFCR) | Diff (FCR - MFCR) |
| FCR | | Modified FCR | |
| EGM96 | EGM2008 | EGM96 | EGM2008 | EGM96 | EGM2008 |
| Degree 180 | Max | 21.22 | 22.02 | 21.22 | 22.02 | 0 | 0 |
| Min | 20.87 | 21.78 | 20.87 | 21.78 | 0 | 0 |
| Range | 0.35 | 0.24 | 0.35 | 0.24 | 0 | 0 |
| Degree 360 | Max | 21.31 | 22.01 | 21.31 | 22.01 | 0 | 0 |
| Min | 20.87 | 21.73 | 20.87 | 21.73 | 0 | 0 |
| Range | 0.44 | 0.28 | 0.44 | 0.28 | 0 | 0 |
| Degree 720 | Max |  | 22.00 |  | 22.00 |  | 0 |
| Min |  | 21.37 |  | 21.37 |  | 0 |
| Range |  | 0.63 |  | 0.63 |  | 0 |
| Degree 1800 | Max |  | 21.88 |  | 21.88 |  | 0 |
| Min |  | 21.48 |  | 21.48 |  | 0 |
| Range |  | 0.40 |  | 0.40 |  | 0 |
| Degree 2190 | Max |  | 21.89 |  | 21.89 |  | 0 |
| Min |  | 21.50 |  | 21.50 |  | 0 |
| Range |  | 0.39 |  | 0.39 |  | 0 |

Table 3: Summary of results of test on computational efficiency based on preferred choice of Ref. Ellipsoid

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | | Reference Ellipsoid | | | | Diff (WGS - GRS) | Diff (WGS - GRS) |
| WGS84 | | GRS80 | |
| EGM96 | EGM2008 | EGM96 | EGM2008 | EGM96 | EGM2008 |
| Degree 180 | Max | 21.22 | 22.02 | 20.28 | 21.08 | 0.94 | 0.94 |
| Min | 20.87 | 21.78 | 19.93 | 20.85 | 0.94 | 0.93 |
| Range | 0.35 | 0.24 | 0.35 | 0.23 | 0.00 | 0.01 |
| Degree 360 | Max | 21.31 | 22.01 | 20.37 | 21.07 | 0.94 | 0.94 |
| Min | 20.87 | 21.73 | 19.93 | 20.79 | 0.94 | 0.94 |
| Range | 0.44 | 0.28 | 0.44 | 0.28 | 0.00 | 0.00 |
| Degree 720 | Max |  | 22.00 |  | 21.07 |  | 0.93 |
| Min |  | 21.37 |  | 20.44 |  | 0.93 |
| Range |  | 0.63 |  | 0.63 |  | 0.00 |
| Degree 1800 | Max |  | 21.88 |  | 20.94 |  | 0.94 |
| Min |  | 21.48 |  | 20.54 |  | 0.94 |
| Range |  | 0.40 |  | 0.40 |  | 0.00 |
| Degree 2190 | Max |  | 21.89 |  | 20.96 |  | 0.93 |
| Min |  | 21.50 |  | 20.57 |  | 0.93 |
| Range |  | 0.39 |  | 0.39 |  | 0.00 |

As observed in table 2, the choice of computational model for computation of the ALF does not affect the numerical stability or results obtained in Long wavelength geoid computation within the study area. This substantiates previous literature that the ALF are numerically stable even at ultra high degrees of expansion within the equatorial region (Holmes and Featherstone, 2002).

However table 3, reveals a numerical variation of 0.93m – 0.94m in geoid computation between the WGS84 and the GRS80 reference ellipsoid. Therefore, users intending to integrate geoid models with local coordinates should ensure that the appropriate reference ellipsoid is specified in the requisite transformation models for such integration else computational errors between the ranges of 0.93 – 0.94m might be accrued.

The haphazard pattern of the discrepancy in geoidal undulation values with increasing degree of expansion of spherical harmonic model especially at high and ultra high degrees (Figure 2 and 3) reveal a non-uniformity in variation. It is therefore expected that users of GGM’s perform their own accuracy and precision verification such as comparing GGM-derived gravity field quantities with local data (Kirby et al, 1998) before determining best degree of expansion within its territory.

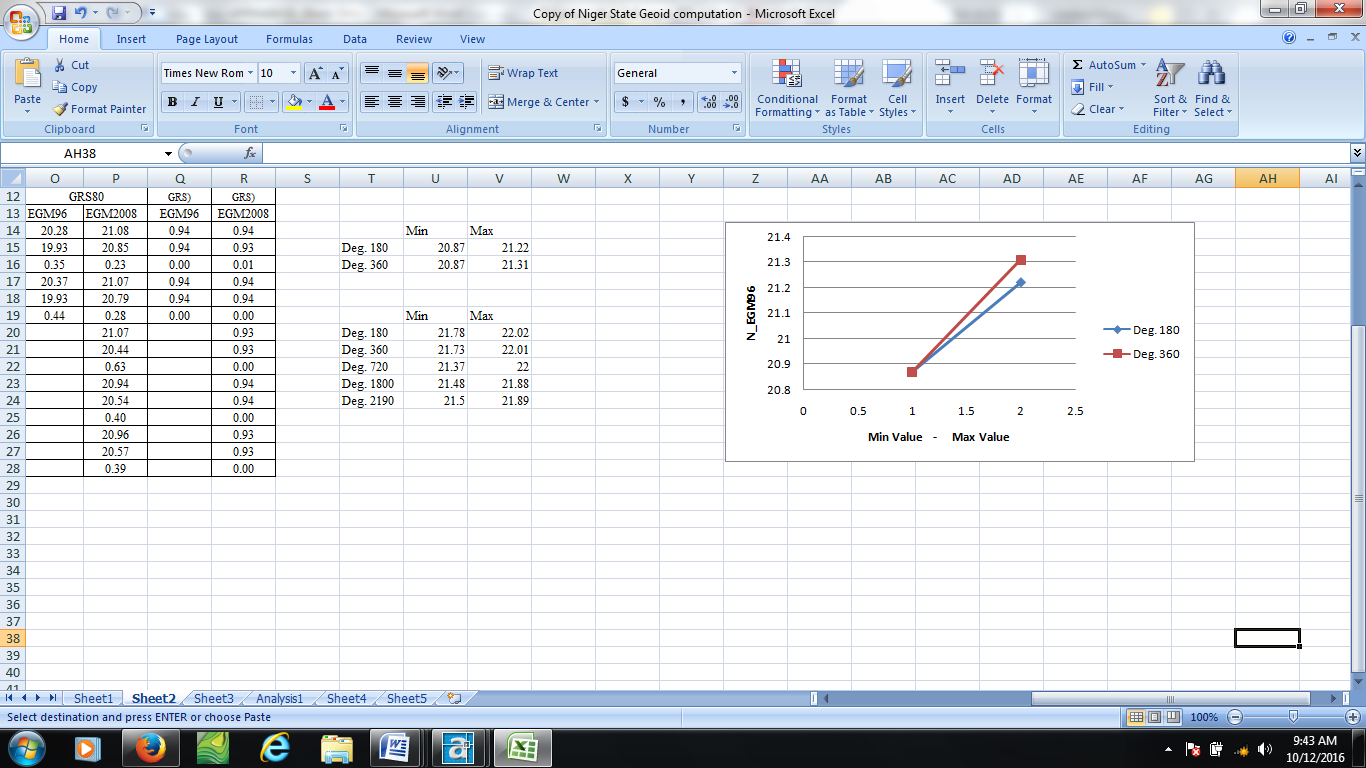
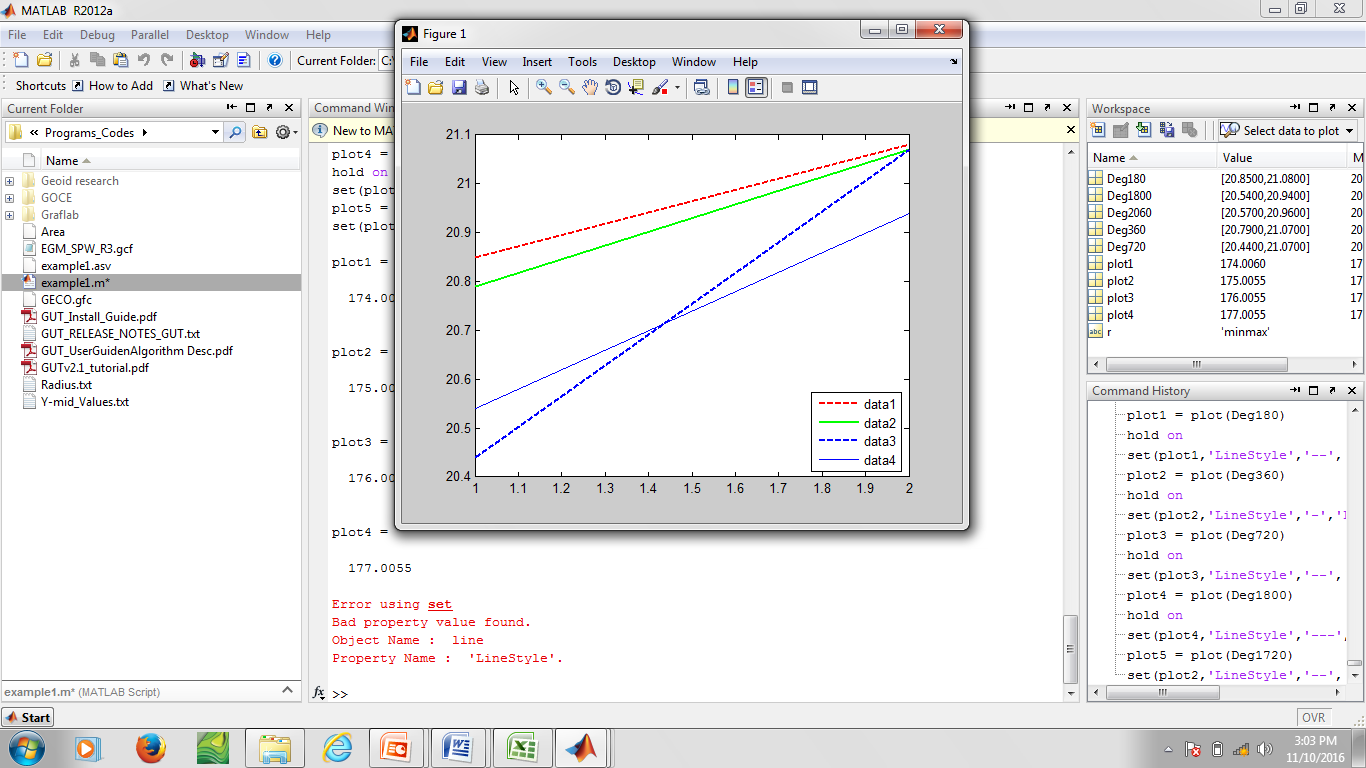


Figure 2: Variation in geoid undulation (EGM96) with increasing degree of harmonic expansion



Where data1 – data 4 are degrees 180, 360, 720 and 1800 respectively.

Figure 3: Variation in geoid undulation (EGM2008) with increasing degree of harmonic expansion

Figures 4 and 5 further show the geoid undulation discrepancy with increasing degrees of expansion for both the EGM96 and EGM2008 model. Figures 6 - 8 show the geoid model of the study area from EGM96 and EGM2008 respectively. The similarity in pattern of the geoid models constructed from EGM96 and EGM2008 at degree 360 indicate that the discrepancy in geoid model caused due to choice of model is minimal within the study area but depends more on the degree of expansion. This suggests the possibility that the additional data used in building EGM2008 does not include the study area.

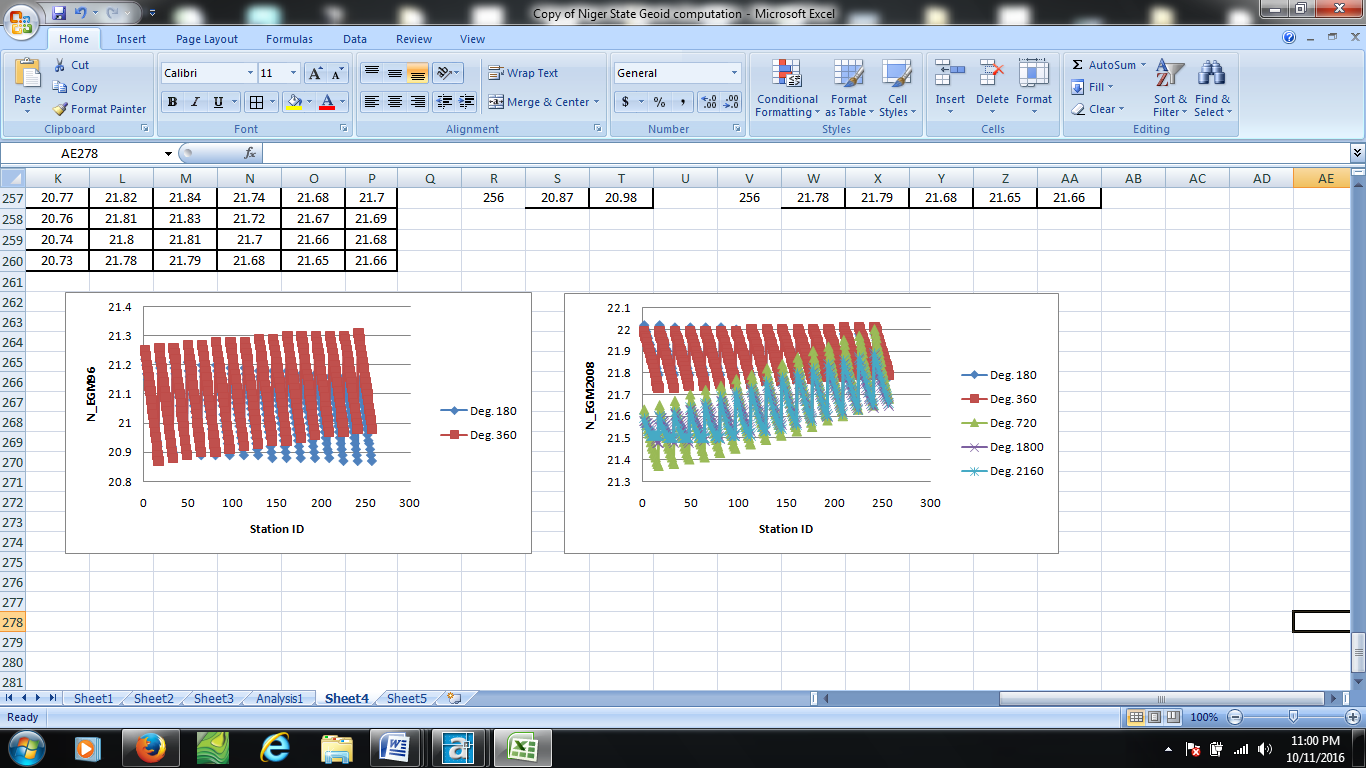


Figure 4: Plot of Long wavelength geoid undulation at degrees 180 and 360 using the EGM96 model

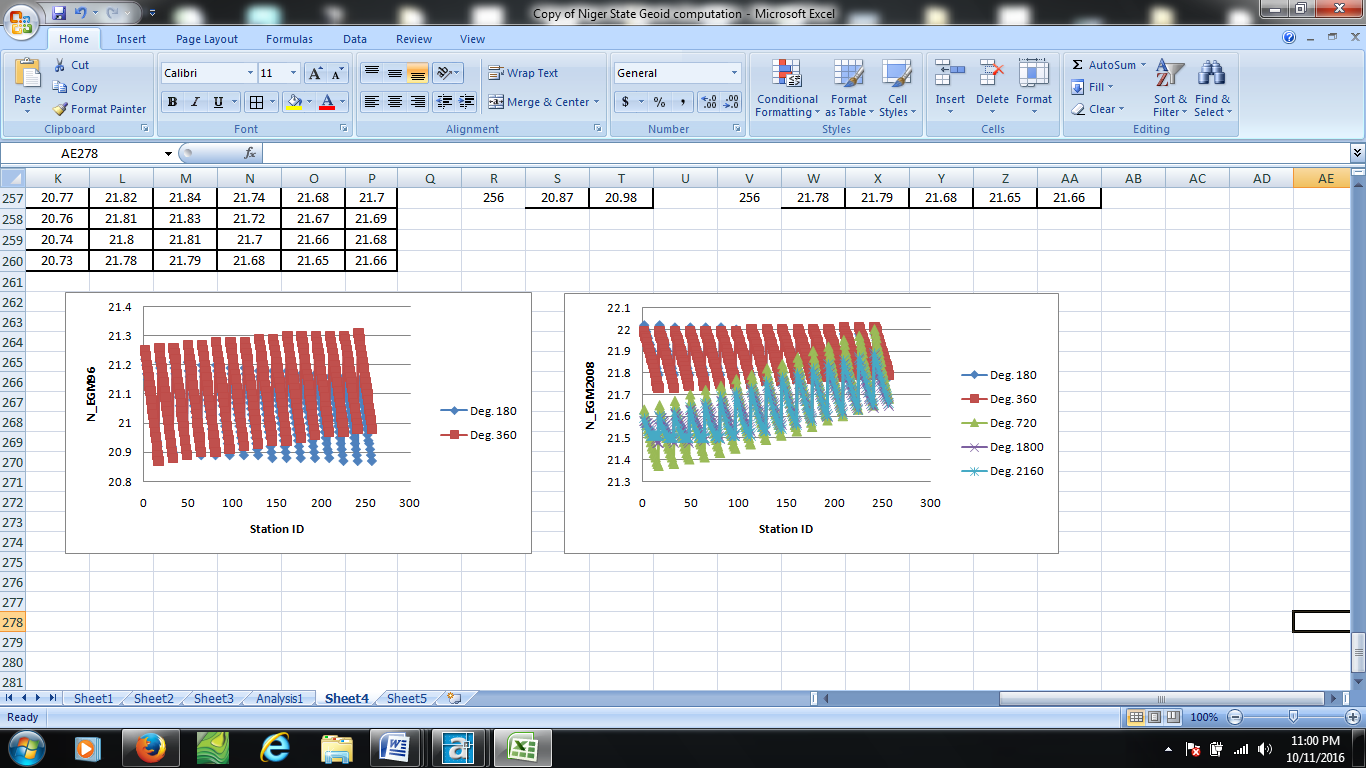


Figure 5: Plot of Long wavelength geoid undulation at degrees 180, 360, 720, 1800 and 2160 using the EGM2008 model

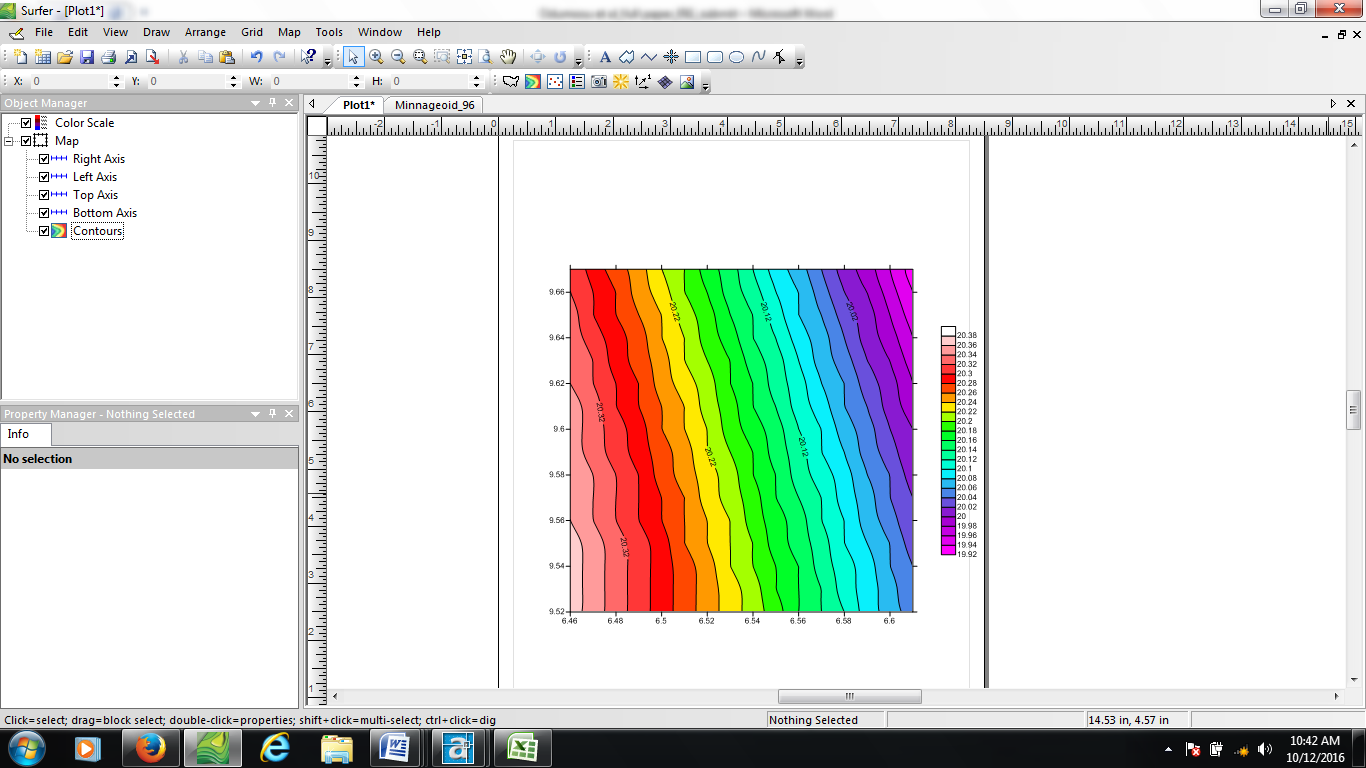


Figure 6: Geoid model of study area from EGM96 model at maximum degree of expansion

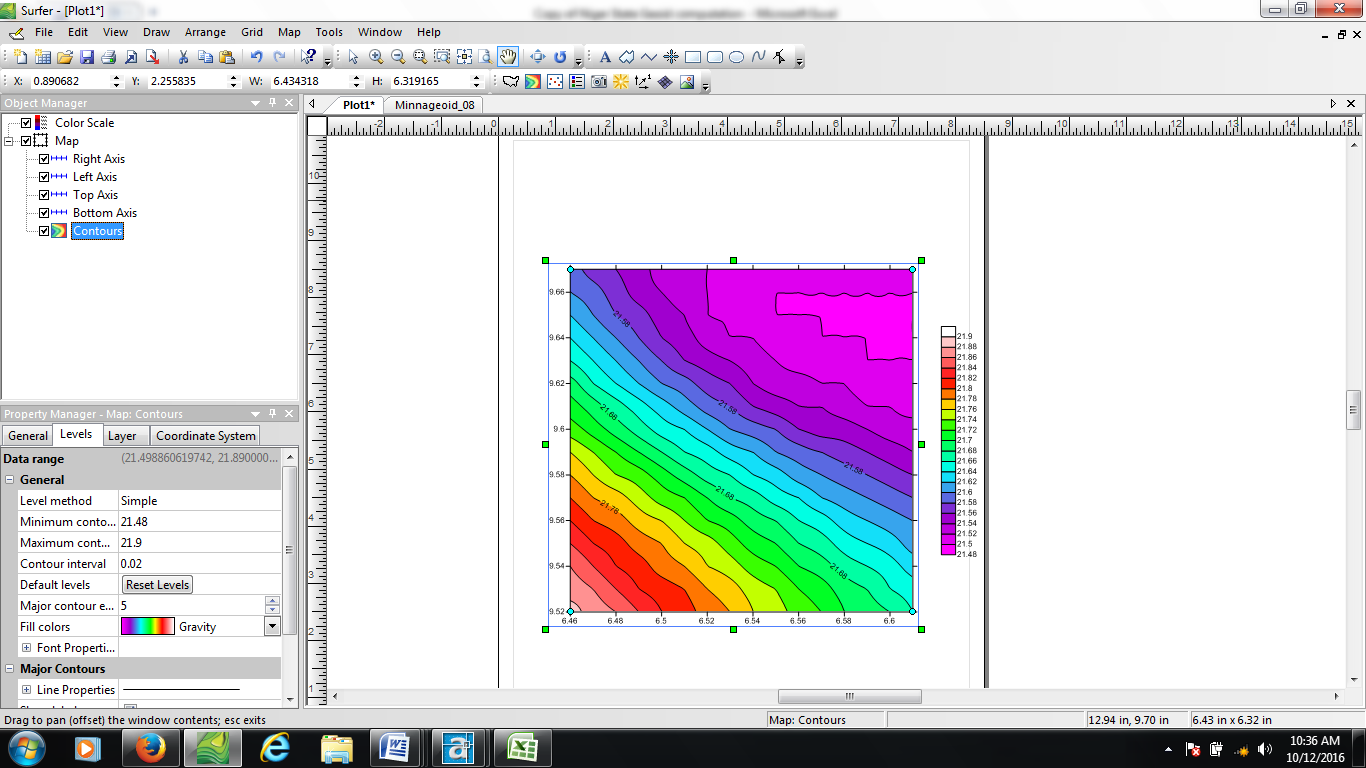


Figure 7: Geoid model of study area from EGM2008 model at maximum degree of expansion

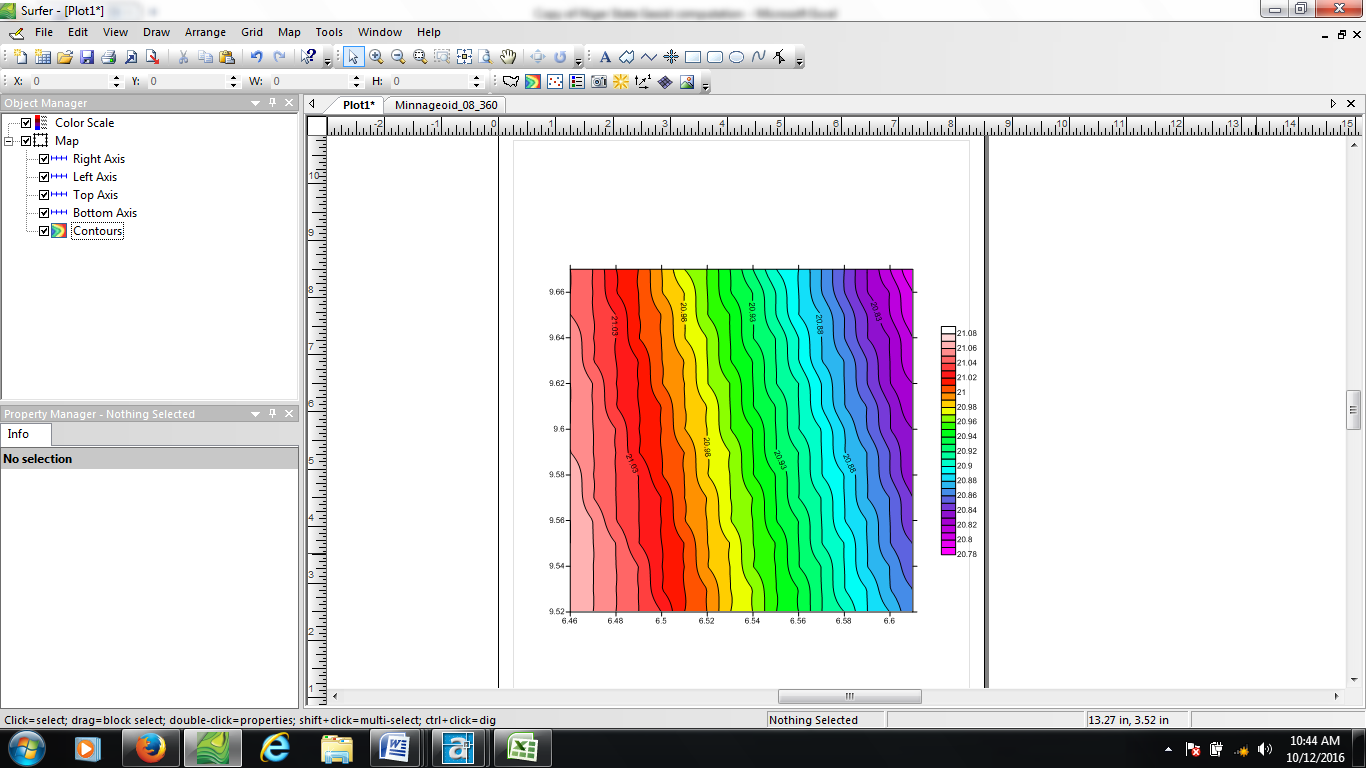


Figure 8: Geoid model of study area from EGM2008 model at degree 360

1. Conclusion

This study confirms the suitability of use of the recursive functions for computation of geoidal undulation even at ultra high degrees of expansion for the equatorial region. The choice of reference ellipsoid has also been discovered to affect results obtained by a variation ranging between 0.93m – 0.94m between WGS84 and GRS80 ellipsoid. The following recommendations are therefore obvious:

1. Although, GGM’s come with commission and omission errors the error estimates for GGMs are global averages and so may not necessarily be representative of the performance of the GGM in a particular region. Hence, the user of a GGM should perform his own accuracy and precision verifications, such as comparing the GGM- derived gravity field quantities with local data.
2. Regional long wavelength geoid derived from GGM’s are based on geocentric co-ordinate system.
3. Users of geoid models intending to incorporate such models with local coordinates (i.e local geoid) should ensure that appropriate transformation models are adopted since errors arising from wrong reference ellipsoid appear significant. This is to ensure that the effect of datum translations between the local coordinate systems and the chosen global system is appropriately taken care of.

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